TRANSACTIONS

AMERICAN GEOPHYSICAL UNION

VOLUME 33 [redacted]

[redacted] 1952

SEISMOLOGY
GEODESY

PUBLISHED BY THE NATIONAL RESEARCH COUNCIL
OF
THE NATIONAL ACADEMY OF SCIENCES
WASHINGTON, D. C.

TRANSACTIONS
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The Transactions are published bimonthly at 1530 P Street, Northwest, Washington 5, D. C.
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TRANSACTIONS
AMERICAN GEOPHYSICAL UNION

Volume 33 FEBRUARY 1952 Number 1

PRECISION LEVELING AT THE WHITE HOUSE

Howard S. Rappleye

Abstract--This paper deals with the planning and execution of a precision-leveling program being carried on during the reconstruction of the Executive Mansion in Washington, D. C. Unusual accuracy requirements and unusual working conditions were responsible for many departures from established practice in precision leveling. Field methods, instrumental equipment, accuracy of leveling, and observed behavior of bench marks are discussed. Special emphasis is on the unusual features of the work rather than the description of just one more leveling project.

The precise-leveling project in connection with the reconstruction of the White House has been carried on by personnel from the Section of Leveling of the Coast and Geodetic Survey and the Public Buildings Service of the General Services Administration. The project entered the planning stage in April 1949.

As outlined at the first conference, the problem was to establish the necessary bench marks and to do the necessary leveling and repeat levelings in such a manner as to be able to detect and prove settlement of one millimeter or more at any mark at any time during the life of the reconstruction project. Such an accuracy specification and the unusual working conditions to be expected on such a project seemed to call for some special methods and special equipment. In general, the project was carried out as planned, but some modifications were made as the project developed and experience showed that changes were necessary.

Ninety-seven bench marks were established around the building and throughout the grounds (see Fig. 1 and 2). Those on the building were for the purpose of detecting settlement, while those out in the grounds were established for control purposes. Seventy-four of them were on the building and adjacent portions of the terrace structures connecting the main building with the east and west wings. The building marks were either brass bolts of special design or standard monel-metal rivets. The ground marks included copper nail-and-washer marks in the roots of trees,

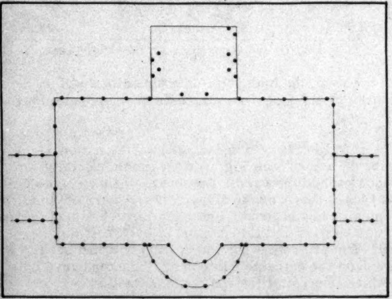

Fig. 1--Locations of bench marks on the White House

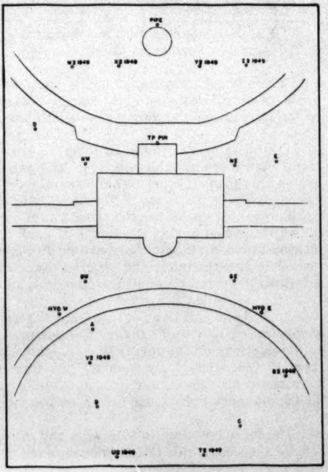

Fig. 2--Locations of bench marks on the White House grounds

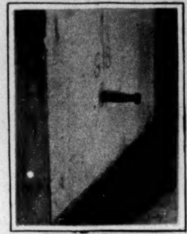

Fig. 3--Special bench mark

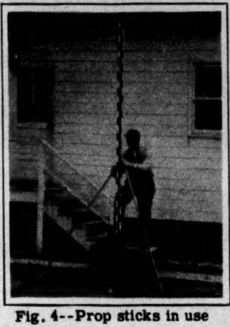

Fig. 4--Prop sticks in use

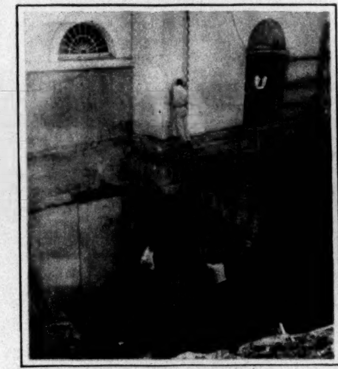

Fig. 5--Reaching high bench marks over excavation

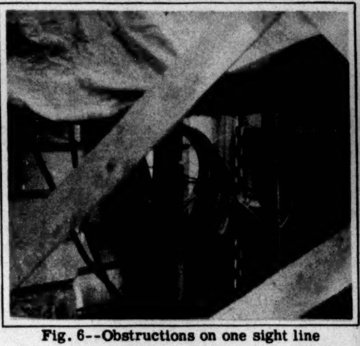

Fig. 6--Obstructions on one sight line

hold-down bolts on hydrants, bolts and pins in pavement, the high point on a pipe elbow at the north-lawn fountain, and eight standard-disk marks in six-foot concrete posts cast in place, four on the north lawn and four on the south lawn.

Of the 74 marks in the building, 14 were the round heads of standard 3/8 × 2-inch monel-metal rivets set in column bases. The remaining 60 marks (see Fig. 3) were made specially for use on this project from either 7/8-inch or 1-inch round brass rod. They were of three sizes because of variations in their locations and the purposes they were to serve. There were 46 small bolts seven inches in length, six medium bolts nine inches in length, and eight large bolts 12 inches in length. About half of the length of each bolt was knurled and this portion of each bolt was cemented into a horizontal hole drilled in the wall. The projecting portion of each bolt had been turned so that the outer end tapered toward a knob on the extreme outer end. This knob gave both a definite high and a definite low point against which the foot of the rod could be held.

These bolts projecting from the wall of the building would have been damaged during the underpinning operations if they had been set at or near the ground level. Those which were not up on the porticos or the terraces were placed in the first course of stone below the water table or about 11 feet above the ground. This immediately posed the problem of leveling to marks which were about six feet higher than the leveling instrument. A thermometer was set in the wall near the bench-mark bolt at each of three locations to determine the temperature of the wall.

The first leveling, in late June and early July of 1949, was done by holding the rod upside down against the under side of the knob on each of the high marks, thus using the bottom of the knob as the bench mark. On the eight large bolts which had to be used as turning points as well as bench marks in passing from low-level to high-level setups and vice versa, the bolt heads or knobs were calipered and the knob heights were allowed for in computing the differences of elevation. Later it was proved that these bolts flexed slightly upwards when the rod was shoved up under them and downwards when the rod was stood on top of them. The sum of these two slight flexures came into the line full force without there being any convenient or practical means of correcting for them. Some means had to be provided for eliminating this trouble.

The particular rod in use on this project is one of the standard Coast and Geodetic Survey geodetic leveling rods selected from a large number of available rods because its index and rod errors were "zero" and this did away with any necessity for index or rod corrections in the computations. By zero we mean that the index and rod errors were less than 0.05 mm, the limit of accuracy to which our standardizations are ordinarily made. The foot piece of a rod of this type terminates in a plane surface of circular shape, 1 1/4 inches in diameter, and normal to the face of the rod.

To provide for hanging the rod from the top of the knob on the end of a brass-bolt mark high in the wall, a steel plate was produced in our shops. This plate was 1 1/4 inches wide, 2 1/2 inches long, and 3/8 inch thick. The two broad surfaces of the plate were ground so that they were both plane and parallel. A stirrup of rectangular shape and with a strong set-screw enabled us to clamp one end of this plate to the foot of the rod. Then, when the other end of the plate was placed on the top of a knob, and the rod was plumbed by means of a second circular level mounted on the rod, the plane representing the zero of the rod was coincident with the horizontal plane tangent to the top of the knob. The stirrup could be folded back out of the way when not needed. Thus, whether the rod was hanging from or standing on the knob, the load on the bolt was the same, except for the weight of the small steel plate (about five ounces) and the flexure was always downward and of practically the same amount.

After provision had been made for hanging the rod from the top of the knob on each high mark, it was necessary to caliper the knobs on all of the high marks so that the previously determined elevations of the bottoms of the knobs could be expressed in terms of the tops of the knobs for comparison purposes.

The instrument in use on this project is a stock Wild N III, except that the pattern of the cross wires was changed to conform with that ordinarily used in the Fischer level. This was done because our rods are graduated in alternate black and white rectangles one centimeter high and do not carry the line graduations for which the forked horizontal cross wire of the stock Wild N III is particularly adapted. This instrument has a plane-parallel attachment in front of the objective. When this is rotated about a horizontal axis, the effect is to raise or lower the exterior portion of the line of sight, parallel to itself, through a maximum range of one centimeter. The screw controlling the rotation of the plane-parallel glass is connected with a scale which is graduated directly in millimeters and tenths and which is open enough so that hundredths may be estimated with considerable certainty.

The observing routine is as follows: (1) Point and focus the telescope on the rod; (2) read the stadia distance; (3) bring the main bubble to the center of the tube; (4) bring the middle horizontal wire to the proper centimeter graduation on the rod; (5) check the centering of the main bubble; (6) repeat (3), (4), and (5) as may be necessary to insure that the wire is exactly on the graduation at the same time the bubble is centered; (7) and read the meters, decimeters, and centimeters from the face of the rod and the millimeters, tenths, and hundredths from the scale in the instrument.

The result is a single reading. The second and third readings are made according to the same routine (omitting the stadia reading) after throwing both micrometer and bubble out of position and bringing them back again, preferably from opposite directions. If the spread of the three readings is not more than 0.1 mm, the mean of the three is taken as the reading for the sight. If the spread is more than 0.1 mm, two additional readings are made and the mean of the five is taken as the reading for the sight.

Early in the development of this project it was found that, when working to the high degree of precision required, the instrument when set up on turf was so unstable that the observer would change the height of instrument by just moving around it. This occurred despite the fact that the legs of the tripod were forced into the ground as far as possible at the time the setup was made. The remedy for this trouble was to make up several sets of turning pins, each of which had a small conical hole drilled in the top. When these were driven into the ground in an appropriate triangular pattern and the points of the tripod legs were placed in the small conical holes, the instrument proved to be much more stable. Even then the observer had to take care not to step within several inches of any of the pins. The pins were driven down so that only enough of their length projected above the surface of the ground to permit their being knocked out and taken on to another setup.

When set up on black-top pavement, especially in warm weather, the points of the tripod legs would settle gradually into the pavement. This difficulty was overcome by using steel plates having a small conical hole in the middle of one broad face of each plate. These distributed the load over a larger area so that the instrument would "stand without hitching."

The recorder was provided with a portable desk which was a great convenience. A plywood top replaced a conventional plane-table board, and on this were mounted the necessary tables for applying certain corrections, a slide rule, a scratch pad, and a pencil sharpener. This piece of equipment is not recommended for straight differential leveling but, under the peculiar circumstances surrounding this job where as many as 14 sights were sometimes taken from a single setup, the table worked out very well.

To keep the rod steady, without undue strain on the rodmen when they were not braced against a wall, a ladder, or a column, we borrowed an idea from the Egyptians and used prop sticks (see Fig. 4). These sticks were a great help, especially when the wind was blowing.

When construction progress was in full swing, there was a rather wide and deep trench all around the main building, in which the underpinning crews worked. To get a rod on a mark 11 feet above the original ground level and over a ditch anywhere from six to eight feet deep and five or six feet wide required some ingenuity at times. Various combinations of ladders and planks were used to get the rodman out to a position from which he could place and hold his rod on the high marks.

When a large excavation was made outside the building and around the northeast corner for the construction of more underground shops, we were faced with the problem of holding the rod on marks 11 feet above the original ground level and over an excavation 15 feet or more in depth. We were using rather long ladders before we finished with some of those marks (see Fig. 5). The excavation also took away the ground on which we had usually set up the instrument and made for quite rugged terrain right on the White House grounds.

Barricades to protect columns from damage during construction caused us considerable difficulty for we had to get the rod down through from the top, break out a board in each barricade, and locate the setups so that we could shoot through the narrow openings. If you do not believe that these are real difficulties, try getting a geodetic leveling rod into and out of such a place without bumping either of two bubble brackets or scratching the graduations. Scaffolding, power cables, and all sorts of construction equipment made it difficult at times to get sights through. In Figure 6, which was taken with the camera held just over the instrument, you may see the mess of obstacles we had to dodge to get one sight through. Now imagine what it was like to pick a setup from which even three or four marks could be observed without shifting the instrument. The rod shown here is on L 1 in the corner at the east side of the south portico. However, despite the difficulties encountered in making the observations, we feel that we have been successful in turning out a good job of precision leveling. Some idea of the accuracy attained in this work may be gained from the following:

(1) The worst circuit closure to date was involved in one of the levelings over the marks on the south lawn. The perimeter of the circuit was about 1000 feet, 12 setups were required, and the closure was under 0.8 mm.

(2) All circuits considered (and there are about 200 of them to date), the perimeters ranged from two or three hundred feet to about a thousand feet; the number of setups ranged from two to thirteen; and the closures varied from 0.00 mm to 0.78 mm, with the average probably somewhere around 0.20 or 0.25 mm. The closures for the leveling of January 27-28, 1951, were as follows (the unit being 0.01 mm): -6, +15, -13, +1, -1, +16, -10, +23, and -30; average (without regard to sign) = 0.13 mm.

And just so that one may not think I have picked out a particularly good sample, I include here also the closures from the last leveling (March 24, 1951), and the unit being the same as before: -10, +19, +14, -15, +17, -31, +20, and -9; average = 0.17 mm.

The original or basic leveling was done in late June and early July of 1949. Some check leveling was done in November 1949, after the eight concrete-post bench marks (Fig. 2, S 2 to Z 2,

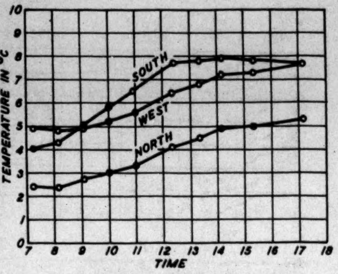

Fig. 7--Typical set of wall temperatures (April 15, 1950)

incl.) were established. In December 1949, another leveling was carried out to include all of the high marks on which the point of reference had been switched from the bottom to the top of the knob. The elevations resulting from the leveling of December 1949 and parts of the previous levelings were used as the basis of comparison with the elevations determined during subsequent levelings for the determination of movement of the bench marks. In all of this leveling, readings were taken to the nearest 0.01 mm and all computations and adjustments were carried out to five decimal places in meters. Then when the adjusted elevations for each leveling and releveling were computed they were rounded to the nearest 0.1 mm for tabulation and publication.

The actual underpinning operations were begun on January 2, 1950, and partial relevelings on January 4, 6, and 10 showed not only that movement of the bench marks was taking place but also that we could not hope to do leveling of the required accuracy while construction activity was in progress. The noise, vibration, dust, and confusion were handicaps which could not be overcome satisfactorily. A conference resulted in a decision to level over all marks in the building and some of the nearby ground marks once a week, the work being done on Saturdays when construction activity was at or nearly at a standstill. Every few weeks the leveling was extended to include all marks and this required work on both Saturdays and Sundays.

Some very interesting things have developed during the progress of this work; for example, it was found that the accuracy of the leveling was such that in addition to measuring the movement of the high marks resulting from the underpinning operations we were also measuring movement of the high marks resulting from the expansion and contraction of the walls with changes in wall temperature. Three thermometers were installed in the walls and temperature readings were taken at suitable intervals during the times when leveling was in progress. These intervals ranged from about 45 minutes to two hours, depending on weather conditions. Curves (see Fig. 7) showing thermometer readings plotted against the times of the observations were drawn and values scaled from these curves were used in computing the necessary corrections to reduce elevations to the standard wall temperature of 50° F (10° C).

These thermometers were hydrometer thermometers and had the zero of the graduations several inches out from the bulbs. This permitted imbedding the thermometer bulbs about 3 1/2 inches into the wall. Lead wool was used to secure a good contact between the bulb and the wall. Cotton was then rammed into the hole for insulation and support. The outer end of the hole was filled with a soft plastic as a weather seal.

Time-movement profiles were drawn and kept current as the work progressed. Results of the leveling on a given week end were usually computed and adjusted so that new elevations, for comparison with the basic elevations, were usually available by the following Tuesday afternoon or Wednesday morning. The group of time-movement profiles in Figure 8(a) shows the results for the eight large bolts located in the reentrant corners and used not only as bench marks but as turning points as well in passing from low-level to high-level setups and vice versa in getting up to and down from the porticos and terraces.

L 1 and L 2 are just east and west, respectively, of the south portico; L 3 and L 4 are just south and north, respectively, of the west terrace; L 5 and L 6 are just west and east, respectively, of the north portico; and L 7 and L 8 are just north and south, respectively, of the east terrace. All of these marks show settlement until about the middle of April or the first part of May 1950, when the effect of digging out the earth to make way for the new basement and sub-basement resulted in a slight but nevertheless real uplift for the next five or six weeks. This is followed by slow and quite uniform settlement which may be either actual settlement of the whole structure, shrinkage of the new concrete foundation, or a combination of both.

The profiles in Figure 8(b) show the results for the marks located along the south and west walls from the south portico around to the south side of the west terrace. Here the settlement is very plainly shown. This continues until about May 6, 1950, and is followed by uplift for five or

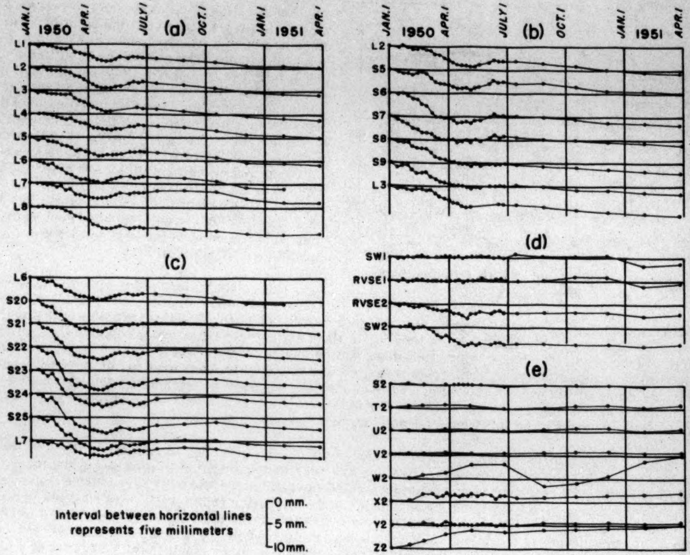

Fig. 8--Typical time-movement profiles

six weeks, after which a gradual and quite uniform settlement again becomes evident. The profiles in Figure 8(c) are for the marks on the north and east walls and extend from the east side of the north portico around the northeast corner to the north side of the east terrace. The maximum settlement observed to date occurred at S 22 on April 22, 1950, and amounted to an even 9.0 mm (0.35 inch). Again settlement is evident until late in April 1950 when the uplift appears, to be followed by slow settlement again. The uplift begins earlier in this group of curves than in the group for the marks around the southwest corner of the building because the digging for the basement and sub-basement was begun at the east end and was completed in that end before it was in the west end of the building.

The profiles in Figure 8(d) are for the four marks on the south side of the east terrace, SW 1 being the farthest from the main building. It will be seen that SW 2 has settled the most and that the settlement decreases as we pass upward through this group of profiles (that is, away from the main building on the ground) and that the movement actually becomes a slight uplift for RVSE 1 and SW 1 until after December 2, 1950, when the underpinning of that portion of the terrace foundation evidently produced considerable settlement. Between January 27 and March 24, 1951, the most easterly of these marks show upward movement again. This is undoubtedly due to the removal of a very large amount of earth from another excavation located south of the east terrace and east of the main building. This excavation was started prior to the leveling of January 27, 1951, and was pretty well completed before the leveling of March 24, 1951.

The profiles in Figure 8(e) show that the four concrete-post bench marks on the south lawn (S 2 to V 2, incl.) have maintained their elevations quite well. This is especially true of S 2 and fortunately so, since an accident to Hydrant E at one time required our depending on S 2 for one of our fixed points for several weeks until the elevation of Hydrant E was redetermined with certainty.

The elevations of the four concrete-post bench marks on the north lawn, particularly W 2 and Z 2, have proved to be quite erratic. The reasons for this are not known and may never be known, though we hope that, when these marks are finally pulled out and we have a chance to examine both the posts and the ground in which they were set, some light may be thrown on the matter.

Bench mark PIPE has remained quite stable. This is the high point of an elbow on a pipe coming up out of a box-like concrete structure at the north side of the north-lawn pool. When we stop to consider that this mark is the one which is farthest from the points held fixed in the adjustment of the various levelings, this time-movement profile not only exhibits the stability of this bench mark but the inherent tightness of the various levelings. This mark was leveled over 25 times in a period of about 64 weeks and the total indicated range of movement combined with any loosenesses in the various leveling networks does not exceed 1.1 mm.

Obviously there is not sufficient space available in a presentation of this sort to discuss in detail the behavior of each of the 97 bench marks involved in this project. However, I believe you will agree that the results show that the time-movement profiles are quite smooth in the tenths of millimeters, and that the upward movements resulting from the excavation of quite large amounts of earth are proved.

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(Manuscript received April 19, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

ON A DIFFERENTIAL EQUATION OF ATMOSPHERIC DIFFUSION

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Abstract--Exact closed solutions of the following equation of diffusion

$$y^m (\partial c/\partial x) = D \partial (y^n \partial c/\partial y)/\partial y$$

where m and n are considered to be independent, are obtained for the following cases: (1) diffusion from a line-source embedded in a smooth surface; (2) diffusion from a smooth surface; (3) vapor concentration in the wake of an evaporating surface. The solutions for Cases (1) and (2) belong to the similarity-solution category. Dimensional analysis has been used in the systematic search for such solutions in as concise a manner as possible. The solution for Case (3) can be obtained from those for Cases (1) and (2). Diffusion in Couette flow is given as an example.

Introduction--O. G. SUTTON [1934] developed a theory of atmospheric diffusion from a smooth surface on the assumption that the distribution of the mean velocity is

$$u = u_1 (y/y_1)^m \quad \text{(1)}$$

where y is the elevation above the ground, and u_1 corresponds to the particular elevation y_1. From the power distribution of the velocity, Sutton was able to show that the exchange coefficient A(y) follows the power law

$$A(y) = A_1 (y/y_1)^n \quad \text{(2)}$$

where $n = 1 - m$, and where again A_1 corresponds to y_1. Sutton's equation of diffusion is then

$$y^m (\partial c/\partial x) = K \partial (y^{1-m} \partial c/\partial y)/\partial y \quad \text{(3)}$$

where $K = A_1 y_1^{2m-1}/u_1$ and where c is the vapor concentration at any point, x being measured downwind from the leading edge of the evaporation surface or from a line source embedded in the smooth surface. Eq. (3) has been extensively treated by O. G. SUTTON [1934], W. G. L. SUTTON [1943], and FROST [1946]. In the present paper m and n are considered to be independent, and the solution of the generalized equation

$$y^m (\partial c/\partial x) = D \partial (y^n \partial c/\partial y)/\partial y \quad \text{(4)}$$

where $D = A_1 y_1^{m-n}/u_1$ is given for each of the three cases cited in the abstract. Since m and n are assumed to be independent, the validity of the solutions contained in the present paper, depends only on the adequacy of the power-function representations of u and A(y), and not on the validity of Sutton's theory.

As a special feature of the present investigation, dimensional considerations will be utilized in searching for a similarity-solution (Ähnlichkeitslösung). These, in conjunction with considerations of the powers of x, A_1, u_1, and y_1, will afford in a systematic way the most adequate transformations to be made in Cases (1) and (2) such that the solutions will be the simplest.

Diffusion from a line source embedded in a smooth surface--Eq. (4) is to be solved with the following boundary conditions:

(a) $\partial c/\partial y = 0$ at $y = 0$
(b) $c \to c_0$ as $y \to \infty$
(c) $c \to c_0$ as $x \to 0$ for $y > 0$

and with the continuity equation

(d) $\int_0^\infty u (c - c_0) dy = Q = \text{constant}$

where c_0 is the ambient vapor concentration, and Q is the strength of the line source per unit length. The first boundary condition stipulates that the ground is impervious to vapor. The pertinent variables are, in this case, the following: c, c_o, Q, A_1, u_1, y_1, x, y. A dimensional analysis yields the relationship

$$(c - c_o)/c_o = F\ (Q/A_1 c_o,\ u_1 x/A_1,\ y_1/x,\ y/x)$$

where $u_1 x/A_1$ is a Reynolds number.

To obtain a similarity solution, assume

$$\psi = (c - c_o)/c_o = (Q/A_1 c_o)\ (u_1 x/A_1)^p\ (y_1/x)^q\ f[(u_1 x/A_1)^r\ (y_1/x)^s\ y/x] \qquad (5)$$

where the exponents p, q, r, and s are to be determined, and where the function f is to be determined from the diffusion equation and the boundary conditions. The last equation is a special form of the general relationship resulting from the dimensional analysis, but has the most general form for a similarity solution. Significantly, the effect of y is felt only through the new variable in the bracket, while the effect of x is felt not only through that variable, but also through the two factors preceding the function f. Substituting (5) in (4) and demanding equal powers in A_1, y_1, u_1, and x, one obtains values of r and s as follows

$$r = 1/(m - n + 2) \qquad s = (n - m)/(m - n + 2)$$

which are independent of the values of p and q. The values of p and q are obtained from condition (d), the satisfaction of which requires that

$$p = (n - 1)/(m - n + 2) \qquad q = (n + m)/(m - n + 2)$$

Thus, a similarity solution is possible with the transformation

$$\psi = (Q/A_1 c_o)\ (A_1^{1-n} y_1^{n+m}/u_1^{1-n} x^{m+1})^{1/(m-n+2)}\ f[(u_1 y_1^{n-m}/A_1 x)^{1/(m-n+2)} y] \qquad (6)$$

Substituting (6) into (4), one obtains the dimensionless equation

$$-[(m + 1)/(m - n + 2)]\,\eta^m f - [1/(m - n + 2)]\,\eta^{m+1} f' = d\,(\eta^n f')/d\eta \qquad (7)$$

where the primes denote differentiation with respect to the new variable

$$\eta = (u_1\, y_1^{n-m}/A_1\, x)^{1/(m-n+2)}\, y \qquad (8)$$

The boundary conditions become

(a) $f\ (0) = 0$
(b) and (c) $f\ (\infty) = 0$ if $m - n + 2 > 0$

and the continuity condition becomes

$$\text{(d)} \int_0^\infty \eta^m f d\eta = 1 \qquad (9)$$

A first integration of (7) yields

$$-[1/(m - n + 2)]\ \eta^{m+1} f = \eta^n f'$$

the constant of integration being zero since both f and f′ are finite at $\eta = 0$. A second integration gives

$$f = K \exp(-\ \eta^{(m-n+2)}/(m - n + 2)^2) \qquad (10)$$

where K is determined from (9) and is given by

$$K^{-1} = \int_0^\infty \eta^m \exp[-\ \eta^{m-n+2}/(m-n+2)^2]\, d\eta = (m-n+2)^{(m+n)/(m-n+2)}\ \Gamma[(m+1)/(m-n+2)] \qquad (11)$$

This integral is convergent since $m > o$. Eq. (6), (10), and (11) then constitute the solution.

Diffusion from a smooth surface--Denoting by c_s the saturated vapor concentration at the evaporation surface, (4) is in this case to be solved with the following boundary conditions:

(a) $c = c_s$ at $y = 0$
(b) $c = c_o$ at $y = \infty$
(c) $c = c_o$ at $x = 0$ for $y > 0$

In this case the parameter containing the unknown c is

$$\theta = (c_s - c)/(c_s - c_o) \quad \text{. . . . .} \quad (12)$$

and the parameter $Q/(A_1\, c_o)$ is eliminated from consideration. Since the vapor flux is no longer constant, the integral condition (d) of the last section no longer exists. Hence, the values of p and q, which are determined by that condition, shall now be chosen to satisfy the boundary conditions listed above. Since condition (b) requires that $\theta = 1$ at $y = \infty$ irrespective of the values of A_1, y_1, u_1, and x, the values of p and q must both be zero.

Substituting therefore

$$\theta = h(\eta) \quad \text{. . . . .} \quad (13)$$

into (4) where c can be replaced by θ, one has

$$-\eta^{m+1}h'/(m - n + 2) = \eta^n h'' + n\eta^{n-1}h' \quad \text{. . . . .} \quad (14)$$

where the primes denote differentiation with respect to η which is defined by (8). The boundary conditions are now

(a) $h(0) = 0$
(b) and (c) $h(\infty) = 1$ if $m - n + 2 > 0$

Eq. (14) may be written

$$-\eta^{m-n+1}/(m - n + 2) - n/\eta = h''/h'$$

a first integration of which gives

$$h' = B\eta^{-n}\exp[-\eta^{m-n+2}/(m - n + 2)^2] \quad \text{. . . . .} \quad (15)$$

A second integration yields the solution

$$\begin{aligned} h &= B\int_0^{\eta} \eta^{-n}\exp[-\eta^{m-n+2}/(m - n + 2)^2]\,d\eta \\ &= B\,\Gamma[(-n + 1)/(m - n + 2),\ \eta^{m-n+2}/(m - n + 2)^2]\,(m - n + 2)^{(-m-n)/(m-n+2)} \quad \text{. . . . . .} \quad (16) \end{aligned}$$

where $\Gamma(a,y) = \int_0^y x^{a-1}e^{-x}dx$ is the incomplete gamma function and where B is determined by $h(\infty) = 1$ and is given by

$$\begin{aligned} B^{-1} &= \int_0^{\infty} \eta^{-n}\exp[-\eta^{m-n+2}/(m - n + 2)^2]d\eta \\ &= \Gamma[(-n + 1)/(m - n + 2)]\,(m - n + 2)^{(-m-n)/(m-n+2)} \quad \text{. . . . .} \quad (17) \end{aligned}$$

where $\Gamma(a) = \Gamma(a, \infty)$ is the gamma function. Eq. (12), (13), (16), and (17) then constitute the solution, so long as $n > 1$ so that the integrals in (16) and (17) exist. Eq. (16) may be written

$$h = \Gamma[(-n + 1)/(m - n + 2),\ \eta^{m-n+2}/(m - n + 2)^2]/\Gamma[(-n + 1)/(m - n + 2)] \quad \text{. . . .} \quad (16a)$$

Vapor concentration in the wake of an evaporating surface--The solution for this case can be obtained from the results of the last two sections by realizing that the evaporating surface can be considered as a collection of line sources the strengths of which are determined from the local rates of evaporation and that both the evaporation surface and the dry surface can then be considered as impervious to vapor.

The strength of the elemental line source at $x = \lambda$ is

$$(c_s - c_o)(A_1/y_1^n) \lim_{y \to 0} (y^n \, \partial h/\partial y) \, d\lambda = (c_s - c_o) \, G \lim_{\eta \to 0} (\eta^n h') \, d\lambda = (c_s - c_o) G B d\lambda \; \ldots . (18)$$

where

$$G = (A_1^{m+1} u_1^{1-n} / \lambda^{1-n} y_1^{m+n})^{1/(m-n+2)}$$

Substituting the expression in (18) for Q in (6), one has

$$d\psi = d[(c-c_o)/c_o] = [(c_s-c_o)/c_o] B (1/\lambda^{1-n} (x-\lambda)^{m+1})^{1/(m-n+2)} f\left\{[u_1 y_1{}^{n-m}/A_1 (x-\lambda)]^{1/(m-n+2)} y\right\} d\lambda \; . (19)$$

as the contribution of the elemental line-source to the value of $\psi = (c - c_o)/c_o$ at any value of x. If the evaporation surface has a downwind length b, then for $x > b$ one has, on integrating 19

$$(c - c_o)/(c_s - c_o) = B \int_0^b (1/\lambda^{1-n} \, (x - \lambda)^{m+1})^{1/m-n+2)} f(\xi) \, d\lambda \; \ldots\ldots\ldots (20)$$

where

$$\xi = [u_1 \, y_1^{n-m} / A_1 \, (x - \lambda)]^{1/(m-n+2)} \, y \; \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

and were f is given by (10).

Diffusion in Couette flow--As an example for the theory developed in the previous sections, one considers the diffusion in Couette flow. Here $m = 1$, $n = 0$, and $A = \alpha$ = constant, where α is the vapor diffusivity.

The solution for the case of a line source is, from (6), (8), (10), and (11)

$$\begin{aligned}(c - c_o)/c_o &= (Q/\alpha c_o) \, (\alpha y_1/u_1 x^2)^{1/3} \, K \exp(-\eta^3/9) \\ &= (Q/\alpha c_o) \, (\alpha y_1/u_1 x^2)^{1/3} \exp(-\eta^3/9)/3^{1/3} \, \Gamma(2/3) \; \ldots\ldots\ldots (22)\end{aligned}$$

where

$$\eta = (u_1/y_1 \, \alpha x)^{1/3} \, y \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The solution for the evaporating surface is, from (12), (13), (16), and (17)

$$(c_s - c)/(c_s - c_o) = B \int_0^{\eta} e^{-\eta^3/9} \, d\eta = \Gamma(1/3, \eta^3/9)/\, \Gamma(1/3) \; \ldots\ldots\ldots (24)$$

where η is given by (23).

From (10), (17), (20), and (21), the vapor concentration in the wake of the evaporating surface of length b is

$$\begin{aligned}(c - c_o)/(c_s - c_o) &= BK \int_0^b [1/\lambda (x - \lambda)^2]^{1/3} \, e^{-\xi^3/9} \, d\lambda \\ &= [1/\Gamma(1/3) \, \Gamma(2/3)] \int_0^b [1/\lambda \, (x - \lambda)^2]^{1/3} \, e^{-\xi^3/9} \, d\lambda \; \ldots\ldots (25)\end{aligned}$$

where

$$\xi = [u_1/y_1 \, \alpha (x - \lambda)]^{1/3} \, y \, . \; \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

It should be noted that since in this case $n = 0$, $A(0) = \alpha$ is different from zero and the singularity at $\eta = 0$ for h' in (15) does not exist, so that there is no objections to the theory based on physical considerations.

Remarks--It should be noted that along any generalized parabola

$$y = \beta x^{1/(m-n+2)}$$

the value for $(c_s - c)/(c_s - c_0)$ is the same for the case of diffusion from a smooth surface, and that on any two generalized parabolas of the above type the values of $(c - c_0)/c_0$ bear the same ratio for any value of x for the case of diffusion from a line source. These facts provide the reason why the related solutions are called similarity-solutions. The solution represented by (20), however, does not belong to the similarity-solution category.

Acknowledgment--This work has been done in connection with an ONR project (Contract No. N90nr-82401) in which the Civil Engineering Department of the Colorado Agricultural and Mechanical College is currently engaged.

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(Communicated manuscript received April 30, 1951;
open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

MOISTURE ACCUMULATION RATES IN THUNDERSTORM CELLS

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Abstract--Limits on the duration and rates of moisture accumulation in thunderstorm cells are estimated from computations based on the empirical results of Byers and Braham applied to idealized cells involving assumptions of radial symmetry, zero divergence of specific momentum, United States standard atmospheric temperature-height curve, no horizontal density difference, initial saturation, and a steady-state flow pattern applied instantaneously. Stationary cells of two sizes in a general area of no wind and another case of a cell in a general horizontal velocity field of ten miles per hour relative to the cell are considered. Maximum moisture-accumulation rates of about one inch per hour are found for all three cells. The rate of accumulation becomes zero in about 90 minutes for the stationary cell of three-mile radius, about five hours for the stationary one of five-mile radius, and becomes constant after 30 minutes for the cell in the horizontal velocity field.

If a thunderstorm cell consists of a circulation somewhat similar to that of the "Bénard cell," namely, a central core of upward moving fluid surrounded by a region of downward moving fluid, and if this flow pattern persists for a sufficiently long time, it is to be expected that condensation-producing processes will sooner or later cease because of the return of air which has already lost its moisture during a previous passage through the system. If, on the other hand, a translational current is added to this circulation, the accumulation process will not cease, but the moisture distribution will become steady state, since new moist air will continuously enter the cell, and the desiccated air will be continuously removed.

In this study the duration and rate of moisture accumulation are computed for three models. Model A is that of a radially symmetrical circular cell with a core of upward moving air of two-mile radius and an annular cylinder, or region, of downward moving air of three-mile outside radius surrounding the core. Model B is the same as model A except that the region of downward moving air has an outside radius of five miles. Model C is model A with a uniform horizontal velocity of ten miles per hour superimposed. The cell height of all three models is eight miles. The vertical velocities in the upward moving air in the core are based on the empirical results given by BYERS and BRAHAM [1948] as typical of a thunderstorm cell in the "cumulus" stage of development (Fig. 1). Because of the simplifying restrictions utilized, the models cannot be considered strictly to represent conditions as found in nature. Assumptions which contribute to the artificiality of the models are:

For models A and B
(1) Radial symmetry of flow about the vertical axis.
(2) Symmetry of flow about a horizontal plane at four-mile elevation.
(3) United States standard atmosphere, initially saturated.
(4) Dimensions of cell remain constant with time.
(5) Computed moisture accumulation expressed as depth of precipitable water distributed uniformly over horizontal cross section of core.
(6) Flow patterns imposed instantaneously at time zero and remain constant with time.

For model C
(1) Bilateral symmetry about a vertical plane through center of cell and parallel to the superimposed horizontal velocity field.
(2) Uniform horizontal velocity field superimposed instantaneously on the velocity field of model A.
(3, 4, 5, and 6) Same as for models A and B.

The vertical velocities in the upward moving air in the core (Fig. 1) are extrapolated to give the vertical velocities in the region of downward moving air. This extrapolation makes use of an assumption that the three-dimensional divergence of specific momentum is zero, which requires that at each level the mass transport of air upward in the core must equal the mass transport downward of the air in the annular region surrounding the core.

Using Green's theorm together with the assumption that $\nabla \cdot (\rho \mathbf{V}) = 0$ (where ρ is the air density), the radial velocities are computed from the values of the vertical velocities. The vertical velocities determined from the airplane data were larger and considered more accurately known than the horizontal velocities. This, together with the fact that horizontal velocities computed from given vertical velocities necessitate internal consistency of the model and the assumptions, prompted the use of the latter only. Particle trajectories are constructed from the radial and vertical velocity components. Vapor density and velocity of particles entering or leaving the core are determined from the trajectories. The product of inflow velocity, moisture density, and the area of the core wall minus the product of the outflow velocity, moisture density, and area of the top of the core gives the rate of accumulation of moisture within the core. Dividing this rate by the horizontal cross-sectional area of the core gives the rate of accumulation expressed in inches of precipitable water. The details of the computation and its results are given below.

From the assumption that

$$\nabla \cdot (\rho \mathbf{V}) = 0$$

or

$$\rho \nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla \rho = 0$$

and if ρ is considered constant in the horizontal, then, using cylindrical coordinates,

$$\rho (V_r/r + \partial V_r/\partial r) + \rho \, \partial V_z/\partial z + V_z \, \partial \rho / \partial z = 0$$

or

$$\nabla_H \cdot \mathbf{V} = - \partial V_z/\partial z - V_z \, \partial \log_e \rho / \partial z$$

where ∇_H is the horizontal component of ∇, and V_r and V_z are the radial and vertical velocities, respectively. At a particular height, h_j, and a radius, r_i, the right-hand side of the last equation will have a fixed value, k_i.

From Green's theorem for two dimensions

$$\int_c V_r \, d\ell = \iint_A \nabla_H \cdot \mathbf{V} \, dA$$

or using average values over a circle of radius r_i

$$2 \pi r_i V_{r_i} = \iint_A k_i dA$$

Divide the area enclosed by a horizontal circle with radius equal to that of the thunderstorm cell into concentric rings whose radii increase in increments of 0.5 mi. The average k for any ring will be one-half the sum of the k_i at the outer edge of the ring and $k_{i-0.5}$ at the inner edge. The area of any ring is $\pi (4 r_i - 1)/4$, where r_i is the outside radius, and the horizontal velocities V_r are computed by

$$V_{r_n} = (1/2\pi r_n) \sum_{i=0.5}^{i=n} [(k_i + k_{i-0.5})/2] [\pi (4r_i - 1)/4] = (1/16 r_n) \sum_{i=0.5}^{i=n} (k_i + k_{i-0.5}) (4r_i - 1).$$

where n = 0.5, 1.0, 1.5, ... Values of k_i (in units per hour) are obtained from the vertical velocities and their incremental variation with height from Figures 1, 2, and 3, and increments of $\log_e \rho$ with height (Table 1) based on density values for the United States standard atmosphere given in the Physical tables of the SMITHSONIAN INSTITUTION [1934].

Table 2 gives k_i for r_i equal zero up to five miles at heights h_j from 0.5 to 3.5 miles.

Table 1--Increments of $\log_e$ with height

Elevation	$\Delta \log_e \rho/\Delta z$[a]
mi	
0.5	-0.16
1.0	-0.16
1.5	-0.16
2.0	-0.17
2.5	-0.17
3.0	-0.17
3.5	-0.18
4.0	-0.18

[a] Δz = one mile.

Table 2--Horizontal divergence in units per hour as a function of height and radial distance

Location	Item	$h_{0.5}$	h_1	$h_{1.5}$	h_2	$h_{2.5}$	h_3	$h_{3.5}$
For models A and B (region of the core	k_0	-3.31625	-3.0218	-2.7015	-2.3018	-4.8694	-13.6329	-11.9495
	$k_{0.5}$	-2.76354	-2.5182	-2.2648	-2.0015	-3.7245	-11.0845	- 9.3506
	k_1	-1.84236	-1.6788	-1.5099	-1.3344	-2.6497	- 4.1946	- 3.1308
	$k_{1.5}$	-0.92118	-0.8394	-0.7549	-0.6672	-0.5748	- 0.9778	- 0.7879
	k_2	0	0	0	0	0	0	0
For model A (downdraft ring)	$k_{2.5}$	+1.2260	+0.9964	+0.8245	+0.6806	+1.8766	+4.3876	+2.5020
	k_3	0	0	0	0	0	0	0
For model B (downdraft ring)	$k_{2.5}$	0.17581	0.1495	0.1559	0.1251	0.1518	0.4021	0.4773
	k_3	0.28478	0.2502	0.1948	0.1235	0.2544	0.7147	0.5523
	$k_{3.5}$	0.32005	0.2438	0.2683	0.2135	0.3388	0.7846	0.4560
	k_4	0.36005	0.3174	0.2869	0.2119	0.3052	0.8159	0.5846
	$k_{4.5}$	0.17581	0.1795	0.1795	0.2051	0.3631	0.5121	0.5210
	k_5	0	0	0	0	0	0	0

Substituting the k values into the summation equation, the horizontal (radial) velocities are computed (Table 3). For the region of the core these velocities are the same for models A and B. (Negative sign indicates velocity toward the center.)

Table 3--Radial velocity in miles per hour

Location	Item	$h_{0.5}$	h_1	$h_{1.5}$	h_2	$h_{2.5}$	h_3	$h_{3.5}$
For the region of the core	$V_{0.5}$	-0.66	-0.69	-0.62	-0.54	-1.07	-3.08	-2.66
	V_1	-1.24	-1.13	-1.02	-0.89	-1.73	-4.42	-3.67
	$V_{1.5}$	-1.43	-1.28	-1.15	-1.01	-1.83	-3.81	-3.26
	V_2	-1.25	-1.14	-1.03	-0.91	-1.50	-3.23	-2.62
Model A (downdraft ring)	$V_{2.5}$	-0.73	-0.69	-0.64	-0.57	-0.77	-1.60	-1.53
Model B (downdraft ring)	$V_{2.5}$	-0.96	-0.88	-0.78	-0.70	-1.16	-2.49	-1.99
	V_3	-0.70	-0.64	-0.58	-0.52	-0.88	-1.82	-1.42
	$V_{3.5}$	-0.46	-0.44	-0.39	-0.37	-0.61	-1.21	-0.98
	V_4	-0.24	-0.25	-0.21	-0.22	-0.38	-0.69	-0.73
	$V_{4.5}$	-0.09	-0.11	-0.08	-0.10	-0.18	-0.30	-0.29

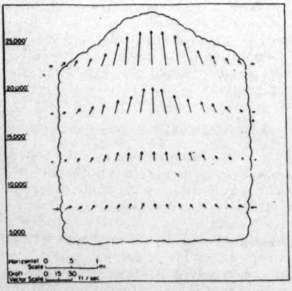
25,000'
20,000'
15,000'
10,000'
5,000
Horizontal Scale 0 .5 1 mi
Draft Vector Scale 0 15 30 ft / sec

Fig. 1--Velocity field within a typical thunderstorm cell in "cumulus" stage of development [after BYERS and BRAHAM, 1948]

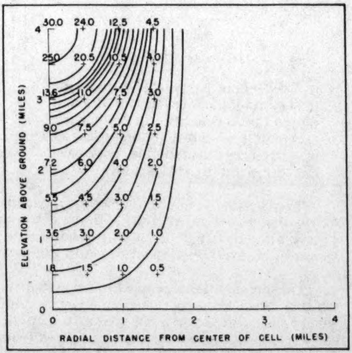

Fig. 2--Vertical velocities in updraft core of thunderstorm-cell models (in mph) from Figure 1

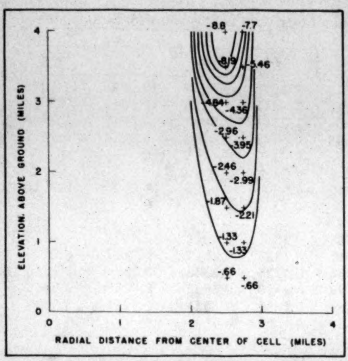

Fig. 3--Vertical velocities in downdraft region of the thunderstorm cell, model A (in mph)

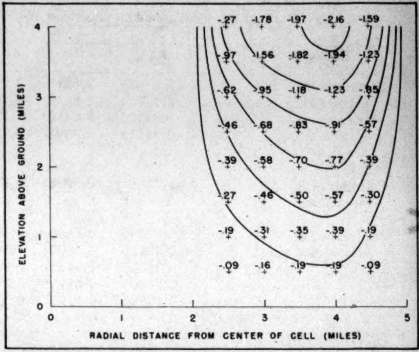

Fig. 4--Vertical velocities in the downdraft region of the thunderstorm cell, model B (in mph)

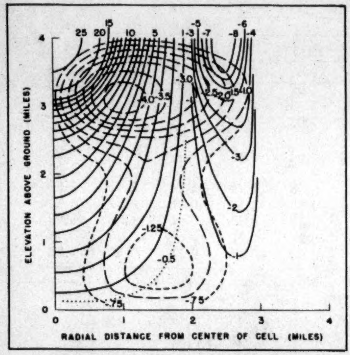

Fig. 5--Field of horizontal (broken and dashed lines) and vertical (solid and dotted lines) velocities in the thunderstorm cell, model A (in mph); because of symmetry assumptions, only one-quarter of the field is shown

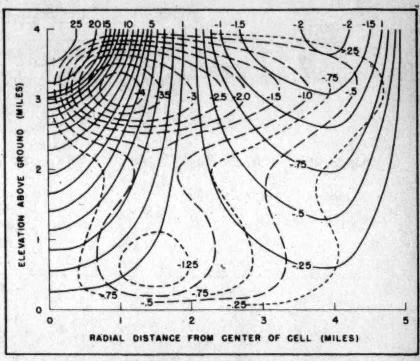

Fig. 6--Field of horizontal and vertical velocities in the thunderstorm cell, model B (in mph) (see Fig. 5 for explanation)

The vertical velocities (smoothed) based on Figure 1 are shown in Figure 2 for the core. The extrapolated vertical velocities in the downflow region are given in Figure 3 for model A and in Figure 4 for model B. Isopleths of horizontal and vertical velocities in the same diagram are shown in Figure 5 for model A and in Figure 6 for model B (in a radial plane).

These velocities are used to construct trajectories based on a time unit of 0.01 hr (0.6 min); the trajectory steps are four units or 2.4 min. long except where lower velocities give steps too short to be plotted. In those cases 16 units or 9.6 min. are used. A particle is moved a horizontal and vertical distance using the velocities from Figure 5 or 6 at its initial position. Then, the velocities from Figure 5 or 6 for the midpoint of the first step are used to give a new (better approximation) step which is then plotted in Figure 7 or 8. The velocities in the core are the same in the two models and are shown in Figure 7. In the downflow region, trajectories for model B are shown

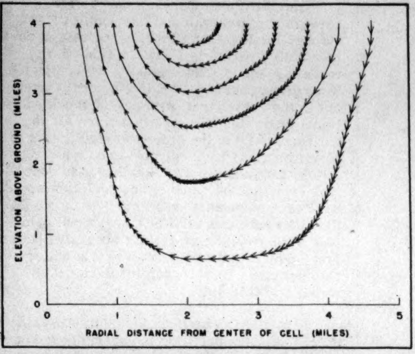

Fig. 7--Particle trajectories in the thunderstorm, model B; because of symmetry assumptions, trajectories in only one-quarter of the field are shown

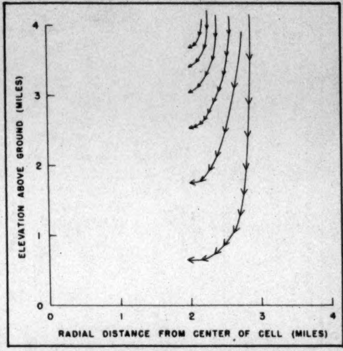

Fig. 8--Particle trajectories in downdraft region of thunderstorm cell, model A; updraft core trajectories are the same as in Figure 7

in Figure 7, those for model A in Figure 8. From the trajectories, the elevation of the particle at any given time previous to that at which it enters the wall of the core is determined. From the elevation and the assumption of a saturated standard atmosphere, the vapor pressure at the starting position is determined. For the moisture passing upward through the four-mile level an initial elevation of four miles (Fig. 7) is used for all particles.

The positions of particles at time t = 0 were determined for particles which in the "inflow" computations arrived at the core wall at t = 0, 10, 30, 60, ... 240 min and for particles which in the "outflow" computations arrived at the four-mile level at t = 0, 10, 30, 60, ... 240 min.

The moisture transport in terms of precipitable water in inches per hour per unit area is

$$P_{in/hr} = 13.77\ V\frac{e}{T}$$

Table 4--Areas of annuli on core top and walls (sq mi) for each trajectory shown in Figure 7

Top	4.020	2.393	2.001	1.610	1.217	1.327
Wall	5.67	4.41	5.67	7.56	11.97	15.12

where V is the velocity (mph) through the core surface, e is the vapor pressure (mb), and T is the temperature (°A). This quantity must, for each of the six trajectories shown in Figure 7, be multiplied by the area represented (Table 4), the boundaries of which are determined by the midpoints between trajectories.

In the computations of inflow rate, the vapor pressure of the particle in its initial position is used. In the computation of the outflow rate, the vapor pressure at the exit point of the trajectory at the four-mile level is used, unless the initial elevation of the particle is higher, in which case the vapor pressure at the higher level is used. The products for the four-mile level of the core are then summed and the total subtracted from the summed products for the wall. The difference is the rate of accumulation of moisture within the lower half of the core. This figure divided by the cross-section area of the core gives the rate of accumulation (storage, condensation, or precipitation) per unit area.

Table 5--Rate of moisture accumulation

Time	Model A	Model B
min	in/hr	in/hr
0	0.99	0.99
10	0.86	0.99
30	0.60	0.91
60	0.25	0.77
90	-0.05	0.66
120	-0.12	0.59
150	-0.09	0.48
180	-0.04	0.40
210	-0.01	0.29
240	0.00	0.22

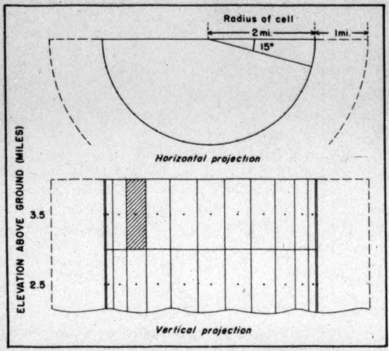

Fig. 9--Illustrating location of points (dots) where trajectories pierce the core wall

The difference between the "inflow" through the core wall and the "outflow" through the four-mile level is the rate of accumulation, which is shown in Table 5 for models A and B. (Since making the computations given in this paper, it has been pointed out to the author that, with not too much additional labor, the accumulation rates for the entire core from the ground to eight miles in elevation could have been obtained instead of for the lower half only. Had that been done, it is expected the rates of accumulation would have been somewhat greater.) For model B the difference drops off at a nearly uniform rate which would give zero at about five hours. For model A it drops off at a nearly uniform rate but more rapidly than for model B as shown in Table 5.

In model C a uniform translational velocity of ten miles per hour is superimposed on the convective cell of model A. Its trajectories are constructed such that there is one exit trajectory for each dot on the exit wall of the core, and one entry trajectory for each dot on the entry wall of the core (Fig. 9). In addition, exit trajectories for the top are prolongations of the entry trajectories for the dots at 3.5 mi (Fig. 10).

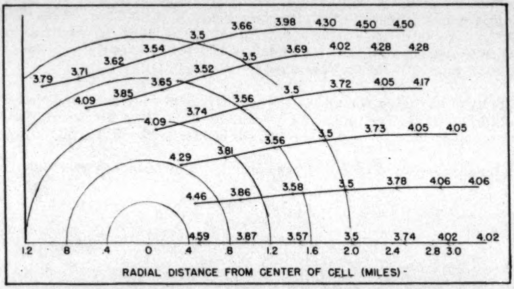

Fig. 10--Plan projection of trajectories which enter the core at an elevation of 3.5 mi above the ground, extended to where they either reach an elevation of four miles or emerge from the side of the core; numbers on trajectories are elevations above the ground in miles

Prolongations of trajectories at other levels have their exit points on the sides. Dot positions are at heights of 0.5, 1.5, 2.5, and 3.5 mi (Fig. 9). They are spaced 15° apart on the circumference (4π mi) of the core as shown in the diagram. The exit and entry trajectories are extended backward until they are outside the region of downdraft. Examples of exit and entry trajectories are shown in Figures 11 and 12, respectively. After a time greater than that required for a particle to go along the longest trajectory from just outside the downdraft region to just outside the core (on the exit side), a steady state thenceforth exists. Each entry and exit trajectory through the wall is representative of a cylindrical surface one mile high and of arc length $4\pi/24$ mi and having an area of 0.5236 sq mi. (One of these is shown cross-hatched in Fig. 9.) Each exit trajectory on the top (Fig. 10) is representative of a strip parallel to the direction of the translational flow. The average exit velocity (vertical) for each strip is determined by plotting vertical velocities for every 0.2 mi along the strip and by averaging graphically (Fig. 13).

Computations similar to those for models A and B give the rates for model C as shown in Table 6. Steady-state conditions exist from 30 min on, and the rate of accumulation is then constant.

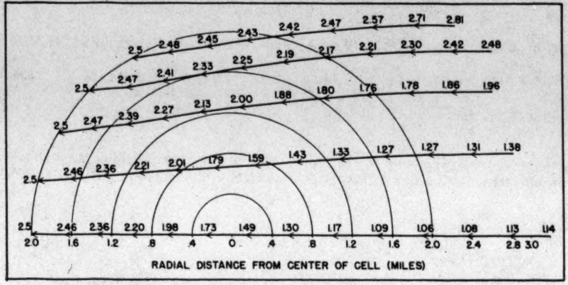

Fig. 11--Plan projection of trajectories which emerge from core at dot positions at elevation of 2.5 mi above ground; numbers in trajectories are elevations above the ground in miles

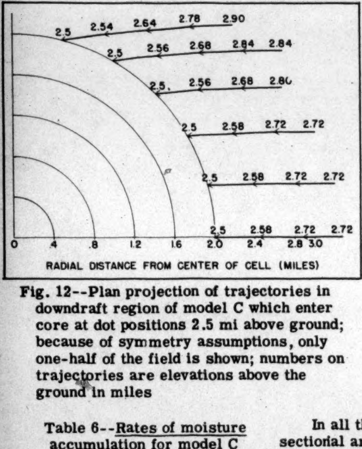

Fig. 12--Plan projection of trajectories in downdraft region of model C which enter core at dot positions 2.5 mi above ground; because of symmetry assumptions, only one-half of the field is shown; numbers on trajectories are elevations above the ground in miles

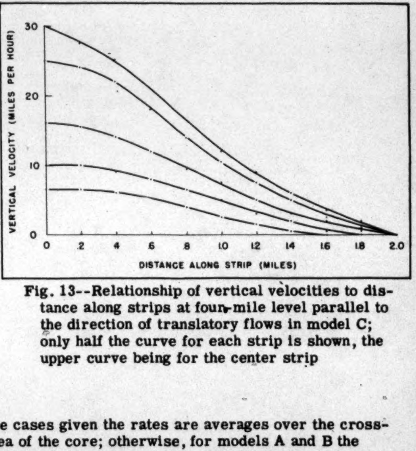

Fig. 13--Relationship of vertical velocities to distance along strips at four-mile level parallel to the direction of translatory flows in model C; only half the curve for each strip is shown, the upper curve being for the center strip

Table 6--Rates of moisture accumulation for model C

Time	Accumulation
min	in/hr
0	0.99
10	0.61
30+	0.66

In all the cases given the rates are averages over the cross-sectional area of the core; otherwise, for models A and B the rates at the center of the core would be greater than at the perimeter, and for model C the highest rate would be along the diameter parallel to the direction of the translational flow.

In conclusion, the reader is reminded that the assumptions upon which the models are based make them highly artificial. Even so, the computations indicate that a possibly important factor in bringing the convective activity to an end is the return to the core of air that has already passed once through the precipitation producing process and has had most of its moisture removed.

Acknowledgment--I thank the other members of the Cooperative Studies Section for their contributed suggestions and advice.

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(Manuscript received June 5, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

PROBABILITIES FROM LIMITED WEATHER DATA

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Abstract--Operational decisions are often based on a "calculated risk." The probability accompanying this risk, when weather elements are the critical factors, can often be estimated from climatological data. If the available data are limited as to quantity and quality, the applied climatological problem is further complicated. Several such problems and a solution to each are presented as illustrations of this phase of applied climatology and of the use of statistical analysis in meteorological problems.

Introduction--Probabilities computed from climatological data have an important niche in the field of applied climatology. Climatology is the historical aspect of meteorology, and applied climatology is the use of this history to plan for the future. That is to say, applied climatology is the analysis of past weather data to obtain an answer to a specific operational problem. This definition excludes analyses of weather data and the relations derived from them unless they are used in the solution of a definite problem, which may or may not be in the field of meteorology.

It is in this respect that climatology in the last decade has departed from the older concept. For the half century prior to 1940, climatologists devoted their main effort to producing simplified versions of inherently complex weather phenomena. It was an age of climatography; climatic statistics were usually presented in the form of generalized maps without consideration of specific needs. It was an age of climatic classification, based on regional variation in mean values of one or more climatic elements. As is usually the case, averages and totals utilized only a small portion of the potential information available in the records.

The demands of military operations during the last war brought into sharp relief the inadequacies of the older climatology. Means and climatic types could not answer the questions: where, when, how often. For example, plans for a paratroop operation might require the probability of simultaneous occurrence of given conditions of cloud cover, wind velocity, visibility, and temperature. Means do not indicate the best time or area for such an operation nor the probability of simultaneous occurrence.

A military operation and a business venture are analogous in many respects. After the conception of the idea, there are three phases: (1) strategic planning phase; (2) tactical planning phase; and (3) operational phase. Climatology contributes chiefly to the first phase, strategic planning. However, it is not without significance in the second and third phases. The military or business man may ask any number of questions. He may request the probability of a certain condition existing as the result of recent weather phenomena. He may ask for the chances of success or failure if certain weather elements are the deciding factors. Both men have definite problems and want specific answers. They may not be interested in, or may not comprehend, meteorological jargon. But they do want an answer that they can understand and use. This is applied climatology.

In such a situation the problem for the climatologist is estimating the *a posteriori* probability of the occurrence of a given event. The climatologist must determine which meteorological elements will affect the military operation or business venture in question, set up the desired criteria for these elements, and study their occurrence in weather records of the past. He can then compute the desired probability together with a confidence limit depending upon the length of record available and the assumption that there will be no major climatic change in the near future.

The procedure in general is straightforward, but often, in the solution of specific problems, the limitations of the data open a new field of application for statistical theory, meteorology, and practical improvisation. It is in these attempts to find specific answers to specific problems that one realizes the imperfection of the available weather records. In the midst of an unbelievably large amount of information, it is not infrequent that there are few or no data to answer the problem. This is not a criticism of any person or any organization. The malady exists everywhere as a result of world conditions, and, moreover, it occurs whenever rapid progress is being made in a field of endeavor.

The limitations referred to exist not only in the cases in which the data are insufficient to draw a reliable conclusion. There are two other situations that occur in climatology: (1) There are no observations of the climatological elements which are needed to answer the problem. (2) The available data are not in a form such that they can be used directly to answer the problem.

However, in many situations, a decision has to be made, often quickly. It is, therefore, necessary for the climatologist to obtain an answer with whatever data are immediately available. He cannot plead that the data are inadequate. A decision must be made. It is better to employ climatological and statistical analysis of a few data than to permit an unsupported guess. In an attempt to illustrate some of the points mentioned above, four problems are presented. They are neither typical nor atypical, but they serve to illustrate the variety of applications of climatology.

Problem 1--It was necessary to determine whether the number of days with precipitation was below normal during June 1950 over a given area. This was an areal problem, but to depict the method only one station, A, will be used. The only available climatic information was the average number of days with precipitation. For this station during June the average was 13.5. The only data available for June 1950 were scattered six-hourly observations with observations missing for 6, 12, and 18 hours, and for full days. From this information a conclusion had to be drawn concerning the number of days with rainfall during June 1950. The information that was available is shown in Table 1. There were five days on which rain was reported in at least one observation.

Table 1--Expected number of days with rain, Station A, June 1950

Description of day	Number of days	Probability of rain	Expected number of days with rain
With rain	5	1.00	5.00
24 hr unknown	6	0.45	2.70
18 hr unknown	2	0.36	0.72
12 hr unknown	6	0.26	1.56
6 hr unknown	6	0.14	0.84
No rain	5	0	0
Total	30		10.82

Climatic average number of days with rain, 13.5
Expected number of days with rain, 10.8

Therefore, these could be counted as days with rain regardless of the reception of other reports. There were six days for which no information was attainable. To these was assigned the climatological probability of rain which was the ratio of 13.5 to 30 or 0.45. There were two days for which one observation indicated no rain but the other three observations were missing. To these days was assigned the climatological probability of rain during an 18-hour period which was computed from the formula $1 - p = (1 - 0.45)^{3/4}$, where p is the probability of rain during that period. There were six days for which two observations indicated no rain and the other two observations were missing. To these days was assigned the probability computed from the formula, $1 - p = (1 - 0.45)^{1/2}$. There were six days with three observations indicating no rain and one observation unknown. These received the probability computed from $1 - p = (1 - 0.45)^{1/4}$. And, finally, there were five days with all observations present and indicating no rain. This method of calculating probabilities for days with missing observations assumes that the weather from one period to another is independent. This, of course, is not true, but in the absence of information as to the degree of dependence, this assumption had to be made. Furthermore, the random manner in which the reports were received partially offsets the bias that might result.

The expected number of days with rain in each category was computed from the first two columns of Table 1, and the sum of these is the expected number of days with precipitation; in this case, 10.8 days. Among the 20 doubtful days, the expected number of days with rain was 5.82 so that the probability of rain during any one of these doubtful days at random is 0.29, (5.82/20). Using this as a binomial probability, the distribution shown in the lower portion of Figure 1 was obtained. It is a cumulative binomial distribution curve of the relative frequency of the various possibilities for 0 to 20 days with rain. The curve is shifted so that it starts at five days since it is known that five days of the month had no rain. By extending the curve beyond 25 days, it combines the 20 days for which incomplete information is available with the five days without rain and the five days with rain. This distribution indicates that the probability that June 1950 had fewer than the normal 13.5 days with rain is 92 pct. The upper portion of Figure 1 shows the normal

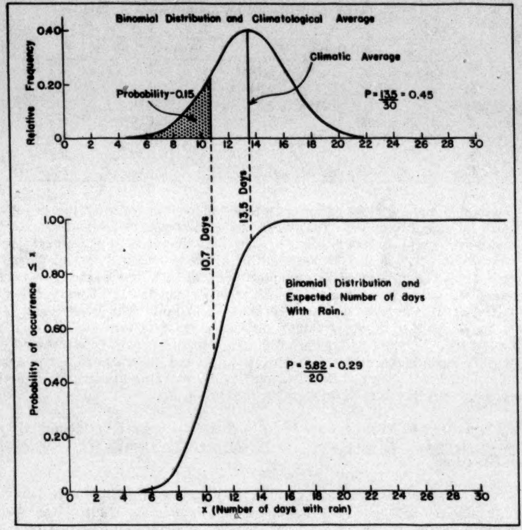

Fig. 1--Probability of number of days with rain, Station A, June 1950

approximation for the binomial distribution based on the long-term climatological average. With only the average number of days with precipitation available, this assumption of a binomial distribution is adequate for an approximation of the true distribution. It is seen that the probable number of rainy days for this particular June, 10.8, has a 15 pct probability of occurrence over a long period of time. In other words, on the average, approximately one June out of seven will have as few or fewer days with rain than this value of 10.8. The conclusion was, therefore, that the number of days with precipitation was almost certainly below normal by a significant amount but probably not by a very unusual amount. It is satisfying to mention that more complete data which were received later verified this conclusion.

Problem 2--One of the many considerations in the selection of an air base, in the air support of ground troops, and in airborne troop operations is the probability or frequency of occurrence of contact flying weather. Contact flying weather requires visibility of three miles or more, and cloud ceiling 1000 ft or more above the ground. Unfortunately, a great number of meteorological stations throughout the world do not record summaries of simultaneous ceiling and visibility observations. All that is available for these stations are separate monthly summaries of the frequency of visibility equal to or greater than three miles and of cloud ceiling at or above 1000 ft. As an expedient, it has been common practice to assume that the two elements are independent, and to multiply the two percentage frequencies together to obtain the percentage frequency of their simultaneous occurrence, that is, the percentage frequency of contact flying conditions. Written as a formula, $K = CV$, where C is the relative frequency of cloud ceilings at or above 1000 ft, V is the relative frequency of visibility equal to or greater than three miles, and K is the relative frequency of contact flying conditions. Since it is evident that there must be some degree of dependence (positive correlation) between these two meteorological elements, this assumption of independence will, on the average, underestimate the correct frequency. This was verified when the regression of K on CV was calculated. It was found that the regression equation was $K = 1.03\,CV$ (see Table 2, Eq. (6)).

It was desired to have a formula which would be simple to use, which would allow for the interdependence of the two elements, which could be applied to all months of the year, and which would apply to all geographic locations within reason when confined to monthly values. With this

Table 2--Regression equations

Equation	Standard error	No.
$K = 0.47 - 0.36C - 0.15V + 1.04CV$	0.013	(1)
$K = 0.15C + 0.37V + 0.48CV$	0.013	(2)
$K = 0.53V + 0.47CV$	0.016	(3)
$K = 0.28C + 0.72CV$	0.019	(4)
$K = 0.15C + 0.85V$	0.025	(5)
$K = 1.03CV$	0.030	(6)
$K = 0.63L + 0.37LU$	0.012	(7)

in mind it was reasoned that, if there were complete dependence between the two elements, K would have its largest possible value. This value would be equal to L, the lower of the two values, C and V. If there were complete independence, then the value of K is equal to the product, CV, which may be written as LU, where L is the smaller of the two values, C and V, and U is the larger of the two. Since there is positive correlation between C and V, the true value of K lies somewhere between the value L and the value LU. The regression for K may, therefore, be written as $K = aL + (1 - a) LU$, where a is an undetermined constant. The least-squares value for a was computed from 780 sets of monthly values, that is, from data for each of the 12 months for 65 stations widely scattered throughout the United States, for which both separate and simultaneous summaries of the elements were available. The equation and its standard error are shown in Table 2 as (7). Note that this regression equation fits the extreme conditions; namely, when L is zero, K is also zero; and when U is 100 pct, K is equal to L.

The first six equations in Table 2 were used to check the degree of acceptability of (7). It is seen that this equation gives a fit as good as or better than the results from any of the other multiple regression equations.

Problem 3--One of many logistic problems is the planning of a flight schedule over an air route to insure the proper amount of supplies at the terminus of the flight. The effect of a head or tail wind over a route, while not the only factor, is very important in determining the pay load, gas consumption, and duration of flight. A knowledge of head or tail winds along a certain route and their frequency is of utmost importance. Generally there are few, if any, adequate upper-air data on a world-wide basis from which the distribution of head or tail winds can be derived. The most recent contribution to the problem, and one of the most valuable, was developed by English meteorologists [BROOKS and Others, 1946 and 1950]. The first step in this contribution was the discovery that the upper-air wind variation above a station could be expressed in terms of a normal distribution. To be more precise, it was found that the distribution of the perpendicular components (x and y in Fig. 2) of the vector difference between the actual wind vector and the mean wind vector could be expressed as normal distributions with equal variability. From this they developed the equation for the probability that a given wind velocity will lie between vector V_1 and vector V_2. This probability is expressed in terms of the vector deviation (v) and the angle (θ) that the vector deviation makes with the mean wind vector (V_R) (see Fig. 2). The equation is

$$P (V_1 < V \leq V_2) = (1/\pi \sigma^2) \int_{v_1}^{v_2} \int_{\theta_1}^{\theta_2} ve^{-v^2/\sigma^2} \, d\theta \, dv \quad \ldots (8)$$

where σ is the standard deviation of v and is called the standard vector deviation. The standard vector deviation was computed for points throughout the world, and charts were drawn so that the standard vector deviation could be estimated at any point along an air route. These charts have been prepared for most of the standard constant-pressure surfaces. From this concept they were able to derive an equation for the mean equivalent head or tail wind over a route ($\overline{w}$) in terms of the components of the mean wind and the standard vector deviation at points along the route [SAWYER, 1950; BRITISH AIR MINISTRY, 1950]. The equation is

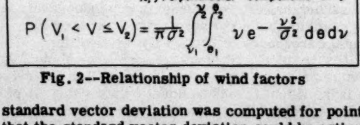
V
ν
y
θ
x
V_R
V_R = Resultant or mean wind
V = Actual wind
ν Vector deviation between actual and resultant wind
θ Angle between vector deviation and resultant wind
x, y Perpendicular components of ν
$P(V_1 < V \leq V_2) = \frac{1}{\pi\sigma^2}\int_{\nu_1}^{\nu_2}\int_{\theta_1}^{\theta_2} \nu e^{-\frac{\nu^2}{\sigma^2}} d\theta d\nu$

Fig. 2--Relationship of wind factors

$$\overline{w} = -\,\overline{u} + (1/2A)\left\{\overline{v^2} + \overline{\sigma^2}/2\right\} \qquad (9)$$

where $\overline{u}$ is the mean of the components of the mean wind along the route, $\overline{v^2}$ is the mean of the square of the components perpendicular to the route, $\overline{\sigma^2}$ is the mean of the square of the standard vector deviations, and A is the air speed of the plane. The standard deviation of the equivalent head or tail wind (w) is given by

$$\sigma_w = m\,(\overline{\sigma^2})^{1/2} \qquad (10)$$

In this equation, m is a factor depending on the length of the air route. Since the distribution of the equivalent head or tail winds is for all practical purposes normal [SAWYER, 1950], the distribution can be computed and drawn from the values w and σ_w. With distributions such as these, the feasibility of different air routes can be compared and the operational schedules prepared when wind is the deciding criterion. As an illustration, the distribution of equivalent head winds is presented in Figure 3 for a flight from Gander, Newfoundland, to New York. This distribution is for a flight level of 10,000 ft. Other distributions may be prepared for different flight levels. From them the frequency of various head winds can be readily computed, so that the practicability of various flight levels and payloads can be evaluated for wind as the deciding factor.

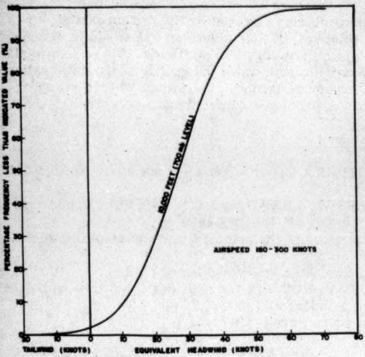

Fig. 3--Percentage frequency of equivalent head winds on flights from Gander, Newfoundland, to New York during December to February

Problem 4--The efficient use of water supply in the West is important to national defense and to the economy of the West. The U. S. Weather Bureau, by providing water-supply forecasts in this area, aids in the formulation of the current operational procedure. These forecasts are an excellent example of the combination of climatological probabilities and forecasting procedures [LINSLEY and Others, 1949]. The forecasts are based on a relationship between the seasonal (October through April) precipitation over a watershed and the runoff from this watershed during the water year (the 12-month period October through September). Precipitation stations in the watershed are used to estimate the precipitation over the area. However, inhabitability requirements and the rugged western topography usually prevent the stations from being in the most representative location. It is therefore necessary to determine how effective each station is in representing precipitation over the watershed. This is done by means of multiple correlation. From the multiple correlation, relative weights for each station are obtained. Using these weights, the monthly precipitation over the basin is estimated for the total period of record. Then the same method of multiple correlation is used to determine the relative effect of each month's precipitation. By applying the relative weights to each station and to each month, an index of the October-April precipitation over the watershed is computed. These values are used to develop a forecasting relation for water-year runoff. But to be of any value, the forecast must be made long before the precipitation over the basin is known. To meet this problem, past records are used to develop partial indices and their distribution for those periods for which the precipitation would be unknown. Thus there would be a partial index and its distribution for the April precipitation, as well as for the March through April precipitation, for the February through April precipitation, and other periods. The known precipitation and the partial indices are combined to form indices from which water-supply forecasts are made for the possible amounts of subsequent precipitation. These forecasts are made in such a way that the probability of various ranges can be computed from them. For example, the forecast for the Flathead River at Columbia Falls, Montana, on February 1, 1950 is shown in Table 3.

For this forecast the known precipitation index for the period October 1949 through January 1950 was combined with the appropriate partial precipitation indices for February through April. With this type of a forecast the user can base his plans for future operations on a calculated risk. Even if the precipitation on the Flathead River from February through April is the minimum of

Table 3--Water-supply forecast for Flathead River at Columbia Falls, Montana, on February 1, 1950

Item	Assuming precipitation February through April is				
	Minimum of record	Lower quartile	Median	Upper quartile	Maximum of record
Forecast (1000 AF)	6480	7300	7720	8210	9300
Per cent of normal	114	128	136	144	163

Normal 5700 (1000 AF) Average error 560 (1000 AF)

record, the water supply will be above normal. Knowledge such as this is of inestimable aid to those who plan the allocation of the water for power, agriculture, etc.

Conclusions--These are but four of the many problems confronting the meteorologist, climatologist, and hydrologist. Although the main object of all four solutions was to aid in planning, this is by no means the limit for the combination of climatology and statistics. There is a wealth of data in the files of the various weather organizations; and, while the quality of the observations may not be comparable to controlled laboratory experiments, they can through statistical and meteorological analysis be used to solve many of the problems confronting meteorology.

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(Manuscript received June 27, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

STABILITY AND MUTUAL COMPENSATION OF RELATIONSHIPS WITH THE ICEBERG SEVERITY OFF NEWFOUNDLAND

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(Contribution No. 574 from the Woods Hole Oceanographic Institution)

Abstract--A stability test has been made of several elements of the North Atlantic air circulation which had previously been concluded by Smith and Groissmayr to influence the severity of the subsequent iceberg season south of Newfoundland. The test indicates that steeper-than-average pressure gradients between Belle Isle (near Labrador) and Ivigtut (in southern Greenland) during December to March continue as before to be associated with above-normal, and less steep pressure gradients with below-normal berg counts off Newfoundland during the following April-June season. On the other hand, steeper-than-average pressure gradients between Bergen in southern Norway and Stykkisholm in Iceland have ceased in recent years to be followed by above-normal berg counts off Newfoundland, probably because of a very marked change in the circulation in the northeastern North Atlantic. Also, the average temperatures from December to March at St. John's (Newfoundland), Bermuda, and Uppsala (Sweden), when taken together, continue as before to show a marked connection with the following ice severity off Newfoundland.

A formula relating the berg count to the relatively stable and mutually compensating series of average December-to-March Belle Isle-to-Ivigtut pressure gradients and St. John's, Bermuda, and Uppsala temperatures was derived using, as before, data up to and including 1926. Employing the recent 25-year period (1927-1951) as a test, the correlation between the computed and actual deviations from the long-term average berg count is quite marked (r = 0.80).

Introduction

Following the preliminary indications obtained mainly by MEINARDUS [1904] and MECKING [1906] that the iceberg severity off Newfoundland in the spring and early summer is influenced by the preceding atmospheric-pressure distribution over the northern North Atlantic and adjacent areas, SMITH [1931] derived a formula employing essentially the pressure differences, Belle Isle-to-Ivigtut and Bergen-to-Stykkisholm, based on data from 1879-80 to 1925-6, inclusive (similar entries are subsequently shown thus: 1880-1926).

Subsequently GROISSMAYR [1939] suggested that the preceding air temperatures over the North Atlantic and elsewhere also bear a marked relation to the following ice off Newfoundland. This might have been expected from the general relation between pressure and temperature in that area.

The probable reason for the apparent control of the ice off Newfoundland in late spring and early summer by the preceding October-January Bergen-to-Stykkisholm and the December-March Belle Isle-to-Ivigtut pressure gradients is the following: An above-normal transport of air into the Arctic, indicated by a large Bergen-to-Stykkisholm pressure difference, may be expected to give rise to an above-normal outflow from the Arctic over some other area. This outflow may be assumed to continue for some time beyond the period of the initial inflow. The outward transport of air probably occurs along several paths. The general area of Baffin Land and Labrador, owing to the normal configuration of the isobars over it, may be regarded as one of the more favored outflow regions. When the outflow from the Arctic actually takes place over this area, it may be assumed to lead to more bergs drifting southward along the west sides of Baffin Bay and Davis Strait and along the Labrador coast. Eventually, bergs appear in greater than normal numbers off Newfoundland. In the same way stronger-than-normal northwesterly winds, associated with large Belle Isle-to-Ivigtut pressure gradients, may help speed the southward drifting bergs and prevent some bergs from getting stranded in the inlets and shallows of the Labrador coast, thus causing more bergs to appear in the south. Similarly, when immediately preceding the Newfoundland iceberg season the temperature is above normal in the general areas of Scandinavia and Bermuda (suggesting stronger-than-normal southwesterly and southerly winds, respectively) and below normal over Newfoundland (suggesting stronger-than-normal northwesterly winds in that area), a greater-than-usual inflow of air into the Arctic and outflow of cold air over the general Baffin Land and Labrador area is also indicated. This would also favor a greater-than-normal drift of bergs along the Labrador coast and eventually off Newfoundland.

A later test by SCHELL [1940] of the pressure-ice and temperature-ice relations indicated above, employing the independent data for the 13-year period 1927-1939, showed fair agreement between the computed and actual deviations. However, it was recently suggested by WALKER [1947], from an overall consideration of the stability of the previously derived relationships by Smith and Groissmayr, that these relationships have undergone a marked change. In place of these relationships, Walker suggested that the pressure distribution to the west of Baffin Bay plays an important part in determining the number of bergs that drift southward in Baffin Bay and Davis Strait. These arrive in late spring and early summer in reduced numbers off Newfoundland.

With only Point Barrow pressures at his disposal, Walker correlated the mean monthly October-November values at that station with the berg count off Newfoundland the following spring and early summer. The coefficient of correlation obtained was r = -0.74 (1922-1939). He assumed that lower-than-normal pressure in the general area of northern Alaska must mean a greater-than-normal south-to-north (Point Barrow) pressure gradient, and hence stronger-than-normal west winds all the way to Baffin Bay. These strong winds in turn lead to a greater movement of bergs into the Bay and thence southward along the Labrador coast toward Newfoundland. The interval of four months (December to March) between the end of the pressure season and the beginning of the ice season is a very rough estimate of the time required for the bergs from Baffin Bay to appear off Newfoundland. However, it was recently pointed out by the author [SCHELL, 1948] that the prevailing wind at Point Barrow during October-November is from the northeast rather than from the west. Also a correlation of the October-November Point Barrow pressures with the ice off Newfoundland during the following season, utilizing the data now available for the periods 1940-1949 and 1900-1921 (subsequent to and preceding, respectively, the period 1922-1939 examined by Walker), gave r = 0.19 and 0.26, respectively. The apparent overall association during the entire period of 50 years, including the 21 available to Walker, was practically nil (r = -0.09, 1900-1949, n = 50).

Stability and mutual compensation of the relationships

North Atlantic pressure gradients and temperatures and subsequent ice--In maintaining that the relationships derived by Smith and Groissmayr have undergone a very marked change, Walker did not consider quite the same combination of months as did Smith. What may be more significant, while some relationships were found by Walker to have weakened in their apparent control of the ice, others appear to have strengthened. This raises the question whether in the combinations of relationships used by the earlier investigators there may not have occurred enough mutual compensation to offset the overall change suggested by Walker's work.

As a test of Walker's findings, correlation coefficients were computed using time intervals in better agreement with those employed by Smith and Groissmayr. The greater body of data now available was utilized (Table 1). The correlation between the ice and the preceding October-to-January Bergen-to-Stykkisholm pressure gradient is 0.30 (1880-1949, n = 70) and that between ice and the December-March Belle Isle-to-Ivigtut pressure gradient is 0.53 (1883-1949, n = 67). Breaking up the periods into halves we obtain for the second half of the period, using the Bergen-to-Stykkisholm pressure gradient, r = 0.08 (1915-1949, n = 35) and, using the Belle Isle-to-Ivigtut pressure gradient, r = 0.58 (1917-1949, n = 33). This indicates that, although the relation of the Bergen-to-Stykkisholm pressure gradient with the ice has virtually ceased in recent years, possibly on account of a northerly shift of the Icelandic Low in the northeastern North Atlantic [SCHERHAG, 1936], still there has been no real change in the marked relation of the Belle Isle-to-Ivigtut pressure gradient with the following ice off Newfoundland (see Table 1).

Table 1--Relation of iceberg count off Newfoundland during spring and early summer to the preceding Bergen-Stykkisholm and Belle Isle-to-Ivigtut pressure difference

With Bergen-Stykkisholm pressure difference			With Belle Isle-Ivigtut pressure difference		
Period	n	Coeff., r Oct.-Jan.	Period	n	Coeff., r Dec.-Mar.
1880-1949	70	0.30	1883-1949	67	0.53
1880-1914	35	0.47	1883-1916	34	0.44
1915-1949	35	0.08	1917-1949	33	0.58

Table 2--Relationship of iceberg count off Newfoundland during spring and early summer to the preceding temperatures at St. John's, Bermuda, and Uppsala

Period	n	Coefficient, r		
		St. John's Dec.-Feb.	Bermuda Dec.-Feb.	Uppsala Mar.
1880-1949	70	-0.45	0.32	0.54
1880-1914	35	-0.64	0.42	0.50
1915-1949	35	-0.23	0.24	0.62

A recomputation of Groissmayr's correlation coefficients between ice and the preceding temperatures at St. John's, Bermuda, and Uppsala, based on data for the 70-year period 1880-1949, show (see Table 2) that, while the individual relations of the Bermuda and St. John's temperatures with the following ice have undergone a very marked change from the first half of the period (1880-1914) to its second half (1915-1949), the overall relation would appear to be rather stable. The lesser association of the ice with St. John's and Bermuda temperatures during the period 1915-1949 apparently was to a marked extent compensated by the greater association with Uppsala temperatures of the same period.

Performance of relationships during 1927-1949--From the above examination of the stability of the relationships between iceberg severity off Newfoundland and the pressures and temperatures of the North Atlantic and adjacent areas, it would appear that, while the pressure gradient, Bergen-to-Stykkisholm, has ceased to indicate an inflow of air into the Arctic followed by an outflow along the west sides of Baffin Bay and Davis Strait, the pressure gradient, Belle Isle-to-Ivigtut, continues to reflect the northerly air flow along the Labrador coast, which in turn materially affects the number and speed of bergs that drift southward to Newfoundland waters (see Table 1).

Similarly, the combined preceding temperatures at St. John's, Bermuda, and Uppsala continue as during 1880-1926 to reflect an influence on the iceberg severity off Newfoundland the following late spring and early summer (see Table 2). This correlation is more stable when employing the full interval December-March for all three temperature terms rather than December-February for St. John's and Bermuda, and March for Uppsala as was done initially. This is shown in Table 3.

Table 3--Relationships between the ice off Newfoundland and December-to-March St. John's, Bermuda, and Uppsala temperatures compared with that using December-to-February St. John's and Bermuda, and March Uppsala values

Period	n	Coefficient, r					
		St. John's		Bermuda		Uppsala	
		Dec.-Mar.	Dec.-Feb.	Dec.-Mar.	Dec.-Feb.	Dec.-Feb.	Mar.
1880-1949	70	-0.49	-0.45	0.40	0.32	0.55	0.54
1880-1914	35	-0.60	-0.64	0.41	0.42	0.54	0.50
1915-1949	35	-0.39	-0.23	0.36	0.24	0.58	0.62

As a test of the relationships of the ice with the Belle Isle-to-Ivigtut pressure gradient and St. John's, Bermuda, and Uppsala temperatures indicated above, the values of the ice departure from normal using a scale of zero to 10 for the recent 25-year period 1927-1951 were computed with the aid of the following formula based on data to 1926 inclusive.

$$\begin{aligned}\Delta\text{Ice (Newfoundland)} = {} & +0.12\Delta\text{ pressure difference (Belle Isle-Ivigtut)} - 0.31 \\ & \Delta\text{ temperature (St. John's)} + 0.19\Delta\text{ temperature} \\ & \text{(Bermuda)} + 0.17\Delta\text{ temperature (Uppsala)}\end{aligned}$$

Here, Δ refers to deviation from the long-term December-to-March averages which are: Belle Isle-Ivigtut pressure difference, 5.52 mm; St. John's temperature, 25.6°F; Bermuda temperature, 63.2°F; Uppsala temperature, -3.0°C.

The computed values of ice anomaly are shown together with the actual departures in Table 4.

As might be expected from the evidence of marked stability and mutual compensation of the relationship between ice and the December-March temperatures and the Belle Isle-Ivigtut pressure gradient (Tables 1 and 3), there is a marked agreement between the computed and actual anomalies in recent years also (1927-1951) (Table 4). Of the 25 cases involved, disagreement as to sign between the computed and observed deviations occurred in only five years (1929, 1944, 1945, 1946, 1949), and of the 12 cases with a computed deviation in excess of ± 1.0 disagreement as to sign

Table 4--Computed and observed departures (Δ) from the long-term normal (1880-1926) in the iceberg severity off Newfoundland (1927-1951) on scale of 10

Year	Computed	Observed
1927	-0.1	0.0
1928	0.3	0.8
1929	-0.2	4.2
1930	2.0[a]	0.6
1931	-1.8[a]	-2.8
1932	0.5	0.8
1933	-0.1	-1.1
1934	1.4[a]	1.0
1935	0.8	2.7
1936	-2.0[a]	-2.8
1937	0.4	0.6
1938	1.1[a]	1.8
1939	1.1[a]	1.8
1940	-2.7[a]	-4.7
1941	-1.5[a]	-4.7
1942	-2.4[a]	-2.4
1943	0.8	2.4
1944	-0.3	2.0
1945	-0.8	3.9
1946	-0.4	0.1
1947	-1.5[a]	-2.0
1948	0.4	1.1
1949	0.4	-2.1
1950	1.1[a]	0.4
1951	-2.0[a]	-4.7

[a] Values in excess of ± 1.0.

occurred in none, while the overall agreement in terms of the correlation coefficient for the entire 25-year period is r = 0.80.

Discussion

The above study indicates that the general premise about changes in the circulation in the North Atlantic and adjacent areas affecting the following iceberg count off Newfoundland is true. Of the variously derived relations by Smith and Groissmayr, only the October-January Bergen-to-Stykkisholm pressure gradient appears to have ceased to be significant, and we may conclude that changes in this pressure gradient in October-January have not continued to lead to changes in the outflow of cold air the following spring over Baffin Land, perhaps because of the marked alteration in the pressure field in recent decades over the northeastern North Atlantic [SCHERHAG, 1936].

The reason for considering only indications of air circulation by the author and previous investigators in developing the formula for foreshadowing the severity of the iceberg season off Newfoundland is due to the small volume of the currently accumulated measurements of the strength and heat content of the Labrador Current. Similarly, the counts of the iceberg population to the north, which may be expected to figure in the number of bergs appearing off Newfoundland, have been too few so far to allow their possible inclusion in the formula.

In making use of the method given here for foreshadowing the iceberg count off Newfoundland, it appears best for each user of the formula to decide for himself the tolerance or limits most suited for the operation in question. The only requirement in each case is that the foreshadowings yield results that are significantly better than chance.

Because the berg count used in our computations is based on a scale of 10, we may wish to use a tolerance of one unit. It should be borne in mind that the probability of a given foreshadowing verifying by chance varies also with the class interval of the computed deviations. The highlighting above of the five cases of disagreement as to sign between the computed and actual deviation out of a possible 25 (Table 4) was to provide a ready comparison with the figure possible by chance (50 pct). Also because a foreshadowing with an indicated 100 pct chance of success has a special meaning, attention was called to the fact that in the 12 cases out of the 25 when the computed deviation exceeded 1.0 on the scale of 10, each agreed as to sign with the observed deviation. A foreshadowing that the number of bergs will fall below 440 (the average value on the scale of 10) may be of little value for some operations, but the information that anything over that number (and possibly over 1000 as in some years) is not likely to happen may be of great value for other operations.

In the original investigation by SMITH [1931], the months with a higher degree of correlation as shown by the coefficient, r, were given twice the weight of the months with smaller values of r. Presumably this was done because a derivation of regression equation giving each coefficient its full weight would imply an undue exactness of the relationships.

It is equally true, however, that where the months in question adjoin each other, the general fluctuation of the relation with time often involves an increase in the association of the months with smaller values of r and decrease for the months with larger values of r. Accordingly, we may desire to treat all months with a significant degree of correlation, but whose coefficients may differ somewhat among themselves, as of equal weight.

Thus it was found that the relation of the unweighted December-to-March monthly data of the Belle Isle-to-Ivigtut pressure gradient with the ice appeared actually closer than the relation of the weighted ones, the obtained values of r (1883-1949, n = 67) being, r = 0.53 and r = 0.47 respectively.

Data and acknowledgments

The data used in this study were obtained from official publications and by personal communication. The author expresses his warm thanks to the officials of the meteorological services of Bermuda, Canada, Denmark, and Sweden for kindly providing the more recent meteorological data used in this study and to the officers of the International Iceberg Patrol for the iceberg counts of more recent years.

I am also indebted to Joseph G. Galway for carrying out the detailed computations of this study.

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(Manuscript received May 31, 1951, and, as revised, November 13, 1951; presented at the Thirty-First Annual Meeting, Washington, D. C., May 3, 1950; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

THE FORCES EXERTED BY WAVES ON OBJECTS

M. P. O'Brien and J. R. Morison

Abstract--A theory is developed and experimental verification is presented for the force exerted by surface waves on a submerged spherical object. The object was small compared to the wave length, and its position was near the bottom. The components of force consist of form drag and inertia forces.

Introduction--Experiments on the forces exerted by oscillatory waves on cylindrical piles have shown that the acceleration forces are of the same order of magnitude as the drag forces associated with the orbital velocities [MORISON, O'BRIEN, JOHNSON, and SCHAAF, 1950]. The movement of sand and gravel on the ocean floor has not been accounted for satisfactorily in terms of the orbital velocities at the bottom, and it appears likely that accelerative forces inherent in the oscillatory motion play an important part in this problem of geomorphology. A second factor also may be involved, namely, the flow into and out of the permeable bed and the related vertical pressure gradient immediately at the bottom [PUTNAM, 1949]. This paper deals with the forces in two idealized situations, namely, a single sphere supported near the bottom, and a single sphere in contact with the bottom. The experimental results obtained in each case showed good agreement with each other. The time history of the measured force showed close agreement to the theoretical time history where the empirically determined coefficients were used in the theoretical calculations. These coefficients were determined at four points of the time history and are shown to apply over the complete cycle.

Terminology--The following symbols are used.

H = Wave height
L = Wave length
T = Wave period
d = Water depth
z = Water depth to object
D = Diameter of object
u = Particle velocity in x-direction
v = Particle velocity in y-direction
w = Particle velocity in z-direction
t = Time
p = Pressure
ρ = Water density
M_o = Mass of fluid displaced by object
M_v = Virtual mass
w_o = Unit weight of water
w_s = Unit weight of object
A = Projected area of object
C_D = Coefficient of drag
C_M = Coefficient of mass
F = Force
R_e = Reynolds number
θ = Angular particle position
α = Geometrical angular relationship between sphere and "tee"
ν = Kinematic viscosity of water
β = Angular position of maximum force
ϕ = Beach slope
g = Gravition constant
γ = Specific weight of sphere

Subscripts on symbols like $C_{D_{Re}}$ mean that the value of C_D used was based on Reynolds number.

Wave forces--In a frictionless, incompressible fluid, the dynamical equation [LAMB, 1945, p. 4] describing the motion of a small element of the liquid is

$$dxdydz\ (X - (1/\rho)\ \partial p/\partial x] = (\partial u/\partial t + u\ \partial u/\partial x + v\ \partial u/\partial y + w\ \partial u/\partial z)\ dxdydz \quad \cdots (1)$$

Here, X is the component of an extraneous force such as gravity, being zero if the x-axis is horizontal in a gravitational field. Oscillatory surface waves are limited in their height-length ratio and it is approximately correct [LAMB, 1945, pp. 19 and 363] to assume that the convective terms $u\ \partial u/\partial x$, $v\ \partial u/\partial y$, and $w\ \partial u/\partial z$ are negligible as compared with $\partial u/\partial t$. The equation of motion becomes

$$-(\partial p/\partial x)\ dxdydz = \rho\ (\partial u/\partial t)\ dxdydz \quad \ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

If the left-hand term is integrated throughout a selected volume, the integral yields the component of the normal pressure in the x-direction exerted on the liquid volume, and this must, of course, equal the integrated acceleration of the liquid mass within the selected volume; but this macroscopic result does not yield information regarding the acceleration of the individual fluid masses within this volume. If the volume chosen is filled with a solid object, the acceleration of this object can be predicted from the pressure distribution provided that the presence of the object does not materially alter the flow, and this condition requires that the object replace a volume of liquid within which the velocities and accelerations are substantially uniform. In terms of objects exposed to oscillatory wave velocities, the object should be small in comparison with the wave length. Assuming this condition to be met, the integrated form of (1a) becomes

$$F_p = M_o \, \partial u/\partial t \quad \dots \quad (2)$$

When a force is applied to an object immersed in a fluid, acceleration will occur. The fluid surrounding the object acquires a velocity and the flow field gains kinetic energy. The force required to produce a given acceleration is greater than that necessary to produce the same acceleration of the same object in vacuo because work must be done on the flow field surrounding the object. This effect may be treated in various ways, the most usual being to regard the immersed object as having an added or "virtual mass," dependent upon the flow pattern around the object. The shape of the object is only one factor in determining the flow field and the virtual mass. The other factor is the state of the flow around the object, dependent primarily on the Reynolds number (the Mach and Weber numbers would not enter this particular problem) and on the turbulence. Experiments on spheres in steady flow show a variation in the drag coefficient with both Reynolds number and turublence which is related to the point of separation of the flow. It is to be expected that the virtual mass would exhibit similar variations; however, the point of separation and the virtual mass may not be the same in accelerated flow as in steady flow at the same relative velocity. Adding a term to represent the virtual mass, (2) becomes

$$F_p = (M_o + M_v) \, \partial u/\partial t \quad \dots \quad (2a)$$

The preceding discussion of the force exerted on submerged objects has dealt with frictionless flow. In any real flow, there is an additional force, due to viscous friction, having a magnitude which will depend upon the Reynolds number. At very low Reynolds number, the drag force will be made up predominantly of the shearing forces over the surface of the object. When the turbulence of the approaching stream and the Reynolds number combine to produce a turbulent boundary layer and separation, the friction force is determined primarily by the shape of the object and the force is usually represented by the equation

$$F_D = 0.5 \, C_D \, (w_o A/g) \, u^2 \quad \dots \quad (3)$$

Here, A is the projected area perpendicular to u, the relative velocity. This equation may be employed to represent the drag force even in viscous flow, where the drag force is proportional to the first power of u, but is not particularly convenient in this flow regime because the coefficient of drag is then an inverse function of u. This discussion is aimed primarily at conditions under which the flow is turbulent and C_D is substantially constant.

Combination of (2a) and (3) to yield the total force on an immersed object rests on the assumption that all dimensions of the object are so small in relation to the wave length that the object produces only a local and limited disturbance of the wave orbital velocities as compared with that which would exist if the object were replaced by fluid.

This analysis merely provides a basis for the correlation of data. Its reasonableness will be tested by comparison of the coefficients of virtual mass and drag obtained in oscillatory flow with those for the same objects under steady flow conditions. Exact agreement is not to be expected but the coefficients of drag and of virtual mass should be of the same order of magnitude as in rectilinear flow.

Experimental procedure--The force on a sphere supported near the bottom of a constant-depth channel was measured by the force balance apparatus shown in Figure 1. The suspension rod was shielded from the wave force. It was pivoted near its center and restrained by springs at the top. The deflection of the suspension rod was recorded on a magnetic oscillograph which recorded simultaneously on the same record the surface wave profile above the sphere as well as three feet in front of the sphere by resistance elements [MORISON, 1949].

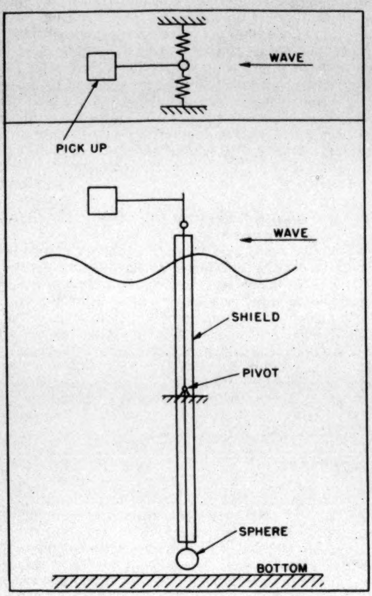

Fig. 1--Schematic diagram of force balance

A second experimental method employed a sphere placed on a narrow slot transverse to the direction of wave motion and the combination of wave height and period which would barely dislodge it was determined experimentally. A vertical section through the sphere and slot is shown in Figure 2. A "tee" was made of wood, one foot wide, two feet long, and 3/8 inch high. It was tapered from the center to each end so that only the center was 3/8 inch high. The slot was made by imbedding two parallel razor blades into the center of the tee. The cross-channel motion of the sphere was blocked by filling all but the center portion of the slot with modeling clay. The wave generator was adjusted by a trial-and-error method until a train of waves would just barely dislodge the sphere. The wave height, length, and period were then recorded six inches in front and behind the position of the sphere. The condition of barely dislodging the sphere was regarded as established when the sphere was moved out of the slot by one of the first 5 to 20 waves. The beach slope was 1:50.

A second series of experiments with the tee was made in a constant depth channel. The waves were set with a constant period but varied in height. Each setting of the wave height was recorded and the effect on the sphere was noted; thus the smallest wave height that moved the sphere was determined. The surface wave profile was measured directly over the sphere and three feet in front of the sphere. Tables 1 and 2 give test conditions and the results of the measurements for the three experiments.

Analysis of data--The wave force from (2a) and (3) required to move the suspended sphere was

$$F = 0.5\ \rho C_D\ (\pi D^2/4)\ u^2 + \rho\ C_M\ (\pi D^3/6)\ \partial u/\partial t \ldots\ldots\ldots\ldots \quad (4)$$

The velocity and the acceleration were computed from the equations for a sinusoidal wave along the center line of the object. These quantities were assumed uniform over the projected area and were computed by the relationships

$$u = \left\{[\pi H \cosh 2\pi (d + z)/L]/[T \sinh 2\pi d/L]\right\} \cos\ (-\ 2\pi t/T) \ldots\ldots\ldots \quad (5)$$

$$\partial u/\partial t = -\left\{[2\pi^2 H\ \cosh 2\pi (d + z)/L]/[T^2 \sinh 2\pi d/L]\right\} \sin\ (-\ 2\pi t/T) \ldots\ldots \quad (6)$$

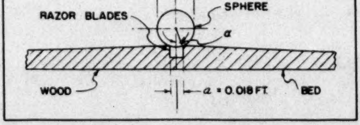

Fig. 2--Sphere and tee

Table 1--Supported sphere

Run	D	H	L	T	d	ϕ	H/L	d/L	C_{D_c}	C_{D_t}	C_{M+}
	ft	ft	ft	sec	ft						
1	0.125	0.281	4.750	0.950	1.315	0	0.0592	0.2769	0.947	2.147	0.997
2	0.125	0.209	5.913	1.150	1.328	0	0.0353	0.2246	1.651	1.929	1.304
3	0.125	0.346	6.085	1.183	1.331	0	0.0571	0.2188	1.919	1.858	1.721
4	0.125	0.249	8.696	1.483	1.328	0	0.0268	0.1527	1.495	0.909	1.273
5	0.125	0.241	3.732	0.850	1.328	0	0.0670	0.3559	2.745	3.093[a]	0.927
6	0.125	0.257	3.468	0.717	1.451	0	0.0738	0.4183	3.129[a]	3.129[a]	0.809
7	0.125	0.217	14.770	2.133	1.328	0	0.0149	0.0899	1.037	0.874	1.042
7A	0.125	0.217	14.770	2.133	1.328	0	0.0149	0.0899	0.938	0.874	0.878
8	0.125	0.273	15.000	2.167	1.328	0	0.0182	0.0885	1.100	0.568	0.905

Table 1--Supported sphere (concluded)

Run	C_{M-}	C_M	C_D	β	β'	β''	F_{max}	$F_{max'}$	$F_{max''}$	R_e	C_D (C_M = 1.59)
	(Aver.)	(Aver.)		°	°	°	lb	lb	lb		(Eq. 17)
1	1.037	1.017	1.547	87	90	90	0.00441	0.00452	0.00443	3883	......[b]
2	1.193	1.248	1.790	83	90	90	0.00458	0.00402	0.00420	3471	......[b]
3	1.546	1.633	1.888	60	50	53	0.00864	0.00886	0.00925	1962	2.985
4	0.791	1.032	1.202	48	40	39	0.00608	0.00454	0.00562	5045	1.134
5	1.052	0.990	2.619	88	90	90	0.00273	0.00282	0.00264	2277	......[b]
6	0.715	0.762	3.129	90	90	90	0.00167	0.00219	0.00232	1915	......[b]
7	0.874	0.958	0.955	54	27	27	0.00467	0.00306	0.00332	6487	0.454
7A	0.868	0.873	0.906	38	26	25	0.00432	0.00283	0.00288	6487	0.542
8	0.951	0.928	0.834	31	23	17	0.00652	0.00539	0.00667	7804	0.799

[a] Measured moment was zero and value of C_D was calculated from the maximum moment and the average value of C_M.

[b] Values indeterminate.

Table 2--Sphere on tee

Run	D	H	L	T	d	ϕ	γ	H/L	d/L	$C_{D_{Re}}$	$C_{M_{Re}}$	C_D (C_M=1.59)	R_e
	ft	ft	ft	sec	ft					(Fig.3)	(Eq.16)	(Eq. 18)	
9	0.0501	0.277	5.700	1.240	0.583	1:50	2.5610	0.0486	0.1023	1.180	0.940	1.140	4715
10	0.0501	0.317	3.860	0.930	0.583	1:50	2.5160	0.0821	0.1511	1.230	1.400	1.201	4507
11	0.1250	0.085	3.177	0.811	0.583	0	1.5444	0.0266	0.2852	....	....	34.957	1302
12	0.1250	0.091	5.753	1.161	0.906	0	1.5444	0.0158	0.1575	2.300	2.030	9.149	2455
13	0.1250	0.096	9.279	1.750	0.906	0	1.5444	0.0103	0.0977	1.800	2.630	6.219	3047
14	0.1250	0.147	9.828	1.800	0.906	0	2.4693	0.0150	0.0922	1.160	3.800	6.878	4834
15	0.0836	0.396	5.129	0.995	0.906	0	2.5203	0.0772	0.1767	0.870	2.034	1.199	7134
16	0.0836	0.295	4.039	0.983	0.708	0	2.5203	0.0730	0.1754	1.050	2.590	2.423	5440
17	0.0834	0.074	3.886	0.950	0.708	0	1.3120	0.0190	0.1823	4.500	2.150	7.624	1335
18	0.0834	0.102	1.884	0.600	0.708	0	1.3120	0.0541	0.3760	....	....	22.938	790
19	0.0300	0.102	3.492	0.989	0.505	0	1.1858	0.0293	0.1447	....	....	21.501	865
20	0.0645	0.080	7.660	1.783	0.505	0	2.5383	0.0104	0.0659	....	....	12.539	1968

Substituting (5) and (6) in (4), the expression for force is

$$F = \pm (\pi^3/8)\ \rho C_D\ (D^2H^2/T^2)\ K_1{}^2 \cos^2 \theta - (\pi^3/3)\ \rho C_M\ (D^3H/T^2)\ K_1 \sin \theta \ \ldots\ldots (7)$$

where

$$K_1 = [\cosh 2\pi\ (d + z)/L]\ [\sinh 2\pi\, d/L] \ \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The position (β) of the maximum force is obtained by differentiating (7) and setting it equal to zero.

$$\beta = \sin^{-1}\ (4 D C_M / 3 H C_D K_1) \ \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

To obtain empirically the C_D and C_M from the experiment, the values of the force were measured along the wave profile at $\theta = 0, 90°, 180°$, and $270°$. The wave profile was trochoidal. Substituting θ into (7) and solving for C_D and C_M, we have

$$\left.\begin{array}{ll} C_{D_{c,t}} = \dfrac{\pm\,(F_{0°,180°})\,8\,T^2}{\pi^3\,\rho\,D^2\,H^2\,K_1^{\,2}} & \begin{array}{l} C_{D_c} \propto F_{0°},\ \theta = 0° \\ C_{D_t} \propto F_{180°},\ \theta = 180° \end{array} \\ & \\ C_{M(+,-)} = \dfrac{\mp\,(F_{90°,270°})\,3\,T^2}{\pi^3\,\rho D^3\,H\,K_1} & \begin{array}{l} C_{M_+} \propto F_{90°},\ \theta = 90° \\ C_{M_-} \propto F_{270°},\ \theta = 270° \end{array} \end{array}\right\} \quad \ldots\ldots\ldots\ldots (10)$$

The empirical values of C_D presented in Table 1 and shown in Figure 3 compared to the flow characterized by Reynolds number. Values of C_M also are shown in Table 1. No relationship was found to exist between C_M and the Reynolds number. The Reynolds number was computed from

$$R_e = Du^2/\nu \quad \ldots\ldots\ldots\ldots (11)$$

where the maximum value of u was taken along the center line of the object.

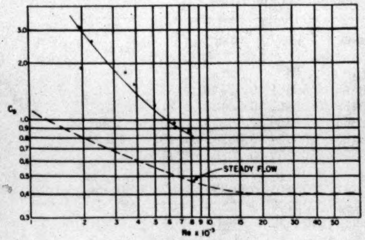

Fig. 3--Coefficient of drag for sphere in oscillatory flow

To compare the time history of the measured force with the theory, the empirical coefficients measured at four points of the force history were used to calculate the theoretical force for a complete cycle (see Fig. 4). Then the theoretical angle β' and β'' and the theoretical maximum force F'_{max} and F''_{max} were calculated and compared to the measured β and F_{max} (see Fig. 5 and 6) where β' and F'_{max} were calculated from average values of C_D and C_M and β'' and F''_{max} were calculated from C_{M_+} and C_{D_c} for one wave length.

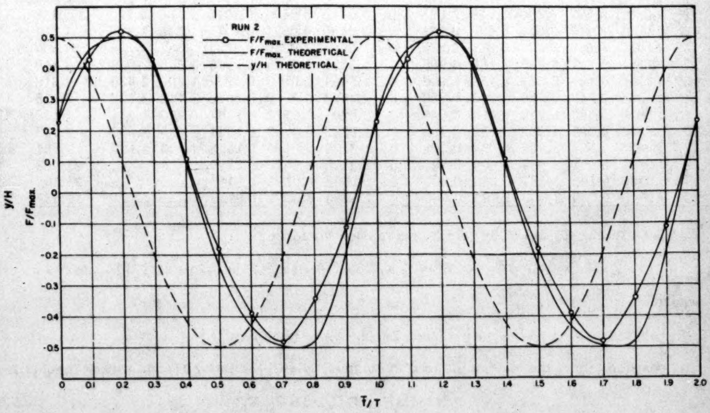

Fig. 4--Time history of wave force on sphere

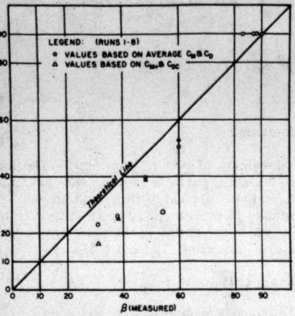

Fig. 5--Phase angle of wave force on sphere

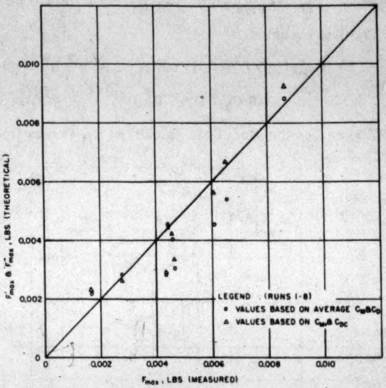

Fig. 6--Maximum wave force on sphere

The experiments with the "tipping" sphere were used to calculate additional empirical values of C_M. The moment required to lift the sphere out of the slot is

$$(\pi D^3/6)\,(w_s - w_o)\,(D/2)\sin\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The moment applied by the wave motion is

$$[C_D\, w_o\, \pi D^2 u^2/8g + (C_M\, \pi D^3\, w_o/6g)\, \partial u/\partial t]\,(D/2)\cos\alpha \quad \ldots\ldots\ldots\ldots (13)$$

The velocity and the acceleration were computed from (5) and (6), the sinusoidal wave equations, where z = -d.

Equating (12) and (13) for the moments, inserting (5) and (6), and rearranging terms, the expression for the equilibrium condition becomes

$$[gDT^2\,(\gamma - 1)\tan\alpha]/BH^2 = (3\,C_D\,B/4)\cos^2\theta + (2\,C_M\,D\sin\theta)/H \quad \ldots\ldots\ldots (14)$$

where

$$B = 1/\sinh\,(2\pi d/L)$$

Assuming that tipping of the sphere occurs at the maximum value of the wave force, differentiation of (14) to obtain the value of θ corresponding to maximum force yields

$$\beta = \sin^{-1}\,(+\,4\,C_M D/3\,C_D BH) \qquad (0° \leq \beta \leq 90°) \ldots\ldots\ldots\ldots (15)$$

Solving (14) for C_M and inserting the values of sin β and cos β

$$C_M = \left\{[3\,gT^2\,C_D\,(\gamma - 1)\tan\alpha]\,4\pi^2 D - 9\,C_D^2 H^2 B^2/16\,D^2\right\}^{1/2} \quad \ldots\ldots\ldots\ldots (16)$$

By calculating Reynolds number for each smallest wave that would tip the sphere, the value of C_D were obtained from Figure 3. The corresponding values of C_M then were calculated from (16) and tabulated in Table 2. From further analysis it appears that C_M stayed relatively constant throughout the range of Reynolds numbers used; hence, the average value of 1.59 which agrees with the value of virtual mass for sphere published by DRYDEN, MURNAGHAN, and BATEMAN [1932] was used, and C_D was calculated from (7) and (16).

Equation (7) was reduced to

$$C_D = [F_{max} - (\pi^3 C_M \rho D^3 H K_1 \sin\alpha)/3T^2]/[(\pi^3 \rho D^2 H^2 K_1^2 \cos^2\beta)/8T^2] \quad \ldots (17)$$

and (16) becomes

$$C_D = [2gDT^2(\gamma - 1)\tan\alpha]/3H^2\pi^2B^2 + \left\{([2gDT^2(\gamma - 1)\tan\alpha]/3H^2\pi^2B^2)^2 - 16\,C_M^2 D^2/9H^2B^2\right\}^{1/2} \quad \ldots (18)$$

Figure 7 is the plot of the calculated C_D versus Reynolds number.

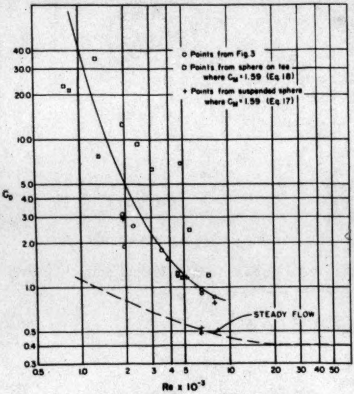

Fig. 7--Coefficient of drag for sphere in oscillatory flow

Movement of bottom materials--The movement of a solid particle resting on a granular bed would correspond to this experiment. The resistance to movement is proportional to the weight while the active forces are drag and pressure. Equating (12) and (13)

$$(g/3)\tan\alpha\,[w_s - w_o)/w_o]\,D^4 = (C_M D^4/3)\,\partial u/\partial t + C_D D^3 u^2/2 \quad \ldots (19)$$

Resistance = Inertia + Drag

The resistance and the inertia terms both include D^4; consequently, neglecting drag forces, large particles would be moved as readily as small ones provided that the shape of the particle and the geometry of the support is the same. The importance of the drag force is inversely proportional to the diameter and becomes the dominant force on small particles.

Conversely, the inertia forces become the dominant factor at large values of D. Streamlined objects having small values of C_D would be influenced predominantly by the pressure-inertia forces. Measurement of the pressure on the bottom in the vicinity of the breakers shows very steep pressure gradients which move with the wave velocity, and these gradients must extend some distance down into a porous bed. Under these circumstances, the inertia term probably dominates. In addition, these pressure gradients cause flow into and out of the bed and the upward flow probably "fluidizes" a large amount of the bed in addition to that which is visibly lifted. This situation may result in movement of the upper portion of the bed en masse, a mechanism fundamentally different from that of bed load in steady flow.

Acknowledgments--The Office of Naval Research sponsored this work. Analysis of data was done by E. K. Rice, R. D. Costello, J. R. Einarsson, and C. L. Bretschneider.

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(Manuscript received December 19, 1950; presented at the Thirty-First Annual Meeting, Washington, D. C., May 3, 1950; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

BREAKING OF WAVES BY AN OPPOSING CURRENT

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Abstract--For deep-water waves propagating from still water upstream against a moving current, complete breaking occurs when the velocity of the opposing current reaches 1/4 of the wave velocity. This was verified by experimental results. Partial breaking took place when the ratio was between 1/7 and 1/4.

Introduction--A wave becomes unstable and breaks when the forward velocity of the water particles at the crest exceeds the velocity of the wave. The usual criterion for wave breaking is given by the steepness of the wave, which is not to exceed 1/7 according to MICHELL [1893]. When the wave propagates upstream against a moving current, its breaking is further related to the velocity of the current. The problem may be studied from a standpoint of energy transmission.

Theory--From an analysis of sinusoidal waves, the velocity G_w at which energy is transmitted by the wave relative to the water is equal to the group velocity C_g of the wave. The absolute velocity of energy transmission G_g is the algebraic sum of the relative velocity and the velocity of the water V; thus

$$G_g = G_w + V = C_g + V \quad \text{(1)}$$

Evidently, if C_g and V are equal in magnitude but opposite in direction, G_g will be zero. In this case, no energy can be transmitted upstream against the current, and the energy is likely to be dissipated through breaking of the wave.

Consider the case in which waves propagate from a region of still water into a moving current having a velocity V. We limit ourselves to deep-water waves. The velocities of the waves in the still region and in the moving current are respectively

$$C_o = \sqrt{gL_o/2\pi} \qquad C = \sqrt{gL/2\pi} \quad \text{(2)}$$

where the subscript o refers to the still region, g is the gravitational acceleration, and L is the wave length. From these we have

$$L_o = 2\pi C_o^2/g \qquad L = 2\pi C^2/g \quad \text{(3)}$$

The period T of the wave should be the same in both regions; therefore

$$T = L_o/C_o = L/(C + V) \quad \text{(4)}$$

Substituting (3) into (4) and solving for C,

$$C = C_o\,[1 \pm \sqrt{1 + 4V/C_o}]/2 \quad \text{(5)}$$

The plus sign in front of the square root should be used, for C should reduce to C_o when V is zero. Since the group velocity for deep-water waves is equal to one-half of the wave velocity, we obtain from (1) and (5)

$$G_g = C_o\,[1 + \sqrt{1 + 4V/C_o}]/4 + V \quad \text{(6)}$$

from which we see that

$$G_g = 0 \quad \text{when} \quad V = -C_o/4 \quad \text{(7)}$$

This simply means that if the current runs against the wave and has a velocity equal to 1/4 of the initial wave velocity, energy cannot be transmitted by the wave into the moving water, and the waves break on meeting such a current. This result was first given by UNNA [1942; see also SCRIPPS INSTITUTION OF OCEANOGRAPHY, 1944].

We now investigate the effect of the current on the wave height, length, and steepness, assuming that there is no breaking of the wave. The absolute velocity of energy transmission of the wave in the still water is simply

$$G_{go} = C_o/2 \qquad (8)$$

Applying the principle of conservation of energy and noting that the energy of the wave per unit area of the water surface is proportional to the square of the wave height

$$H_o^2 G_{go} = H^2 G_g \qquad (9)$$

where H_o and H are the respective wave heights in the still region and in the moving current. Substituting G_{go} and G_g from above, we have

$$H/H_o = [2/(1 + 4V/C_o + \sqrt{1 + 4V/C_o})]^{1/2} \qquad (10)$$

which gives the effect of the current on the wave height. The effect on the wave length is obtained from (3) and (5) directly

$$L/L_o = (C/C_o)^2 = [1 + \sqrt{1 + 4V/C_o}]^2/4 \qquad (11)$$

The effect on the wave steepness is therefore

$$\frac{H/L}{H_o/L_o} = \frac{[2/(1 + 4V/C_o + \sqrt{1 + 4V/C_o})]^{1/2}}{[1 + \sqrt{1 + 4V/C_o}]^2/4} \qquad (12)$$

If we put H/L equal to the theoretical maximum value of 1/7, we can obtain for each value of V/C_o a corresponding value of H_o/L_o, which is the theoretical limiting value of the original steepness of the wave without breaking.

It should be noted that since the theory of sinusoidal waves is used in the present treatment, the wave is considered to be of rather small amplitude.

Experiment--Experiments were conducted with the general set-up as shown in Figure 1. Water from a constant-head tank flowed down a flume having a width of eight inches, which consisted of two sections of different depths. The upstream section was shallower and had a length of 18 ft. The downstream section was nine feet long and was installed in a big tank, the bottom of which also served as the bottom of this section. The water was partly discharged over a weir at the end of the tank, but most of it was drained away through a hole at the bottom of the tank near the transition of the two sections; therefore, the water in the deeper section was practically still. Waves were created by a paddle, which was run by a variable speed motor through a set of belts and pulleys. Water was collected in an underground ditch and finally discharged into a weighing tank.

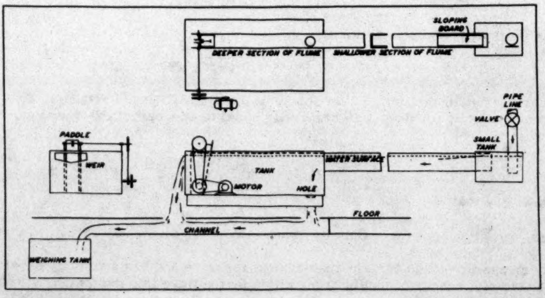

Fig. 1--General setup

The total discharge was measured by means of the weighing tank and covered a range from 0.24 to 0.55 cu ft/sec. The depth of water in the shallower flume was maintained at a value around 0.8 ft and was read directly from a scale. From these the velocity V of the current was computed. The period of the wave T was given various values between 1/3 and 1 1/3 seconds and was timed with a stop watch. Given the period, the wave velocity C_0 and the wave length L_0 could be calculated by means of (3) and (4). The wave heights H_0 and H were measured with a scale to an accuracy of 0.05 inch, with their highest values up to about 1.60 inches.

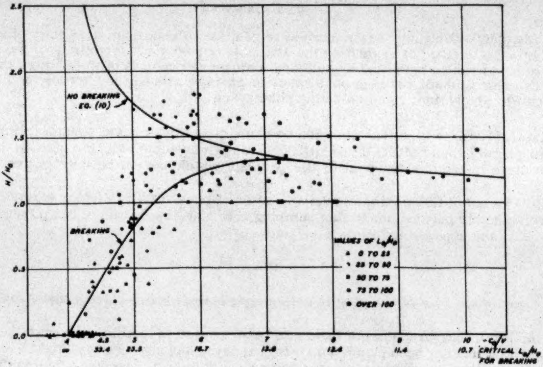

Fig. 2--Change of wave height

Results--Figure 2 shows the results. The ratio of the wave heights is plotted against the negative value of the ratio of the initial wave velocity to the current velocity, that is, H/H_0 versus $-C_0/V$. Equation (10) is also plotted showing the change in wave height when there is no breaking. It is seen that the waves broke completely at the value of $-C_0/V$ equal to 4.0. The water surface in the shallower section of the flume was observed undisturbed, and the theoretical relation given in (7) was verified. For values of $-C_0/V$ greater than about 7 there was practically no breaking; that is, the change in wave height followed the theoretical curve. For values of $-C_0/V$ between 4 and 7, the actual curve deviates continuously from the theoretical curve of no breaking, indicating that there was partial breaking of the wave to various degrees.

Reciprocals of the theoretical limiting values of the original wave steepness as calculated by means of (12) are also shown on the abscissa. These can be used as a guide for determining whether there will be any breaking in a particular case. The change in wave height can then be determined from the curves.

This paper is taken from a thesis submitted to Northwestern University in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering, June 1950, and was presented at the meeting of the Fluid Dynamics Division of the American Physical Society, Urbana, Illinois, May 12-13, 1950. An abstract appeared in Physical Review, v. 79, p. 546, 1950.

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(Communicated manuscript received June 27, 1951;
open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

ON THE STORAGE OF WATER IN ROCKS IN SITU

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Abstract--Plotting maximum yield in relation to maximum drawdown of a pumped well offers a method of evaluating the available porosity in the vicinity of the borehole. The process can be reversed by pumping a measured volume of water down the borehole. A distinction is made between pore space or storage around the borehole and the more difficult to determine, communicating pore space.

TOLMAN [1937, chap. 8] has summed up some of the theoretical considerations of the flow of water through rocks, and MORLEY-PARKER [1932] gives some valuable data on the yields of wells, but there is another approach to the problem which has not been extensively used.

Measurements of the maximum pumping rates of boreholes in various formations indicate that the relationship between maximum pumping rate and time is one of diminishing returns [DIXEY, 1931] and appears to follow the equation

$$R = a^{-ct}$$

where R = maximum rate of pumping in gallons per hour, t = time, and a and c are constants.

The method of carrying out the tests has been consistent, and in each case the newly-drilled hole, or one that has not been pumped for several days, has been pumped and the rate of flow measured at various time intervals. In order to ensure that at any one stage the maximum is being withdrawn, the rate of pumping is purposely kept high, and in the case of a force and lift pump a small amount of air must be sucked in, if possible, at each stroke. Several tests were also carried out with turbines, and these were run at constant speed. In each case the turbine capacity greatly exceeded that of the hole, for example, the turbine capacity was about 8000 gal/hr (Imperial gallons used throughout) as against 3000 gal/hr for the hole. (To prevent air being sucked when turbines were used, the delivery was controlled by means of a gate-valve.)

In Figure 1 the results of three boreholes, all near the contact of the diabase of the Bushveld

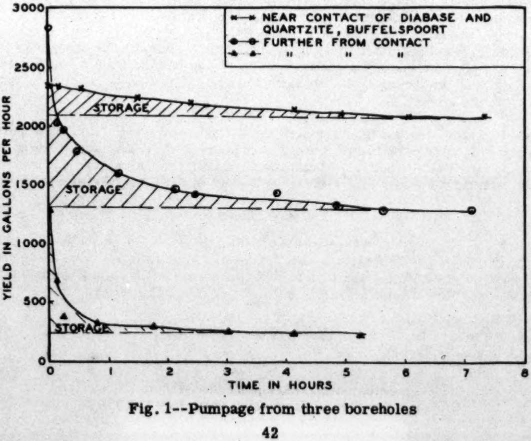

Fig. 1--Pumpage from three boreholes

Complex on Magaliesburg quartzite [HALL, 1932], are given. It will be seen that the points agree very well with the smooth curve drawn through them, and that the asymptote represents the maximum constant yield at any time of testing. There is an annual variation depending upon the rainfall, but tests carried over periods of weeks give a very similar constant yield value.

The shaded area under the curve represents the storage capacity in the rocks plus the storage in the borehole. This latter figure is not always easily obtained in the case of boreholes which have pumps installed, as the data given by owners are not always accurate. The rock storage volume is given by the equation

Rock storage volume = area under curve (water stored from suction of pump to rest level)

Table 1 was calculated for the three boreholes given in Figure 1.

Table 1--Values computed from data on Figure 1

Maximum constant yield	Total storage	Rock storage
gal/hr	cu ft	cu ft
2100	184	156
1200	109	81
247	43	17

These quartzites are dense rocks with a porosity of less than 0.5 pct total, so that the storage must be all in the form of fractures either due to tension or shear stresses. When the rock storage volume and communicating passages are zero, then the flow of water must be nil, and while we now only have three points to plot a graph, this is the best that can be done as there were no other boreholes in the immediate area of these boreholes from which to obtain further data. In Figure 2 it will be noted that the plot of maximum rate of yield against storage is one that requires some interpretation. The line of best fit does not pass through the origin. Another point is that it is extremely unlikely that the relationship between maximum constant yield and rock storage will be linear except within a limited range. Hence the curve B has been drawn to fit the points and also diminish at a greater rate in the upper ranges.

Data obtained from other boreholes have also been plotted as indicated but, except in the lowest range, there is complete disagreement, and in the lower ranges the agreement is probably due to the small scale of plotting relative to the differences. While it is possible that for any one set of relationships between a suite of rocks, that is, for same strike, dip, and general arrangement and frequency of joints and fracture cleavage, there is some mathematical relationship, it is not likely to be of any general application.

In Figure 3 the maximum rates of yield against time are given for some boreholes in the Karroo System near Benoni, Witwatersrand. The formation drilled through was, from the surface downwards,

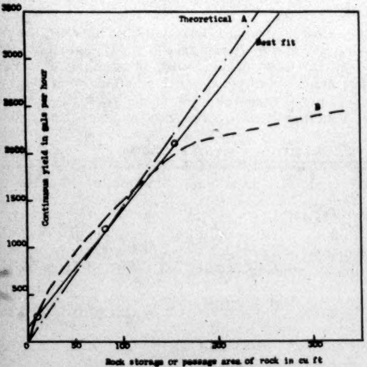

Fig. 2--Relationship of yield to storage

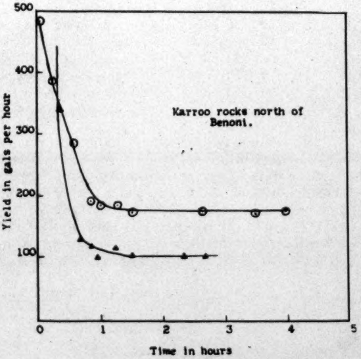

Fig. 3--Relationship of time to yield

shales, sandy shales, sandstones, and grits. The water was found on the slightly decomposed surface of a dolerite sill in the coarse sandstones and grits. The very small rock storage will be noticed as shown in Table 2.

Table 2--Values computed from data on Figure 3

Maximum constant yield	Total storage	Rock storage
gal/hr	cu ft	cu ft
175	32	23
103	21	11

In view of the sandy nature of water-bearing zones, it was considered that, to improve the yields, wells should be sunk. The first consideration is, in what proportion of the well diameter will the yield increase? MORLEY-PARKER [1932, p. 264] indicates that, for a uniform formation and depending upon pore flow, the yield increase is very small with increase in the diameter of the well. His data have been plotted in Figure 4. This means that for a borehole of six-inch mean diameter the yield will be about 1.02 units, and for the same site the yield for a six-foot diameter well (that is, 12 times as great) the yield will only be in the ratio (1.46/1.02) × yield of six-inch diameter borehole. In the case of a borehole of 100 gal/hr the well dug on the same spot and to the same depth will only give 143 gal/hr. This is, of course, a very disappointing result and, when the success of "aqueducts" in North Africa and Kanat's [TOLMAN, 1937] in Persia are taken into account, of doubtful economic application.

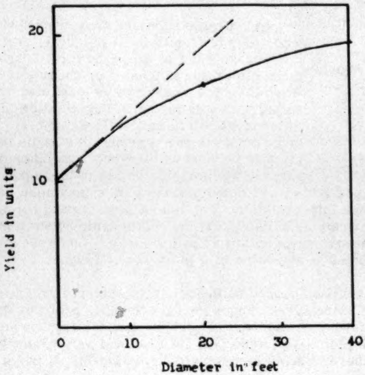

Fig. 4--Relationship of yield to well diameter

For example, in a trench or aqueduct system 1360 ft long, about three feet wide at water level, about seven feet deep, and about 30 miles west of Tobruk (prepared partly by the 4th Field Company of South African Engineering Corps and the Italian authorities), the quantities of potable water shown in Table 3 were pumped. This is in an area of very low rainfall (about 6-10 inches per annum, mainly in the winter) so that it was considered that the idea might be worthy of application in higher rainfall but poor underground water yields in South Africa.

Table 3--Pumpage from a trench system

Date	Yield
1942	gal/day
June 6	26,000
10	28,780
11	24,500
12	17,900
13	22,000

In an area to the north of Benoni, where the best borehole was about 80 gal/hr, a well six feet in diameter was sunk. This gave a constant yield of 67 gal/hr, thus demonstrating that the increase in yield with increase of diameter is not a linear function. However, it would take a great deal more research to show the exact relationship between diameter of well and constant yield for any one formation.

In many cases, shafts or wells have been sunk and increased yields obtained, but these have been mainly in the dolomitic limestones, and what has happened is that the number of fissures intersected has been increased and this is not easily reduced to a mathematical relationship [DU TOIT, 1921].

The data for boreholes in very similar formation to those given in Figure 1 show that there is no relationship between rock storage and maximum constant yield for rocks of the same type but of different structure (see Fig. 5). The data obtained on three boreholes are shown in Table 4.

Table 4--Values computed from data on Figure 5

Maximum constant yield	Total storage	Rock storage
gal/hr	cu ft	cu ft
1850	86.3	76.9
1650	20.0	13.0
390	45	33

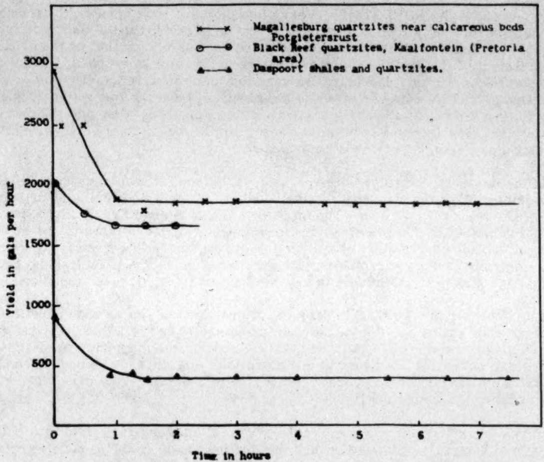

Fig. 5--Pumpage from three boreholes in various formations

Qualitative data--An attempt has been made to study the openings in water-bearing formations from the results of shot and diamond drill cores as well as inspecting wells, shafts, and other excavations in rocks. The results of drilling observations, not only of cores but during the progress of drilling with jumper and core drills, is that the water-bearing fissures and dry inclined fissures are very common in rocks varying from dolerite sills to quartzites and even to granites. Shot drilling often presents the phenomenon of the shot being washed away either by the downward stream of water going into a dry opening or by an underground stream. In many formations, such as the dense Dwyka Tillite of the Cape Province, shot drill cores reveal the fissures to be commonly of about 1 inch to 1 1/2 inches wide. Many of these, in the course of underground water flow, become sealed by the deposition of silica (Fig. 6). This follows from the geochemistry

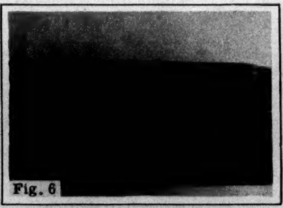

Fig. 6

Fig. 7

Fig. 6--A vertical fracture in the Dwyka Tillite from the Douglas District, Cape Province; note that the fracture has been filled with quartz and fault breccia by circulating water; water-bearing openings in rocks must be regarded from a dynamic point of view

Fig. 7--Joints in granite as shown by shot drill cores (six-inch diameter); on the left the open joint is about 8° from the vertical; note the small parallel joint to it with a very small and not even continuous opening; on the right a low angle joint 33° from the horizontal can be seen; with the exception of these and similar joints, the granite is dense, with an apparent porosity of less than one per cent; the aquifers must be, therefore, joints or similar fissures of the waters, as shown by BOND [1946]. Very often the core is interrupted by fissures and, while some reconstruction is possible, it must be borne in mind that the thickness of such fissures cannot be measured with the accuracy obtained on a lathe, and that as the driller cannot keep a measurement of the depth of the crown before the fissure is reached, a great deal depends upon acuteness of observation. In Figure 7 a very low angle fissure has been intersected by the drill. In the Dwyka Tillite, however, most of the fissures, from the inspection of many cores, appear to vary from a dip of 30° to nearly vertical. The impression of some drillers is that the water is generally in the fissures between these two limits, but it must be stressed this is not a matter of exact measurement but one of impression.

Caves in the dolomitic limestones are, of course, extremely common, there being three types: (1) in joints, (2) in faults, and (3) in the bedding. In the Transvaal and northern Cape Province all three are very common. The Sterkfontein Caves near Krugersdorp, famous for their humanoid remains, have a small bedding development of caves in which the humanoid remains have been found, but they are mainly of the vertical joint type. In the Pretoria district the bedding type is more common, that is, narrow caves of wide dimensions, while fault plane caves are best seen at Taungs and Makapan. However, in any one cave system, all three types are found.

It is not so generally known that, in frequent alternations of shales and quartzites, caves develop by subterranean decomposition of the shales and subsequent erosion. A case of this type containing a small lake in one part and a dry stream course was mapped in the Potgietersrust district. Here, the shale intercalations in the Black Reef quartzites have been decomposed to a ferruginous kaolinite clay and then eroded, giving rise to a bedding-type cave. The cave is subject to inundation from water during heavy rains.

If such caves are intersected by a borehole, the maximum constant yield may only be obtained after many days of pumping. Hence to ensure that the maximum constant yield is obtained, the pumping tests must show a diminishing return, or the curve shown in Figure 8 will be obtained, which is an actual case in the Rustenburg district, in the lower portion of the Magaliesburg quartzites.

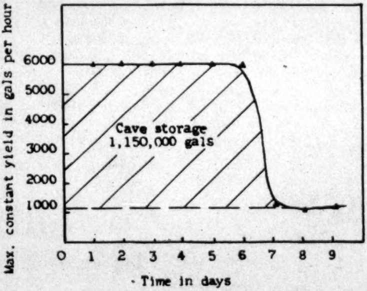

Fig. 8--Typical cave-storage yield curve

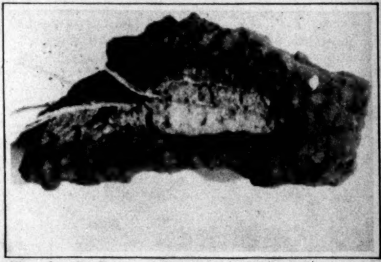

Fig. 9--Kaolinite rich cores (cylindrical) in red plastic Karroo clays; taken from a well north of Benoni

It has been shown by Du Toit and Frommurze that, except for certain formations, porous rocks have very little importance as aquifers in South Africa [FROMMURZE, 1937]. Although basins of decomposition have some importance in the granites and amygdaloidal andesites [ENSLIN, 1943], in general, FROMMURZE's [1943] contention that fissures in granite are important must stand.

It has been shown that in the Karroo formation very often the water is derived from the clays and not the sandstone layers, although the latter have a much higher porosity and permeability, but in the clays there are kaolinite pipes lying in the horizontal plane which carry the water (see Fig. 9).

At most rock contacts there are small fissures and caverns. Many of these develop during the course of pumping a borehole. A borehole in the Pretoria iron-bearing shales dried up during a severe drought, and after withdrawal of the pump it was found that there was only a slight muddy layer at the bottom of the borehole. It was considered that flushing out the mud in a manner similar to that carried out during shot or diamond drilling might reopen the water-bearing fissures or other openings. Clean water was therefore pumped down the hole with the results shown in Table 5. The capacity of the borehole at start was 67 cu ft, equivalent to 413 Imperial gal. A total volume exceeding (13,760/6.24) ± 2200 cu ft of cave or fissure space must exist at the bottom of this borehole at completion of pumping. This borehole had previously given a yield of 350 to 500 gal/hr for over ten years.

Table 5--Results of pumping clean water into a borehole

Date		Amount of water	Depth of hole
1949		Imperial gal	ft
January	7	1440	286
	10	6230	.....
	11	3190	291.8
	11	2800	286.0
		13760	

The reverse of the pumping method is the recharge method [MEINZER, 1923]. It is rare that the opportunity is afforded in South Africa to test the ground-recharge method adequately. In most cases there is not enough water available for such a test. In the Karroo shales and sandstones a very limited test was carried out, but there was insufficient water to obtain a saturation value. In a six-inch-diameter borehole 62 ft deep in sandstones and shales the quantities of water shown in Table 6 were pumped down and the observations made. The borehole was left for 12 hr and the level then fell to the "rest level" at 50 ft from the surface. A pump was then installed and the borehole tested as shown in Table 7. Then the water was exhausted. (Note that Fig. 10 shows that the graph does not conform to the boreholes which are in equilibrium with nature and that a volume of 415 gal of water was recovered beginning five minutes after the end of pumping water down hole.)

Table 6--Recharge into a six-inch borehole

Time	Quantity of water pumped	Water level
hr	gal	
1	1380	Ten ft from surface rapidly falling to 30 ft in five min.
3/4	1400	Similar results

Table 7--Pumpage from recharged borehole

Time	Rate
min	gal/hr
0	1120
15	965
30	181
45	132

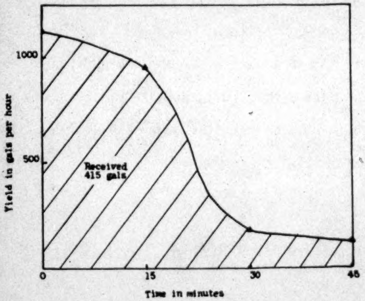

Fig. 10--Pumpage from a recharged borehole

From these rather meager data, the following deductions can be made.

(1) The underground passages are capable of taking 1300 gal/hr at least for several hours. The reservoir is by no means full as the water pumped down is rapidly disseminated to the general water table lying on the dolerite, as shown by the rapid exhaustion on immediate pumping.

(2) A head of 40 ft in a one-foot-diameter hole is sufficient to force the water to the underground reservoir.

(3) The infiltration of rainfall in this very level terrain is not sufficient to maintain the underground water. The total annual rainfall of the area is 25 to 30 inches, and owing to the level terrain the runoff is very small. The infiltration through the Karroo rocks to the dolomite, calculated by DU TOIT [1921] from the withdrawals is three per cent of the total annual rainfall, which is of the same order as that deduced from the hydrological balance worked out for other parts of the Transvaal where the runoff and estimates of the transpiration and evaporation losses have been made.

A rainfall of 26 inches per annum means that an annual total of 2.17 acre feet of water fall on each acre of ground. An infiltration of three per cent means that 0.068 acre feet feed to underground per annum per acre, that is, 18,600 Imperial gallons of water per acre per year go to the underground reservoir. It is evident that the yields must be small unless the water from many hundreds of acres are concentrated against a dyke or a fault. Thus one acre will yield 50.8 gal/day, or 2.12 gal/hr. The water from 500 acres will give 1060 gal/hr. However, we know that this borehole can take over 1000 gal/hr until the reservoir is saturated. It is obvious that this area is one that is admirably suited to ground recharge and that 60 ft boreholes appear to be sufficient to effect this.

Conclusions--The data given here are too inadequate for any far-reaching conclusions to be reached, but the method of both extracting water and measurement of the flow, as well as reversing the procedure and pumping water down boreholes, can give valuable information on rock openings from a quantitative point of view. These preliminary experiments show that it can be a fruitful approach to the manner in which water is stored underground, and that its basis can be quantitative.

Note that these data only give the storage around the borehole and not the volume of the flow passages. To calculate this we have only one known factor, namely, the flow of water. The head is not known but could be estimated from the point of intersection of the water and the rest level. For each rock the friction coefficient for water flow through that particular rock will have to be found by experiment for both smooth and rough passage surfaces.

Acknowledgments--I am indebted to members of my staff, H. G. Trevelyan and J. D. Moore, and to my wife, for assistance in carrying out many of the tests and for preparation of the paper.

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(Communicated manuscript received March 9, 1951;
open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

LABORATORY EVALUATION OF DESERT SOILS FOR IRRIGATION

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Abstract--A simple laboratory technique of evaluating desert soils for suitability to irrigation is outlined. Such laboratory observations are for the purpose of augmenting a field examination of physiographic and pedological limitations of the land. The method is based upon the premise that soil moisture retained against a tension of 100-cm water tension, less the moisture retained against 15,000-cm tension, gives an index of total available water in the soil. The importance of the relative distribution of available moisture within the tension range of 100 and 15,000 cm is indicated.

Introduction--Laboratory evaluation of desert soils is of material value in augmenting a field investigation of land suitability to irrigation. It is self-evident that limiting factors such as unsuitable terrain, micro-relief, depth of effective soil profile, and broad soil textural groups, must take precedence over segregation of land by laboratory techniques. However, laboratory evaluation of soils may establish significant limitations to preclude suitability for irrigation, regardless of favorable field indications.

Soils of the Mojave and Colorado deserts are predominantly coarse to sandy in texture. Medium to fine-textured soils are relatively inextensive. This fact suggests that low water-holding capacity of the soil is a significant factor of limitation. Salinity and fertility are important points for consideration, but frequently these conditions may be corrected. There are no known practical methods of changing the inherent water-holding characteristics of soils. Consequently, this makes an excellent yard stick in establishing suitability of land for irrigation.

All normal-growing crops are dependent upon water available between field capacity and wilting percentage. Soils having very little available water between field capacity and wilting percentage require frequent irrigations, or crops are subject to repeated periods of moisture stress. This will result in greatly reduced productivity. A simple laboratory method of estimating maximum available moisture under field conditions is well adapted to evaluation of coarse-textured desert soils for irrigation.

Procedure--Soil samples are collected at representative locations in the area under study. Sampling is, of course, restricted to areas not eliminated by obvious site limitations. Approximately one-kilogram samples are taken of one or more soil horizons depending upon the character of the profile.

Upon arrival at the laboratory, the samples are air-dried, screened through a standard 20-mesh screen, and the per cent of coarse material computed. Three samples of approximately 90 gm each are separated out by means of a standard sample splitter. Each sample is carefully packed into a small soil can prepared especially for this work. The can, one inch deep and two inches in diameter, previously prepared by coating with a heat-resistant spar varnish, and by punching a pin hole in the bottom, is used in processing the samples. A small piece of scotch tape is placed over the pin hole while filling. Each sample is intermittently tamped with a rubber stopper while filling. The replicated samples are checked for uniformity of volume weight (apparent specific gravity). The open end of the sample can is covered with a coarse-textured filter paper and secured to the can by means of a rubber band.

Sample saturation is approximated by placing the can with sample, top down, into a large desiccator. Atmospheric pressure is reduced one-third. While maintaining vacuum, the water level in the desiccator is allowed to rise, wetting the sample. At the end of one-half hour, the vacuum is released slowly and the sample is placed in a large tray, containing water slightly less than one inch deep, and remains there for 24 hours.

The samples are then placed on a LEAMER-SHAW [1941] tension table at 60-cm water tension for 24 hours. This step is for two reasons: first, to establish the percentage of large pores; and, second, to reduce the total water extracted at higher tensions. Samples are removed from the tension table, weighed, and placed on tension plates maintained at 100-cm tension. The tension plates consist of 11-inch ceramic discs. The entire unit is enclosed in a covered tray to prevent evaporation. After 24 hours, the samples are weighed and returned to the plates for a second period of 24 hours with 330-cm water tension. Samples are again weighed and oven-dried at 105°C and finally reweighed. Per cent of water retained against each tension is computed on a dry-weight basis.

A second set of samples is split from the original screened samples for the purpose of establishing the wilting percentage, using pressure membrane plates and 15-atmosphere pressure. The technique described by RICHARDS [1941] is used.

Moisture percentages on a dry-weight basis are now established for 60-, 100-, 330-, and 15,000-cm tension. It is apparent that volume weight and moisture percentages obtained from sieved samples are not representative of field conditions. The 15-atmosphere value is only slightly affected by sieving. In order to process a large number of samples, utilizing relatively small containers, it is mandatory to remove coarse material from the samples. Many samples contain gravel ranging in size from 2 to 50 mm in diameter. A comparison of volume weight and moisture percentages at several tensions was made of 20 representative sieved and unsieved desert soil samples. Figure 1 illustrates a comparison of water-holding capacity of sieved and unsieved soil samples and relates these differences to the per cent of material passing the 20-mesh sieve. By utilizing this relationship and a similar type of curve for volume weight, it is possible to adjust the laboratory data derived from sieved samples so as more nearly to reflect field conditions.

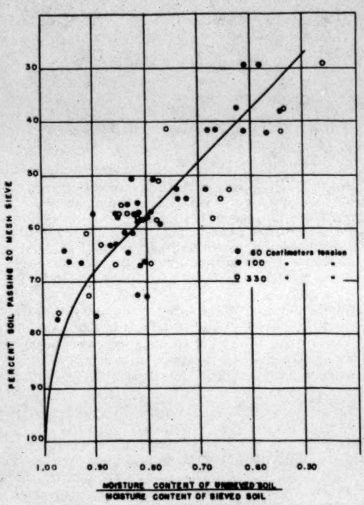

Fig. 1--Comparison of amounts of water retained by sieved and unsieved laboratory soil samples, Mojave Desert

Discussion--The foregoing procedures have established soil-moisture percentages on a dry-weight basis for 60-, 100-, 330-, and 15,000-cm water tension. The maximum limits of available moisture fall between field capacity and wilting percentage. Moisture per cent of soil in equilibrium with 15 atmospheres has been rather convincingly established by RICHARDS and WEAVER [1944] and others as providing a good estimate of permanent wilting percentage. Consistent correlation of field capacity with a given tension has been far less successful. The Bureau of Reclamation found, in its studies of coarse-textured soils of the eastern Colorado Desert, good correlation between field capacity and water retained at 100-cm water tension [HORETZ and Others, 1947]. COLMAN [1947] found good correlation between field capacity and 330-cm tension in the San Gabriel Mountain region of southern California. BROWNING [1941] found soils of West Virginia had a ratio between field capacity and moisture equivalent of approximately unity when moisture equivalent was about 21 pct. However, for soils of very coarse texture, field capacity may be twice as great as moisture equivalent. RICHARDS and WEAVER [1944] have shown that 1/3 atmosphere tension most closely approximates moisture equivalent.

It is evident from the above divergence of results that there is apparently no single tension that can be used as an index of field capacity. As pointed out above, soils usually encountered in the deserts are coarse-textured. Consequently, soil moisture retained against 100-cm tension has been set as comparable to field capacity. This is substantiated by the findings of HORETZ and Others [1947]. The difference between this value and that at 15-atmospheres tension is then an index of total available moisture. Consequently, from these determinations, soil may be classified on the basis of an estimated total available moisture.

In addition to determining the total available moisture of a given soil, it is desirable to know through what range of tensions the bulk of the water is available. In the majority of coarse and light-textured soils, the bulk of water is available at relatively low tensions. Figure 2 illustrates three curves relating soil moisture to tension. Note that sample A has a very small balance of

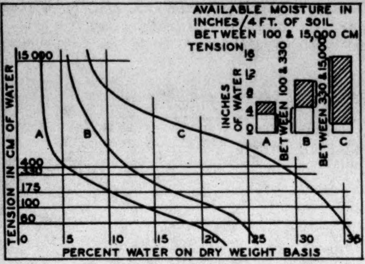

Fig. 2--Three soil-moisture curves indicating character of moisture availability between 0 and 15,000-cm water tension, Mojave Desert, California

available moisture above 330-cm tension and there is very little moisture available above 400-cm tension. A soil of this type may not be adaptable for irrigation use, not only because it has a low total available moisture, but because of the sharp transition from available to non-available moisture. Consumptive use of two per cent moisture computed on a dry-weight basis, will raise moisture stress from 400 to 15,000 cm. In the case of curve B, the reserve of available moisture is somewhat larger while C indicates a uniform rate of increase in tension with decrease in available moisture. These moisture characteristics are important modifying factors in evaluating soils for irrigation. It would be desirable to make complete soil-moisture desorption curves for each sample. Unfortunately, this procedure is time-consuming.

It was found that the moisture percentage at 330-cm tension indicated the general trend of the desorption curve between 100- and 15,000-cm tension. Thus the 330-cm tension values provided a sufficiently accurate means of modifying the basic classification of soils established by the water-holding capacity between 100- and 15,000-cm tension.

The separation of soils, based upon their water-holding characteristics, into classes of suitability for irrigation, must of necessity be arbitrary. Classification of desert soils is made difficult by the lack of experimental data. Studies by the Bureau of Reclamation [HORETZ and Others, 1947] in the Yuma and East Mesas of the Colorado Desert have shown that soils having available moisture of less than 2.7 inches per four-foot profile are unsuitable for irrigation. However, the values of total available moisture are computed on the assumption that total available moisture equals one-half the moisture percentage retained against 100 cm water tension. Actually, this value is somewhat less than measured total available moisture between 100-cm tension and wilting percentage. Other workers have indicated that irrigation is practical where the total available moisture is more than one-half inch per foot of soil. These observations were made under less severe climatic conditions than found in the desert. The reader is cautioned to keep in mind that values of total water-holding capacity based upon the moisture range between 100- and 15,000-cm tension may be twice the values determined by utilizing moisture equivalent and wilting percentage. Table 1 summarizes the results of a representative group of Mojave Desert soils.

Table 1--Soil-moisture characteristics of a representative group of desert soils, Mojave Desert, California

Sample No.	Field texture	Soil passing 20-mesh sieve	Volume weight ratio	Computed total available water		Class	Sub-class
				100-15,000-cm tension	330-15,000-cm tension		
		pct		in/ft	in/ft		
633	Sandy loam	84.2	1.38	2.70	2.04	I	A
575	Gr. sandy loam	69.0	1.59	2.46	1.45	I	A
573	Gr. sandy loam	61.2	1.61	1.97	0.97	I	B
45	Sandy loam	72.9	1.40	2.34	1.26	I	B
61	Gr. sandy loam	62.3	1.26	1.69	0.78	II	B
6416	Sandy loam	85.0	1.53	1.85	0.86	II	B
62	Sand	66.3	1.73	2.27	0.33	I	C
561	Gr. sandy loam	79.6	1.63	1.47	0.70	II	C
656	Loamy sand	84.0	1.56	1.37	0.51	II	C
663	Sand and gravel	44.8	1.82	0.76	0.48	III	C
6712	Sand	61.0	1.81	1.13	0.56	II	C
685	Sand and gravel	37.5	1.79	0.82	0.56	III	C

Table 2 outlines a suggested classification of desert soils based upon total available water as determined by the method described in this report. This classification is suggested as a guide and should be adjusted upward or downward depending upon local conditions and special problems.

Table 2--Suggested classification of desert soils based upon total water-holding capacity between 100-, 330, and 15,000-cm water tension

Description	Class	Subclass	Water retained between 100-15,000-cm tension	Water retained between 330-15,000-cm tension
			in/ft	in/ft
Good	I	A	2.0	1.5
	I	B	2.0	0.75-1.5
Fair	II	B	1.0-1.9	0.75-1.5
	I	C	2.0	0.75
Poor	II	C	1.0-1.9	0.75
	III	C	1.0	0.75

Conclusion--A rapid laboratory method of estimating total available moisture of desert soils has proved to be a valuable tool in determining suitability of large areas of uncultivated desert lands for irrigation development. The simplicity of the technique makes possible the processing of very large number of samples, utilizing relatively simple laboratory equipment.

Evaluation of the total available moisture as measured by the 100-15,000-cm tension range does not give a complete picture of soil-moisture characteristics as related to irrigation. The author believes that it is highly desirable to introduce a factor of moisture availability through a range of tensions rather than any single value. This is particularly true because there is no certainty that soil moisture in equilibrium with 100 cm is an accurate measurement of field capacity.

Classification of land on the basis of the 100-, 330-, and 15,000-cm tension as presented is necessarily tentative. Field irrigation trials and careful consideration of specific localities will result in more reliable separations.

Acknowledgment--The writer thanks the Department of Public Works, Division of Water Resources, State of California, for its cooperation in collecting soil samples and Clayton Wintermute for his assistance in processing them in the laboratory.

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(Manuscript received March 23, 1951; presented at the Twenty-Sixth Meeting, Fresno, California, February 10, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

SOME EVALUATIONS OF SUSTAINED CLOUD-SEEDING OPERATIONS

I. P. Krick and T. B. Smith

Abstract--Evaluations of several cloud-seeding projects are discussed. These projects included portions of northeastern New Mexico, southeastern Colorado, and eastern Oregon, and were conducted during July to December 1950. These evaluations are statistical in nature and involve comparisons with the precipitation occurring in nearby unseeded areas. A multiple regression equation was developed from past data relating precipitation in seeded and unseeded areas. In six of the seven months of operation, actual rainfall in the seeded area exceeded the estimate provided by the regression equation. A suggestion of a possible smoothing effect in precipitation patterns due to seeding is also presented.

Extensive cloud-seeding experiments have been conducted by the Water Resources Development Corporation in various western sections of the country during the past year. Sufficient precipitation data have now become available for analysis from a portion of these operations and certain preliminary evaluations are presented in the following paragraphs.

These seeding experiments differed in a number of major respects from those reported in the literature to date:

(1) Seeding was conducted over extensive regions amounting to a maximum of nine million acres in the largest operational area. Seeding over these broad areas was carried out solely through the use of silver-iodide generating equipment located at ground levels. In this way the airstream passing over the operational area was impregnated with silver-iodide crystals in contrast to the more common procedure of seeding individual clouds.

(2) Operation of the equipment was under close meteorological supervision in order to target the seeding effects over the desired area as effectively as possible. This required a considerable amount of flexibility in operating locations under the wide variety of weather conditions experienced.

(3) Seeding was conducted only at every favorable weather opportunity over a period of several months for each operating area. As indicated by previous investigators, favorable weather opportunities usually coincide with those meteorological conditions which might reasonably be expected to produce some rainfall without seeding.

From the nature of the experiments during the past year, consisting of seeding during each favorable opportunity over an extended period of time, the results of the operations lend themselves to an investigation of the cumulative effect on monthly precipitation values which may be produced by sustained cloud seeding. Since the extent of this effect is currently a controversial question, the evaluations presented in this report have been designed primarily to indicate the possible magnitude of the effect of seeding on total monthly precipitation.

During July to December 1950, experiments were conducted in northeast New Mexico (July, August, and a portion of September), southeast Colorado (August, September, and a portion of July), and in the Tri-Counties area of Oregon (Morrow, Sherman, and Gilliam Counties - October, November, and December). As a consequence, seven monthly evaluations were available for analysis, representing the only experimental results when seeding operations were conducted for a major portion of the month. Due to the delay in receipt of complete precipitation data, no experiments in 1951 have been fully analyzed.

In the evaluation of total monthly precipitation amounts falling in the seeded area, an attempt has been made to segregate rainfall increases due to seeding from the amounts which might have occurred from natural causes. This has been accomplished through comparisons with outside or control areas. In each case the chosen control area was upwind from the generators affecting the seeded area in order to eliminate any possible influence on the rainfall of the control stations. Selection of rainfall stations in the control area which would provide proper comparisons with the seeded area was accomplished by correlation techniques. Separate correlations were calculated between monthly rainfall amounts at various stations in the control area with an average of the monthly precipitation amounts occurring at all long-record rainfall stations within the seeded or target area. Final selection of the control stations to be used was determined entirely from the magnitude of the computed correlation coefficient for each individual station. Total years available for the correlation analysis ranged from 23 to 37.

In order to smooth out a portion of the random fluctuations in rainfall amounts and thereby improve the correlation between control and target areas, it was considered desirable to group the rainfall in the control area into a single parameter which might then be similarly correlated with average monthly rainfall amounts in the target area. This was accomplished in two separate ways, (1) by using an arithmetical average of the rainfall values at the various control stations, and (2) through the use of multiple correlation techniques. The latter method provided a computed rainfall value which would be expected to occur in the target area on the basis of the established interrelationship between the control and target areas. Since the multiple correlation analysis provided a slightly higher relationship between target and control areas, that method has been adopted in the following analysis. Both methods yielded similar results when applied to the 1950 data. Having established the interrelationship between target and control areas for past years when no seeding occurred, 1950 rainfall amounts are investigated and compared with the occurrences noted in past years.

Table 1 gives an example of the geographical distribution of the target and control stations as described in the foregoing paragraphs.

Table 1--Locations of target and control stations, Tri-Counties, Oregon, December 1950

Target stations	Control stations
Arlington, Oregon	Winthrop, Washington
Condon, Oregon	Seattle, Washington
Heppner, Oregon	Salem, Oregon
Kent, Oregon	Medford, Oregon
Mikkalo, Oregon	Lakeview, Oregon
Morgan, Oregon	
Moro, Oregon	
Wasco, Oregon	

Figures 1-7 show the results of the multiple correlations for all of the months when such analyses could be undertaken. All past years used in the correlations are plotted on the charts in order to show the variability about the regression line. Any point falling to the right of the regression line indicates an actual rainfall amount in the target area higher than computed through the use of the control station precipitation amounts. Note that the 1950 data fell to the right of the regression line on six of the seven charts. For any particular rainfall value computed from the control-station amounts, the rainfall amount indicated by the regression line should be considered as the best estimate of the rainfall which would have occurred in the target area. The term "best estimate" is here used in the sense that approximately a 50 pct chance exists that the rainfall in the target area would have been less than indicated by the regression line and a 50 pct chance that more rainfall might have occurred than indicated. Con-

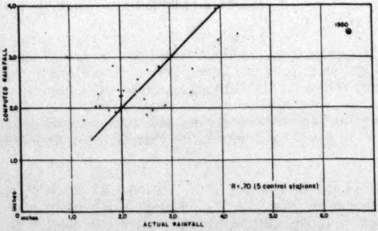

Fig. 1--Rainfall relation between target and control areas, July 1950, northeast New Mexico

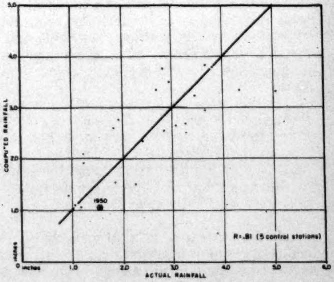

Fig. 2--Rainfall relation between target and control areas, August 1950, northeast New Mexico

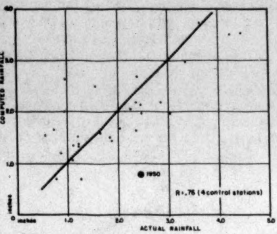

Fig. 3--Rainfall relation between target and control areas, August 1950, southeast Colorado

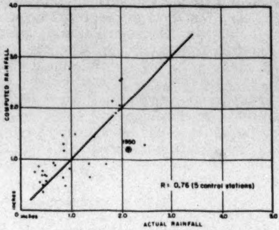

Fig. 4--Rainfall relation between target and control areas, September 1950, southeast Colorado

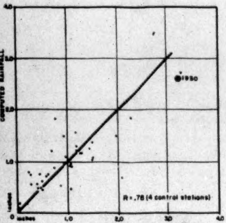

Fig. 5--Rainfall relation between target and control areas, October 1950, Tri-Counties, Oregon

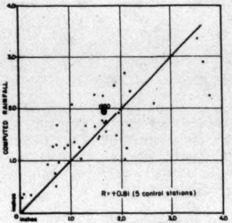

Fig. 6--Rainfall relation between target and control areas, November 1950, Tri-Counties, Oregon

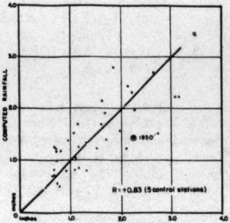

Fig. 7--Rainfall relation between target and control areas, December 1950, Tri-Counties, Oregon

sequently, the extent of the deviation of the 1950 data to the right of the regression line is to be considered as a probable increase due to cloud seeding with the understanding that there remains approximately a 50 pct chance that the deviation due to seeding might be more than the indicated value and roughly a 50 pct chance of the deviation being less than indicated.

Since Figure 6 shows a unique position for the 1950 datum to the left of the regression line, some attention has been given to the month of November 1950 in the Pacific Northwest. It has been noted that the usual analysis method applied to the precipitation data from November 1 to 9, 1950, yields results comparable to those shown in the remaining six charts. As a consequence, it appears that the storminess during the latter portion of the month was unusual to the extent that seeding effects comparable to those experienced in other months were not noted. Further study is being undertaken to determine the effects of seeding during the latter portion of November.

Figures 1-7 are not intended to offer proof of the effects of seeding on monthly rainfall totals. However, the fact that 1950 data during six of the seven months fell to the right of the regression line is suggestive of important modifications in the monthly precipitation patterns. In addition, pending further analysis, the extent of the deviation to the right of the regression line may be taken as a measure of the possible increase in rainfall due to cloud seeding during the month.

It should be noted that, percentagewise, any rainfall increases shown in Figures 1-7 do not appear to be a function of the average rainfall occurring in the area. There appears to be no important distinction between the percentage increase of rainfall possible in a dry month and in a wet month.

A logical analysis of Figures 1-7 would appear to be an investigation of the statistical significance of the 1950 data as compared to the occurrences of past years. The problem is complicated considerably by the non-normal distributions of monthly rainfall amounts and by the dependence of the deviations from the regression line (in terms of both inches and percentages) on the actual rainfall amounts. Considerable time has been devoted to this problem and it has not been considered desirable to include a discussion of the statistical significance until such time as adequate parameters can be developed which are less dependent on rainfall amounts.

In addition to suggestions of rainfall increases which have been mentioned in the preceding paragraphs, there have been certain visual and analytical indications that seeding as conducted during the experiments previously mentioned may distribute rainfall more uniformly over wide areas than is usually the case under natural conditions. Table 2 presents a suggestion of this effect.

Table 2--Total rainy days in September, southeast Colorado

Line	Area	Item	Rainfall amounts, inches				Average rainfall
			T - 0.09	0.10-0.29	0.30-0.69	0.70 or more	
1	Target	20-yr average, inch	17.7	7.8	6.3	3.7	1.19
2	Target	Average wettest 40 pct of years, inch	20.4	9.9	11.1	6.6	2.12
3	Target	1950, inch	40	19	18	5	2.11
..	Target	Ratio:item 3/item 2	1.96	1.92	1.62	0.76	1.00
4	Control	21-yr average, inch	12.9	7.0	3.1	1.2	1.10
5	Control	1950, inch	13	13	3	0	1.06
..	Control	Ratio:item 5/item 4	1.00	1.85	0.97	0.00	0.96

In Table 2 total rainy days per month in various ranges have been summed for the entire eight stations in the target area and four stations in the control area for all past Septembers. Since the number of rainy days is a direct function of the rainfall occurring in the area, 1950 has been compared to an average of past years having comparable rainfall amounts. Also included is a tabulation of the percentage change in total rainy days in 1950 compared to the average of past years with comparable rainfall amounts.

Sizable percentage increases are shown in the target area for rainfall ranges below 0.70 inch with a comparable increase noted in the control area only in the 0.10-0.29 inch range.

There have also been numerous independent visual reports from the seeded areas indicating that rainfall during seeding periods has been more evenly distributed and gentler than has been customary in past years. If further data substantiate these indications, it is believed that this modification of rainfall patterns will be fully as significant as possible increases in rainfall totals. Any such effects which may occur as the result of seeding are attributed to the impregnation of a broad airstream with silver-iodide crystals in contrast to local seeding of individual clouds.

Conclusions--This report has attempted to indicate the possibility of certain significant modifications in rainfall patterns as a result of sustained cloud seeding. If further data substantiate the indications mentioned previously, two important conclusions will have been demonstrated:

(1) Significant increases in monthly rainfall totals can be achieved by sustained cloud-seeding activities.

(2) Operation of the seeding equipment can be regulated to target the rainfall increases over a localized area.

It is hoped that additional data will become available in the future for further analyses along similar or more refined lines.

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(Manuscript received April 30, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

SYMPOSIUM ON PHREATOPHYTES

PHREATOPHYTES AND THEIR RELATION TO WATER IN WESTERN UNITED STATES

T. W. Robinson

Abstract--Phreatophytes, plants that depend on ground water for their water supply, cover about 15 million acres in the 17 western states, and may waste as much as 20 to 25 million ac ft of water into the atmosphere annually. There are many variable factors that affect the growth and use of water by the plants, but present-day knowledge of them is limited. The water used by these plants probably represents the largest source of reclaimable water in the arid western United States. Although it appears feasible to salvage a part of this water, methods for doing so have not passed the experimental stage. In Nevada, it is estimated that it would be practical to salvage about 25 pct of the water wasted annually, or about 400,000 ac ft. This would be sufficient to irrigate about 133,000 acres of alfalfa having a crop value of $10,000,000.

In the arid and semiarid regions of western United States, the close association of certain plants with areas in which the water table is at shallow depth is well known. MEINZER [1927] gave the name "phreatophytes" to this group of plants. He defined phreatophytes as "plants that habitually grow where they can send their roots down to the water table, or the capillary fringe immediately overlying the water table, and are then able to obtain a perennial and secure supply of water." The term was derived from two Greek roots and means a "well plant."

It may be desirable to point out here that phreatophytes do not belong to any particular genus or even any one family of plants. Indeed, they are made up of many species belonging to different plant families. They have only one common characteristic and that is their typical dependence on ground water for their water supply. The list of known phreatophytes is quite long and, although probably not complete, includes about 50 plant species and several times that number of subspecies. The most important and widespread species are saltcedar, baccharis, greasewood, rabbitbrush, saltgrass, willow, cottonwood, mesquite, and alfalfa. Of this group only alfalfa is important in agriculture. The rest are all indigenous to the United States, with the exception of saltcedar which is a native of the Mediterranean region, whence it was imported to this country about 35 years ago.

Through transpiration these plants discharge large quantities of ground water into the atmosphere. In areas where the water table is so shallow that the capillary fringe extends to the land surface, there is discharge of ground water by evaporation from the soil also. Transpiration and soil evaporation are so closely associated that it is difficult to determine separately the quantity of water discharged by each. For this reason the two are commonly referred to as a single process, evapotranspiration.

Phreatophytes may be contrasted with xerophytes (from the Greek root meaning "dry plant"), desert plants that are not associated with the water table and do not depend on it for their water supply. Xerophytes maintain themselves on the small amount of water that gets to them from infrequent rains. During prolonged drought periods they are nearly dormant.

Phreatophytes use many times the quantity of water used by xerophytes. Unfortunately, however, most of the ground water, except that used by alfalfa, is not beneficially used or is only slightly so. In western United States the area of phreatophytes, excluding alfalfa, is reckoned in millions of acres. The water used by these plants probably represents the largest source of reclaimable water in that region. It is to the economic interest of this region that, where feasible, at least a part of the ground water now virtually wasted be utilized efficiently. This is especially true in this day of high water costs.

The quantity of water used by the different species varies widely. As an example, WHITE [1932] estimated from experimental data that the annual ground-water discharge by an association of greasewood, rabbitbrush, and shadscale in Escalante Valley, Utah, where the water table was 8 to 30 ft below the land surface, was about two inches per year. At the other extreme, it was estimated from experimental data that the annual discharge by saltcedar growing in areas of shallow water table under very favorable conditions, in Safford Valley, Arizona, was about seven feet per year [GATEWOOD and Others, 1950].

Therefore, before attempts are made to salvage or to utilize more efficiently the water discharged by phreatophytes it is important to know within reasonable limits the quantity of water used by the plants annually in the area concerned. The determination of this quantity is based upon two primary factors, namely, the annual rate of use by the different phreatophytes and the area covered by the plants. The area usually is not difficult to ascertain, but the annual rate of use varies not only with the species but with variable factors that control the growth of the species.

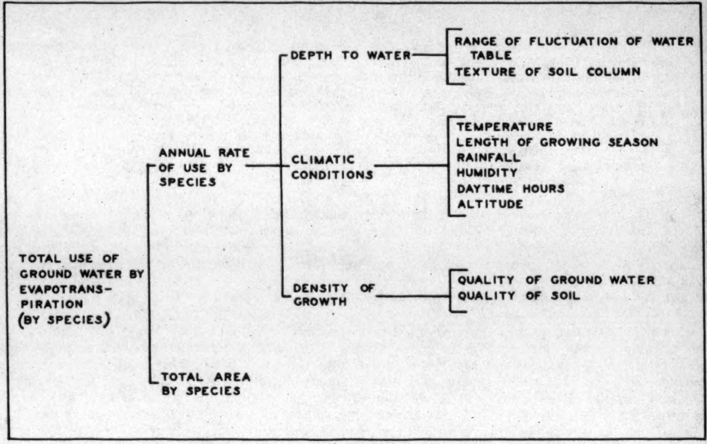
TOTAL USE OF GROUND WATER BY EVAPOTRANSPIRATION (BY SPECIES)
ANNUAL RATE OF USE BY SPECIES
TOTAL AREA BY SPECIES
DEPTH TO WATER
CLIMATIC CONDITIONS
DENSITY OF GROWTH
RANGE OF FLUCTUATION OF WATER TABLE
TEXTURE OF SOIL COLUMN
TEMPERATURE
LENGTH OF GROWING SEASON
RAINFALL
HUMIDITY
DAYTIME HOURS
ALTITUDE
QUALITY OF GROUND WATER
QUALITY OF SOIL

Fig. 1--Factors that may affect the evapotranspiration of ground water for each species of phreatophytes

Figure 1 is an attempt to show graphically the factors that may affect discharge of ground water by evapotranspiration in areas of phreatophyte growth. The following discussion will outline briefly the effects of these factors as they control or influence the growth of plants and the use of ground water by them. Not all the species, however, are necessarily affected by all the factors. Present knowledge of these factors is limited and, although some progress has been made, a great deal remains to be learned.

In general, the greater the depth to water the lower the rate of water use. For some species the depth of the water table is the controlling factor in the growth of the plant. Tules grow only in water or where the water table is shallow, usually less than one foot below the land surface. Saltgrass, as a rule, does not thrive where the depth to water exceeds six to eight feet, although it has been observed growing where the depth was 11 to 12 feet [BLANEY, 1933, p. 150]. On the other hand, deep-rooted plants such as mesquite and alfalfa have been known to send their roots 40, 50, and in a few instances more than 100 ft in search of ground water. Greasewood usually thrives best where the depth to water does not exceed 15 ft, although it grows in some places where the depth is greater. An extreme depth of 80 ft was noted in Goshute Valley, Nevada, where greasewood was observed growing. Saltcedar, baccharis, willow, and cottonwood appear to prefer localities where the depth to water does not exceed 20 ft. Although no surveys have been made, observations indicate that the bulk of the ground water discharged by phreatophytes occurs in areas where the depth to water is less than 20 ft, and possibly less than 15 ft.

In areas where the water is shallow the texture of the soil affects the rate of evaporation from the soil by controlling the height of the capillary fringe. In soils of fine texture the height of the capillary fringe is greater than in soils of coarse texture. Thus for a given depth to water the capillary fringe may extend to the land surface where the soil texture is fine but fail to do so where the texture is coarse. All other factors being equal, the opportunity for evaporation from the soil is greatest where the soil texture is fine.

Climatic conditions exert a controlling influence on the occurrence of some species but not of others. Saltcedar, for instance, prefers a moderate to low altitude and a warm climate and is confined almost entirely to southwestern United States. It thrives best south of the 37th parallel and below 5000 ft. Saltcedar plants several years old in the vicinity of Carson City and Fallon, Nevada (lat. 39°N, and altitude a little over 4000 ft), are not aggressive in spreading over the land, nor do they exhibit the vigorous junglelike growth so common to the plant in the valleys of central and southern Arizona and New Mexico. Yet at this locality in Nevada the ground-water conditions are otherwise favorable for saltcedar growth. Other phreatophytes, particularly cottonwood, willow, saltgrass, and greasewood, grow abundantly.

In addition to saltcedar, mesquite and baccharis also do not grow much farther north than the 37th parallel or at an altitude above 5000 ft. In contrast to these three plants is greasewood, whose southern limit of growth is at about this latitude. Species that apparently are relatively unaffected by climatic conditions include cottonwood, willow, saltgrass, and alfalfa. In western United States, from Canada to Mexico, where ground-water conditions are favorable these plants thrive at altitudes ranging from 8000 ft to sea level.

The other climatic factors that affect rate of use of ground water are length of growing season, daytime hours, rainfall, and humidity. It has been found that the draft on ground water is greatest when the growing season and daylight hours are long, when the rainfall is scanty, and when the relative humidity is low.

Density of growth is a controlling factor in the use of water by phreatophytes. Field studies in Safford Valley, Arizona, in 1943-44, indicated that there are two variables with respect to density of growth, areal density and vertical density, and their product is termed "volume density." Areal density relates to the numerical density of the plants in a unit area with respect to the maximum possible, and vertical density relates to the vertical depth of fronds with respect to the maximum possible. As plant growth is a function of sunlight, density of growth and depth of frondage are apparently limited by the amount of sunlight that penetrates to the most remote foliage. It was found in the Safford Valley that use of water by saltcedar, cottonwood, and baccharis varied directly as the volume density. The amount of water used by phreatophytes, then, depends upon the thickness of growth and the amount of foliage on individual plants.

Data obtained in the Safford Valley indicate also that the quality of the water has an effect on plant growth and rate of use of ground water. In general, plants tend to use less water and to grow less vigorously as the mineral content of the water increases. Some species of phreatophytes, particularly willow, cottonwood, rabbitbrush, and alfalfa, do not grow where the mineral of the water is high. On the other hand, saltcedar, greasewood, and saltgrass have a high tolerance for mineralized water.

An inventory of the water resources of a region to be complete must include the discharge of ground water by evapotranspiration. In order to include it a working knowledge of the factors discussed is necessary. The process of evapotranspiration affects the flow of the streams and the storage of water in underground reservoirs. It is a well-known fact that during the growing season there is a daily fluctuation in the flow of western streams not affected by diversion or other regulation. These fluctuations are a measure of the rate at which water is withdrawn from the stream during a 24-hour period by riparian vegetation. In ground-water studies the quantity of water discharged by evapotranspiration is especially important. In closed basins the quantity so discharged marks the upper limit of the average annual amount of water that may be safely recovered from the valley. In open basins it is an index to the quantity that may be salvaged in addition to that which may be recovered by pumping that reduces the surface outflow or underflow from the basin.

Until 1948 no attempt had ever been made to obtain an overall picture of the area covered by phreatophytes or of the use of water by them in the 17 western states. In the autumn of 1948 the writer assembled all available data for this purpose. In 1951 estimates for two States, California and Utah, were revised on the basis of later information. The estimates are incomplete; nevertheless, they are believed sufficient to show the order of magnitude of the quantities involved. For the most part the figures for the quantity of water discharged by evapotranspiration are based on knowledge of the area, the plants, and the limited data on rate of use. These were compiled by men of the U. S. Geological Survey who were familiar with conditions in the various states. In this survey only those phreatophytes known to have a low beneficial use were included. Alfalfa, which has a high beneficial use, was excluded. The available estimates summarized by States are shown in Table 1.

Table 1--Data on occurrence of phreatophytes

State	Area	Annual use
	ac	ac ft
Arizona	400,000	1,375,000
California[a]	317,000	1,150,000
Colorado[a]	737,200	1,056,200
Idaho	500,000	1,000,000
Kansas	No data	No data
Montana	1,600,000	3,200,000
Nebraska[a]	515,000	709,000
Nevada	2,801,000	1,500,000
New Mexico	441,000	870,000
North Dakota	1,035,000	1,660,000
Oklahoma	No data	No data
Oregon[a]	40,800	21,200
South Dakota	850,000	1,240,000
Texas[a]	262,000	436,500
Utah	1,200,000	1,500,000
Washington	No data	No data
Wyoming	No data	No data
Totals approximate[a]	10,700,000	15,700,000

[a] Partial data.

It must be emphasized that the data in the table are incomplete. For about one-fourth of the States there are no data and for another one-fourth only partial data. As a result, the grand total is a minimum that may be in error on the low side by 50 to 75 pct. Probably the total area of phreatophytes is of the magnitude of 15,000,000 acres. If this figure is reasonably correct, the total use of water may be estimated. Assuming that the ratio of acres to acre-feet in the grand totals is representative, then the total use would be between 20 and 25 million acre feet annually. In order to visualize a little more clearly the magnitude of this amount of water, it may be stated in terms of flow or volume. It is equivalent to about twice the average annual flow of the Colorado River at Lees Ferry for the 25-year period 1921-45, or, expressed in another way, it is equivalent to about 75 pct of the total storage capacity of Lake Mead.

In the arid and semiarid regions of western United States, where water is the ‘‘bread and butter’’ of the land, the virtual waste of this immense quantity is certainly ‘‘food’’ for thought. What, then? Obviously it is not possible to salvage all the water wasted, but the potentialities and the rewards of success are sufficient to warrant a thorough investigation and study of the problem of ground-water salvage.

Here again is a dearth of information. As far as the writer is aware, it is only within the last five years that attention has been given to developing practical methods for salvaging on a large scale the ground water lost in areas of phreatophyte growth. Salvage operations may be divided into (1) reduction of waste, (2) increased efficiency of use, and (3) a combination of these methods. Each has its problems.

One obvious way of reducing waste would be to destroy the vegetation. Destroying some species of vegetation, however, is not a simple task. In the Safford Valley of Arizona attempts were made to destroy saltcedar by burning, using flame throwers. The operation was successful in killing the trunk and limbs but not the root, for by midsummer of the following year new growth had attained a height of five to six feet. Uprooting also was tried, using heavy mechanical equipment to rip the plants out of the ground. The operation would have been successful if all the roots could have been removed. This was not the case and soon new growth appeared from the end of each severed root, so that two plants grew where one had grown before. In one test plot the saltcedar was cut off just above the ground. Three months after cutting the new growth had reached a height of 2 1/2 ft. Experiments by the Bureau of Reclamation in 1948 and 1949 above McMillan Reservoir in New Mexico were quite successful. They involved spraying tracts of salt cedar with the chemical 2, 4-D. Thus this method is a promising and inexpensive one for destroying saltcedar, and perhaps willow also, which is similar to saltcedar in resisting destruction.

Even when the plant is successfully destroyed there is a question whether a saving of water will be effected under certain conditions. In areas where the capillary fringe is at or near the land surface the rise of the water table resulting from the decrease of evapotranspiration may bring the water so close to the surface that the discharge by soil evaporation will increase and possibly equal the former discharge by evapotranspiration. On the other hand, where the water table is sufficiently deep that the capillary fringe does not extend to the land surface and the plants intercept only a part of the ground water as it moves down gradient to a point of ground-water discharge, the method should be quite successful.

Lowering the water table in the area of evapotranspiration discharge either by pumping or by drainage is another method of reducing waste. To be successful, however, the drainage should be rapid so that the plants will die for lack of water; otherwise, the plant roots will keep pace with the declining water table and keep the plant alive until conditions are again stable. In materials of low permeability, where drainage is slow, it is doubtful if this method would be successful, although the draft by the plants would be reduced.

Intercepting ground water up gradient from the area of plant discharge also reduces waste. In fact, this method is unwittingly practiced in some of the closed valleys in Nevada. In many of these valleys, where the lowest part of the valley floor is covered with phreatophytes, the soil is too alkaline for agricultural development. Up slope from the area of phreatophytes, where the soil is suitable for the raising of crops, wells have been drilled and the water pumped for irrigation. As the ground water migrates from the sides toward the lowest part of the valley, some of the water that would otherwise have been discharged by the plants is thus diverted for irrigation. In Nevada, this practice is quite recent, and has resulted from a desire to put more land under irrigation, but with no realization that it was also a salvage operation. Although limited to areas where conditions are favorable, this method holds a great deal of promise.

The efficiency of use of ground water may be increased by substituting, in the area of discharge, plants having higher economic value. Generally this means substituting one phreatophyte for another. Forage crops, particularly alfalfa and grasses, seem best adapted for this purpose. In the Escalante Valley of Utah, alfalfa was successfully substituted for an association of greasewood, rabbitbrush, and saltgrass. There are doubtless other areas, particularly on the flood plains of some rivers and on the floors of open valleys, where the method would prove successful. The advantage of this method lies in the fact that, once the substituted plant has established itself, irrigation cost becomes negligible for the plant pumps its own water.

An indication of the economic value of salvaging the ground water wasted by non-beneficial vegetation is provided by the State of Nevada. The estimate of evapotranspiration discharge in Nevada is 1,500,000 acre feet per year. Of this amount about 25 pct, or 400,000 acre feet, is estimated to be salvageable. If this water were utilized in growing alfalfa, which has an irrigation requirement of about three acre feet per acre per year, it would be sufficient to irrigate about 133,000 acres. It is conservatively estimated that the average yield of alfalfa in Nevada is about three tons per acre. At the average price of $25 per ton, prevailing for the past three years, the gross value of 400,000 acre feet of water is $10,000,000.

In Arizona, where it is estimated about 700,000 acre feet of water is salvageable, and where crops of higher value may be grown, the gross value of the salvageable water could easily be 20 to 25 million dollars. Much the same situation exists in the other western states. It seems that the reward for salvaging at least a part of this wasted water would justify intensive effort involved.

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(Manuscript received May 21, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

DETERMINING EVAPOTRANSPIRATION BY PHREATOPHYTES FROM CLIMATOLOGICAL DATA

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Abstract--Before the available water resources of a drainage basin in arid and semi-arid regions can be satisfactorily ascertained, careful consideration must be given to the consumptive water requirements of phreatophytes (or water-loving vegetation) and other types of natural vegetation and agricultural crops. The moisture requirements of natural vegetation are usually satisfied before water becomes available for irrigation and other purposes. Although evapotranspiration (consumptive use) by phreatophytes has been measured at various times and places during the past 40 years, very little data are available in most sections of the United States. This paper presents data on measured evapotranspiration by phreatophytes and describes a method of determining rates of water consumption in areas where no measurements except climatological data are available. Briefly, the procedure is to correlate existing evapotranspiration data with monthly temperature, monthly percentages of yearly daytime hours, and growing period. The coefficients so developed for different kinds of vegetation are used to transpose the evapotranspiration data for a given area to other areas for which only climatological data are available.

Introduction--In many parts of the west the natural water supply is exceedingly limited, and the demands for water, already great, are constantly increasing through the requirements of more intensive farming. In considering the water supply of a river basin, water consumed by phreatophytes, such as cottonwoods, saltcedar (tamarisk), willows, baccharis, tules, sedges, and salt grass growing in irrigated valleys, moist areas, and along streams, becomes of increasing importance as greater land areas are irrigated, especially during periods of drought. The value of data on evapotranspiration (consumptive use) by these non-crop plants is recognized by administrators and engineers in regions where water rights are in dispute, or where interstate water supply and water use are not in balance. The term "evapotranspiration" is considered synonymous with the term "consumptive use" by engineers, and is defined as the quantity of water evaporated and transpired from an area.

In planning new irrigation projects, consideration must often be given to difference in consumptive uses of water utilized by irrigated crops and by the natural vegetation replaced by the crops. Thus the quantity of water available for plant use and effect of plant growth on supply is of great interest to irrigation engineers and hydrologists. Measurements of evapotranspiration indicate that water-loving natural vegetation uses from 50 to 100 pct more water than most crop plants. Tules and saltcedar growing in irrigation canals and drainage ditches and on their banks are exposed in narrow strips to sun and wind so that their consumption of water is unusually high. Under such circumstances phreatophytes along a mile of canal or ditch may consume enough water to irrigate five acres of alfalfa or a greater acreage of other crops [BLANEY and Others, 1933].

Measurements of water use--Evapotranspiration losses by natural vegetation growing in areas of high water table have been measured by means of tanks, lysimeters, inflow-outflow ground-water fluctuations, and other methods by the Division of Irrigation and Water Conservation of the Soil Conservation Service and the U. S. Geological Survey at various times and places [BLANEY and Others, 1933; GATEWOOD and Others, 1950; TURNER and Others, 1941; YOUNG and BLANEY, 1942]. The unit values thus determined may be used to compute valley consumptive use by the integration method described by BLANEY and Others [1938]. Results of typical studies are presented in Table 1.

Correlation of water use and climatological data--Meteorological conditions influencing evaporation from water surfaces likewise affect evaporation from soils and transpiration from vegetation. Both evaporation and transpiration freely respond to changes in temperature, wind movement, and humidity so that evaporation from water may, under certain conditions, be used as an index of evapotranspiration losses for areas in which there is ample water to take care of evaporation and transpiration.

The effect of sunshine and heat in stimulating transpiration was studied as early as 1691. Measurements of transpiration of various kinds of plants by BRIGGS and SHANTZ [1916] indicate a close correlation between transpiration and evaporation from free-water surfaces, air temperature, solar radiation, and wet-bulb depression readings.

The results of investigations in California, New Mexico, and other areas indicate that observed evaporation data may be used as a means of estimating evapotranspiration by vegetation having access to an ample water supply and when the relation of the two is known for a particular area [BLANEY and Others, 1933; BLANEY and Others, 1938]. As an example, for two locations in California, for tules growing in large tanks within the confines of a swamp area, the percentage of consumptive use, with reference to evaporation from a nearby exposed Weather Bureau pan, had a two-year average of 95 pct. The percentage varied from month to month, increasing during the summer and becoming smaller in the cooler months.

From long-period periods of evaporation, temperature, and humidity in New Mexico and Texas, together with consumptive-use measurements at Carlsbad, New Mexico, empirical formulas were developed by BLANEY and MORIN [1942] for computing evaporation and consumptive use where temperature and humidity are available. In 1945 this method was modified so that estimates of

Table 1--Mean annual or seasonal evapotranspiration by phreatophytes as determined by by tank and lysimeter studies in several western states

Locality	Type	Period	Depth to water table	Evapo-transpiration	Authority
			inch	inch	
Arizona					
Safford	Salt cedar	May-Dec. 1940	24	61.1	Turner and Halpenny [1941]
Safford	Salt cedar	May-Dec. 1940	48	47.9	Turner and Halpenny [1941]
Safford	Baccharis	May-Dec. 1940	24	52.0	Turner and Halpenny [1941]
Safford	Baccharis	May-Dec. 1940	48	39.7	Turner and Halpenny [1941]
California					
Santa Ana	Salt grass	May 1929-Apr. 1932[a]	24	35.3	Blaney and Others [1933]
Santa Ana	Salt grass	May 1929-Apr. 1932[a]	48	13.4	Blaney and Others [1933]
Santa Ana	Wire rush	Aug. 1930-July 1931	24	78.9	Blaney and Others [1933]
Victorville	Tules	Jan. 1931-Dec. 1932	0	78.4	Blaney and Others [1933]
King Island	Tules	Jan.-Dec. 1932	0	90.0	Stout [Young and Blaney, 1942]
San Luis Rey	Cottonwood-willows	Apr. 1941-Mar. 1943[a]	48	62.5	Muckel and Blaney [1945]
Owens Valley	Salt grass	Jan.-Dec. 1911	46	24.6	Lee [1912]
Colorado					
San Luis Valley	Meadow grass	June-Nov. 1936	0	36.3	Blaney and Others [1938]
San Luis Valley	Tules	June-Nov. 1936	0	38.8	Blaney and Others [1938]
Ft. Collins	Sedge grass	May-Oct. 1930	18	53.6	Parshall [Young and Blaney, 1942]
Ft. Collins	Rushes	July-Oct. 1930	...	52.6	Parshall [Young and Blaney, 1942]
Idaho					
Grays Lake	Tules	June 22-Sept. 30, 1942-1944	0	42.4	Criddle and Marr [1945]
Bonner's Ferry	Cattails	May-Sept. 1934-1944	0	61.6	Criddle and Marr [1945]
New Mexico					
Los Griegos	Salt grass	Oct. 1927-Sept. 1928	26	22.7	Elder [Young and Blaney, 1942]
Isleta	Sedge grass	June 1936-May 1937	3	76.9	Blaney and Others [1938]
Carlsbad	Tamarisk	Jan.-Dec. 1940	36	57.3	Blaney and Others [1942]
State College	Cattails	July-Dec. 1936	0	44.2	Blaney and Others [1938]
State College	Salt grass	July-Dec. 1936	14	29.3	Blaney and Others [1938]
Utah					
Escalante Valley	Salt grass	May-Oct. 1927	23	22.6	White [1932]
Escalante Valley	Greasewood	May-Oct. 1927	26	25.2	White [1932]

[a] Average yearly for period of record.

consumptive use could be made for areas where humidity records are not available [BLANEY and EWING, 1946; BLANEY and CRIDDLE, 1950].

Actual measurements of evapotranspiration under each of the various physical and climatical conditions of any large area are expensive and time-consuming. Therefore, some rapid method of transferring the results of careful measurements made in several areas to other areas of similar conditions is needed. Several years ago the Soil Conservation Service developed such a method to estimate rates of water consumption [BLANEY and EWING, 1946; BLANEY and CRIDDLE, 1949; BLANEY and CRIDDLE, 1950]. Briefly the procedure is to correlate existing consumptive-use data with monthly temperature, per cent of daytime hours, precipitation, and frost-free (growing) period. The coefficients so developed for different plants are used to translocate or transpose consumptive-use data from one section to other areas where only climatological data are available.

Although it is recognized that consumptive use of water is affected by numerous independent and related variables, records of temperatures and precipitation are by far the most universally available throughout the western states. Research studies indicate that of all the climatic factors, these elements, together with daylight, undoubtedly have the greatest influence on plant growth.

Evapotranspiration formula--Disregarding the unmeasured factors, consumptive use varies with the temperature, daytime hours, and available moisture (precipitation, irrigation water, or natural ground water). By multiplying the mean monthly temperature t by the monthly per cent of daytime hours of the year p, there is obtained a monthly consumptive-use factor f. It is assumed that the consumptive use varies directly as this factor when an ample water supply is available. Expressed mathematically, $U = KF$ = sum of kf, where

U = consumptive use (or evapotranspiration) in inches for any period
F = sum of the monthly consumptive-use factors for the period (sum of the products of mean monthly temperature and monthly per cent of daytime hours of the year)
K = empirical consumptive-use coefficient (growing period)
t = mean monthly temperature, °F
p = monthly per cent of daytime hours of the year
$f = t \times p/100$ = monthly consumptive-use factor
k = monthly consumptive-use coefficient
$u = kf$ = monthly consumptive use in inches

The consumptive-use factor F for any period may be computed for areas for which monthly temperature records are available. Then by knowing the consumptive-use coefficient K for a particular plant in some locality, an estimate of the use by the same vegetation in some other area may be made by application of the formula $U = KF$.

Coefficients K for the more important phreatophytes have been developed from actual measurements of evapotranspiration by water-loving plants under field conditions in several western areas [BLANEY and Others, 1942; YOUNG and BLANEY, 1942]. These data have been correlated with temperature, per cent of daytime and the growing season, and the coefficient K has been computed by the formula $K = U/F$. These coefficients vary with the kind of plants, density of growth (areal density and vertical density), and the depth to water-table. Examples of computed coefficients for phreatophytes are shown in Table 2. Figure 1 shows the relation of consumptive use U, consumptive-use factor F, and coefficient K for various kinds of natural vegetation in the west under semiarid climate. Monthly per cent of annual daytime hours may be computed from sunshine tables published by U. S. WEATHER BUREAU [1905].

Table 2--Examples of consumptive-use coefficients for phreatophytes based on measured consumptive use and temperature records, and per cent of daytime hours and growing season

Location and classification	Year	Depth to water table	Growing season or period	Consumptive use factor F	Consumptive use U	Coefficient K growing season
		ft			inch	
Carlsbad, New Mexico						
Salt Cedar	1940	2	Apr. to Sept.	40.72	48.61	1.19
Salt Cedar	1940	3	Apr. to Sept.	40.72	43.95	1.08
Sacaton	1940	2	Apr. to Sept.	40.72	37.98	0.93
Sacaton	1940	4	Apr. to Sept.	40.72	32.82	0.81
San Luis Rey, California						
Cottonwoods and grass	1940-43	4	Apr. to Sept.	36.27	48.99	1.35
Tules	1940-43	0	Apr. to Sept.	41.89	47.03	1.12

Application of method--In connection with a water utilization investigation in 1945 by the California State Division of Water Resources, the Division of Irrigation and Water Conservation of the Soil Conservation Service was requested to determine the evapotranspiration losses from moist areas containing willows, cottonwoods, tules, baccharis, and salt grass in Salinas Valley [BLANEY and EWING, 1946]. Since no measurements had been made of evapotranspiration in this Valley and the information was needed within a limited time, estimates of water use were made by analyses of climatological data for Salinas Valley in comparison with similar data for San Luis Rey Valley, Table 3--Estimated normal monthly evapotranspiration by tules growing in swamp areas in the lower Salinas Valley, California

Month	San Luis Rey Valley, Calif.			Salinas Valley, Calif.			
	Water use u	Factor f	Coefficient[a] k	Mean temperature t	Daytime hours p	Factor f	Water use[b] u
	inch			°F	pct		inch
January	1.82	3.91	0.47	49.1	6.95	3.41	1.60
February	1.90	3.81	0.50	51.3	6.83	3.50	1.75
March	3.09	4.79	0.65	53.3	8.35	4.45	2.89
April	4.56	5.12	0.89	55.6	8.86	4.93	4.39
May	7.07	6.22	1.14	58.6	9.85	5.77	6.58
June	7.46	6.44	1.16	60.7	9.87	5.99	6.95
July	8.70	6.89	1.26	61.8	10.03	6.20	7.81
August	7.76	6.68	1.16	61.9	9.43	5.84	6.77
September	6.41	5.51	1.16	62.0	8.37	5.19	6.02
October	5.07	5.03	1.01	59.0	7.83	4.62	4.67
November	3.18	4.02	0.79	54.2	6.89	3.73	2.95
December	1.92	3.74	0.51	50.4	6.74	3.40	1.73
Totals	58.94	62.16		56.5	100.00	57.03	54.11

[a] Based on average measured evapotranspiration and temperature 1940-1943, k = u/f.

[b] Computed normal evapotranspiration (consumptive use) where u = kf, and f = t × p/100 = normal use factor at Salinas.

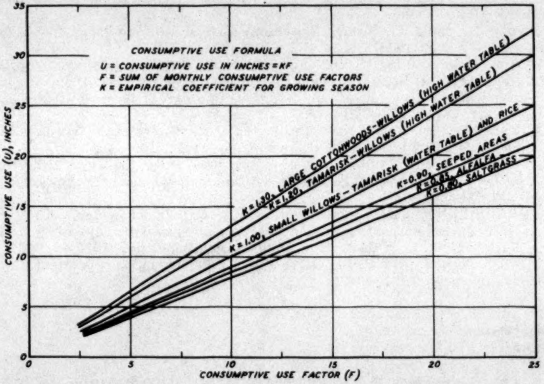

Fig. 1[illegible]--Relation of consumptive use to temperature and daytime hours

California, where measurements of evapotranspiration had been made. Climatic conditions in the lower San Luis Rey Valley are somewhat similar to those in the lower Salinas Valley and the formula, U = KF, was used to compute the evapotranspiration losses in Salinas Valley. Native vegetation was mapped by the State as swamp, dense trees-brush-grass, and sparse brush-grass. Table 3 illustrates the procedure employed to compute the evapotranspiration of tules growing in swamp areas.

In 1948, at the request of the Upper Colorado River Basin Compact Commission, the Research Division of the Soil Conservation Service was confronted with the problem of determining the unit normal consumptive use of water, natural vegetation, and agricultural crops in valley areas extending several thousand miles in the Upper Colorado River Basin which includes parts of Arizona, Colorado, New Mexico, Utah, and Wyoming. The procedure heretofore described was used and the results are given in a report by BLANEY and CRIDDLE [1949].

After the unit values of evapotranspiration have been computed, the total water consumption by phreatophytes and other vegetation in a given area may be obtained by the summation of the products of the unit water use for each kind of vegetation by its area. The areas of various classes of vegetation may be determined by field studies and from areal photographs.

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(Manuscript received May 14, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

USE OF WATER BY PHREATOPHYTES IN 2000-FOOT CHANNEL BETWEEN GRANITE REEF AND GILLESPIE DAMS, MARICOPA COUNTY, ARIZONA

S. F. Turner and H. E. Skibitzke

Abstract--The following is a summary of data obtained in a study of transpiration by phreatophytes in the channels of the Salt and Gila Rivers, between Granite Reef and Gillespie dams, Maricopa County, Arizona. The study was made in the spring of 1950 by the U. S. Geological Survey in cooperation with the Corps of Engineers, United States Army. The kind and amount of phreatophyte growth was mapped in the area covered by a proposed flood-control channel, 2000 ft wide, extending between the two dams. Most of the mapping was done from the air, using recent aerial photos. Transpiration by the phreato-

phytes was computed using the results of the mapping combined with water-use factors developed by experimental work done by the Geological Survey in Safford Valley, Arizona, during 1943 and 1944. Estimates of future phreatophyte use and of the amount of water that might be saved by clearing and maintaining the channel area were based on extensions of water-level graphs to include the next 50 years, and on the experimental work at Safford. The estimates were as follows: (1) total estimated transpiration from ground water at time of investigation - 29,000 ac ft per year; (2) estimated average transpiration from ground water, within the channel area, during period 1950-1999 (a) without Colorado River water - 22,200 ac ft per year, (b) with Colorado River water in 1960 - 29,900 ac ft per year; (3) estimated average water saving effected by channel clearing, 1950-1999 (a) without Colorado River water - 16,600 ac ft per year, (b) with Colorado River water in 1960 - 22,400 ac ft per year.

Introduction--An investigation of the use of water by phreatophytes along the Salt River and Gila River channels between Granite Reef Dam and Gillespie Dam in Maricopa County, Arizona, has been made by the U. S. Geological Survey in cooperation with the Corps of Engineers of the United States Army. The purpose of the investigation was to determine various effects that would result from clearing the phreatophytes from a proposed 2000-foot-wide flood-control channel between the two dams, a distance of approximately 70 mi. Field work and computations were carried on during February and March 1950.

Aerial photographs and maps--The Corps of Engineers furnished two complete sets of contact prints of aerial photographs on a scale of approximately 1500 ft to the inch, and a set of aerial mosaics to the same scale. The aerial photographs were made during October 1948 and May 1949. The mosaics were used to delineate the areas of different density of phreatophyte growth, and the contact prints were used in the field for the purpose of recording phreatophyte types, heights, areal densities, and frondage densities. The Corps of Engineers also furnished prints of base maps showing the natural river channel and the proposed 2000-ft flood-control channel. These maps are drawn to the same scale as the aerial mosaics, and were used to show the outlines of the various phreatophyte areas.

Methods used in investigation--In obtaining the data desired by the Corps of Engineers, the following steps were taken: The areas of different apparent phreatophyte densities were outlined on the aerial mosaics and on the contact prints. The areas were measured by planimeter and numbered consecutively, beginning at Gillespie Dam. The contact prints were then taken into the field, and notes were made as to the types, heights, and densities of the phreatophytes in each numbered area. Most of the observation was done from a light plane at altitudes of 50 to 100 ft above the ground. Checks were made on foot along numerous typical cross sections (see Fig. 1) throughout the length of the channel, and these ground checks were compared with the aerial observations. The comparison between results obtained by air and by ground mapping led to the conclusion that the former offers a much faster and more accurate method, particularly in areas of dense growth.

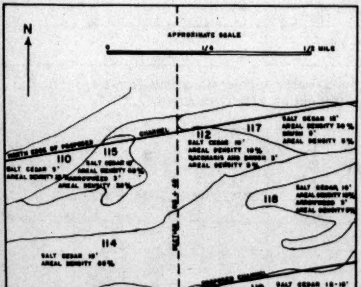

Fig. 1--Section of proposed Gila River flood-control channel showing plant types and densities

The data obtained from the field work were tabulated and evaluated as to total water use by phreatophytes in each individual area, based upon acreage, growth types, use factors, and densities. Use factors for different types of phreatophytes were taken from the results of experimental work done by GATEWOOD and Others [1950] at Safford, Arizona, in 1943 and 1944. It was recognized that, because of higher temperatures and a longer growing season in Maricopa County, water use would be somewhat higher than at Safford. However, because the actual difference in water use is not known, the Safford figures were used and therefore the results are conservative. Water use according to plant types and on the basis of 100 pct areal and frondage density was taken as follows: for salt cedar, 7.2 acre feet per acre per year; cottonwood and willow, 6.0 acre feet; baccharis, 4.7 acre feet; arrowweed and miscellaneous brush, assumed same as baccharis; mesquite, 3.3 acre feet; and palo verde, assumed same as mesquite. From each of the above figures there was deducted 0.65 acre foot of rainfall, leaving only the factor applicable to the use from ground water. The amounts of ground water used in the individual areas were combined to give the total water used within the 2000-ft channel.

In estimating the future average annual transpiration from ground water by phreatophytes within the proposed channel area, the following steps were taken. Profiles (Fig. 2) were drawn to show water levels along the line of the channel for the years 1923, 1939, and 1949. The profiles extended from a point in the channel near Mesa downstream to a point 1 1/2 miles below the junction of the Salt and Gila Rivers. Graphs (Fig. 3 to 5 are examples) were constructed to show water-level fluctuations between 1905 and 1949 at selected places along the channel. Estimated

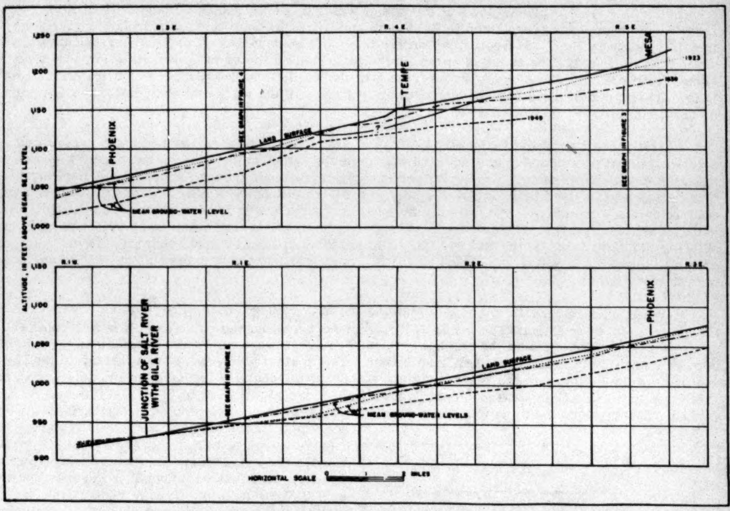

Fig. 2--Profile of land surface and ground-water levels along Salt River from north of Mesa downstream to junction with Gila River

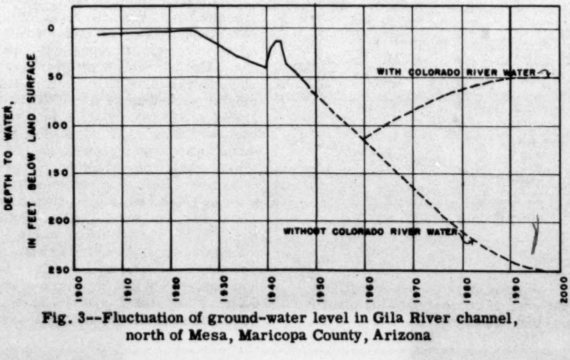

Fig. 3--Fluctuation of ground-water level in Gila River channel, north of Mesa, Maricopa County, Arizona

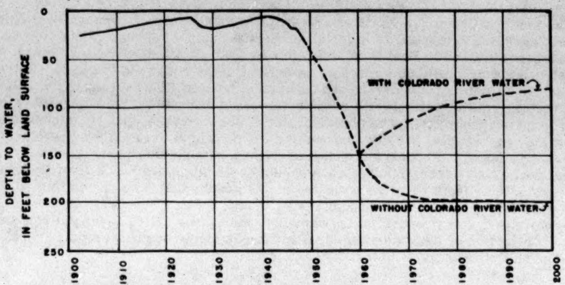

Fig. 4--Fluctuation of ground-water level in Gila River channel, about three miles east of Phoenix, Maricopa County, Arizona

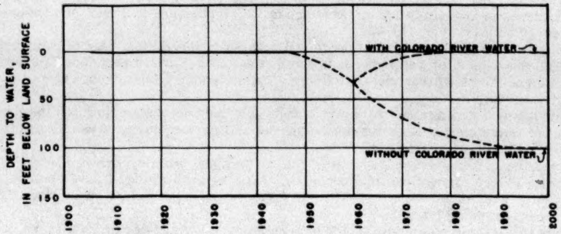

Fig. 5--Fluctuation of ground-water level in Gila River channel, about two miles above junction of Salt and Gila Rivers, Maricopa County, Arizona

trends during the period 1950-2000 are shown on the same graphs. These estimated trends are based both upon the continuation of present conditions and upon the possible introduction of Colorado River water in 1960. The graphs and the following factors were used in estimating future phreatophyte growth and ground-water use along the channel.

Factors considered in forecast of water use--In any attempt to forecast water-level fluctuations and the corresponding changes in water use by phreatophytes along the Salt and Gila River channels during the next 50 years, numerous factors must be taken into consideration. Some of these factors can be evaluated and used in the calculations. Others, of equal or even greater weight, are recognized but cannot be transposed into mathematical values with any degree of accuracy. All the factors listed below were given consideration.

Factors tending to raise water levels--(1) Increase in salt content of ground water. A continuing increase in salt content of the ground water will cause presently developed land to be taken out of cultivation and will decrease pumping for irrigation purposes.

(2) Lower prices for agricultural products. This is a factor that would have an important effect on future water use. However, no attempt was made to forecast economic trends, and current price levels were assumed in estimating water use during the period 1950-2000.

(3) Possible introduction of Colorado River water. It was felt that the forecast should not ignore the effects that would be caused by bringing Colorado River water into the area. There is considerable uncertainty regarding this matter and also uncertainty as to the actual data when additional water might be brought in. For the purposes of this estimate, 1960 was arbitrarily taken as the year when the factor might become operative, and any change in that date would mean either a hastening or retarding of the effects that would be felt.

(4) A prolonged wet cycle similar to the period 1905-20. It is believed that the possibility of a wet cycle should be seriously considered in any water-use forecast. Rainfall and runoff records available for the period 1905-20 were carefully studied in order to determine the nature, sequence, magnitude, and duration of the effects of another such wet cycle. Taking into account the storage

dams on the Salt and Verde Rivers, it was determined that the effects would be reflected almost immediately along the Salt and Gila River channels and that continuous surface flow probably would occur between Granite Reef Dam and Gillespie Dam within three or four years. Recharge from the streams would build up a steep ground-water ridge under the channels and the slopes would gradually flatten as the water moved outward into depleted areas within the valley. It is estimated that the rise in water levels in the basin would average not less than five feet per year during a wet cycle of this sort. The phreatophyte problem would be greatly intensified by the rise of water levels, and an areal density approaching 100 pct might eventually be expected over the entire channel. Although it appears reasonable to anticipate another protracted wet cycle at some time during the next 50 years, the date when such a cycle might begin is a matter of speculation. For that reason, the effects were not indicated on the graphs that show the estimated water levels during the period from 1950 to 2000, inclusive.

(5) Higher power rates. Increased power costs would automatically curtail pumping and thus prevent further decline of the water table in some areas that are now near the borderline between profitable and unprofitable agricultural development.

Factors tending to lower water levels--(1) Additional agricultural development. All available surface water is now being used and additional development would necessarily entail increased use of ground water.

(2) Lining of canals in the Salt River Valley. This would greatly reduce recharge from the canals. As the time of installation and the amount of canal lining could not be predicted, the effect was considered only in a general way.

(3) Construction of a storage dam on the Gila River at the Buttes, east of Florence. This would reduce the amount of water that now enters the channel area as surface flow and underflow at the junction of the Salt River and Gila River. This is another factor that cannot be predicted as to date and quantitative effects.

(4) Development and mining of the San Manuel ore body near Mammoth, on the San Pedro River. The amount of water used will be considerable, and will correspondingly reduce the flow of the San Pedro River, a tributary of the Gila River.

(5) Lowering of power rates. This would allow pumping from deeper levels.

(6) Higher prices for agricultural products. This, as in the case of lower prices, is an economic factor with an unknown time element. The forecast, therefore, was based upon the continuation of present price levels.

(7) Increased efficiency in application of irrigation water and use of fertilizers, and increased mechanization of crop harvesting. This will lower overall production costs and leave money available for deeper pumping.

Indeterminate factors--In addition to factors that will exert definite influence toward raising or lowering water levels in the basin, there are a few that will have effects in each direction, and the final net effect of their influence cannot be forecast. An example is the probable increased use of sprinkler systems to replace present methods of irrigation. Sprinkling will decrease the amount of water used per acre of crops, but it will also increase evaporation losses and will decrease recharge to the ground-water reservoir. Whether the widespread adoption of sprinkling systems in the area would eventually raise or lower the water table is debatable.

Interpretation of water-level graphs--In constructing the graphs shown in Figures 3 to 5, all available water-level records of a group of wells within the selected area were combined to show average fluctuations in the area. In some cases only partial records, particularly for the years preceding 1925, were available. Water levels as shown on the graphs may not be strictly accurate for any individual year, but the general trends are as shown.

Recharge from irrigation raised water levels in the area represented by Figure 3 (between Secs. 3 and 4, T1N, R5E, approximately three miles north of Mesa) until 1925. At that time increased pumping caused a sharp decline which continued through 1939. The abrupt rise after 1939 was caused by the wet winter of 1940-41. Increased development and pumping during and after the war has lowered water levels at an accelerated rate.

Without the introduction of Colorado River water the decline in water levels will continue. If Colorado River water is brought in about 1960, water levels in this area will rise, probably to a level about 50 ft below the land surface.

From 1902 to 1925 the water levels in the area represented by Figure 4 (between Sec. 13 and 14, T1N, R3E, approximately halfway between Phoenix and Tempe) rose owing to recharge from irrigation. Drought and increased local pumping for drainage purposes caused a sharp decline between 1925 and 1927. After 1930 the rise in water levels was resumed and reached a peak during the wet winter of 1940-41. Then the water levels receded slowly until 1944, when increased pumping started the rapid decline.

Without Colorado River water, the present rapid rate of decline will probably continue and may increase owing to lessened recharge after the main canals have been lined. The rate of decline will probably decrease at depths greater than 150 ft owing to reduced pumping. If water from the Colorado River is brought into the area about 1960, the ground-water levels would rise continuously to within about 80 ft of the ground surface, where they probably would be stabilized. The water table could not be stabilized at a much greater depth in this area without an adverse effect upon the salt balance in the basin.

The water levels in the area represented by Figure 5 (Sec. 33, T1N, R1E, approximately two miles east of junction of Salt and Gila Rivers) were practically at the surface of the ground, with flow in the Salt River, until 1947. Then a gradual decline started, and it is expected that the rate of this decline will increase as the effect of pumping in the Maricopa area is transmitted along the Gila River to the junction with the Salt River. It is probable that the water table cannot be lowered more than 100 ft below the ground surface without increasing the salt content sufficiently to make the water unfit for use. If Colorado River water is brought into the basin, the ground-water levels probably will rise to the surface of the ground in from 10 to 20 years.

Results of investigation--The river-bottom vegetation in parts of this area has deteriorated rapidly during the past few years. Some of this deterioration was caused by the rapidly lowering water levels and, in the lower part of the area, some may have been caused by a change in the dissolved-solids content of the ground water. A part of the deterioration was caused by the burning of large areas during the past two years. Where the depth to water was shallow, the phreatophytes in the burned areas recovered rapidly, but in areas where the depth to water was more than 12 to 15 feet and the phreatophytes had to struggle for their existence, they made little or no recovery after burning.

The total transpiration within the channel limits was estimated as 29,000 acre feet per year under present conditions. This is certainly a minimum amount because the rates of water used, determined in Safford Valley, were applied unchanged to the phreatophytes in this locality. No allowance was made for the fact that plants in Maricopa County must use more water than those at the higher elevations in Safford Valley. The estimate must also be considered a minimum figure because the survey was made at a time of year when the phreatophytes were practically bare of foliage. It is likely that a survey made during the summer would indicate a higher density of growth and consequently a higher rate of water use.

Using the water-level profiles and graphs, and considering the factors previously described, another graph (Fig. 6) was constructed to show the estimated annual transpiration from ground water by the phreatophytes during the next 50-year period. This estimate was made to cover two different conditions: (1) a continuation of the present conditions, and (2) the possible introduction of Colorado River water in 1960. In neither case was any allowance made for the effects of wet cycles which will probably occur during the 50-year period. The estimate, therefore, represents minimum values. During the next 50 years there will probably be several short wet cycles and

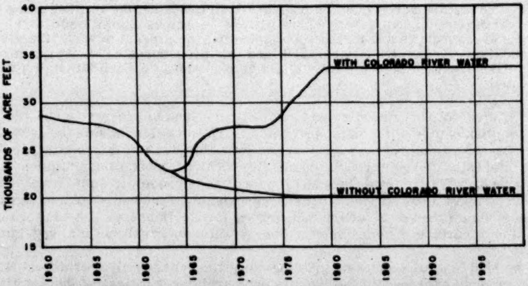

Fig. 6--Estimated future annual transpiration from ground water by phreatophytes in proposed flood-control channel between Granite Reef and Gillespie Dams, Maricopa County, Arizona there may be a long wet cycle similar to the period 1905-20. The effect of the short wet period of 1940-41 is shown on Figure 3.

The results of the foregoing work indicate that the minimum values for the average annual transpiration by phreatophytes within the channel limits during the period 1950 to 2000 inclusive would be about as follows: (1) without the introduction of Colorado River water, 22,200 acre feet; (2) with Colorado River water introduced in 1960, 29,900 acre feet.

The amount of water that will be saved if the channel area is cleared and maintained in a reasonably clear condition was obtained by using data from the experimental work in Safford Valley. Clearing the phreatophytes from the channel area will greatly decrease the use of water by transpiration, but will allow some increase in evaporation. No experimental data were available on the evaporation of water from wetted soil surfaces in the Maricopa County area. A careful study was made of the Safford data and, considering all the factors involved, it was estimated that the water saved in the Maricopa County area would amount to about 75 pct of the total water transpired by the river-bottom growth. The full saving could be obtained only by constant maintenance of the channel area. This maintenance would also be required to make the channel effective for flood control.

Applying the factor of 75 pct to the estimated present transpiration of 29,000 acre feet, the amount of water saved by clearing the phreatophytes would be approximately 21,700 acre feet annually. In like manner, the average annual water saving effected by keeping the channel cleared during the next 50 years would be: (1) without the introduction of Colorado River water, approximately 16,600 acre feet; (2) with Colorado River water introduced in 1960, approximately 22,400 acre feet. These amounts are minimum savings because the computed basic figures for present and future use of water represent minimum amounts.

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(Manuscript received May 18, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

WATER CONSERVATION THROUGH ELIMINATION OF UNDESIRABLE PHREATOPHYTE GROWTH

Curtis W. Bowser

Abstract--A point is now being reached in our agricultural planning, especially in the Southwest, that all measures must be considered in extending the limited water supply The removal of phreatophytes, which annually use enormous quantities of water in a very non-beneficial manner, is one method of augmenting the present supply. Existing eradication methods, post-treatment control, and land-use procedures need additional refinement and further development before a fully effective phreatophyte elimination program can be consummated.

Eliminating wasteful phreatophytes and restoring the land to productive use are not easy tasks. Owing to the complex nature of the many problems, such work lies outside the scope of any single organization. Considerable progress has been made in this work by correlating field activities of the numerous interested groups through organization of two important committees, known as the Salt Cedar Interagency Task Force, New Mexico, and the Subcommittee on Phreatophytes, Pacific Southwest, each composed of representatives of governmental land-management agencies. The function of these committees is to advise and counsel for the interests of all in formulating a practical solution to curbing the consumptive loss of water by non-beneficial vegetation.

It cannot be said that all vegetation growing along the constant or intermittent flowing streams, in the ravines, or playas throughout the West is undesirable regardless of the quantity of water used. These plants may be highly beneficial for soil-erosion control, for browse and cover to both domestic and wild animals, as a source of limited wood supply, and from many other aspects even to aesthetic effects in the otherwise treeless countryside. The value of these waste areas to wildlife is of paramount importance. With the ever-shrinking frontier, the natural habitat for our wildlife is very limited. The river-bottom areas, often covered with phreatophytes, are one of the last strongholds as a natural sanctuary for wild animals and birds. They must be regarded as part of the Nation's resources, and worthy of consideration.

Within the life span of many men now living there has been observed a complete transition from one type of phreatophyte growth to another. These conditions are often brought about through the intervention of man in construction for river regulation, by overgrazing, or from a combination of causes. The following remarks will describe one change in the plant community which is occurring at the lower elevations in the Southwest, although in other sections of the country similar conditions involving different plant groups are being evidenced. Specifically an important change in the plant community throughout the Southwest has been from the mesquite, baccharis, arrowweed, and cottonwood cover to that of the non-indigenous salt cedar, a plant of limited usefulness for soil-erosion protection, and of little or no value to wildlife.

Salt cedar, botanically Tamarix gallica, T. pentandra, T. parviflora, and subspecies, probably was introduced from southern Europe over a hundred years ago, and in this relatively short period has spread with alarming rapidity until it is now found to some extent in almost every drainage channel throughout the Southwest. In some areas, especially the Salt and Gila Valleys in Arizona, and the Pecos and Rio Grande Valleys in New Mexico and Texas, this highly aggressive species has thrived at the expense of other vegetation so that the infested lands are nearly valueless for grazing or wildlife. Further, these jungle-like growths often have encroached into the stream channels to such extent that a flood problem is created. It is folly to believe that further spread of the plant to higher elevations, into the stream flood plains, and along reservoirs, where it will drink even more of the available moisture, is not probable.

Salt cedar plants are entrenched so firmly and scattered so widely that their complete eradication is virtually impossible. However, control or suppression of salt cedar and the restoration of the now valueless land to a productive capacity appears to be feasible.

Salt cedar control by chemical and mechanical methods--Salt cedar is a very tenacious plant. Its ability to survive periodic cutting or burning and regrow perhaps more luxuriously than before has been demonstrated. Many acres of infested lands have been cleared but, unless the ground was farmed or periodic follow-up suppression measures were practiced, infestation from seed produced on undisturbed adjacent-growing trees soon nullified the control measures.

The general inaccessibility and thick growing habit of the plant have made it impractical until recently to consider elimination of salt cedar by chemical means. Following the discovery and accepted usage of the systemic phenoxyacetic herbicides, such as 2,4-D and related products, there was offered to the investigators a practical method of broad-leaved plant eradication at comparatively low cost. No comprehensive experimental studies have been conducted with these chemicals to control salt cedar. However, limited investigations by technicians of the Bureau of Reclamation in cooperation with other interested parties have revealed this plant to be responsive to certain herbicides. Field applications, especially when applying chemical by aircraft, should be fully investigated.

Applications by ground-operated equipment of 1 1/2 pounds acid 2,4-D per 100 gallons water applied in sufficient volume to provide thorough wetting of the plant's numerous small scale-like leaves have been quite effective, although two or more sprayings, each to a lesser number of living plants, have been required to eradicate salt cedar on ditchbanks in the Southwest. Comparable and higher quantities of chemical applied as a low-volume spray by aircraft have not proved entirely effective, probably owing to the lack of complete foliage coverage. The recent developments in herbicides and more advanced application techniques remain to be tried. No doubt, a practical "spray-away" technique to suppress salt cedar will be developed. However, spraying to remove river-bottom vegetation is not the final answer and is applicable only in certain areas. Investigations also are under way pursuing a course of mechanically clearing the lands, then practicing suppression measures, and encouraging establishment of competitive plants which will use less water and be of some economic value in addition to soil stabilization.

Even a program of phreatophyte eradication by mechanical clearing, chemical spraying, or methods yet to be considered may not be the complete or even the practical solution. More on-the-ground, independent initiative of the land manager is required in order to release State and Federal action agencies from the necessity of constant surveillance and periodic maintenance.

Land-management practices to control undesirable vegetation--There are large areas of government-owned lands in the western states that are too rugged in terrain and of such poor soil or so located as to be of little productive value. Dense water-consuming vegetation frequently infests these lands. Examination of these tracts is desirable to determine the practicability of clearing the selected phreatophyte-infested areas and the subsequent development of salvaged water. On privately controlled lands the situation would be comparable, stimulate the incentive of clearing and development of land to utilize the accrued saving in water. Similar programs, such as land drainage and contouring, are functioning, and it cannot be assumed that a comprehensive policy of reclaiming phreatophyte-infested lands would not be equally effective. Such a policy must envision a sound program of land use for the cleared areas so that no agency will be bound forever to continued maintenance.

Hydrologic investigations--There are indications in some critically water-short areas that deep pumping may be lowering the water table so rapidly that the river-bottom vegetation is dying because the plant's roots are incapable of following the receding moisture supply. Along some reaches of the Gila and Salt Rivers in Arizona where this condition is pronounced there now remains less than eight per cent of living plants. These areas formerly supported a luxuriant stand of phreatophytes with, in some cases, emergent growth thriving less than ten years ago.

The possibilities of siphoning away the water supply of river-bottom vegetation by pumping, channelization of normal flow, deep drains, or other methods are worthy of field study at certain specific locations.

Biological control of salt cedar--The control by biological means of various plants, such as gorse, prickly pear cactus, St. Johnswort, and other noxious species, has been accomplished. However, considering the hardy nature of salt cedar, it is believed that only dim hope exists of enlisting natural enemies such as parasitic organisms to prey upon this plant.

Summary--The removal of these rank-growing riparian plants, or phreatophytes, which annually use enormous quantities of water in a very non-beneficial manner, is one source of augmenting the existing water supply. One of the most wasteful water user, and certainly a plant of limited importance, is the salt cedar, a shrub which is not indigenous to this hemisphere. Clearing of salt cedar invested areas would salvage a certain amount of potential irrigation water. The cleared lands would also be available, where feasible, for productive use.

Practically no consideration has been given to investigating in a comprehensive manner just how phreatophytes and especially salt cedar, can be suppressed and what problems will be encountered following their removal from the river-bottom lands. Tests with chemicals, particularly the phenoxyacetic acids, have indicated these compounds to be of value, with the possibility of their practical use over large areas to suppress salt cedar growth. However, application of chemicals in all places is not the answer to the problem. Other possibilities, such as mechanical clearing and the replacement of phreatophytes with economic plants using less water, or the channeling away of available subsurface water, thereby causing natural thinning of the existing plant cover, are being considered in an effort to control river-bottom vegetation for conservation of needed irrigation water. The landholders should assume the responsibility for phreatophyte control, whenever practical. A program of proper land use on the cleared areas should be initiated so that no one will be bound forever to practice constant surveillance and periodic maintenance.

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(Manuscript received April 27, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

PHREATOPHYTE CONTROL ON IRRIGATION PROJECTS IN NEW MEXICO

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Abstract--The problem of increased growths of non-beneficial phreatophytes on operating irrigation projects in New Mexico and the relationship with the water supply available is shown to be one of increasing concern. The methods employed and the results of eradication programs are shown, indicating that additional experimentation and treatments must be performed prior to complete elimination of the phreatophytic growths.

In New Mexico salt cedar has been of particular concern to the Bureau of Reclamation because of the large amounts of water believed to be transpired by it in the Rio Grande and Pecos River basins. Willows have been troublesome on the Rio Grande Project near El Paso, Texas, and control of mixed stands of salt cedar and cottonwoods are an important consideration in the future development of the Middle Rio Grande Project near Albuquerque, New Mexico. The most important problem areas in this region are in New Mexico east of the Continental Divide and are as shown in Table 1.

Table 1--Phreatophyte problem areas in New Mexico east of the Continental Divide

Basin	Location	Area
		ac
Grande	Middle Rio Grande Valley, Cochiti to Elephant Butte Reservoir	51,000
	Above Caballo Reservoir	5,000
Pecos	Above Alamogordo Reservoir	500
	Above McMillan Reservoir	15,000
	Above Avalon Reservoir	5,000
Canadian	Above Conchas Reservoir	650

With the advent of the new hormone-type weed killers and a greatly expanded soil and moisture conservation operations program in 1948, the Bureau was in a position to set up (1) a project for extensive field tests for the control of salt cedar and (2) supply technical assistance to the irrigation projects in this region for the control of weeds and woody plants.

Salt cedar control, Carlsbad Project--The salt cedar growth at the head of McMillan Reservoir, which is now firmly established on the sediment delta, was first reported in 1914. As the salt cedars spread, they formed an obstruction to the passage of flood waters, resulting in the deposition of sediment and the formation of a delta barrier. The salt cedars spread rapidly through the delta area. By 1925, it is estimated that about 12,000 ac were covered with salt cedars. At the present time the salt cedar area is estimated to cover 15,000 ac in the flood plain between Artesia and the head of the reservoir.

The sediment delta and salt cedar barrier,through which the waters of the Pecos River must pass before entering McMillan Reservoir, results in a large amount of water being non-beneficially consumed. The National Resources Planning Board made a study of the apparent water depletions by the salt cedar area during the period from 1934 to 1940. The results of their study indicate an average annual depletion by salt cedars for that period amounting to about 54,000 acre feet.

The same condition exists or is developing at the head of many other reservoirs and in river channels throughout the southwestern part of the United States.

To officials of the Bureau of Reclamation the McMillan delta area appeared to be an excellent area in which to study salt cedar control and means of effecting water savings in similar areas in the Pecos, Rio Grande, and other major drainage basins. In addition, any control measures accomplished in a field-trial program carried out in this area would have a beneficial effect on water supply for the Carlsbad Irrigation Project. For these reasons the Carlsbad Salt Cedar Project was authorized as a soil and moisture conservation project in the fiscal year 1949.

In addition to the salt cedar control trials, the program includes sedimentation, water conservation, non-beneficial consumptive use, and other hydrologic studies closely related to the salt cedar work. Since the sedimentation and hydrological studies are just now getting under way, the following discussion will be confined to trials and accomplishments in the salt cedar control program.

Field trials--One hundred acres of salt cedar on the McMillan Reservoir delta was sprayed twice, once in 1948 and again in 1949, by airplane using one pound per acre of 2,4-D as the sodium salt for the application in September 1948, and two pounds per acre of the same material for the retreatment in June 1949. In both cases the chemical was applied in five gallons of spray per acre consisting of four gallons of water and one gallon of diesel oil. By October 10, 1950, regrowth varied from as little as one per cent in some areas to 100 pct in a few small more or less circular patches. The overall kill was estimated at 85 pct. The total cost of the two airplane applications was $12.50 per acre.

In the fall of 1948 an additional 100 acres of salt cedar was sprayed by airplane using one pound per acre of 2,4-D as the sodium salt. An estimated 30-40 pct kill was obtained from that spraying with considerable regrowth present in 1950. Also, in the spring of 1949 another 100 acres was sprayed with two pounds of 2,4-D per acre which produced better results than the single fall application of one pound per acre but not nearly as good as the two repeated fall and spring treatments produced on the 100-acre tract sprayed in 1948-1949.

The results secured from single and repeated low-volume airplane application of 2,4-D on salt cedars on the McMillan Reservoir delta are reported to have been considerably better than the results secured from similar applications near Yuma, Arizona. The much drier climate and the greater depth to ground water at Yuma as compared to the McMillan Reservoir delta are believed to be important factors in this difference in results.

Willow control, Rio Grande Project--Through the war years, in spite of a fairly intensive oil-burning program, willows (Salex s.p.p.) spread along project irrigation canals and laterals until they had become unsightly, hindered normal operations, and were responsible for the nonbeneficial consumptive use, by transpiration, of relatively large amounts of water. A solution for the willow problem was developed during the past year and it now appears probable, unless drastic retrenchment becomes necessary during the present emergency, that willow control on a large scale will materialize.

In 1949 a new Oliver tractor, equipped with Davco hydraulically powered and operated mower, was purchased. The mower is a heavy-duty unit and is used to cut ditch bank willows along project canals and laterals. Willows which have grown to a height of 14 ft with stems up to and including two inches in diameter have been removed with it along 80 to 100 mi of canal.

The first step in the willow-control program was to select a 20-mi stretch of canal heavily infested with willow growth and mow and slash down all of the willows. In mid-April, after spring regrowth had started, this stretch was given a spray application of the sodium salts of 2,4-D in solution containing 1 1/2 pounds of acid equivalent, one pint of Triton X-100, five gallons of diesel oil, and 95 gallons of water at the rate of 100 gal/ac. The average cost of this treatment was $12.98 per bank mile.

Marked results were evident within 15 days or by May 1 along the entire treated area. About 90 pct of the willows were killed. The small amount of regrowth which has occurred has been gradually eliminated by repeated applications of fortified oil which is used for the control of Johnson grass and other weeds in the regular maintenance program.

Cattails and tules, Rio Grande Project--While awaiting full results from the application of the 2,4-D sprays in the willow-control program, our spray facilities were directed to the increasing menace of cattails, rushes, and sedges which were spreading rapidly throughout the canal and lateral system, creating great berms occluding the channels in some of the laterals as much as 50 pct.

During the latter part of the 1949 irrigation season some kill had been noted in the cattails in the drains which had been treated for emergent water weeds and some along the canal and lateral banks which had been sprayed with fortified oils for miscellaneous weeds and grasses. As a result of these observations it was decided to work out a system of procedure and spray treatment sequence which would control cattails as well as various types of weeds and woody plants which occur on canal and lateral banks. The following is the general procedure: (1) Clear channels, banks, and roads during the winter maintenance season by mowing, slashing, burning, or blading, using one or more of the four methods; (2) early spring spraying of roads and inside banks with a solution of sodium salts of 2,4-D for control of return growth of willows and weeds which would interfere with equipment in obtaining proper coverage of cattails and tules in the channels.

When the new cattail and tule plants have made a good growth in the late spring or early summer, all water is cut out of the channel to be treated, and all growth is sprayed thoroughly with undiluted diesel oil fortified with pentachloraphenol. This treatment is applied twice and is followed each time by burning in order to remove the canopy of dead material which interferes with complete spray coverage for the plant and exposed root.

After another period of from 10 to 15 days, another application is made using diesel oil with the regular amount of penthachloraphenol but still further fortified with 1 1/2 pounds of acid equivalent of the sodium salts of 2,4-D. The results were successful since approximately 84 pct of the bank growth and better than 60 pct of the cattails were killed from the first series of treatments.

Precautions in the use of herbicides--Precaution should be exercised in the use of the phenoxyacetic acids such as 2,4-D to prevent destruction of susceptible crops or plants. Many crops and plants, among which are cotton, beans, peas, ornamentals, and most broad-leafed plants, are damaged or destroyed by all forms of 2,4-D. Application by airplane on a large scale should not be made near susceptible crops but when ground spray rigs are used and precautions taken, damage can be minimized or completely eliminated. Weeds and woody plants may be sprayed prior to the emergence of susceptible crops and again after the crops have matured. Migration of the herbicide may be held to a minimum by the use of low nozzle pressures, large size droplets (300 to 500 microns), and non-volatile types of 2,4-D such as the sodium and amine salts or non-volatile esters. Good supervision of the spray program and careful handling of the chemicals and equipment are essential in preventing crop damage.

Results achieved during the year have been most encouraging, with complete relief effected, at least temporarily insofar as heavy plant growth on banks and in channels is concerned. The full extent of actual kill cannot be determined at this time but it is believed that faster and more economical methods of control than were known two years ago have been established as a result of the experiments.

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(Manuscript received April 27, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

ACTIVITIES OF THE SOUTH PACIFIC DIVISION, CORPS OF ENGINEERS, IN ERADICATION AND CONTROL OF PHREATOPHYTES

Seldon F. Cramer

Abstract--As early as 1939, the Corps of Engineers provided funds to enable the U. S. Geological Survey to perform investigational work to determine the consumptive use of water by river-bottom growth in the Gila River Valley near Safford, Arizona. Since that time, the South Pacific Division of the Corps of Engineers has taken an active part in fostering other investigational work by fact-finding agencies on eradication and control of phreatophytes in southwestern United States. These activities culminated in the recent work of the Pacific Southwest Federal Inter-Agency Technical Committee, which recommended that a 77-mi flood channel, under study by the Corps of Engineers on Salt and Gila Rivers in Arizona, be used as a proving ground for self-financed cooperative studies by all interested State and Federal agencies.

The eradication and control of salt cedar and other phreatophytes growing in and along stream channels poses a special problem to the Corps of Engineers in the South Pacific Division, where these growths result in choking channels and in wasting water. Although the body of knowledge about this problem is far from adequate, fortunately much information has been made available to us by other agencies. The primary purpose of this paper is to summarize this information, to indicate our use of that information in solving our phreatophyte problem, and to indicate our present and future activities in this matter.

The late E. C. LaRue of the Corps of Engineers was one of the first in the Southwest to foresee that the water used by phreatophytes would some day be sought after as an additional source of water in certain areas and began a study of the problem in the early thirties. In 1939 the Corps provided funds to enable the U. S. Geological Survey to perform investigational work to determine the consumptive use of water by river-bottom growth in the Gila River Valley near Safford, Arizona. This pioneer work was accomplished under the supervision of S. F. Turner, U. S. Geological Survey, Tucson, Arizona. Information on the investigation is included as Enclosure 13 to our flood-control report on Gila River above Salt River dated December 1, 1945.

The Corps of Engineers is primarily a construction agency. As such, we sometimes are faced with the problem of clearing and maintaining flood channels through heavy infestations of phreatophytic growth. The Corps has done little or no investigational or research work on eradication or control of this growth and does not anticipate doing any in the immediate future. Instead, we are relying upon other fact-finding agencies who have done investigational work in the matter for data on eradication and control of phreatophytes, for data on use of water by phreatophytes, and for data on the amount and quality of water that can be utilized from the water saved by eliminating these water-loving plants.

I cite and commend some of these agencies who are presently doing investigational work on eradication and control of undesirable vegetation. The U. S. Geological Survey has compiled and published excellent data on consumptive use of water by river-bottom growth in the Southwest in reports titled Use of water by river-bottom vegetation in Lower Safford Valley, Arizona, by Gatewood, Robinson, Colby, and Hem; and Geology and ground-water resources of the Salt River area, Maricopa and Pinal Counties, Arizona, by Wolcott and Hem. The Branch of Operation and Maintenance of the U. S. Bureau of Reclamation at Boulder City, Nevada, has compiled a report titled Salt cedar problem in Region 3, which is an abridgment of their files on the subject. This report contains an excellent bibliography and list of references besides factual data on their investigational work on eradication and control of salt cedar. The Bureau of Reclamation is presently engaged in an investigational program in the Southwest and will undoubtedly publish their findings in future publications. The Albuquerque, New Mexico, office of the Soil Conservation Service, which has been cooperating with conservation districts and other agencies in the eradication and control of undesirable river-bottom growth, has compiled an authoritative experience record. The Bureau of Plant Industry of the Department of Agriculture is cooperating with State and other Federal agencies in studying the phreatophyte problem. The National Resources Planning Board made excellent studies in the Pecos River Basin and on Upper Rio Grande River and published its findings as Regional Planning Part VI - Upper Rio Grande, and Regional Planning Part X - Pecos River Basin.

The Agricultural Experiment Station of the Agricultural and Mechanical College of Texas has conducted extensive experimental work on chemical and mechanical control of woody plants on Texas ranges and has published their findings in Progress Report No. 1056 entitled Present information on the mesquite problem, in Miscellaneous Publication No. 21 entitled Brush problems on Texas ranges, and in Progress Report No. 1034 entitled Brush control on south Texas pasture land. The Imperial Irrigation District of California has compiled a good experience record of data acquired in controlling growth on their canal banks. Many other agencies have interested themselves in this matter and are actively engaged in trying to resolve the problem. We have utilized all of these sources of information and others in our studies on eradication and control of phreatophytes, principally salt cedars.

Chief among the phreatophyte problems presently under study by the Los Angeles District of the Corps of Engineers is the clearing and maintaining of a flood channel 2000 ft wide on Salt and Gila Rivers in Arizona, between Granite Reef Dam on Salt River and Gillespie Dam on Gila River, a distance of about 77 mi through an infestation of salt cedar of varying density. Cost for mechanically clearing, grubbing, and disposal of phreatophytes has been estimated at about $150 per acre for very dense growth and $20 for sparse growth. Mechanical methods of clearing and grubbing now appear to be best for the initial clearing, but if subsequent investigations prove that other methods are better, we will use them.

Information supplied by the agencies previously mentioned indicates that the best method of maintaining flood channels after the initial clearing may be by mechanical means combined with revegetation by selective planting of grasses. Chemical methods of control are not recommended at this time because of the chance of possible contamination of the water supply, making it undesirable for irrigation or domestic use, and the lack of conclusive evidence that the channel can be maintained free of undesirable growth by chemical means of control.

Studies have been made to determine the consumptive use of water by phreatophytic vegetation. The resulting figures are rather staggering. On the basis of the U. S. Geological Survey study in Safford Valley, the transpiration waste by salt cedar in the Salt River Valley is estimated at about seven acre feet of water per year for each acre infested at 100 pct density of growth. The part of that wasted water that could be utilized for productive purposes if these plants were removed is a matter of much concern to all of us.

Because economic justification of all works undertaken by the Corps of Engineers is prescribed by Congressional enactments, we must consider all benefits that will accrue to a project. In removing phreatophytic vegetation from flood channels, increased protection would be provided against floods and the ensuing benefits can be claimed in justification for doing the proposed work. Because flood control is a major consideration of the Corps in civil works, it is usually the benefits from flood control that we look to in justifying construction of flood-control projects.

If, however, it can be shown that removing phreatophytes from flood channels will make an additional definite quantity of water available for productive consumptive uses, then an additional benefit will accrue to the project for the value to local interests of that additional water. In studying the Salt and Gila Rivers problem in Arizona, the Corps of Engineers has asked the U. S. Geological Survey to make an investigation from which the quantity of usable salvaged water may be conservatively estimated. By carefully transposing the data obtained in the Safford Valley study to the Salt-Gila area, it was estimated that an average of about 16,000 acre feet of water would be saved annually by removing phreatophytes from the proposed floodway, 2000 feet wide and 77 miles long, and keeping it cleared during the next 50 years. There is no apparent reason why the entire volume of water saved cannot be beneficially used for irrigation or to carry more dissolved salts out of the area.

Another benefit that may be attributed to removal of phreatophytic growth from flood channels is that of improved quality of water by a reduction in concentration of salts dissolved in the available waters in the vicinity of the channel. The amount of dilution to be expected is a controversial matter and further investigation will be required to arrive at a definite answer.

Another factor that we must consider is the possibility of increasing erosion by removal of the channel vegetation. Actually, the phreatophytes left growing on the land marginal to cleared channels will serve to stabilize the banks. Any channel degradation that may take place as a result of channel clearing will be accompanied by a general lowering of the drainage outlet in the main channel. The resultant improvement in drainage will be welcomed by local interests in most instances.

Presently available data indicate that the annual cost of maintaining cleared channels free from salt cedar or other phreatophytic growth will be high. We have considered the possibility of reducing maintenance costs by partial utilization of the cleared channel as stock pasture. If parts of the channel, where conditions permit, were to be planted to pasture with salt sacaton or similar grasses, which may inhibit growth of phreatophytes, the worth of the pasture may pay for installation and maintenance of stock fence, and grazing or cropping the planted pasture may reduce the amount of required maintenance. In any event, it appears that some type of selected revegetation may be desirable because a completely denuded channel in this area will contribute large quantities of sediment to excessive surface flows, and create a dust menace in dry seasons.

Section 3 of the Flood Control Act of 1936, governing construction of flood channels, requires that they be maintained by responsible local agencies after completion, in strict compliance with regulations prescribed by the Secretary of the Army. Before local interests will obligate themselves for the necessarily high annual maintenance costs, we must be able to show them that they will receive comparably high average annual benefits.

Our study of the phreatophyte problem in the Salt and Gila Rivers flood channel, made in conjunction with the U. S. Geological Survey, indicates that a conservative estimate of the average annual benefits that will accrue from eliminating undesirable vegetation from the river channels are estimated to be in the magnitude of about $96,000 for the value of the water salvaged that may be put to productive uses. This value for average annual benefits exceeds the estimated average annual channel maintenance costs of $80,000 by a comfortable margin.

The studies of the phreatophyte problem in the South Pacific Division of the Corps of Engineers indicate that a great need exists for further investigational work on the matter. Continuing investigational work by fact-finding agencies is justifiable in order that all agencies may have the use of reliable and accurate data in the future. Consequently, at a meeting of the Pacific Southwest Federal Inter-Agency Technical Committee held at Boulder City, Nevada, on March 15, 1950, the Corps of Engineers representative suggested that our project on Salt and Gila Rivers in Arizona could be used advantageously as an experimental area for self-financed cooperative studies by all interested State and Federal agencies. To this end a temporary subcommittee was appointed to draw up a statement for the parent committee which outlined the problem and recommended responsibilities of those agencies then known to be interested and desirous of cooperating in a study of the phreatophyte problem. This statement was adopted and attached to the minutes of the parent committee's 8th meeting, held in San Francisco on June 7, 1950.

On the basis of the recommendation of the Pacific Southwest Federal Inter-Agency Technical Committee, several fact-finding agencies, notably the Geological Survey, the Bureau of Reclamation, the Soil Conservation Service, and the Bureau of Plant Industry, have begun studies on Salt and Gila Rivers that must be made before the 2000-ft flood channel is cleared.

Formation of a Phreatophyte Subcommittee was undertaken at the 9th Meeting of the Pacific Southwest Federal Inter-Agency Technical Committee to correlate agency activities, particularly the research phases, on phreatophyte control studies. We will all look to this group to supply factual data on phreatophytes in the future and it is the sincere belief of the South Pacific Division of the Corps of Engineers that through the cooperative efforts of this subcommittee with all interested State and Federal Agencies, the phreatophyte problem in the Southwest may be economically resolved in the foreseeable future.

South Pacific Division,
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(Manuscript received April 27, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

EARTHQUAKE MAGNITUDE DETERMINATION FROM THE VERTICAL COMPONENT OF SURFACE WAVES

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Abstract--An earthquake magnitude formula, giving the closest possible fit to magnitudes of GUTENBERG and RICHTER [1949], is developed for the vertical component of surface waves with periods of about 20 sec from records at Pasadena. The magnitude is obtained as the sum of five terms. The first term takes account of the amplitude, the second accounts for the amplitude variation with epicentral distance, the third accounts for depth of earthquake focus (down to 100 km), the fourth is the regional correction, depending on the properties of the path of the surface waves, and the fifth corrects for the fact, verified here, that the ratio of vertical to horizontal amplitudes of surface waves varies with the magnitude. It is not possible to obtain the magnitude from the records at one station to a higher accuracy than ± 1/4.

Introduction--The earthquake magnitude scale was originally defined by RICHTER [1935] for local shocks in Southern California by means of maximum trace amplitudes recorded by standard torsion seismometers (horizontal). It was soon extended to teleseisms by GUTENBERG and RICHTER [1936], using the horizontal component of surface waves with periods of about 20 sec. A more detailed study of the same problem was later given by GUTENBERG [1945a]. By further studies, GUTENBERG [1945b] made possible the magnitude determination also from the body waves P, PP, and S; this was followed by an extension of the magnitude determination to deep-focus earthquakes [GUTENBERG, 1945c], for which surface waves cannot be used.

So far only the horizontal component of surface waves has been used for magnitude determination. The present study is concerned with the development of a magnitude formula for the vertical component of surface waves of periods around 20 sec. The use of the vertical component has a practical advantage as only one record needs to be measured, whereas for the horizontal component always two records (N-S and E-W) have to be measured and the amplitudes combined. The vertical component has also a theoretical advantage as it represents the vertical component of the Rayleigh waves alone, whereas the horizontal component generally is a combination of Rayleigh and Love waves. On the other hand, disadvantages of the vertical component are that the number of stations with data for this component is smaller than for the horizontal components, and that often the accuracy of the constants seems to be less than for the horizontal instruments.

Materials used--The magnitude formula to be developed in the next section is based on the records at Pasadena (lat. 34° 08.9′ N, long. 118° 10.3′ W) of 305 earthquakes (141 normal earthquakes, 126 with foci in the depth range $h = 40$ to 100 km, and 38 with $h > 100$ km) during the ten-year period 1940-1949. In every case the maximum amplitude in the period range 18-22 sec was measured together with a few of 17 and 23 sec, but no measurements were made outside the period range 17-23 sec. It is evident that this is a practical method but not always the most accurate. A more elaborate method to obtain the magnitude and energy would be an integration over the whole surface phase.

Measurements were made, whenever possible, on all the records of the vertical component. The instruments are Benioff reluctance-transducer vertical seismometers, and the following combinations of seismometer free period T_o and galvanometer free period T_g were in use: $T_o = 1.0$ sec and $T_g = 112$ sec, $T_o = 1.5$ sec and $T_g = 20$ sec, $T_o = 1.0$ sec and $T_g = 0.23$ sec. The first combination ($T_g = 112$ sec) gave records measurable in almost every case, and it is the actual basis for our formula. The second combination ($T_g = 20$ sec) was in existence only part of the time investigated. The third combination ($T_g = 0.23$ sec) gave measurable amplitudes of the surface waves only when the ground amplitudes were large.

If the assumed magnification of an instrument within our period range (17-23 sec) is wrong by a constant factor, this is of no great consequence for the magnitude formula, as it will be taken into account by an added constant. It was, however, considered preferable to use the actual ground amplitudes, and therefore a complete determination of the constants was made for the long-period combination. The results were: $T_o = 0.95$ sec; $T_g = 112$ sec (corrected for air damping); damping critical both for seismometer and galvanometer. The magnification was determined by means of the test-weight method, introduced at the Seismological Laboratory, Pasadena, by H. Benioff and described by CHAKRABARTY [1949]. Formula (28) (p. 214) in that paper gives the equation of the recorded curve, when the test weight is lifted off the pendulum, for the case of critical damping of both seismometer and galvanometer

$$\alpha_o'(t) = -\left[2\sigma_g \ell_o\, d\, \omega_o^2/(\omega_g - \omega_o)^3\right]\left\{[(\omega_g - \omega_o)t - 2]\, e^{-\omega_o t} + [(\omega_g - \omega_o)t + 2]\, e^{-\omega_g t}\right\} \quad \ldots (1)$$

where

$\alpha_o'(t)$ = trace amplitude at time t
$\omega_g = 2\pi/T_g$ (galvanometer)
$\omega_o = 2\pi/T_o$ (seismometer)
d = pendulum displacement, when test weight is lifted off
ℓ_o = distance of galvanometer lens to recording drum
σ_g = a constant, depending on the electromagnetic properties of the system (same as b defined by BENIOFF [1932]

Table 1--Relation of t to $2\sigma_g\ell_o$

t	$2\sigma_g\ell_o$
sec	sec^{-1}
10	2.493×10^4
20	2.499×10^4
30	2.486×10^4
40	2.444×10^4
50	2.305×10^4
60	2.079×10^4

If this formula were a true representation of the trace amplitude versus time, it should give a constant value of $2\sigma_g\ell_o$, regardless of t. Table 1 gives the results, where t is measured from the moment when the test weight was lifted off. The formula gives t = 18.2 sec for the time of maximum of $\alpha_o'(t)$, which agrees very well with the observations. It is quite clear that $2\sigma_g\ell_o$, instead of being constant, varies slightly in unison with the amplitude. This evidently requires further experimental investigation as well as theoretical refinement. The author used the value $2.49 \times 10^4\ \text{sec}^{-1}$ for $2\sigma_g\ell_o$ in the subsequent computation of the magnification (incidentally, it ought to be pointed out that the initial condition $\dot{x} = 0$ given by CHAKRABARTY [1949, p. 210] evidently is not correct; this does not affect the calculation of the magnification). The amplitudes used in deriving our magnitude formula represent the true ground displacements as nearly as possible.

Data about the location of epicenters, the depth of foci, and the magnitudes were obtained for 1946 and earlier years from GUTENBERG and RICHTER [1949], for 1947-1949 from the determinations by B. Gutenberg given at the end of each year in the Pasadena seismic bulletin, and for some of the earthquakes in 1946 from an unpublished file of B. Gutenberg. The earthquakes used cover the magnitude range from about 5 1/2 to 8 1/2 and all distances from about 30° upward.

Development of the magnitude formula--Our task is to develop a formula containing the vertical component of surface waves and giving the closest possible fit to the given magnitudes. GUTENBERG [1945a] gives the following expression for the magnitude M

$$M = \log A_H - \log B + C + D \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where

A_H = horizontal component of the maximum ground movement in microns for surface waves of about 20-sec period
B = the same quantity for a shock of magnitude zero
C = a constant for each station, depending on ground at the station and on instrument
D = a term depending on the depth of focus, energy distribution in azimuth, and particular properties of the path epicenter-station

All logarithms are to the base 10. B is a function of the epicentral distance Δ given by the expression

$$-\log B = 5.04 + (1/2)\,[48.25\,k\,(\Delta - 90) + \log\sin\Delta + (1/3)\,(\log\Delta - 1.954)] \quad \ldots\ldots (3)$$

where k = the absorption factor per km. Table 4 of GUTENBERG [1945a, p. 7] contains numerical values of - log B for various distances, assuming $k = 0.0003\ \mathrm{km}^{-1}$ (this table is a result of a comparison of theoretical and observed values).

As a first approximation we write the following formula containing the vertical amplitude A_Z

$$M = \log A_Z - \log B + \delta \qquad (4)$$

The term log B, taking account of the amplitude variation with distance, is assumed to be the same as for the horizontal component (the constant term in the expression of log B is certainly different, but that is taken into account by δ). The value of δ has been calculated for each earthquake from the known values of M, A_Z, and Δ. The ground amplitudes have all been reduced to their values for 20 sec under the reasonable assumption of equal energy, that is, constant A_Z/T, where T is the period of the earth vibration. Figure 1 (full curve) gives the variation of δ with epicentral distance Δ for normal shocks. There is an apparent general decrease of δ with increasing Δ. The question arises whether the reason is that the variation with Δ is not duly represented by the expression for log B, that is, if δ has some functional relation to Δ, or whether the reason may be the particular locations of the earthquakes for the different distances. It is already known [GUTENBERG, 1945a, p. 9] that certain paths, especially those tangential to the Pacific boundary, show higher attenuation than other paths. To answer the question above, the normal shocks were divided into three groups according to path (Pacific, continental, and boundary paths), and the mean values of δ for various distances were computed (see Fig. 1). The data for the curves in Figure 1, together with the number of observations on which they are based, are given in Table 2. These data refer only to normal shocks.

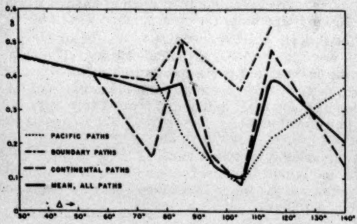


Fig. 1--Variation of δ with epicentral distance Δ for various paths (see Table 2)

Table 2--Variation of δ with epicentral distance Δ for various paths

Δ	All paths		Pacific paths		Boundary paths		Continental paths	
	n	$\bar{\delta}$	n	$\bar{\delta}$	n	$\bar{\delta}$	n	$\bar{\delta}$
≤ 40°	9	+0.46	...	....	9	+0.46	...	....
40 1/2- 70	13	+0.41	...	....	9	+0.41	4	+0.41
70 1/2- 80	17	+0.37	10	+0.41	4	+0.39	3	+0.17
80 1/2- 90	21	+0.38	10	+0.23	8	+0.51	3	+0.51
90 1/2-100	19	+0.14	15	+0.14	...	....	4	+0.17
100 1/2-110	35	+0.10	21	+0.08	3	+0.36	11	+0.08
110 1/2-120	13	+0.39	5	+0.22	3	+0.55	5	+0.48
>120	14	+0.20	4	+0.37	1[a]	(+0.02)	8	+0.14

[a] One of the paths is mixed and is not used here.

It is clear from Figure 1 that these partial curves show no definite variation of δ with Δ. This means that the Δ-expression contained in log B adequately takes account of the functional relation of M to Δ. This is certainly also the case for the deeper shocks. The detailed shapes of the δ-curves depend on the different paths and distribution of earthquakes. We are therefore allowed to calculate mean values of δ over all distances for shocks of given depth.

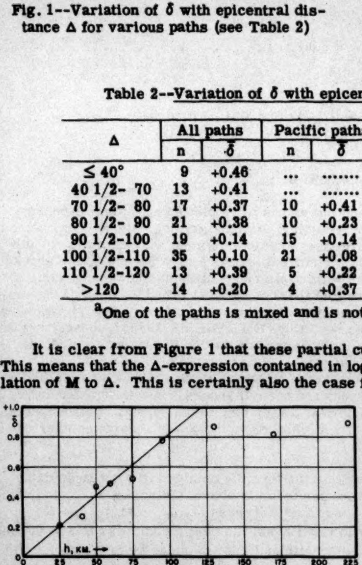


Fig. 2--Relation between $\bar{\delta}$ and focal depth h (see Table 3)

Figure 2 gives the mean values $\bar{\delta}$ (h) of δ over all distances plotted against the depth h of the foci. In calculating the mean values of δ the earthquakes with boundary paths were excluded, as they are to be regarded as disturbances which are quite different at different stations. The observations for h ≥ 100 km are well represented by the straight line in Figure 2.

$$\bar{\delta}(h) = 0.0082\,h \qquad (5)$$

Table 3--Relation of observed and calculated $\bar{\delta}$ to focal depth h

h	Number of observations	$\bar{\delta}$ (h)		
		Observed	Calculated	Observed - calculated
km				
25	103	+0.21	+0.21	±0.00
40	8	+0.27	+0.33	-0.06
50	32	+0.47	+0.41	+0.06
60	20	+0.49	+0.49	±0.00
70-80	13	+0.52	+0.62	-0.10
90-100	11	+0.78	+0.78	±0.00

Table 4--Observed values of $\bar{\delta}$

h	Number of observations	$\bar{\delta}$
km		
110-150	13	+0.87
160-180	9	+0.82
210-230	5	+0.89
300	1	+0.83

A linear relation is also to be expected from theory. Table 3 gives a comparison between observed $\bar{\delta}$ (h) and $\bar{\delta}$ (h) calculated from the formula above. The mean depth of normal shocks was assumed to be 25 km. The amplitudes of surface waves decrease with focal depth h corresponding to the factor $e^{-2\pi ah/L}$, where L = the wave length [GUTENBERG, 1945c]. The slope of the straight line in Figure 2 corresponds to a value of a = 0.21 for L = 70 km, in good agreement with the mean value of 0.28 for the horizontal component calculated from Table 2, p. 128, in the paper just mentioned.

On the other hand, we find for h > 100 km a remarkably constant value of δ (δ = 0.85). Details are given in Table 4. The reason is most probably that the observations here to a large extent do not refer to surface waves but are SS or other waves, which in a given record often have the largest amplitudes at about 20-sec period for h > 100 km.

Defining the regional correction m_R by

$$m_R = \delta - \bar{\delta}(h) \qquad (6)$$

we can write our magnitude formula as follows

$$M = \log A_Z - \log B + \bar{\delta}(h) + m_R \qquad (7)$$

In this formula we have

$$\bar{\delta}(h) = 0.0082\, h \qquad (8)$$

The value m_R was calculated for each shock, and the values were plotted on maps. It became evident that the values of m_R show a definite geographical distribution, the values of m_R being the same within relatively large areas. This permitted us to calculate mean values of m_R for different regions. The results are given in Figure 3a for normal shocks and in Figure 3b for shocks with h = 40 to 100 km. It was found that all shocks within this range of depth showed the same distribution of m_R. On comparing the two maps we find that not only the broad outlines of the limits but also the values of m_R are essentially the same, indicating that the effect of depth h has been adequately accounted for by the term $\bar{\delta}$ (h). The result by GUTENBERG [1945a], especially the high attenuation of surface waves passing along the boundary of the Pacific, is confirmed. It seems to be an indication of higher attenuation along the American side of the Pacific than along the Asiatic side. The New Guinea area is characterized by giving abnormally large amplitudes for shallow shocks. Some of the positive m_R-areas for normal shocks, especially around the Kermadec and Tonga Islands, may be due to somewhat larger depth than normal. The negative m_R-area over Central Asia and to the south of it corresponds to the lower absorption over continental paths.

The next thing to do is to investigate how closely the formula obtained fits the given data. Therefore, M_{calc} was computed in each case from our formula above, taking A_Z, Δ, h, and region into account. M_{calc} was compared with M_{given} and the differences $M_{given} - M_{calc}$ plotted against M_{calc} (see Fig. 4). For earthquakes with h > 100 km the best method was to use the same m_R as for h = 40-100 km for epicenters along the Asiatic and the American sides of the Pacific and to disregard m_R for other epicenters. Figure 4 reveals the remarkable fact that the residuals

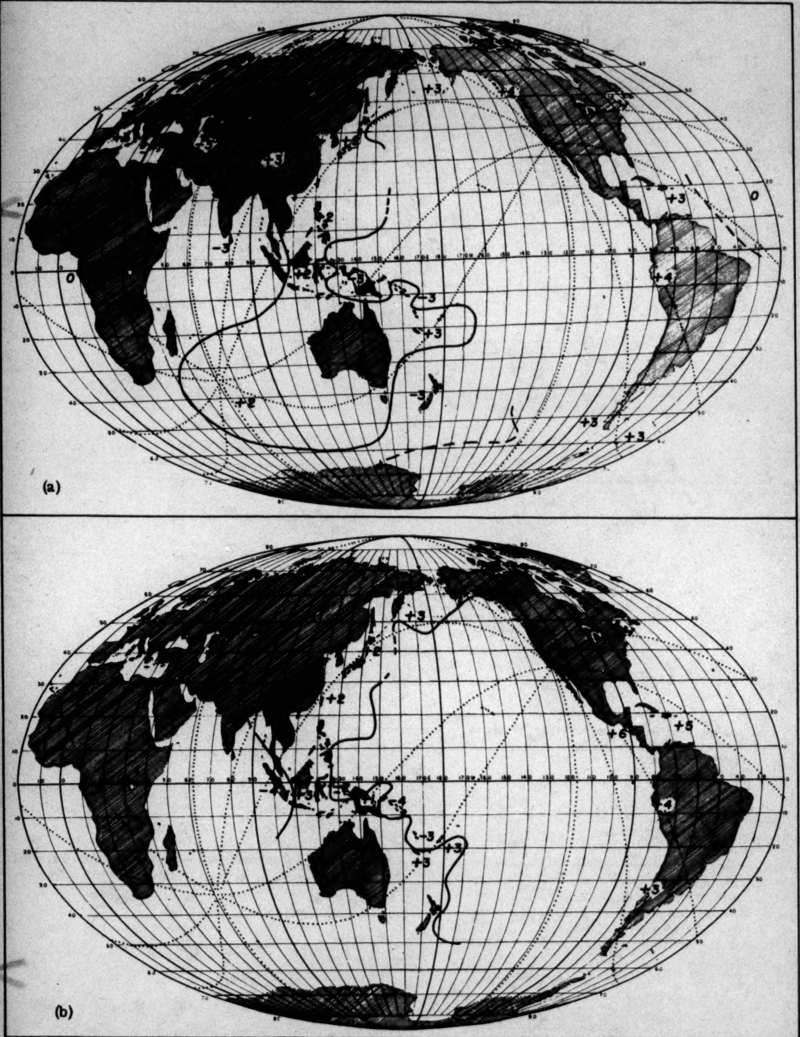

(a)
+3
+4
-3
+2
-3
0
0
+3
+2
-3
+3
-3
+2
+4
+3
+3
(b)
+3
2
+2
+6
+5
-4
+5
-4
-3
+3
+3
+3

Fig. 3--Regional corrections m_R in units of 1/10 of a magnitude; solid (and dashed) curves correspond to $m_R = 0$; dotted curves are great circles through Pasadena; (a) normal shocks; (b) h = 40 to 100 km

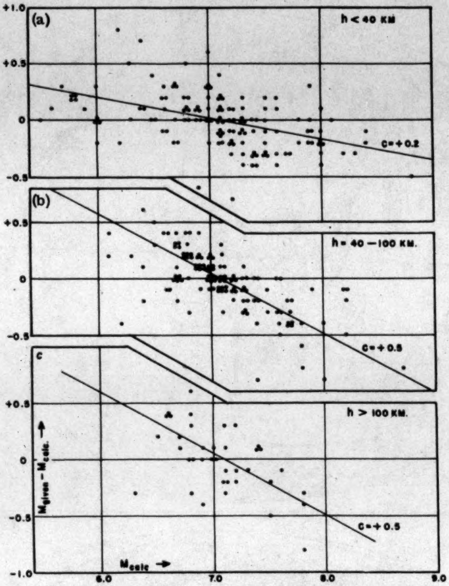


Fig. 4--$M_{given} - M_{calc}$ plotted against M_{calc}; (a) normal shocks; (b) h = 40 to 100 km; (c) h > 100 km

Table 5--Relation of M_{calc} to $M_{given} - M_{calc}$

Normal shocks		h ≥ 40 km	
M_{calc}	$c\,(M_o - M_{calc})$	M_{calc}	$c\,(M_o - M_{calc})$
5.5-5.7	+0.2[a]	6.0	+0.6
5.8-6.8	+0.1[a]	6.1-6.2	+0.5
6.9-7.3	±0.0	6.3-6.4	+0.4
7.4-7.8	-0.1	6.5-6.6	+0.3
7.9-8.4	-0.2	6.7-6.8	+0.2
8.5-8.8	-0.3	6.9-7.0	+0.1
		7.1	±0.0
		7.2-7.3	-0.1
		7.4-7.5	-0.2
		7.6-7.7	-0.3
		7.8-7.9	-0.4
		8.0-8.1	-0.5
		8.2	-0.6
		8.3-8.4	-0.7
		8.5-8.6	-0.8
		8.7-8.8	-0.9

[a] Account has been taken of the fact that the corrections are here slightly smaller than according to the straight line.

are not evenly distributed around the zero line, but along sloping lines. This means that our magnitude formula has a too wide range compared with the given magnitudes. This is clear for normal shocks and even more pronounced for the deeper shocks. We can clearly write

$$M_{given} - M_{calc} = c\,(M_o - M_{calc}) \;.\;.\;.\; (9)$$

where c = 0.2 for normal shocks and = 0.5 for h ≥ 40 km. M_o = 7.1 in all cases. Introducing this correction in the expression for M_{calc} we obtain the following formula which gives the closest possible fit to the given M

$$\begin{aligned} M_{given} &= M_{calc} + c\,(M_o - M_{calc}) \\ &= \log A_Z - \log B + \bar{\delta}(h) \\ &\quad + m_R + c\,(M_o - M_{calc}) \\ &= (1 - c)\,(\log A_Z - \log B \\ &\quad + \bar{\delta}(h) + m_R) + c\,M_o \;.\;.\;.\;.\; (10) \end{aligned}$$

Of the last two expressions for M_{given} the first one is simpler in practical use. The numerical values of the term c $(M_o - M_{calc})$ are given in Table 5.

Our next question concerns the reason for this last correction term. There is a possibility that A_Z has a larger range of variation than A_H.

We find from (10) that the coefficient of log A_Z is no longer 1.0 but 1.0 - c. This was further investigated by plotting log A_Z against M for pairs of earthquakes with the same foci (see Fig. 5). For such pairs of earthquakes Δ, h, and m_R are the same, and the equation becomes

$$M_{given} = (1 - c) \log A_Z + \text{constant} \quad . \; . \; . \; . \quad (11)$$

The slope of the lines in Figure 5 determines c, given in Table 6.

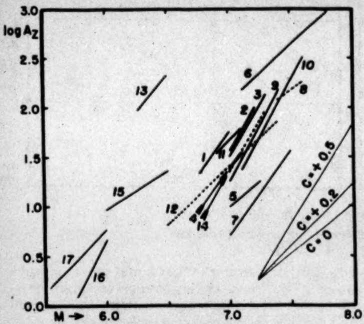


Fig. 5--Log A_Z versus M for pairs of earthquakes with the same hypocenters; each straight line connects the relevant points for a pair of earthquakes with the same hypocenter (see Table 5)

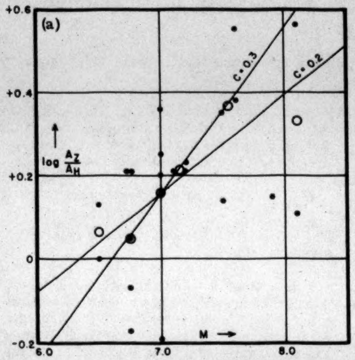
It is clear that the slopes are not evenly distributed around c = 0, but definitely show that c > 0. The agreement with the findings from Figure 4 is also very good quantitatively. The mean value of c from Figure 5 is 0.2 for normal shocks (in excellent agreement with Figure 4a) and 0.3 for h ≥ 40 km. In addition, there seems to be some dependence of c on region: c = 0.4 - 0.5 for Japan and vicinity; c = 0.1 - 0.2 for inner Asia; and c = 0.0 - 0.2 for the southwest Pacific.

The result implies that the ratio A_Z/A_H depends on M. If the reason were instrumental, this would mean that, for instance, for the vertical component the magnification for a given period of the earth vibration should depend on the amplitude. Considering the theory of the seismograph [BENIOFF, 1932], this seems excluded. Also, the fact that c depends on depth h and on region is against an instrumental explanation.

Other reasons could be either some inconsistency of the given magnitude scale or a real variation of A_Z/A_H with M. To decide this question the simultaneous A_Z and A_H for surface waves of about 20 sec were determined from the Pasadena records for 22 earthquakes in various regions

Table 6--Values of c from Figure 5

Reference number in Figure 5	Epicenter	Depth	c	Remarks
		km		
1	57°N, 164°E	Normal	0.4	
2	22 N, 145 E	50	0.5	
3	42 N, 143 E	50	0.5	
4	5 S, 151 E	60	0.3	
5	22 S, 170 E	60	0.0	
6	6 S, 150 E	Normal	0.0	
7	19 S, 169 E	170	0.3	
8	1 S, 101 E	50	0.0	Dotted in Figure 5
9	42 N, 142 E	70	0.5	
10	30 S, 178 W	(75)	0.5	
11	22 S, 169 E	Normal	0.2	
12	45 N, 26 E	150	0.1	Dotted in Figure 5
13	67 N, 136 W	Normal	0.2	
14	15 N, 145 E	80	0.4	Dotted in Figure 5
15	43 S, 171 E	Normal	<0	
16	46 N, 7 E	Normal	0.5	
17	40 N, 41 E	Normal	0.2	



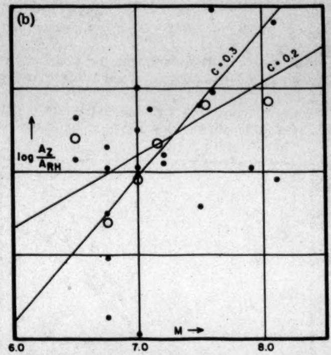


Fig. 6a--Log (A_Z/A_H) versus M for h = 50 to 60 km

Fig. 6b--Log (A_Z/A_{RH}) versus M for h = 50 to 60 km

with h = 50 to 60 km. A_H was measured on all records available from the horizontal seismographs (Benioff reluctance-transducer seismometers, N-S and E-W; Wood-Anderson torsion seismometers, N-S and E-W; Benioff strain seismometer, N-S) and the computed ground amplitudes were compared with each other. In Figure 6, log (A_Z/A_H) and log (A_Z/A_{RH}) respectively are plotted against M_{given}; $A_H = (A_N^2 + A_E^2)^{1/2}$ is the total horizontal amplitude; $A_{RH} = A_N \sin \gamma + A_E \cos \gamma$, where γ = the azimuth of the epicenter measured from the E-W line at the station, is the horizontal component of the Rayleigh wave alone. The dots refer to individual earthquakes and the circles to mean values for given M. The scatter is relatively large and beyond the limits of observational errors, but nothing else is to be expected as the amplitude ratio certainly does not depend on magnitude alone, but also on earthquake mechanism and other factors. In comparing A_Z and A_H for the same earthquake, the magnitude is eliminated. By comparing the expressions for M in A_H and A_Z above we find

$$M = \log A_H - \log B + c_1 = (1 - c)(\log A_H - \log B + c_1) + c\,M$$

$$= (1 - c)(\log A_Z - \log B + c_2) + c\,M_0 \quad \ldots\ldots\ldots\ldots \quad (12)$$

where c (already defined), c_1, c_2 (for given h), and M_0 are constants. From this we obtain

$$\log (A_Z/A_H) = c_1 - c_2 + c\,(M - M_0)/(1 - c) \quad \ldots\ldots\ldots\ldots \quad (13)$$

that is,

$$\log (A_Z/A_H) = \text{constant} + Mc/(1 - c) \quad \ldots\ldots\ldots\ldots \quad (14)$$

Therefore, the slopes of the lines in Figure 6 give $c/(1 - c)$ and c. Deviations between theoretically expected and observed values of A_Z/A_H could be explained by a constant factor of A_H. As a mean for all regions and all magnitudes we obtain for normal shocks $A_H = 1.35\,A_Z$. The ratio depends also on h. Two straight lines have been drawn in each of Figures 6a and 6b; the lines of smaller c correspond to equal weight of all points, and the lines of greater c correspond to smaller weight for the extreme left and right points, as they are based on a smaller number of observations. The latter c corresponds very well to what is to be expected for $h \simeq 55$ km from earlier results. It is clearly a real variation of A_Z/A_H with magnitude. This could have two explanations: Either that for smaller earthquakes a larger proportion of the energy goes into Love waves than for larger earthquakes and the ratio A_Z/A_{RH} is constant, or that there is a variation of the ratio of vertical to horizontal amplitudes for the Rayleigh waves alone with magnitude. Figure 6b clearly demonstrates that the second explanation is the right one. Our last correction term in (10), the magnitude formula, is thus explained even quantitatively to a good approximation. And the given magnitude scale is consistent.

In subsequent searching of the literature for measurements of A_Z/A_H the only similar arrangement of the data was found in a paper by MAINKA [1915], who used data for Pulkovo from 1912 to 1914. There is also a clear indication of the same result. It is obvious that this phenomenon requires further extensive investigations, especially observational but also theoretical.

For the sake of clearness, we summarize the results in a form for use in practical applications. We have

$$M = \log A_Z - \log B + \bar{\delta}(h) + m_R + c\,(M_o - M_{calc}) \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (15)$$

M is evidently given as a sum of five terms. The first term takes account of the ground amplitude (in microns); the second term contains the epicentral distance [GUTENBERG, 1945a, Table 4, p. 7]. The third term takes account of the depth (see Table 3); the fourth term is the regional correction (see Fig. 3); and the fifth term corrects for the variation of A_Z/A_H with M (see Table 5). As above, by M_{calc} we mean the sum of the first four terms in the formula for M. If instead we prefer a formula containing A_Z/T, proportional to the square root of the energy, this is immediately accomplished by replacing $\log A_Z$ by $\log (A_Z/T) + 1.30$. In the latter expression the value of A_Z/T shall be directly inserted for any period in the range 17-23 sec without reduction to 20 sec.

The final formula for M gives the closest possible fit to the observations. The deviations between given M and those computed from the final formula were determined. They are now evenly distributed around the zero line. The standard deviation of a single observation is ± 0.2 (0.21) both for normal shocks (141 observations) and for h = 40 to 100 km (126 observations), and is ± 0.3 (0.26) for $h > 100$ km (38 observations). It is evidently possible by this method to obtain the magnitude from surface waves with undiminished accuracy for earthquakes with foci at least down to h = 100 km. In practice it is, however, better to work the other way round, that is, to determine the depth (for $h < 100$ km) from a comparison of the magnitudes obtained from surface waves and from body waves. As a further test of the accuracy of the final formula it was applied to 29 earthquakes from normal down to h = 100 km recorded at Pasadena during 1938 and 1939, which were not included in the material on which the formula is based. The standard deviation of a single observation was in this case ± 0.3, taking all 29 observations together.

The given M are mean values of determinations by means of observations at several stations over the world. In comparison with the Pasadena records it is immediately clear that inconsistencies exist in these given M-values. But it can also be taken the other way, that the given M-values are more truly representative of the shocks, and the records at one station (here, Pasadena) show clear inconsistencies. They may be due to the earthquake mechanism, giving unequal energy in different directions, etc. These effects are impossible to correct for, as they do not show any obvious or predictable regularities, and they put a definite limit to the accuracy of the M-determination by means of the records at a single station. One can with only one station always expect errors of about 1/4 of a magnitude.

A further undertaking might be to extend the magnitude formula for vertical amplitudes to other stations.

Acknowledgments--My most sincere thanks for valuable suggestions and discussions are due to B. Gutenberg, C. F. Richter, and H. Benioff of the Seismological Laboratory, Pasadena, where this research was carried out. My fellowship from the Sweden-America Foundation, Stockholm, which made by seismological studies at Pasadena possible, is gratefully acknowledged. My thanks are also due to J. M. Nordquist of the Seismological Laboratory, who has drawn the figures and made suggestions regarding wording of the text.

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(Communicated manuscript received March 20, 1951;
open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

ACTIVITY OF PARICUTIN VOLCANO FROM JANUARY 1 TO JUNE 30, 1951

Carl Fries, Jr. and Celedonio Gutiérrez

(Publication authorized by the Director, U. S. Geological Survey)

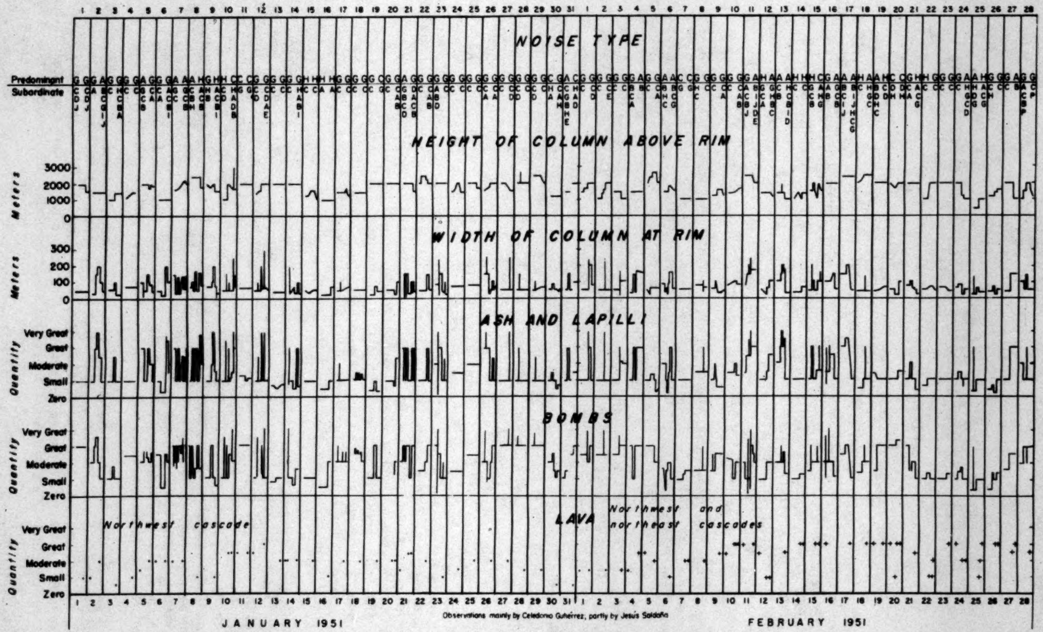
The present report describes the activity of Parícutin Volcano during the first half of 1951, in a form similar to reports published earlier in these Transactions. Continuous observations of activity were made by the junior author and, when he was absent from the observatory station, by his assistant, Jesús Saldaña. The senior author visited the Volcano on March 10-12, May 24-28, and July 10-12 for a total of 11 days. On these visits, measurements were made of changes in position and elevation of points on Parícutin cone and the observational data of Gutiérrez and Saldaña were reviewed and discussed. All this information was evaluated and synthesized for formal presentation by the senior author, who prepared also Tables 2-4 and Figures 4-7.

Eruptive activity--Continuous records of eruptive activity during the first half of 1951 are given in the diagrams in Figures 1-3. For a description of the lettered noise types used in these diagrams, readers are referred to FRIES and GUTIERREZ [1951a, p. 216]. No marked change was noted in the maximum intensity of eruptions, although the duration of strong eruptions seems, on the whole, to have been less than in the second half of 1950, and periods of weak eruption were longer and more frequent than formerly. The height of the eruptive column exceeded 2000 m on only three to five days of each month, and electric discharges were heard on only one or two days of each month— as usual, during the strongest ash eruptions.

The most notable difference in eruptive activity was a gradual increase in frequency of the periodic explosive type of eruption. These explosions were usually preceded by one to five minutes of complete quiescence and each was accompanied by a loud report, earth tremors, and a strong emission of ash and bombs. They were usually succeeded by a period of several minutes to more than one hour of the silent type of eruption, in which large quantities of ash were emitted without much noise. It is presumed that the normal eruptive activity was too weak to keep the crater throats continuously open. After sufficient pressure had been built up, great blasts suddenly blew out this accumulated material, the strongest blasts hurling bombs as far as one kilometer from the base of the cone. The accompanying noise was comparable to that of several simultaneous thunder claps heard at close range, so intense that the needle of the continuous-recording barometer at Cuezeño station, five kilometers from the Volcano, was sent skidding across the record sheet. Such blasts occurred as often as 15 to 20 times during January and February, somewhat less frequently in March and April, and again with increased frequency during May and June. An appreciation of the force of some of these blasts is obtained from Figure 4, a photograph of a large block of solid lava blown out of the crater in May.

A vent that blew open in the lower northeast flank of the cone on May 12 erupted much like the main crater for several days, while a small flow of lava oozed out of its bottom (see Fig. 5). At times the eruptions from the vent and the crater were simultaneous, but at other times they were apparently unrelated to each other. By May 18 the eruptions from this vent, which we have named the No. 1 eruptive vent, had become less frequent and were noted only three to five times an hour. The frequency of eruptions continued to decrease during the rest of the month, although on some days as many as six blasts occurred in an hour. The blasts from the vent were generally followed by a series of weaker gas eruptions, which sounded much like the puffing of a slow-moving locomotive, a noise that has been characteristic of explosive vents filled with lava. By the end of June this No. 1 eruptive vent had become stopped up and was partly covered by pyroclastics.

A second vent that blew open higher in the northeast flank of the cone on June 9 erupted also much like the main crater. In fact, eruptions from it were generally stronger than those from the main crater itself, as illustrated in the photograph in Figure 6. Occasionally the eruptions from the crater and No. 2 vent were simultaneous, but generally they were unrelated; at times the two vents and the main crater were in eruption simultaneously. With the formation of No. 2 vent, the activity of No. 1 vent declined markedly, and within two weeks the large volume of bombs and ash from No. 2 vent had nearly buried the lower No. 1 vent. By the end of June, No. 2 vent had settled down to a nearly constant rhythm of periodic eruptions heavily laden with ash and bombs,



Observations mainly by Celedonio Gutiérrez, partly by Jesús Saldaña

Fig. 1--Eruptive characteristics of Parícutin Volcano from January 1 to February 28, 1951

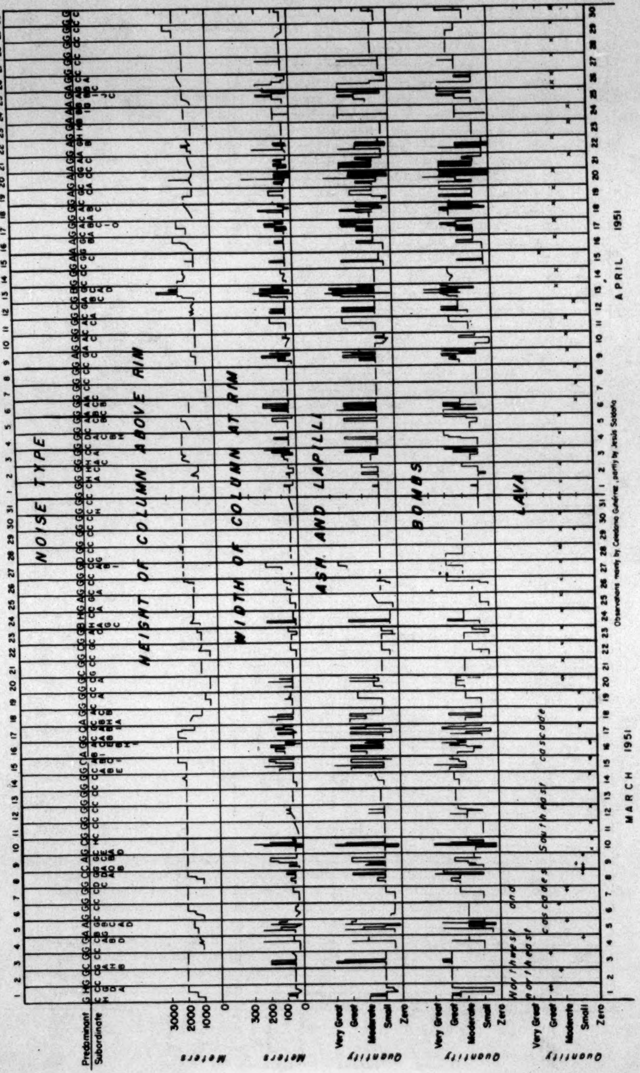
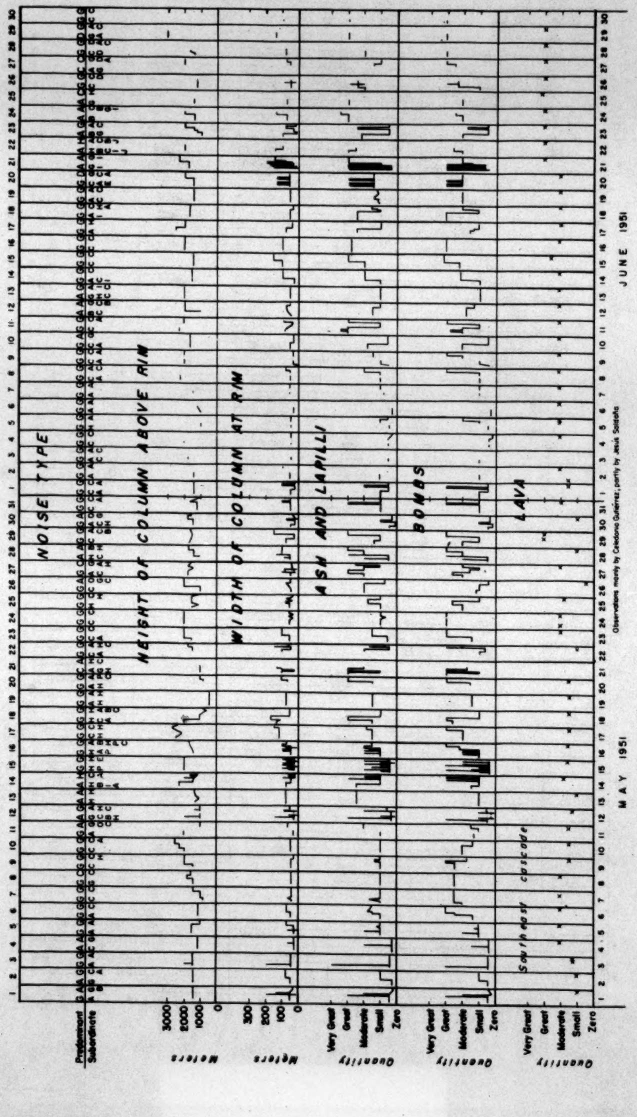


Fig. 2--Eruptive characteristics of Parícutin Volcano from March 1 to April 30, 1951



Observations mainly by Celedonio Gutiérrez; partly by Jesús Saldaña

Fig. 3--Eruptive characteristics of Parícutin Volcano from May 1 to June 30, 1951

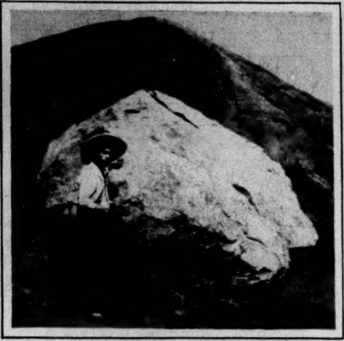


Fig. 4--Large block of lava erupted from crater in May and dropped 100 m from west base of cone; Cerro de Canicjuata in background

Fig. 5--Bottom of No. 1 eruptive vent; small lava flow issues from vent, which was later covered by pyroclastics when higher No. 2 eruptive vent blew open; moist gases blow intermittently through walls of scar; photograph taken from Nuevo Juatita lava-vent mount on May 25

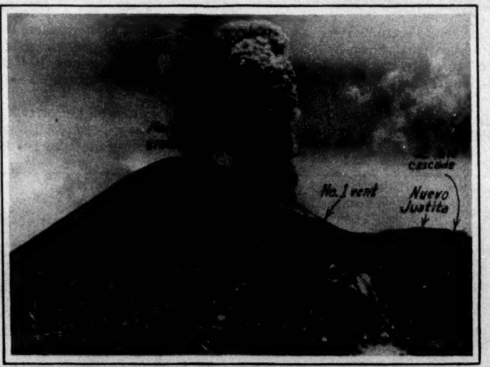


Fig. 6--Main cone and Nuevo Juatita lava-vent mound, showing weak eruption from crater, strong blast from No. 2 eruptive vent, position of buried No. 1 eruptive vent, and lava-vent mound with active southeast cascade; photograph taken from the east on July 11

as shown in Figure 6. These came every 5 to 20 minutes and were followed for one to three minutes by a continuous eruption of the silent type. On the whole, the volume of material erupted from this vent was probably greater than that from the main crater.

Ashfall has continued to decline in volume, only 1.1 mm having been recorded at Cuezeño station and 5.2 mm at Equijuata station during the first half of 1951 (see Table 1), in contrast to

Table 1--Monthly summaries of weather observations, rainfall, and ashfall near Parícutin Volcano

Month	Cuezeño station							Average direction of upper winds	Equijuata station	
	Temperature		Rainfall		Ashfall		Minimum humidity		Rainfall (total)	Ashfall (total)
	Max.	Min.	Total	Max.	Total	Max.				
1951	°C	°C	mm	mm	mm	mm	pct		mm	mm
January	25.8	1.1	1.2	0.7	0.4	0.3	...	SW	3.0	0.7
February	27.2	0.3	0.0	0.0	0.2	0.2	...	SE	0.0	1.5
March	28.1	2.2	2.5	1.5	0.5	0.3	...	SW	3.0	2.0
April	30.8	3.3	0.0	0.0	tr	tr	...	WSW	0.0	0.6
May	30.0	5.5	92.8	54.0	tr	tr	...	E and SW	88.5	0.2
June	26.6	9.2	220.3	41.0	0.0	0.0	52	E	184.8	0.2

Table 2--Rainfall and ashfall at Cuezeño and Equijuata stations for the yearly periods from July 1946 to June 1951[a]

Period	Cuezeño station		Equijuata station	
	Rainfall (total)	Ashfall (total)	Rainfall (total)	Ashfall (total)
	mm	mm	mm	mm
July 1946 to June 1947	1707.0	64[b]	?	?
July 1947 to June 1948	1455.0	30.0	?	?
July 1948 to June 1949	1584.3	12.2	1843[c]	27.7[c]
July 1949 to June 1950	1456.6	9.1	1671.9	12.7
July 1950 to June 1951	1206.1	3.2	1446.5	9.1

[a] Compiled in part from earlier published reports.

[b] Includes estimate for second half of 1946, based on unpublished graph.

[c] Includes estimate for July 1948.

5.1 and 7.7 mm at these stations during the first half of 1950. The continuous decline in ashfall over the past five years is shown by the figures in Table 2. In general, the ashfall in each year appears to have been only about half that of the preceding year. It is now so light that accurate records are difficult to obtain at distances of more than three kilometers.

Form of the cone and crater--On February 20, the Volcano's eighth birthday, the cone and crater were much like the description given in the last activity report [FRIES and GUTIERREZ, 1951b]. The left photograph in Figure 7 shows the typical form of the cone for the past year or

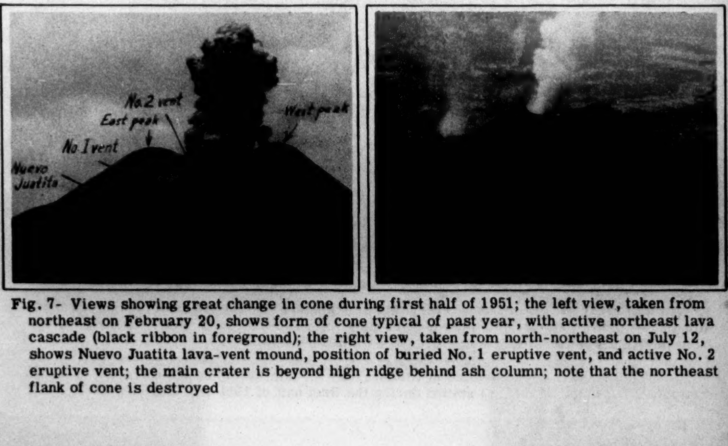


Fig. 7- Views showing great change in cone during first half of 1951; the left view, taken from northeast on February 20, shows form of cone typical of past year, with active northeast lava cascade (black ribbon in foreground); the right view, taken from north-northeast on July 12, shows Nuevo Juatita lava-vent mound, position of buried No. 1 eruptive vent, and active No. 2 eruptive vent; the main crater is beyond high ridge behind ash column; note that the northeast flank of cone is destroyed

Table 3--Horizontal distances from Cuezeño station, and elevations above sea level of points on Parícutin Volcano

Point[a]	Horizontal distance[b]			Elevation[c]		
	March 11	May 24	July 12	March 11	May 24	July 12
	m	m	m	m	m	m
West peak	5200	5200	5200	2807.0	2807.0	2810.1
Lowest point on northeast rim	5220	5220 ┐	......	2783.0	2773.8 ┐	....
Ridge between No. 2 eruptive vent and main crater[d]	np	np	→ 5200	np	np	→ 2765.7
Northeast rim of No. 2 eruptive vent	np	np	5110	np	np	2717.8
Outer east peak	5330 }	......	......	2786.5 }	....	....
Inner east peak	5300 } ┐	......	......	2800.6 } ┐	....	....
East peak[e]	......	→ 5310	5310	....	→ 2798.5	2798.5
Bottom of No. 1 eruptive vent	np	5100	5100	np	2686.9	2686.9
Lava-vent mound (Nuevo Juatita)	4970	4970	4970	2678.7	2675.7	2675.7

[a] See map, Figure 8, for location of present points; for location of earlier, destroyed points, see FRIES and GUTIERREZ [1951b, Fig. 4].

[b] Estimated accuracy of ± 10 m.

[c] Determined by alidade from Cuezeño station; estimated accuracy of ± 2 m.

[d] Lowest point on northeast rim has become ridge between main crater and No. 2 eruptive vent.

[e] Outer and inner east peaks have become a single east peak.

np = not present.

more, as seen from the air from the northeast. The junior author ascended the cone on February 27 and found the crater to be about 30 m deep relative to the northeast saddle, and to have a roundish bottom about 12 m in diameter bounded by a circular fracture. The bottom consisted of reddish, somewhat pasty material through which the eruptions pushed their way, but it could not be considered as representing a lava lake, for it did not have a flat surface. On March 10 a double eruptive column indicated that a second throat had formed in the crater, but it lasted only a day or so, apparently, for a single column was noted on the following day. On a second ascent to the crater on May 7, the junior author found the crater bottom to be about 50 m below the northeast saddle and to contain a throat 12 to 14 m long and eight meters wide, oriented northeast-southwest.

On May 12 at 08h 30m a depression began to form in the lower northeast flank of the cone and soon became an eruptive vent (No. 1). A small flow of lava issued from this vent and kept the lower side open, much like the small cone of Sapichu in 1944. The bottom of this vent was only 11 m higher than the top of Nuevo Juatita lava-vent mound (see Fig. 7, right picture). No open throat was ever noted, as lava occupied the bottom and the eruptions pushed their way through the lava. On May 18 the northeast saddle of the crater rim began to break up and slump. On May 25, when the senior author visited the cone and crater, eruptive activity in No. 1 vent had declined to the extent that the bottom could be seen and photographed (see Fig. 5). The main crater was found to be radically altered, as all the walls were broken up into segments which were gradually slumping into the bottom. The northeast saddle was about ten meters lower than formerly, and a large slice of the inner east peak had slumped into the crater, causing the inner and outer east peaks to form a single east peak. The bottom of the crater was elongated northeast-southwest. A throat about one meter in diameter, surrounded by material at red heat, occupied the southwest side of the bottom. The northeast side of the bottom showed a narrow vent about eight meters long oriented northeast-southwest. Strong eruptions came from this throat, whereas the small southwest throat erupted mainly gases. The bottom of the crater was only about 30 m below the northeast saddle.

On the morning of June 9 a second depression began to form in the northeast flank of the cone, at a point some 20 m higher than No. 1 vent. This depression grew rapidly by frequent eruptions that blew out large volumes of the material forming the flank of the cone. By the end of June this No. 2 eruptive vent rivaled the crater in diameter and activity (see Fig. 7, right picture, and Fig. 6). A high, firmly cemented pyroclastic ridge separated No. 2 vent from the main crater, in a position roughly equivalent to the former northeast saddle. The lower rim of No. 2 vent was only 31 m higher than the top of Nuevo Juatita lava-vent mound. Material erupted from No. 2 vent had covered No. 1 vent and was rounding out the lower flank of the cone. If No. 2 vent should continue to coalesce with the main crater, the two together would have a nearly circular rim about 250 m in diameter. Even this dimension would be much less than the size of the crater in February 1946, when the maximum diameter was 400 m.

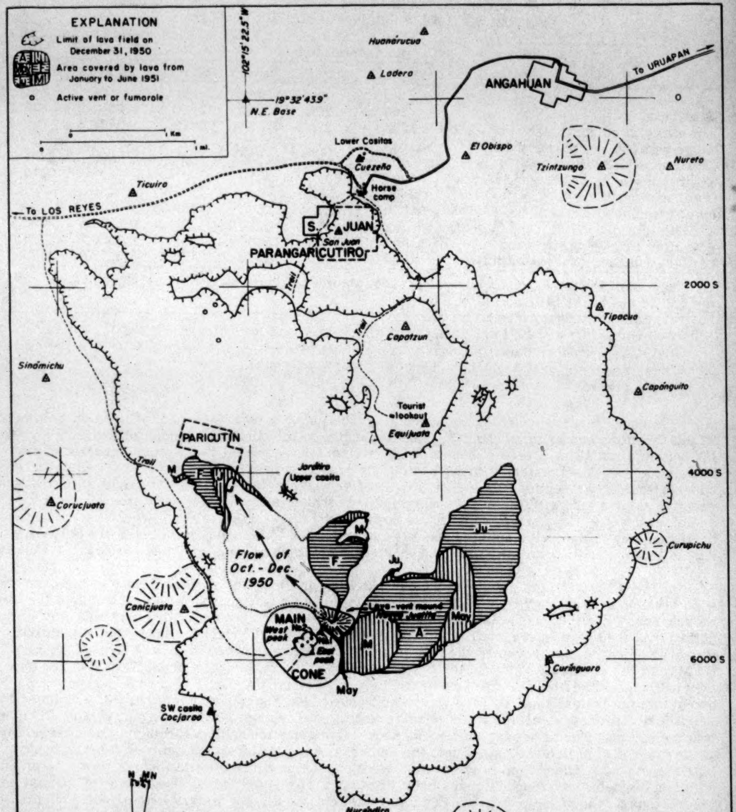

EXPLANATION
Limit of lava field on December 31, 1950
Area covered by lava from January to June 1951
Active vent or fumarole
0
1 Km
0
1 mi.
102°15'22.5"W
19°32'43.9"
N.E. Base
Huanárucua
Ladero
ANGAHUAN
To URUAPAN
Lower Casitas
Cuezeño
Horse camp
El Obispo
Tzintzungo
Nureto
Ticuiro
To LOS REYES
S. JUAN
San Juan
PARANGARICUTIRO
Trail
2000 S
Tipacua
Capatzun
Sinámichu
Capónguito
Tourist lookout
Equijuata
PARICUTIN
M
F
Ju
Jarditiro
Upper casita
4000 S
Corucjuata
Flow of Oct.-Dec. 1950
M
F
Ju
Ju
Curupichu
Lava-vent mound
Mayor Juatita
May
A
M
MAIN
West peak
No. 2
East peak
CONE
May
Canicjuata
6000 S
Curíngaro
SW casita
Cocjarao
N
MN
Nurehdiro Spring
Tzirapan
8000 S
2000 W
0
2000 W
4000 W
Outline of new lava by Celedonio Gutiérrez

Fig. 8--Areas covered by Parícutin lava flows from January 1 to June 30, 1951

Because of drastic changes in the form of the cone during the period, all points had to be retriangulated in order to determine their elevations. The results of horizontal and vertical measurements are given in Table 3, and the points named can be identified in Figure 8. The lowest point on the northeast rim became the ridge between No. 2 eruptive vent and the main crater; it was about 15 m lower at the end of the period than at the beginning. The outer and inner east peaks became a single east peak about two meters higher than the inner east peak was at the beginning of the period. New points are the northeast rim of No. 2 eruptive vent and the bottom of

No. 1 eruptive vent, which was nearly buried by pyroclastic material by the end of the period. Only the west peak and the lava-vent mound continued to rise in height, although the latter lost about three meters by slumping when the lava shifted its direction of flow in March.

Activity of the lava vents--Lava flowed from No. 1 eruptive vent for only a week or so after the vent formed on May 12. Gas eruptions from this vent were vigorous and frequent for the first two weeks and then tapered off and ceased by the end of June. These eruptions were at first accompanied by large volumes of ash and bombs, but after three weeks very little ash was erupted and the material consisted mainly of pasty lava blown out of the vent by the force of the gas escape. At times the lava was lifted four or five meters, appearing as a fountain. After June 9, when No. 2 eruptive vent formed, No. 1 vent ceased to erupt ash and bombs and gave off only sporadic belches of gas.

Nuevo Juatita vent, however, supplied lava during the entire six-month period. The northwest cascade, which had formed in October 1950, continued to be active until February 3, when freezing of the lava surface and a break-through initiated a second cascade to the northeast. These two cascades were alternately active until March 9, either one or the other carrying the major part of the lava flowing from the vent. On that day, however, freezing of the lava at the vent caused a break-through at a somewhat lower level to the southeast, with the consequent beheading of the northwest and northeast cascades. The southeast cascade was continuously active until the end of June, when it still showed a normal rate of flow. Brief flooding at several times during the period caused overflows toward the main cone and also to the east, broadening and rounding out the vent mound. Weak gas eruptions from Nuevo Juatita vent were noted on at least one day of each month.

Activity of the lava flows--The new lava moved over older flows to the northwest until the early part of March, when it had reached the position of buried Parícutin village (see Fig. 8). A small flow from the northeast cascade moved about 1200 m north, also over older flows. The tiny flow from No. 1 eruptive vent formed a tongue about 20 m wide and 600 m long against the northeast and east base of the main cone. The largest flow, from the southeast cascade, had moved 2500 m northeast of the main cone by the end of June. The total area covered by these four flows was about 2,560,000 sq m, and the average thickness of the flows is estimated to have been about seven meters, which is less than it has been for the past three years because the flows were fairly near their source and had not yet piled up notably at their fronts. The total volume of lava for the first half of 1951 is thus estimated to have been about 18,000,000 cu m, which is equivalent to an average daily volume of 99,000 cu m. The total thickness of the lava field around the northeast and north base of the main cone is now between 200 and 250 m and reaches a maximum of at least 275 m below the top of Nuevc Juatita vent mound.

Table 4 gives the volume of lava erupted at Parícutin at different periods since the birth of the Volcano and shows the decline in lava volume over the past three years. The volume appears to have remained nearly constant at about 190,000 cu m daily, on the average, until the end of 1948.

Table 4--Areas covered by, and estimated volumes of, Parícutin lava for different periods

Period	Area covered by lava[a]	Estimated average thickness[b]	Estimated total volume for period	Estimated average daily volume for period
	sq m	m	cu m	cu m
Feb. 20, 1943-Jan. 8, 1944	?	?	61,000,000[c]	190,000[c]
Jan. 9, 1944-July 31, 1948	60,460,000	5.2	316,000,000[d]	189,000[d]
Aug. 1-Dec. 31, 1948	4,050,000	8	32,000,000	209,000
Jan. 1-June 30, 1949	2,440,000	10	24,000,000	133,000
July 1-Dec. 31, 1949	2,640,000	9.5	25,000,000	136,000
Jan. 1-June 30, 1950	2,260,000	10	23,000,000	127,000
July 1-Dec. 31, 1950	2,080,000	8	17,000,000	92,000
Jan. 1-June 30, 1951	2,560,000	7	18,000,000	99,000

[a] Determined by means of planimetric measurements on published maps.

[b] Estimated from personal observations or from published descriptions of lava flows; highest estimates correspond to periods when lava piled up near ends of long flows.

[c] Straight estimate; not based on actual measurements because no accurate maps were made to cover period.

[d] Compiled from planimetric measurements and lava-thickness estimates for 11 consecutive periods.

In the first six months of 1949 the average daily volume dropped to 133,000 cu m and held at nearly that level until the middle of 1950. In the second half of 1950, however, it dropped below 100,000 cu m for the first time, and in the first half of the present year it has continued to hold at about that level. Thus the average daily volume of lava issuing from Parícutin is now only about half the daily volume of the first five years of the Volcano's existence. This small decline is in marked contrast to the great decline in eruptive activity and volume of ashfall.

Although the lava volume for 1943 could not be determined from published maps and reports, the average daily volume is believed to have been roughly equal to that for the period 1944-1948. Assuming that this daily volume averaged 190,000 cu m, the total volume for the period from the birth of the Volcano to January 8, 1944, would be about 61,000,000 cu m. The total volume of lava extruded at Parícutin up to the end of June 1951 was therefore about 500,000,000 cu m, or half a cubic kilometer.

Weather data--Monthly summaries of weather observations are given in Table 1. The rainfall during the first half of 1951 was considerably less than during any equivalent period for the past four years, and that at Equijuata station was less than that at Cuezeño station for the first time. The yearly rainfall figures given in Table 2 show that the period 1950-1951 was by far the driest yearly period since at least 1946, when records began to be maintained, and probably since the Volcano's birth.

Acknowledgments--The present report is a result of a cooperative project being carried on by the United States Geological Survey under the auspices of the Technical Cooperation Administration of the Department of State and the Instituto Nacional de Investigación Científica of the Mexican Government. The salary of the authors is borne by the U. S. Geological Survey and that of Jesús Saldana, who assists the junior author in the field, is paid by the Instituto through Ricardo Monges López, whose enthusiastic support is much appreciated. The Instituto also pays rental on the land occupied by the observatory cabins and shares other expenses. The writers are indebted to José Lauro Ramírez for help in preparing the illustrations, and to Ray E. Wilcox for reviewing the manuscript.

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(Communicated manuscript received September 28, 1951;
open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

SCINTILLATION ALPHA COUNTING OF ROCKS AND MINERALS

J. Laurence Kulp, Heinrich D. Holland, and Herbert L. Volchok

(Lamont Geological Observatory Contribution No. 13)

Abstract--Scintillation counters have been developed which have advantages over parallel-plate ionization chambers for the low-level alpha counting of rocks and minerals. The design, construction, and calibration of the counters is described. Two types of counters have been studied. In one type a lucite cone is used to gather light flashes from a much larger surface area of sample than is possible with a photomultiplier tube alone. However, the gain in count rate is overbalanced by decreased pulse height, loss of plateau, and additional correction factors. In the second type of counter no cone is used, but the sample area is essentially infinite with respect to the sensitive area of the photomultiplier tube. In this situation a count rate is obtained which is 119 pct of that of the theoretical count of a thin source. Under these conditions the background can be reduced to 0.35 counts per cm^2 per hour. Experiments to define the source of the background of the counters are described.

Introduction--Many types of radiation cause emission of light when they impinge on suitable substances. In most instances the flash thus produced has a duration of 10^{-7} to 10^{-5} seconds and an intensity which is a function of the energy absorbed by the fluorescing substance [HOPKINS, 1950]. The development of modern photomultiplier tubes permits the precise detection of such flashes of light [MORTON, 1949; ALLEN, 1950].

Since 1940 considerable advances have been made in the technique of this type of particle detection. Several types of photomultiplier tubes which are superior to older models in amplification, signal-to-noise ratio, and photosensitive area have been developed, extensive improvements of the phosphors have been made, and electronic circuits have been simplified and stabilized. A rather complete list of references covering these developments is given by GOLDSMITH [1949] and HANLE [1951].

The quantitative measurement of the alpha activity of geologic materials has many uses, including age determination and earth-heat study. Most sediments, rocks, and minerals are of low activity and require equipment of high efficiency, low background, and good stability for the accurate determination of their alpha activity. It appeared that for these measurements scintillation counting would have certain advantages over the older parallel-plate ionization chamber methods used by FINNEY and EVANS [1934], HURLEY [1950], and others. The scintillation counter is not subject to microphonics, has low background, and can be operated at high efficiencies. No waiting time is required for the adjustment to equilibrium of nitrogen filling gas. Another feature is the wide range of activity which can be measured with the scintillation counter, for while the parallel-plate technique may be limited to some 300 counts per hour, the scintillation counter can easily record thousands of counts a minute.

The purpose of this paper is to describe two types of scintillation counters which have been studied and to show characteristics in thin and thick source alpha counting of rocks and minerals. Low-level scintillation alpha counters which differ somewhat from those described here have been reported recently in the literature [REED, 1950; GRAVESON and Others, 1950; BAURMASH and DIETRICH, 1951].

Description of apparatus

General description--A scintillation counter consists of a photomultiplier tube in a suitable housing, a regulated high-voltage supply, an amplifier, scaler, and register. An arrangement for holding a sample close to the phosphored photomultiplier tube completes the device.

Housing and cone--Figure 1 shows the housing of a low-level scintillation alpha counter. Magnetic shielding may be accomplished by a cylinder of mu-metal inside the housing. This did not appear critical under our conditions of operation. The sample tray is placed in a closely machined depression under the phosphor at a distance of about 0.7 mm. Light is excluded during

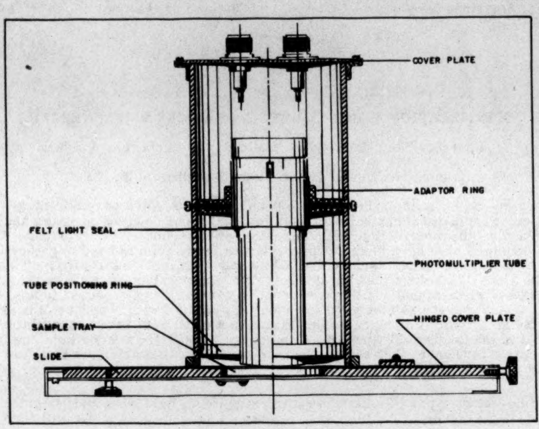
COVER PLATE
ADAPTOR RING
FELT LIGHT SEAL
PHOTOMULTIPLIER TUBE
TUBE POSITIONING RING
HINGED COVER PLATE
SAMPLE TRAY
SLIDE

Fig. 1--Low-level scintillation alpha counter

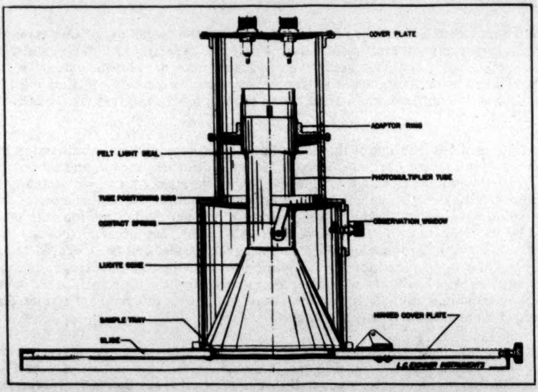
COVER PLATE
ADAPTOR RING
FELT LIGHT SEAL
PHOTOMULTIPLIER TUBE
TUBE POSITIONING RING
CONTACT SPRING
OBSERVATION WINDOW
LUCITE CONE
SAMPLE TRAY
HINGED COVER PLATE
SLIDE

Fig. 2--Low-level scintillation alpha counter employing lucite cone

sample changing by the external cover. Light is prevented from entering from the top of the housing by light-tight seals at the cap and by the clamping ring which holds the photomultiplier tube. Once the desired distance between the phosphored end of the photomultiplier tube and the sample is achieved, the photomultiplier tube is clamped firmly in place. The size of the sample tray (42 cm^2) is chosen to offer "infinite area" geometry (that is, the source is sufficiently extensive that the alphas from the outermost radius are not detectable) to the photomultiplier tube which has a sensitive area of 12 to 13 cm^2.

In order to study the efficiency of counting when a collimating cone is employed, several housings were built to accommodate lucite cones with highly reflective metal-coated walls. Figure 2 shows a typical counter of this type. A sample tray equal in area to the base of the cone is placed just beneath the phosphor which is coated on the base of the cone. A drop of oil with a

high index of refraction is placed on the top of the cone to eliminate the air space, and hence light loss by refraction between the cone and the photomultiplier tube.

The phosphor--In the present work two commercially available, silver-activated, zinc sulfide phosphors, the Patterson type D, manufactured by a DuPont subsidiary, and the RCA No. 33-Z-20A phosphor, have been used. Both have proved to be satisfactory, qualitative tests indicating a slight superiority of the RCA product in light evolution, but inferior mechanical properties. ZnS-Ag phosphor has been found to have an efficiency of 28 pct in converting the energy of alpha particles into light energy [KALLMANN and WARMINSKY, 1948]. This exceeds the efficiency of other commercially available phosphors for alpha-particle detection [JORDAN and BELL, 1949]. The light emitted by ZnS-Ag phosphor has an intensity maximum at a wave length of 4500 A, not far from the peak-response wave length of 4750 A of the RCA No. 5819 photomultiplier tube. The phosphor is readily available in the form of small crystals which show relatively little self-absorption of green light.

Figure 3 shows the count rate as a function of time after a long exposure of the phosphor and photomultiplier tube to light. This is probably due to the phosphorescent effect in the phosphor, but some may be due to excitation in the photomultiplier tube. These data indicate that the majority of the excitation disappears in three hours, and after eight hours normal background is obtained. Since all sample changing is accomplished without the admission of light, this effect is only present after rephosphoring the photomultiplier tube.

The phosphor is applied to the base of the photomultiplier tube or the cone by placing phosphor crystals on the surface freshly coated with a thin layer of silicone high-vacuum grease. The excess phosphor is shaken off, leaving a thin, even, nearly monocrystallite layer. Each time a new coat of phosphor is applied to the tube or the cone, the counter is calibrated. Variations, however, are small and the procedure quite reproducible.

The photomultiplier tube and electronic components--For low-activity counting an end window tube with a large photosensitive area is most suitable. These features are combined in the RCA No. 5819 photomultiplier. The Nos. 4588 and 5311 photomultiplier tubes manufactured by Electrical and Musical Industries, Great Britain, should also prove useful for this work [MORTON, 1949]. The voltage for the individual dynodes of the No. 5819 photomultiplier tube is obtained by means of the voltage divider shown in Figure 4.

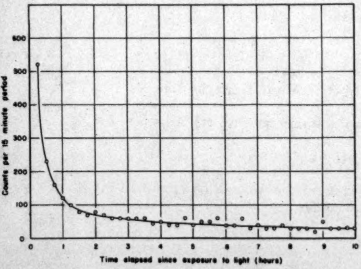

Fig. 3--Count rate as a function of time after a long exposure of the phosphor to light

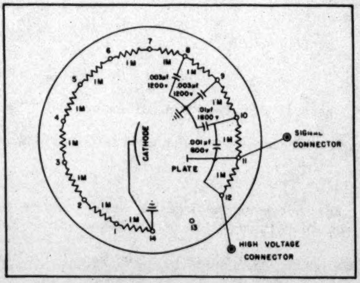

Fig. 4--Voltage-divider circuit for RCA No. 5819 photomultiplier

The electronic components required are available commercially. A Nuclear Instrument and Chemical Corp. Model No. 162 scaler contains sufficient amplification, adequate discrimination, and high-voltage-supply regulation for most use but requires an external register. It was used directly in this work. The Atomic Instrument Co. Model No. 1040X contains the register and has a few features which give more accurate discrimination and high-voltage regulation. GRAVESON and Others [1950] describe electronic circuits which include a cathode follower stage mounted directly on the housing. This is essential for beta and gamma counting but not for alpha counting. However, it decreases the operating voltage which is desirable.

Sample preparation--For thick sources the sample is crushed so that it will all pass a 60-mesh screen. Sufficient material is taken to fill a sample tray to a depth of 2.7 mm, which is considerably greater than the critical value for a "thick" source.

For thin sources a new rapid technique has been developed which is superior in results to the alcohol slurry method described by EVANS and GOODMAN [1944]. The brass or stainless-steel sample trays used for thick-source counting are simply inverted, a thin film of silicone grease applied, and the sample, which has been reduced to greater than 300 mesh, is dusted on. Sources ranging from 0.5 mg/cm^2 to 2.5 mg/cm^2 may be readily obtained. The preparation is very uniform even when viewed with a binocular microscope and provides activity measurement reproducible within the statistical error to one or two per cent.

Theory

The theory of solid alpha counting has been developed by FINNEY and EVANS [1934], NOGAMI and HURLEY [1948], and BEHARREL [1949]. If a homogeneous thick source contains a single alpha emitter surmounted by absorbers, the counting rate in alpha/cm^2/min. is given by

$$n = (Nu/4)\,(R - \rho - a)^2/(R - \rho) \ldots \tag{1}$$

For a thin source, it is

$$n' = (Nu\,\tau/4)\,[2\,(R - \rho - a) - \tau]/(R - \rho) \ldots \tag{2}$$

where N = alphas/cm^3 min emitted from the source, R = mean range of alphas in air (generally stated at 15°C and 760 mm), u = range in substance/range in air, ρ = residual range in air, cm, required to activate the counter, a = total thickness of absorber or absorbers in air, cm, and τ = thickness of the source in air in cm. If $\tau \geq R - \rho - a$, the source is thick.

If, by internal standardization, with similar material containing the same alpha emitters with the same average range it is possible to state the count rate in terms of alphas/cm^2 min leaving the surface of the source, then the terms a and ρ can be dropped so that (1) and (2) become

Thick source

$$n = (Nu/4)\,R = (N''ud/4)\,R \ldots \tag{3}$$

Thin source

$$n' = (Nu\,\tau/4)\,(2 - \tau/R) = (N''ud\,\tau/4)\,(2 - \tau/R) \ldots \tag{4}$$

where d = density of mineral gm/cc, and N'' = alphas/gm min from the source.

Now since $\tau = m/udA$ where m is the mass of the thin source, (4) becomes

$$n' = (N''m/2A)\,(1 - m/2udAR) \ldots \tag{5}$$

For sources on the order of one mg/cm^2 or smaller, m/2udAR will be less than 0.1; thus any uncertainty in ud will be only of second-order importance.

For U-238 where UI = gms U_{238}/gm mineral

$$N'' = 6.02 \times 10^{23} \times 4.83 \times 10^{-18} \times 60 \times UI/238 \ldots \tag{6}$$

Hence for the U-238 series the alphas/cm^2 min will be

Thick source

$$n_{UI} = 1.83 \times 10^5\, UI \sum_i UI\,(ud)_i\, R_i \ldots \tag{7}$$

Thin source

$$n'_{UI} = 3.66 \times 10^5\, UI\,(m/A)\,[8 - (m/2A) \sum_i 1/(ud)_i\, R_i] \ldots \tag{8}$$

Similar relationships hold for the Th_{232} and U_{235} series.

In order to obtain a simple usable relation for the total emission from all sources it is necessary to assume either that ud is independent of R or that an average value may be used. NOGAMI and HURLEY [1948] assumed ud constant, estimated the value of ud by the Bragg-Kleeman rule, and found that the predicted values for emission agreed with the experimental values within the experimental error of about five per cent. BEHARRELL [1949] uses some data of BRIGGS [1927] in an attempt to show that the Bragg-Kleeman rule gives values for ud ten per cent too low. However the argument is inconclusive for a number of reasons, including the uncertainty in the composition of the mica used. If the mica actually used had considerable iron or titanium content, instead of the idealized formula assumed by Beharrell, the apparent discrepancy would be removed. From the same data it can be shown that $\sum_{I} (ud)_i R_i / \overline{ud} \sum_{I} R_i = 1.00 \pm 0.02$, where $\overline{ud}$ is the average value of ud. Since the accuracy of the absolute standards is within two per cent, the use of an average ud is justifiable. Obviously, the error will be entirely negligible in the case of thin sources (Eq. 8) because the term involving the summation is already of second-order importance.

Therefore, assuming ud constant, the total emission in α/cm^2 min becomes

Thick source

$$n_T = n_{UI} + n_{AcU} + n_{Th} = ud \times 10^6\,[6.07UI + 1.80Th] \;\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Thin source

$$\begin{aligned} n'_T &= n'_{UI} + n'_{AcU} + n'_{Th} \\ &= m/A \times 10^6\,[3.05UI + 0.75Th] - (m^2/2AUd)\,[UI\,(3.66 \times 10^5\ \Sigma 1/R_{UI} \\ &\quad + 0.16 \times 10^5\ \Sigma 1/R_{AcU}) + 1.25 \times 10^5\,Th\ \Sigma 1/R_{Th}] \\ &= m/A \times 10^6\,[3.05UI + 0.75Th] - m^2/A^2 ud \times 10^5\,[4.13UI + 0.87Th] \;\ldots\ldots\ldots \quad (10) \end{aligned}$$

where Th is gm Th_{232}/gm mineral.

Another useful equation which can be derived for the particular conditions of the scintillation counter relates the total number of alphas produced per gram in the source per unit time to the number of alphas omitted from the surface of a thin source of the same material per unit time. This is

$$8\,N_{UI} + 7\,N_{AcU}\,6\,N_{Th}/2(n'_{UI} + n'_{AcU} + n'_{Th} = 1 + \tau\,(0.115 + 0.02\,F) + \tau^2(0.013 + 0.006\,F) + \ldots$$

where the N-symbols refer to the number of alphas emitted from U-238 per gram of source per unit, and F is the fraction of the alphas produced in the source by members of the U-238 and U-235 series. It can easily be shown that for sufficiently thin sources, variations in F do not significantly affect the ratio.

One other point requires attention. All of the derivation above assumes the radioactive isotopes to be evenly distributed in a source of uniform density. In a rock this is seldom the case. If very thin sources are used, this correction is still one of second order. If thick sources are measured, the problem becomes very serious. If the distribution of most of the activity can be well defined, for example, if it is largely contained in zircon in an acid-cleaned granite, and the mineral grains containing the activity are essentially "thick" sources, it is still possible to arrive at a reasonably accurate value of specific alpha activity with a thick-source count.

Counting characteristics without cone--Figure 5 shows the relation of count rate to voltage for a thick source of uranium metal and the background at various discriminator levels. (Zero discrimination means that all pulses and noise pass.) The plateaus obtained increase in length and decrease in slope with greater selectivity of the pulses. The plateau at a discriminator setting of 30 is at least 100 volts long while that at 80 is 200 volts long. It is clear that it is advantageous to operate at as low voltages as possible to keep the background to a minimum. Normal operating voltage under these conditions is about 870 volts. The length and slope of the plateau is affected by the characteristics of the photomultiplier tube, the electronic circuits, and the nature of the source. A thin source of one alpha emitter gives a better plateau than a thick source containing

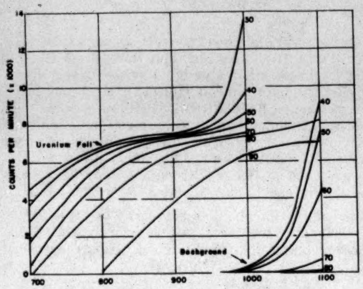

Fig. 5--Relation of count rate to voltage, no cone, uranium metal thick source

several alpha active isotopes. A cathode follower described by GRAVESON and Others [1950] was constructed. The operating potential dropped to 600 volts and the plateau had similar slope and length to that obtained without the cathode follower.

The residual energy required of an alpha particle to produce enough light to yield a pulse that will be counted is apparently less than one per cent of the range. Thus for practical purposes $\rho = 0$ (Eq. 1, 2). This was demonstrated in several ways. If a known thick source of small but precisely measured area was placed below the center of the photomultiplier tube, at a known short distance from the phosphor, the loss of alphas due to counting geometry and air absorption, plus the number actually counted, equaled the known emission. Another experiment, in which the sample was wrapped around the bottom of the tube in contact with the phosphor, yielded similar results.

The geometry used in these counters is that of the "infinite-plane" source. The diameter of the tray holding the thick or thin source is sufficiently large so that the outer few millimeters of sample contribute negligibly to the count rate. The constant distance from the bottom of the phosphored tube to the top of the sample is made as small as possible (generally about 0.7 mm). Since the end of the photomultiplier tube is slightly convex in an irregular manner, it is not feasible to calculate precisely the number of alphas from such an infinite-plane source which will reach the phosphor by correcting for geometry and air absorption. Therefore, calibration is done empirically with known sources.

If a standard producing a known emission of alphas per cm^2 is counted, the ratio of total count rate for such a source to that expected if the source were molded on the bottom of the phosphor is a factor (the "efficiency") which can be used to correct any other sample of similar composition to absolute alphas/cm^2 min leaving the surface of the sample. This number can be placed in (9) or (10) to obtain activity per gram per unit time.

The possible variation of efficiency with alpha range and source thickness requires evaluation. The effect of variations in alpha energy can be deduced theoretically by calculating the ratio of actual count to the number of alphas leaving the surface of a point source under an infinite detector. Table 1 gives these results for two conditions of a and ρ. In the counters described in this paper the situation is essentially that of $\rho = 0$, a = 0.15.

It appears that the variation in count detected from low-energy U^{238} to high-energy RaC′ alphas is considerable. However, as long as the uranium or thorium series in equilibrium are being measured, a calibration with either a uranium or thorium series standard will be entirely adequate (within one per cent) for any mixture of these series in a rock or mineral. Further, since $\rho = 0$ in these counters, the maximum variation in the extreme cases of a RaC′ thin source

Table 1--Fraction of alphas leaving a surface which are counted for infinite plane detector

Type of source	$\rho = 0$, a = 0.15	$\rho = 0.50$, a = 0.15
U^{238} in uranium metal; thin ($\tau = 0.204$)	0.94	0.92
U^{238} in dunite; thin ($\tau = 0.531$)	0.94	0.92
RaC′ in dunite; thin ($\tau = 0.531$)	0.98	0.97
U^{238} in uranium metal or dunite; thick	0.89	0.71
RaC′ in dunite; thick	0.96	0.89
Average U series range; thin	0.96	0.94
Average Th series range; thin	0.97	0.95
Average U series range; thick	0.93	0.83
Average Th series range; thick	0.94	0.84

Table 2--Data of thorium standard BS-11

Sample (1)	Amount (2)	Air (3)	Observed count rate (4)	Corr. factor (5)	Corrected count rate (6)	Calculated emission (7)	Efficiency (8)	Experimental determination (9)
	mg/cm^2	cm	cph/cm^2		α/cm^{2}hr	α/cm^{2}hr	pct	α/mg hr
1	0.480	0.176	2.83±0.08	0.97	2.92±0.08	2.40	122	12.16±0.33
2	0.644	0.228	3.97±0.14	0.97	4.09±0.14	3.16	129	12.38±0.46
3	0.910	0.322	5.68±0.20	0.97	5.85±0.21	4.35	134	13.00±0.46
4	1.046	0.370	6.06±0.15	0.97	6.26±0.15	4.92	127	12.32±0.29
						Average	128	

and U^{238} thick source is only ten per cent. Thus it can be concluded that as far as the range effect is concerned internal standardization by a sample containing a known concentration of either the uranium or thorium series in equilibrium is adequate for the measurement of any naturally occurring rock or mineral sample. The effect is independent of ud.

In order to assess the effect of source thickness, four samples of various thicknesses using the National Bureau of Standards No. 11 (0.01 pct Th, 0.0004 pct U as monazite in dunite) thorium standard were prepared and counted. The data for these samples are set forth in Table 2.

Column 4 gives the observed count rate. Column 6 gives the corrected values of column 4 which may be expressed as alphas/cm^2 hr. Column 7 shows the calculated emission in alphas/cm^2 hr from (10). Column 8 shows the efficiency as defined above, while column 9 gives the experimentally determined alphas/mg hr as calculated from the data of columns 3 and 6 using (11).

It can be seen that for sources well down in the "thin-source" range the cph/cm^2 is proportional to the thickness of the sample. For sources in range of thickness used, these counters operate at about 128 pct efficiency.

Such a definite statement of efficiency can not be made regarding thick sources unless the ud of the source is known, since the efficiency varies strongly with ud as is seen from (9). Consider this same monazite-dunite standard as an example. If the monazite grains are so large as to be individual "thick" sources, the ud will be that for monazite, that is, 2.82×10^{-3} gm cm^3, and the efficiency turns out to be 63 pct. If the monazite is present as extremely "thin" particles, then the effective ud is that of dunite, that is, 1.88×10^{-3} gm/cm^3 whence the efficiency is determined to be 94 pct. Thus to obtain five per cent precision in a thick source, some knowledge of the distribution of the radioactivity and the particle size is required.

Precision on the same mounted sample is determined by counting statistics to at least 0.5 pct. For the same sample mounted successively the results for thick source are within one per cent; for thin source, within two per cent. The absolute values can be no better than the standards which are certified to within two per cent. If a two per cent uncertainty is added for the calibration procedure described above, the total probable error for the thin source in the absolute value is five per cent or better.

A comparison of the characteristics of this scintillation counter with other low-level alpha counters seems desirable. For thin sources it operates at about 128 pct efficiency as defined above which is equivalent to an "apparent geometry" of 66 pct as compared to the 32 pct geometry of REED [1950] and 40 pct of GRAVESON [1950] for low-level alpha scintillation counters. The background can be reduced to 0.35 /cm^2 hr which is comparable to that obtained by Reed and Graveson. Parallel-plate ionization chambers such as have been used by EVANS [1944], NOGAMI and HURLEY [1948], and others allow larger areas of sample to be used and thus permit a more rapid decrease of the statistical counting error. The ratio of sample count to background is similar for the parallel plate and the scintillation counters. For example, FINNEY and EVANS [1934] describe a parallel-plate system of 78.5 cm^2 which yielded a background of about ten cpH or about 0.13 α/cm^2 hr. Most rocks would yield 20-200 α/50 cm^2 hr so that for a granite the count rate/background would be about 250/10. Under similar conditions this scintillation counter would give the same ratio. For these conditions a 24-hour count would yield a probable error of 1.4 pct with a parallel-plate system and 3.0 pct with the scintillation counter. However, the scintillation counter is much simpler to operate, is not subject to mechanical noise, does not require nitrogen filling gas, and can be used to measure high levels of activity while the parallel-plate system is limited to activities below 400 cpH. Thus for most counting of natural materials the scintillation counter is superior but for the extremely low activity materials the faster rate of accumulation of counts with the parallel-plate system makes its use preferred. If scintillation counters can be made with lower background at the same efficiency, they would be preferred for all purposes.

Counting characteristics with cone--An attempt was made to increase the total count rate of the scintillation counter by counting a larger area of sample and thus to overcome the advantage which the parallel-plate technique holds in the matter of counting statistics. To do this an aluminum-plated lucite cone was introduced between the phosphor and the end of the photomultiplier tube (Fig. 2). Photons produced in the phosphor by interaction with an alpha particle can be considered as entering the cone at a point. The fraction of the photons which passes from the cone into the photomultiplier tube is a function of the half-angle of the cone, of the ratio of the area of the base to the top of the cone, and of the solid angle which the photomultiplier tube subtends at the point of entrance of the photons. This latter angle is somewhat larger for photons entering the cone at the center of the base than for those entering near the outer surface. Thus an alpha particle impinging on the phosphor at the center of the cone will produce an electronic pulse in the photomultiplier tube greater than that produced by an alpha particle of equal energy impinging on the phosphor near the edge of the cone. This limits the feasible base area of the cone, since, when the solid angle subtended by the photomultiplier tube for photons entering the cone at the edge is very small, the pulse produced in the photomultiplier can not be differentiated from the background noise pulses generated in the tube. The most efficient cone tested (Fig. 1) had a height of four inches and a base diameter of five inches. A larger cone four inches high and with a base diameter of nine inches was constructed and yielded more net total counts; however, the efficiency decreases so rapidly with increasing background beyond a diameter of five inches that there is no practical gain.

Figure 6 shows the type of curves obtained with a cone four inches high and five inches in base diameter with a thin source of small area at the center of the cone placed within two mm of the phosphor. The expected numbers of alphas reaching the phosphor is about 13,000 per minute. It can be seen that, as the voltage across the photomultiplier assembly is increased, the flat central portion of the curve becomes more pronounced, the curve becomes steeper at high discriminator level settings, and the inflection point in the lower range of discriminator level settings moves toward the right. Since the polonium source used was extremely thin, alpha particles leaving the source into the air space between the source and the phosphor must have been essentially monoenergetic except for straggling and backscattering. However, since the source was not in direct contact with the phosphor, alpha particles emitted at the same point in the source but at

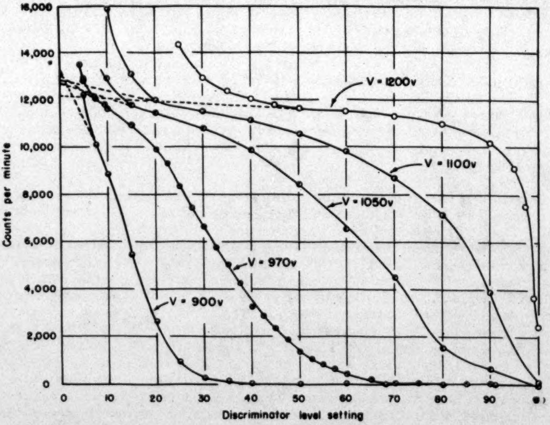

Fig. 6--Count rate of a thin polonium source under a cone as a function of the discriminator level setting at various photomultiplier assembly voltages

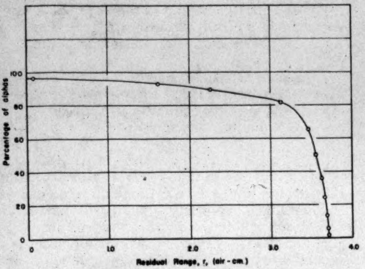

Fig. 7--Theoretical distribution of residual alpha-particle ranges from a thin source emitting monoenergetic alphas

different angles with the vertical traveled different air distances before impinging on the phosphor. The residual range on reaching the phosphor was therefore not the same for all of the alphas emitted by the source. This effect on the residual range distribution for polonium alpha particles impinging on the phosphor was calculated for a = 0.2 cm of air, and is shown in Figure 7. From this it can be seen that 97 pct of the alpha particles emitted by the source reach the phosphor and that 80 pct of the alphas arrive with more than 86 pct of their original energy. Since the light energy liberated in the phosphor is proportional to the residual energy of the alpha particles impinging on the phosphor, the distribution of the intensity of the light flashes should follow approximately the distribution of the residual range of Figure 7, provided that a sufficient thickness of phosphor is available to absorb all the residual energy of the alpha particles. The difference in the percentage of light which enters the photomultiplier tube from flashes of identical intensity at different parts of the base of the cone will cause the pulse-height distribution curve in the photomultiplier tube to have a less pronounced curvature than the theoretical residual-range distribution curve of Figure 7. The discriminator level setting on the scaler indicates the minimum pulse height necessary for registering a signal in the recording system. Thus curves of count rate versus discriminator level setting indicate the pulse-height distribution of signals from the photomultiplier tube. In terms of these concepts the curves of Figure 6 can be interpreted. The relationship of count rate to discriminator level setting at 1200 volts approximates the expected distribution except at very low and at very high discriminator level settings. The sharp rise at low discriminator level settings seems to be due to photomultiplier tube noise and secondary flashes of light produced in the phosphor after the main burst. At very high discriminator-level settings the count rate drops more rapidly than is predicted on the basis of the simple theory.

For a typical tube the plateau progressively decreases in length at voltages lower than 1200 volts. Since the light distribution reaching the photomultiplier tube is not affected by the voltage applied to the photomultiplier assembly, changes in the curve of count rate versus discriminator-level setting must be due to changes in the sensitivity of the photomultiplier tube. As the voltage is increased, the tube sensitivity increases until a level is reached such that the amount of light from more than 95 pct of the alphas will produce enough photoelectrons to register a pulse. Unfortunately operation at this voltage is not feasible with low-activity samples since the background count rate increases rapidly with voltage.

If secondary pulses are neglected, the extrapolation of all the curves to zero pulse height yields almost the same value of the count rate. This indicates that the extrapolated count rate at zero discriminator-level setting is the actual rate at which alpha particles are impinging on the phosphor. This number is independent of the voltage applied to the photomultiplier tube.

A more positive proof of this suggestion was obtained by determining the close correspondence of the extrapolated count rate (2070 ± 50 cpm) of a primary AEC alpha standard with the certified count rate of 2043 ± 38 cpm.

The use of a thick source of monoenergetic alphas, a thick or thin source of a sample containing a series of alpha emitters, or an increase in distance from a sample to phosphor, all decrease length and increase the slope of the plateau.

Thus for normal rock or mineral samples, the low voltage required for an adequately low background yields a curve so steep that (a) the counting can only be done at low efficiencies, that is, 10 to 20 pct, and (b) the high-voltage supply must be controlled over long periods of time to within five volts. For example, from Figure 6 it can be seen that at satisfactory operating conditions of 950 volts and a discriminator-level setting of 30, the count rate varies by more than 100 pct per 100 volts or at least one per cent per volt. This prohibits the use of most commercial equipment.

Another difficult correction required is that of a variation in efficiency from point to point

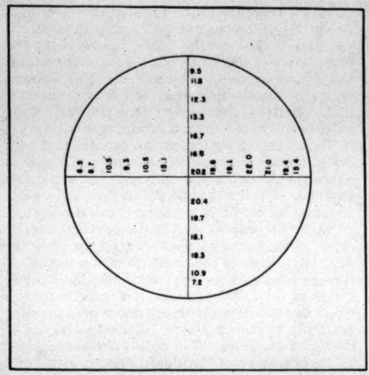

Fig. 8--Counting efficiency for a thick source square of uranium metal at various positions under a cone for normal operating conditions

below the cone. This efficiency generally decreases from the center to the edge but it is by no means a smooth variation. Figure 8 shows a typical calibration diagram for one cone.

Loss of alphas occurs at the edges of the source, but in this large sample size at close distance to the phosphor this is a minor correction of one to two per cent.

To calibrate such a source, it is necessary to count a known silicate source with the natural radioactive series in equilibrium. This can be accomplished successfully. Figure 9 shows the type of agreement between the experimental data and the theoretical curve for a sediment source of various thicknesses prepared by the Evans technique [EVANS and GOODMAN, 1944].

In summary, the use of a cone does increase the rate of accumulation of counts but it does not increase the count/background ratio, and it introduces several additional factors which require elaborate correction or control. It was found that the high-voltage fluctuations commonly produce errors of five to ten per cent. It is concluded that the system without the cone described in previous pages is the superior of the two.

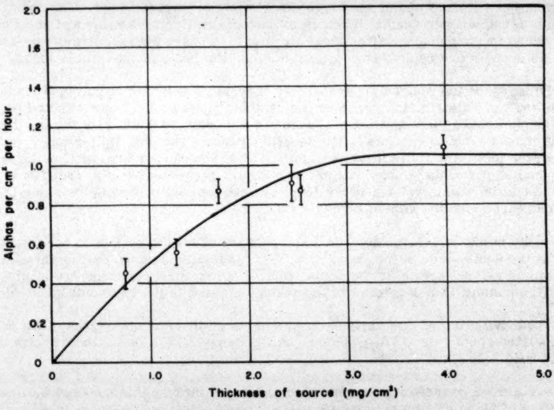

Fig. 9--Count rate as a function of thickness of a sample of ocean-bottom sediment

Background--Optimum operation calls for a maximum sample to background, count ratio. As previously indicated, this occurs at lower voltages and discriminator levels, that is, at the lower end of the plateau of Figure 5. Under typical operating conditions the background of the scintillation counters, operating without a cone, was 0.47 to 0.67 cpH/cm^2 of active tube area, while 0.35 cpH/cm^2 is attainable. The possible sources of the background count include electronic components, metal parts of housing, dunite used for background measurements, glass of photomultiplier tube, grease which is applied to the glass, the phosphor, and the radon contamination which may be present in the air.

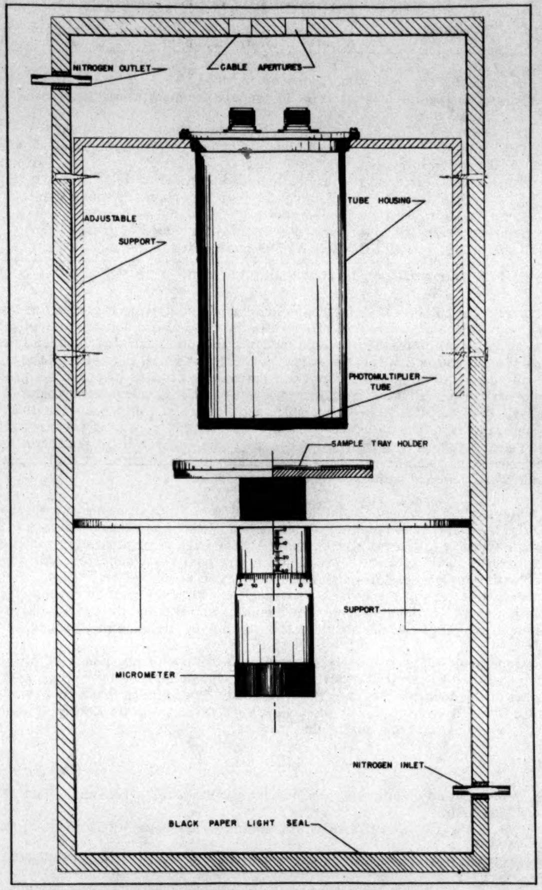
NITROGEN OUTLET
CABLE APERTURES
TUBE HOUSING
ADJUSTABLE
SUPPORT
PHOTOMULTIPLIER
TUBE
SAMPLE TRAY HOLDER
SUPPORT
MICROMETER
NITROGEN INLET
BLACK PAPER LIGHT SEAL

Fig. 10--Scintillation counter apparatus for the study of background and absorber effects

To study these factors a light-tight box, shown in Figure 10, was constructed such that any combination of any of the above listed possible sources of contamination might be eliminated. The micrometer and sample holder are removable. Their removal leaves the sensitive area of the tube surrounded by air at a greater distance from any solid than the range of the longest natural alpha. Thus the walls and housing may be eliminated as possible contaminants. Similarly the air may be replaced by radon-free nitrogen to determine the contribution of air contamination to the background. To get the activity of the photomultiplier tube glass, the grease, and the phosphor, another tube was mounted just below the counting tube. The results as tabulated in Table 3

Table 3--Determination of residual background

Number	Experimental arrangement	Count rate[a] cpH/cm^2
1	Entire apparatus except no phosphor on tube	< 0.01
2	Dunite, in regular housing (Fig. 1), freshly phosphored with Patterson type D ZnS-Ag	0.63 ± 0.043
3	Dunite, in box (Fig. 9) on sample holder, micrometer adjustment	0.53 ± 0.038
4	Box as in No. 3 without sample, sample holder, micrometer; phosphor is 10 cm from sides	0.58 ± 0.044
5	Same as No. 4, except that air is replaced by Rn free N_2	0.63 ± 0.045
6	Another P.M. tube No. 5819 placed beneath phosphor at same distance	0.71 ± 0.048
7	Same as No. 6, with lower tube greased	0.71 ± 0.054
8	Same as No. 7, with Patterson type D phosphor added to grease	1.29 ± 0.064
9	Same as No. 7, with RCA ZnS-Ag phosphor added to grease	1.24 ± 0.063

[a] Based on the active area of the photomultiplier tube.

indicate clearly that noise in the electronic components contribute negligibly to the background. Similarly the presence of radon-free nitrogen in the counter did not alter the count; thus air contamination is also negligible. The sample holder and counter housing apparently contain insufficient alpha contamination to affect the background. Some small part of the background about 0.10 cpH/cm^2 seems to be due to the glass of the photomultiplier tube, but the grease makes a negligible contribution. Finally, it is seen that most of the residual background is coming from the phosphor. Both the RCA ZnS-Ag phosphor and Patterson type D gave identical residual background counts, which in both cases double that of the normal operating background count as would be expected with the present apparatus. Thus the alpha activity of the phosphor seems to be the most likely source of about 85 pct of the total background of these scintillation counters. Similar backgrounds were obtained with or without the cathode follower.

Conclusions--It may be concluded that for low-level alpha counting of fairly low activity rocks and minerals, the scintillation counter as described, without cone, may be used to advantage in place of other types of low-level alpha counters. The ease of manipulation and calibration, good precision, stability, and excellent reproducibility are positive advantages. Absolute activities may be obtained with thin sources for which the efficiency is about 128 pct. This efficiency decreases as the source gets thicker. For "thick" sources the efficiency may be down as far as 60 pct for samples with high ud. If the distribution of the alpha activity and the particle size is known, reasonable approximations of the absolute activity may be made on thick sources.

Acknowledgments--The research reported here has been made possible through support and sponsorship extended by the U. S. Navy, Office of Naval Research, under N6-onr-271 Task Order 18. It is part of the doctoral thesis of H. D. Holland. The authors thank H. Levine and R. Graveson of the Health Physics Instrument Division, New York Office, Atomic Energy Commission, for giving valuable advice at the start and during this project.

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(Communicated manuscript received January 8, 1951, and, as revised, October 12, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

REPORT OF THE COMMITTEE ON DYNAMICS OF STREAMS, 1950-1951

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Ralph W. Powell

A glancing examination of the National Bureau of Standards annual review of "Hydraulic Research in the United States" impresses one with the wide variety of projects which fall under the general subject of stream dynamics. Undoubtedly there are other projects in progress that are not listed in this publication. Obviously the report of this Committee should not summarize the activities in this entire field. Instead, it appears desirable to present in some detail the results of investigations that are nearing completion, have been completed, or present valuable data or experimental techniques even though work is in a continuing phase. In addition to the restriction that each investigation be more or less completed, there is the space restriction necessitating the narrowing of the field even further. This report, therefore, is confined to a summary of a number of investigations on sediment transportation and related fields. Eventually the studies summarized herein perhaps will be published, at least in part. A preview, however, of this information in a single report is of interest.

Bed-load studies by the U. S. Geological Survey and the Bureau of Reclamation--Bed-load studies, based on data obtained by the Geological Survey at the turbulent sill on the Middle Loup River near Dunning, Nebraska, are being developed by the Sedimentation Section of the Hydrology Division, Bureau of Reclamation [PEMBERTON, 1950]. This turbulent sill or flume, which was developed from the model studies completed in the hydraulic laboratory of the COLORADO AGRICULTURAL EXPERIMENT STATION [1948; SERR, 1951; ALBERTSON, 1951], creates sufficient turbulence to force all of the sediment load carried by the river into suspension, thereby enabling the measurement of the total sediment load with the ordinary suspended sediment sampler. The total load measured at the turbulent sill was to serve two purposes: (1) provide the opportunity to apply various bed-load formulas where a check on the results could be made and (2) to determine the total sediment load of the Middle Loup River that can be expected at the proposed Milburn Diversion Dam.

Several cross sections of the river at which basic data could be obtained to define the hydraulic conditions of the river were selected above and below the flume. The hydraulic conditions include such data as discharge, mean depth or hydraulic radius, width, slope, velocity, bed-material size analyses, suspended-load size analysis, and water temperature. These data could then be applied to the various bed-load formulas. In the initial studies, the hydraulic conditions for the individual sections were used to compute the bed load carried by the river at that particular section. These computed bed-load values were added to the suspended load at the particular section to obtain the total load and were then compared with the total load measured at the flume. The results obtained in this comparison varied from good to poor. One by one the sections which were originally selected have been dropped when when it became evident that they did not represent a normal section until at present only one section, located 500 ft below the flume, is being used in the intensive investigations.

The data obtained at this one section are being applied to the bed-load function as presented by EINSTEIN [1950]. The values determined by use of this bed-load function represent the bed-material load that is transported by the river. All of the data available to date, which includes 18 days of sampling from August 1949 to July 1950 [VICE and SERR, 1950], have been applied to this bed-load function and the results from day to day were not consistent. When the hydraulic conditions for all of the available data were averaged and then applied to the bed-load function, the result was within reasonable limits of accuracy. The mean measured total load at the flume for the available data was 1155 tons per day whereas the mean computed total load was 1031 tons per day. This computed load is corrected by a correlation based on size analyses of the suspended and bed material. In other words, the measured suspended load contains some of the load that is computed as bed-material load and the direct addition of the two values is incorrect. When the data obtained during the spring break-up period are eliminated in the averaging of the data, the results are even more satisfactory with 860 tons per day measured total load and 850 tons per day computed total load. Based on these figures, the unmeasured sediment load carried by the Middle Loup River near Dunning is approximately 56 pct of the total load.

In the course of these latest studies, it has become evident that the hydraulic conditions at any one section do not necessarily define the hydraulic conditions of the river in that area. One particular section may be in aggradation or degradation at the time of measurement. It is necessary, therefore, to consider a reach of the river of sufficient length to have points of both aggradation and degradation; and yet when considered as a whole, the reach is substantially in equilibrium. The hydraulic conditions are needed at varying discharges for several sections along the river reach in order that when the conditions are averaged, they will adequately define the values to be used in the bed-load determinations. This desired end was obtained to some degree in the computations based on the averaging of all the data for the one section. Unfortunately, the Middle Loup River at Dunning is a sand-hill stream which has a small fluctuation in daily discharge with a minimum of 331 cfs and a maximum of 432 cfs for the days that sediment data were obtained. Though the bed-load function does give satisfactory answers at Dunning, it will be necessary to apply data to the function from streams that have a large discharge range before the validity of the use of the function can be fully determined.

The hydraulic principles involved in the determination of the bed load by formula have been applied by the Hydrology Division in developing a total-load sediment-rating curve for the Middle Loup River at Arcadia, Nebraska [SCHROEDER and PEMBERTON, 1950]. Studies are contemplated on the Niobrara River near Sparks and Cody, Nebraska, when the necessary basic data required by the Einstein bed-load function are obtained. It will be possible to check the computed total load for the Niobrara River near Cody against the total load measured at the Cody natural flume [SERR, 1950]. A much greater range in discharge is encountered in this river than was the case on the Middle Loup River, although the Niobrara is still a sand-hill stream. It is hoped that total-load sedmient-rating curves can be developed that will aid in determining the amount of sediment which will deposit in the various proposed reservoirs on the Niobrara River.

Study of fluvial morphology and stable channels--A quantitative study is being made by the Bureau of Reclamation of the forms of rivers, such as meandering and braided, to the principal variables influencing them, such as discharge, slope, and bed material. The purpose of the study is to enable predictions to be made of the changes in the stream action which will result from the changed conditions brought about by the development of the water resources of this country. A study is also being made to develop better methods of design of earth channels for carrying water. A tentative method for designing canals in coarse, non-cohesive material for carrying clear water is being developed. It is planned that this will be followed by the development of analysis for canals carrying sediment.

Public Roads research on sediment--Basic research on the mechanics of scour around bridge piers and abutments is proceeding at the Iowa Institute of Hydraulic Research. The project is jointly sponsored by the Iowa State Highway Commission and the U. S. Bureau of Public Roads. Initial tests were directed toward discovering the effect of pier and abutment geometry upon the relative depth and pattern of scour, with velocity and depth of flow in the five-foot wide flume being held constant. A progress report on this phase of the investigation has been published [LAURSEN, 1951].

The second phase of the project is an investigation of the effect of general bed-load movement upon localized scour around bridge piers. These tests are being conducted in a flume equipped with a sand-feeding mechanism. A series of closely spaced electrodes on the nose of the pier enables a measurement of depth of scour at any time during actual flow by noting the marked change in electrical capacity between successive pairs of electrodes at the level of the sand. In this manner it has been found possible to record the passage of a sand wave. Equilibrium of bed-load transport is obtained for each combination of velocity and depth of flow.

Basic research on transportation of sediment in a closed conduit, and on devices for entrapping sediment carried in a closed conduit, is being conducted at the Iowa Institute of Hydraulic Research under sponsorship of the Bureau of Public Roads. Tests have been made on transportation of three sizes of sediment in a two-inch and a six-inch lucite pipe. The problem is complex but progress is being made toward discovering significant parameters. The loss of head appears to vary as the square root of the sediment-concentration ratio, other variables being fixed. Surprisingly, there is strong indication that the scope of the pressure gradient in a pipe-flowing, full-carrying water and sand at a given rate is the same for the small pipe as for the large pipe. This is due to the latter filling up until the unobstructed area is about the same as the unobstructed area of the small pipe. The loss of head in a pipe-carrying sediment can be many times the loss of head in the same pipe carrying the same water discharge without sediment. The practical significance of these findings, if confirmed by continuing tests, is obvious, since at the present the sole criterion for design of storm drain pipes likely to carry sediment is an arbitrary minimum velocity.

The other phase of the project, trapping sediment, follows logically, since provision must be made to trap sediment before it enters a section of pipe where sufficient head is not available for transport to the entire load. Tests on a two-dimensional rectangular trap in a rectangular conduit are in progress. The rate of entrapment appears to be constant for a given sand and water flow until the trap is almost full, and is higher for low values of V/w where V is the mean velocity in the pipe and w is the mean fall velocity of the sand being tested. Only one size of sediment has been used to date.

Echo sounding of Missouri River channel--Supersonic echo sounding has been done periodically, about five times yearly, on about 70 cross-section ranges on Missouri River in the Kansas City District of the Corps of Engineers. The ranges are irregularly distributed in three groups from the vicinity of Kansas City to about 200 river miles downstream. Each group extends up and downstream from a channel cut-off, the uppermost group extending upstream from the Liberty Bend cut-off through the Kansas City section where there has been extensive levee and flood-wall construction within the past few years. Similar periodic soundings are to be initiated soon on about 20 additional ranges, which will extend the uppermost group downstream past another cut-off. The purpose is to obtain data for (1) detailed studies of hydraulic effects of the cut-offs, and (2) studies of bed-sediment transportation.

One by-product has been a record of temporary scouring to a depth of about 65 ft below the flood plain surface near Kansas City at a stage slightly below bankfull. Generally the flood plain alluvial deposits are about 80 ft thick in this part of the valley, although some parts of the underlying bedrock floor are known to have been eroded to nearly twice this depth at some past time. Test holes have shown that the alluvial deposits generally become coarser downward, but to a depth of about 80 ft they are not known to be coarser than some of the river bed sediments. From this it appears that at least the upper 80 ft of alluvium is subject to comparatively frequent scour and replacement, within the zone of lateral channel migration of this part of the Missouri.

Construction has begun within the past few years on a program of levees to protect most of the Missouri River flood plain, and thus for the first time restrict overbank flows to a comparatively narrow floodway. Sedimentation on the berms between the levees and stream channel, which is being extensively altered by open-channel training works for navigation, may have considerable effect on future hydraulic conditions. A group of nine cross-section ranges has been established as a basis for future measurements in a two-mile reach where the river has first been confined between levees on each side, between St. Joseph, Missouri, and Atchison, Kansas; and another group of nine ranges in a two-mile reach below Kansas City where overflows are not now restricted by levees, and a period of comparative records is expected to be obtained before levees are built some years hence. It is anticipated that this program will be expanded by establishment of some additional groups of ranges within the next few years.

Retrogression study on Smoky Hill River below Kanopolis Dam--In 1946 the Kansas City District Corps of Engineers established a series of 17 cross-sectional ranges in a reach of about 68 river miles below Kanopolis Dam on Smoky Hill River in central Kansas as a basis for checking stream-bed retrogression. Partial flood detention began by diversion of the river through the outlet works in July 1946, after which flows in excess of about 4000 cfs were temporarily impounded; storage for filling the conservation pool began in February 1948, and subsequently the reservoir has been operated for flood control. Bankfull channel capacity below the dam is about 8000 cfs, but flows were limited to about 4000 cfs during the 1946-1948 period of partial detention, and since February 1948 the maximum discharge has been slightly over 3000 cfs for a period of six days in September 1950, except for a discharge slightly above 4000 cfs for a six-hour period in December 1950.

Resurveys of the ranges in January 1951 show retrogression of about two feet at the uppermost range, about one-half mile below the outlet, and about one foot at the next range, about 1 1/2 miles farther downstream; no significant changes were found farther downstream.

Stream-bed samples were obtained on about half the ranges to depths of about three feet in July-August 1949. The bed materials are mostly medium and coarse sands, gravelly in some places, generally similar to older stream deposits now forming the adjacent banks as found by test drilling. There did not appear to be any appreciable coarsening of the bed material at the uppermost range, although retrogression of about one foot appeared to have occurred at that place at the time of sampling.

Bed-sediment segregation in a degrading bed--Flume studies made in the Fluid Mechanics Laboratory of the University of California [HARRISON, 1950] have yielded certain basic data on the details of sediment motion on a degrading bed such as involved in the retrogression of stream beds below dams. It appears from these studies that in an equilibrium bed in which non-moving particles are present the non-moving particles segregate from the layer of moving bed material and form a coarse sublayer. This sublayer is well defined, smooth in profile, and approximately the thickness of the average non-moving particle. The sublayer appears to interfere with the equilibrium transport of the moving grain sizes; it is believed that it tends to equalize the relative rate of transport for all moving sizes. The EINSTEIN [1950] relationship for the rate of transport predicts very well the limiting grain size ($\psi_* = 27$) for transport. When a bed containing non-moving particles is allowed to degrade, the non-moving particles accumulate at the bed surface reducing the area of the bed from which particles are scoured. The reduction in the rate of transport is found to be proportional to the accumulation of non-moving particles. Non-moving particles in a partial pavement, or a pavement, arrange themselves in a characteristic "shingled" formation. If this characteristic shingled arrangement is disturbed, the non-moving particles will move until they are rearranged into a new shingled pattern. Non-moving or extremely slow-moving particles in a degrading bed move downstream as they move downward. The resultant direction of this motion can be expressed as a slope with the bed surface; the order of magnitude of that slope for the non-moving particles was found to be 0.01. The motion of the non-moving particles may prevent the formation of a complete pavement in a short flume if this concentration in the original bed is small, but this effect has only local significance in a river. The accumulation of non-moving particles on the surface causes an increase in its effective roughness, and, in experiment, may cause the limiting grain size to be increased if the flow velocity is held constant. It may be concluded that in a river this increase in roughness will not increase appreciably the limiting grain size for non-movement, because the river has a tendency to hold its energy gradient constant during the process of degradation. It has been found that with a layer of non-moving particles the thickness of one particle is effective in preventing scour. It is believed that a complete layer of non-moving particles is not necessary in all cases, for in a layer of non-moving particles with values of ψ_* ranging from 27 to 40 [EINSTEIN, 1950], only 50 pct completeness appears to be required; while a complete layer appears to be necessary if all particles are on the verge of movement ($\psi_* \sim 27$).

Investigation of the maximum equilibrium rate of bed-load movement--This investigation at the University of California [CHIEN, 1951] was an outgrowth of the study by HARRISON [1950]. The purpose of this study was to find experimentally whether there exists a maximum equilibrium rate of bed-load movement, and how it differs quantitatively from the minimum equilibrium rate. The minimum equilibrium rate is obtained while the bed is being scoured. Three possible methods were tried in this study to explore the maximum equilibrium rate.

It was found that the answer to the questions above depends on the choice of what is considered as the bed material. If the active surface layer of the bed is selected as the bed material, then the rate of transport is unique for all the grain sizes. But this definition leads to difficulties in actual sampling of bed material in the field. On the other hand, if the bed material is defined as that of a thick layer which can easily be sampled in the field, and the composition of which remains more or less constant for a reasonably long period, then (1) for the particles which constitute the bulk of the bed there remains one and only one rate of transport, but (2) for the fine particles which constitute only a few per cent of the bed there does exist a range of equilibrium rates, depending on the availability for transport of the sediment from the upstream watershed.

The Einstein bed-load function is found to be applicable to the wide range of particle size from one to 0.005 mm in the experiment if only the correction factor ξ for mixtures is somewhat changed from that given by EINSTEIN [1950].

Basically there is no difference between the so-called "wash load" and "bed-material load." It is due to their relative abundance in the bed that bed-material load develops a relationship between rate of transport and flow. The rate of transport of the wash load and its availability in the bed depend only on the rate with which these particles become available in the watershed. The bed composition generally follows the logarithmic-probability distribution as found in this investigation. From the findings of this study, an estimate of either the bed composition of an aggrading river or of its rate of sediment motion at the time at which the bed is accumulated can be made if the other is known.

Linearity of friction in open channel flow--In the past, considerable research has been conducted to determine the effect of certain types of roughness elements on a fluid flow. Some of these studies have been limited to the case in which there was a single type of roughness on the flow boundary; Nikuradse's work serves as an example. Other studies, however, introduced in effect two types of roughness elements on the same flow boundary; an example of the latter might be the case in which small sills are attached to the channel periphery. Invariably the concern of the latter type of work has been restricted to expressing the overall or composite roughness in terms of established friction parameters. In many of these investigations the cooperating roughness types were of such a nature that the role which each played individually might have been considered. Recently a study was made on the fluid resistance of composite roughness in connection with a sediment-load study of the Salinas River in California [EINSTEIN and BANKS, 1950]. To extend this scope of the problem a subsequent study was made [BANKS, 1951] to determine how the composite roughness can be described, in the quantitative sense, utilizing the concept of resolution of resistances. This presupposes that the friction of the roughness types acting individually can be described by any of the well-known friction parameters. On the basis of experimental investigation, it has been found that the resistance which the composite roughness exerts on a flow can be expressed as the sum of the resistances which each of the roughness types exerts individually. By resistance is meant, of course, the actual force per unit of boundary area. This "linearity of friction" is valid over a wide range of the variables concerned. No attempt was made to extend the range of such linearity into regions where mutual interference among the roughness elements is introduced.

Such linearity among the friction elements offers a fundamental approach to the practical aspects of certain river and canal problems. This method, for example, can be employed readily in problems concerned with grade stabilization in rivers and canals with movable beds.

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(Manuscript received August 27, 1951; presented by title at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

REPORT OF THE COMMITTEE ON EVAPORATION AND TRANSPIRATION, 1950-1951

A. R. Croft, Chairman

1951	1952	1953
E. G. Dunford	Henry W. Anderson	J. M. Aikman
A. H. Hendrickson	Robert W. Gerdel	Harry F. Blaney
Adolph F. Meyer	C. W. Thornthwaite	Carl Rohwer

The report of the Committee this year consists of statements by members covering their work in the field of evaporation and transpiration. In two instances research collaborators have been listed at the suggestion of Committee members.

H. W. Anderson and T. G. Storey (California Forest and Range Experiment Station, Berkeley, California)--How evaporation and transpiration are affected by changes in forest cover and other land-use conditions has immediate practical importance in how water yields from watersheds are affected. In some preliminary studies of the problem in the Willamette basin of Oregon, investigations were made to see whether annual stream flow as an index of water yield has varied with changes in land use throughout the period of measured stream flow. The double-mass method suggested by LINSLEY, KOHLER, and PAULHUS [1949, pp. 435 and 219] was employed.

The steps were: (1) Testing of precipitation at a key station, Albany, Oregon, against the mean of five stations to see if any unique time changes had occurred at the key stations--none had. (2) Developing relations between the annual stream flow from each watershed and meteorological variables measured at the key station. The annual stream flow as a function of the annual precipitation, the mean annual temperature, and the antecedent annual precipitation was obtained by regression analysis. (3) Plotting, double-mass, the measured annual stream flow against the flow calculated from the meteorological variables. (4) Inspecting the records to see that no changes in the discharges claimed for a given stage and channel and no changes in the general climate had occurred. Then (5) inferring that if changes in stream flow had occurred because of changes in the land use, these would be detectable and measurable by changes in the slope of double-mass curve.

The results of tests for two watersheds for which cover changes had been drastic (the Mollala River near Canby, Oregon, and the Mohawk River near Springfield, Oregon) showed no significant change in annual stream flow.

It is possible, of course, that more sensitive methods or better application of the same method would have indicated some differences. However, the method seems fairly sensitive since, in other studies, use of the method shows marked increases in the peak discharges over the same periods of time for these watersheds. If further tests on more watersheds in this area show agreement with these results, then no large changes in annual stream flow due to current land-use practices are to be expected.

In this area forest cutting is followed generally by the re-establishment of some kind of vegetation on the ground within a year or two. Tree seedings or dense stands of bracken fern and vine maple soon cover the soil. The annual precipitation is great so that rarely is interception loss a large fraction of the precipitation. Snow accumulation and melt play a complex role, with the trees shading the snow and the snow tending to keep the soil and air temperatures low for several weeks in the spring. These factors acting together may result in quite different effects of forest cutting on annual yields than experience elsewhere or over-simple hypotheses might indicate. Double-mass analysis seems to be one promising way of studying the problem.

Harry F. Blaney (Division of Irrigation and Water Conservation, Soil Conservation Service, Los Angeles, California)--Measurements of evaporation and evapotranspiration have been correlated with climatological and irrigation data and a formula developed for estimating water consumption for areas where no evapotranspiration measurements are available. Briefly, the procedure is to correlate existing evapotranspiration data with monthly temperature, monthly percentages of yearly daytime hours, and growing period. The coefficients so developed for different kinds of vegetation are used to transpose the evapotranspiration data for a given area to other areas for which only climatological data are available. Disregarding the unmeasured factors, consumptive use U varies with the temperature and the daytime hours, and with available moisture (precipitation, irrigation, and ground water). By multiplying the mean monthly temperature t by the monthly per cent of daytime hours of the year p, there is obtained a monthly consumptive-use factor F. Then it is assumed that the consumptive use varies directly as this factor when an ample water supply is available. Expressed mathematically, $U = KF$ = consumptive use for any period, where F = sum of f, or $(t \times p)/100$, and K = empirical coefficient for the period = U/F.

The consumptive-use coefficients K for the more important irrigated crops grown under normal conditions in the West, and for natural vegetation, are shown in Table 1. These coefficients were developed from actual measurements of consumptive use in tank and soil-moisture field studies and inflow-outflow measurements made throughout the West over a period of years by the Division of Irrigation and Water Conservation and other agencies. Monthly per cent of annual daytime hours p may be computed from sunshine tables of the U. S. Weather Bureau.

Table 1--Tentative normal consumptive-use coefficients for the more important irrigated crops and natural vegetation of the West

Crop	Length of growing season or period	Consumptive-use coefficient, K
Irrigated alfalfa	Frost-free	0.85
Irrigated beans	Three months	0.65
Irrigated corn	Four months	0.75
Irrigated cotton	Seven months	0.62
Irrigated citrus orchard	Seven months	0.55
Irrigated deciduous orchard	Frost-free	0.65
Irrigated pasture, grass	Frost-free	0.75
Irrigated small grains	Three months	0.75
Dense natural vegetation[a]	Frost-free	1.20
Medium natural vegetation[a]	Frost-free	1.00
Light natural vegetation[a]	Frost-free	0.80

[a] Ample moisture available from ground water or water table.

E. G. Dunford (Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colorado)--In the past year the Rocky Mountain Forest and Range Experiment Station has undertaken a study in three small watersheds to follow the trend of soil moisture under a variety of conditions through the summer. The watersheds are part of a series at Manitou Experimental Forest on which we are investigating the relation of vegetative changes to runoff and erosion. Two of the three areas used in this study have had all the ponderosa-pine cover removed and the third remains in its original condition.

Samples were taken with a soil tube at weekly intervals during July, August, and early September. Ten randomly chosen sample points were selected within each watershed every week. Depth of measurement was 18 inches from which two moisture subsamples of 0-6 inches and 6-18 inches were obtained. Added information for each sampling point also included: (1) compass direction of exposure; (2) slope of ground in per cent; (3) number of live plants in a square foot centered over the sample point; and (4) estimated per cent of litter cover in the same square-foot area.

The results showed a significant difference in moisture content between watersheds for the entire nine-week period. In addition, they pointed to a very strong relationship between soil moisture and two of the above factors; direction of exposure and litter density on the ground surface. The number of live plants per square foot was only weakly associated with soil moisture in the upper zone and showed no influence at all on moisture in the lower zone. Slope in per cent was not associated with moisture content of soil in either zone.

Litter density showed a positive relationship with soil moisture, that is, as litter density increased more moisture was retained in the soil mantle, indicating the protection offered by litter against drying. In the lower zone this protective influence was less pronounced.

Live plants had the opposite effect as indicated by a negative regression coefficient. The transpiration draft of live plants tends to dry the soil, more than offsetting whatever effect the foliage may have in reducing evaporation.

R. W. Gerdel (Snow, Ice, and Permafrost Research Establishment, Corps of Engineers, 1215 Washington Avenue, Wilmette, Illinois)--The attention of all who are interested in evaporation should be directed to the Selected Annotated Bibliography on Evaporation which comprises part 2 of no. 11, v. 1, Meteorological Abstracts and Bibliography, November 1950. The 210 annotated references covering the period between 1910 and 1950 constitute an exceptionally fine piece of work. When combined with the earlier annotated bibliography on evaporation for the period from 1670 to 1909, presented in the editions of the Monthly Weather Review for 1908 and 1909, the researcher in this field has very exceptional reference material at his finger-tips.

The Snow, Ice, and Permafrost Research Establishment has assumed responsibility for the support of some of the basic research on snow at the Central Sierra and Upper Columbia Snow Laboratories. The Establishment has made a special effort to inaugurate a very intensive study on the influence of micrometeorological phenomena on snow metamorphism. Four meteorological masts have been instrumented so as to provide a wide range of data. It is hoped that when the instrumentation is completed with a set of Foxboro dew-cells, the humidity-gradient data so obtained will be adaptable to the analysis of evaporative ablation from, and metamorphic changes in, the snow pack. This type of basic research is of particular value to the forecasting of maneuverability and trafficability over snow.

John R. Mather (Laboratory of Climatology, The Johns Hopkins University, Seabrook, New Jersey)--During the period, the concept of potential evapotranspiration discussed by THORNTHWAITE [1944, 1948] has been further investigated by means of evapotranspiration measurements at Seabrook and at several other experiment stations. Recent publications of interest on the subject of potential evapotranspiration are listed in the References. (1) MATHER [1950] summarizes much of the basic potential evapotranspiration information to the present, and discusses the installation of evapotranspirometers, the evaluation of potential evapotranspiration, and some of the limitations of these measurements and of the instrument itself. (2) SANDERSON [1950a] discusses the comparison between the measured and computed values of potential evapotranspiration at Norman Wells, N.W.T. (64.5°N) and shows that, although the empirical evapotranspiration formula was devised for the mid-latitudes, its extrapolation to the Arctic Circle is permissible. (3) SANDERSON [1950b] sums up potential evapotranspiration measurements over the past three years at various Canadian stations and discusses the relation between the empirically computed and the measured values of potential evapotranspiration.

In addition to these publications, an investigation of potential evapotranspiration in tropical regions was carried out. The results [THORNTHWAITE, 1951] were presented at the Caribbean Soils Conference, San Juan, Puerto Rico, April 1950. The investigation provides an insight into the role of moisture in tropical climates. The use of the empirical evapotranspiration formula in tropical regions appears to be justified, for the resulting climatic pattern agrees well with the limited available observations.

Investigations of potential evapotranspiration at Seabrook during the period were designed to test the evapotranspiration formula and the evapotranspirometer described by THORNTHWAITE [1946]. To study the effect of different amounts of soil moisture on water use, the watering regimes of the six evapotranspirometers in operation at Seabrook were varied. Three tanks were left with a water table permanently maintained 50 cm below the soil surface, two more had a water table only 35 cm below the surface, while the sixth had no water table at all but was irrigated from above enough each day to insure a small overflow from the tank. In spite of these different watering practices, the total yearly values of water use differed at most by only about ten per cent from one tank to another. This would indicate that water use does not vary greatly within a considerable range of soil-moisture values as long as the soil moisture is maintained above the field capacity. The fresh-cut weight of the grass cover from the various evapotranspirometers was determined during the growing season. No consistent variation in the weight of the vegetation removed from the different tanks was found. The different watering practices seem to be equally efficient for crop growth.

Other tests of the relation between the computed evapotranspiration amounts and the measured values of evapotranspiration [SANDERSON, 1950ab] have shown remarkably good agreement between the two. It is significant that in these tests only the summer and some of the spring and fall months were used. From year-round measurements of potential evapotranspiration at Seabrook, it has been found that the empirical formula underestimates wintertime water use while overestimating summertime evapotranspiration. These discrepancies do not appear to be serious.

Considerable interest has been shown in the concept of potential evapotranspiration during the period and it has been used to aid in the solution of agricultural, climatic, ecologic, and soil tractionability problems. Plans have been formulated for the establishment of evapotranspiration stations in Canada, Mexico, Argentina, Israel, New Zealand, the British West Indies, and Pakistan.

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(Manuscript received July 30, 1951; presented by title at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union   Volume 33, Number 1   February 1952

RELAXATION TECHNIQUE FOR THREE-DIMENSIONAL FLOW NET

Chia-Huan Chien

Abstract--All of the existing methods for solving flow net are applicable only to two-dimensional problems. Moreover, they are either limited for further refinement (such as electric analogy, model test in glass flume, and trial-and-error method) or impossible for complicated boundary conditions (such as mathematical solution). As a consequence, a relatively new technique, relaxation method, developed by SOUTHWELL [1945], has been employed for hydraulic design during the past few years. It is still confined to the problems with two dimensions. The relaxation procedure for two-dimensional flow net was described by LUTHIN and GASKELL [1950] and by the writer [CHIEN, 1950].

It is the purpose of this paper to present a generalized relaxation formula for three-dimensional flow and to interpret its basic principles of relaxation.

Derivation of fundamental formulas--The Laplace equation applies to steady and irrotational flow. For three-dimensional flow problems,

$$\nabla^2 \phi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where ϕ = velocity potential and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$.

Let us draw an arbitrary flow net so that it subdivides the three-dimensional flow field into a number of small cubes and partial cubes if the boundary is irregular. For every intersection corner of the cubes, a possible value of velocity potential ϕ is assumed. If the side length a of every cube in the network is small as compared with the dimensions of the hydraulic structure under consideration, it is accurate enough to assume that the values of ϕ vary linearly along the length a.

Referring to Figure 1,

$$\partial\phi/\partial x\Big|_{x_0 + a/2} = (\phi_1 - \phi_0)/a \qquad \partial\phi/\partial x\Big|_{x_0 - a/2} = (\phi_0 - \phi_3)/a$$

The rate of change of $\partial\phi/\partial x$ is

$$\partial^2\phi/\partial x^2\Big|_{x_0} = [\partial\phi/\partial x\Big|_{x_0 + a/2} - \partial\phi/\partial x\Big|_{x_0 - a/2}]/a = (\phi_1 + \phi_3 - 2\,\phi_0)/a^2$$

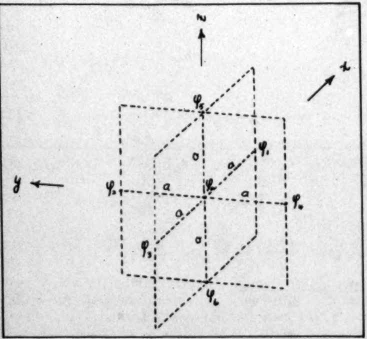

Fig. 1--Three-dimensional flow field

Similarly,

$$\partial^2\phi/\partial y^2\Big|_{y_0} = (\phi_2 + \phi_4 - 2\,\phi_0)/a^2$$

$$\partial^2\phi/\partial z^2\Big|_{z_0} = (\phi_5 + \phi_6 - 2\,\phi_0)/a^2$$

Hence,

$$a^2\nabla^2\phi\Big|_{x_0 y_0 z_0} = (\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 - 6\,\phi_0) \;. \quad (2)$$

Equation (2) is the fundamental formula of relaxation for any intersection corner not near the boundary. The derivation of the most general formula for boundary conditions is shown in Figure 2.

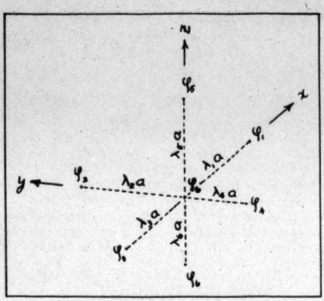
z
φ_5
$\lambda_5 a$
$\lambda_1 a$
φ_1
x
y
φ_2
$\lambda_2 a$
φ_0
$\lambda_4 a$
φ_4
$\lambda_3 a$
$\lambda_6 a$
φ_3
φ_6

Fig. 2--Three-dimensional boundary conditions for Equation (3)

Referring to Figure 2,

$$\partial\phi/\partial x\Big|_{x_o+\lambda_1 a/2} = (\phi_1 - \phi_o)/\lambda_1 a$$

$$\partial\phi/\partial x\Big|_{x_o-\lambda_3 a/2} = (\phi_o - \phi_3)/\lambda_3 a$$

The rate of change of $\partial\phi/\partial x$ is

$$\partial^2\phi/\partial x^2\Big|_{x_o} =$$

$$2[\partial\phi/\partial x\Big|_{x_o+\lambda_1 a/2} - \partial\phi/\partial x\Big|_{x_o-\lambda_3 a/2}]/(\lambda_3 a + \lambda_1 a)$$

$$= 2[\phi_1/\lambda_1 + \phi_3/\lambda_3 - (\phi_o/\lambda_1\lambda_3)(\lambda_1 + \lambda_3)]/(\lambda_1 + \lambda_3)a^2$$

Similarly,

$$\partial^2\phi/\partial y^2\Big|_{y_o} = 2[\phi_2/\lambda_2 + \phi_4/\lambda_4 - (\phi_o/\lambda_2\lambda_4)(\lambda_2 + \lambda_4)]/(\lambda_2 + \lambda_4)a^2$$

$$\partial^2\phi/\partial z^2\Big|_{z_o} = 2[\phi_5/\lambda_5 + \phi_6/\lambda_6 - (\phi_o/\lambda_5\lambda_6)(\lambda_5 + \lambda_6)]/(\lambda_5 + \lambda_6)a^2$$

Hence,

$$a^2\nabla^2\phi/2 = (\phi_1/\lambda_1 + \phi_3/\lambda_3)/(\lambda_1 + \lambda_3) + (\phi_2/\lambda_2 + \phi_4/\lambda_4)/(\lambda_2 + \lambda_4)$$

$$+ (\phi_5/\lambda_5 + \phi_6/\lambda_6)/(\lambda_5 + \lambda_6) - \phi_o(1/\lambda_1\lambda_3 + 1/\lambda_2\lambda_4 + 1/\lambda_5\lambda_6) \;\ldots\ldots \quad (3)$$

All values of λ are measured from network diagrams.

For regular cubes, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 1$, the most general equation (3) reduces to (2). For irregular boundary at one side only, say $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 1$, then

$$a^2\nabla^2\phi\Big|_{x_o y_o z_o} = 2(\phi_1/\lambda_1 + \phi_3)/(\lambda_1 + 1) + (\phi_2 + \phi_4 + \phi_5 + \phi_6) - 2\phi_o(1/\lambda_1 + 2) \;\ldots\ldots \quad (4)$$

For two-dimensional flow, $\partial\phi/\partial z = 0$, namely, no variation of ϕ along z-axis direction, so the relaxation formulas become much simpler

$$\nabla^2\phi = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (1a)$$

where

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$$

$$a^2\nabla^2\phi\Big|_{x_o y_o} = (\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o) \;\ldots\ldots\ldots\ldots \quad (2a)$$

$$a^2\nabla^2\phi/2\Big|_{x_o y_o} = (\phi_1/\lambda_1 + \phi_3/\lambda_3)/(\lambda_1 + \lambda_3) + (\phi_2/\lambda_2 + \phi_4/\lambda_4)/(\lambda_2 + \lambda_4) - \phi_o(1/\lambda_1\lambda_3 + 1/\lambda_2\lambda_4) \;\ldots \quad (3a)$$

$$a^2\nabla^2\phi\Big|_{x_o y_o} = 2(\phi_1/\lambda_1 + \phi_3)/(\lambda_1 + 1)$$

$$+ (\phi_2 + \phi_4) - 2\phi_o(1/\lambda_1 + 1) \;\ldots\ldots\ldots \quad (4a)$$

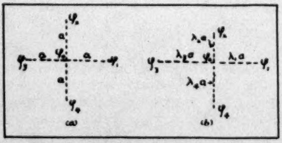
φ_2
a
φ_3
a
φ_0
a
φ_1
a
φ_4
(a)
$\lambda_2 a$
φ_2
φ_3
$\lambda_3 a$
φ_0
$\lambda_1 a$
φ_1
$\lambda_4 a$
φ_4
(b)

Fig. 3--Boundary conditions for two-dimensional flow

Relaxation procedure--By comparing (1) and (2), we know there is a difference of residual amount $a^2\nabla^2\phi$. In other words, (2) does not satisfy the Laplace equation until the residue $a^2\nabla^2\phi = 0$ or $\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = 6\phi_o$.

The theory of relaxation is based upon an idea that (2) is applied successively to every corner of each cube. If ϕ_0 is increased by one unit, the residue $a^2\nabla^2\phi$ is decreased by six units. Hence, if ϕ_0 is reduced by one-sixth of the residue, the Laplace equation will be satisfied, or $\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = 6\,\phi_0$ (see Fig. 4). However, relaxing ϕ_0 will affect the residues of the six nearest adjacent points and their subsequent relaxation will upset the balance at $(x_0\ y_0\ z_0)$. If all corners of the cubes are relaxed repeatedly by the order of the magnitude of residues, all the residues will decrease to negligible amounts. Then smaller cubes are drawn (usually one-eighth of original cubes) and relaxation is repeated if a high degree of accuracy is required.

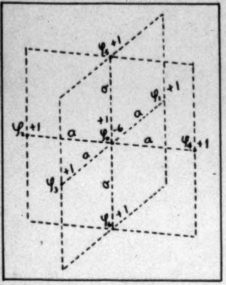

Fig. 4--Conditions to satisfy the Laplace equation

Equation (4) may be applied in case of irregular boundary at one side only. But the distribution factor as shown in Figure 5 is not as simple as regular cubes. The quantities $(1/\lambda_1 + 2)/2$, $2/(\lambda_1 + 1)\,\lambda_1$, and $2/(\lambda_1 + 1)$ should be computed before relaxation begins.

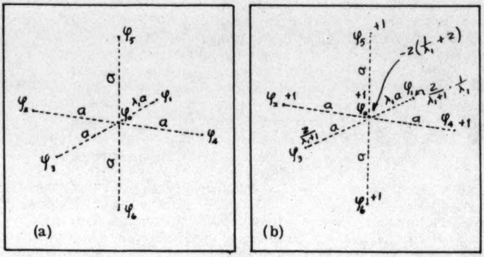

Fig. 5--Complexity of distribution factor in non-regular cubes

For two-dimensional problems, the computation was shown by LUTHIN and GASKELL [1950]. For three-dimensional problems, the writer introduces three sets of section planes: the first perpendicular to the flow, the second and third planes parallel to the flow, respectively horizontal and vertical. Simultaneous computations are necessary on three planes for relaxation of one corner.

Conclusions--After the residues of all intersection corners are relaxed to negligible amounts, a flow net is drawn by interpolation. For the three-dimensional flow net consisting of three sets of curves in space, the net obtained is only the projection of any two of the curves on the nearest parallel plane. Fortunately, the result required is usually the flow net on boundaries.

The advantage of the new method is the possibility of refining the results to the accuracy desired in three-dimensional flow. Depending on the complexity of the problem, the time required may be from a few days to several months. However, the time and expense are less than those for hydraulic model test.

Acknowledgment--The writer desires to express his acknowledgment to C. T. Lee, Professor of Hydraulic Engineering, University of Chekiang, for carefully reading the manuscript.

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(Communicated manuscript received February 23, 1951; open for formal discussion until July 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 1 February 1952

DISCUSSION OF

"ON THE ORIGIN OF THE CAROLINA BAYS"

by WILLIAM SCHRIEVER

[Trans., v. 32, pp. 87-95, 1951]

John S. Rinehart (U. S. Naval Ordnance Test Station, Inyokern, China Lake, California)--Prediction of the shape of the crater that will be excavated by an impacting meteorite is a more complex problem than Schriever has indicated. The well-established meteorite craters are too few in number to provide a sound basis for generalization, particularly in view of the fact that there are scant data regarding the masses, velocities, or angles of impact of the meteorites that formed the craters.

During the last few years considerable data that bear on meteorite impacts have been acquired through studies of the impact of high-velocity missiles [RINEHART, 1950]. More recently, the craters produced in rocklike material by 8,000 to 15,000 ft/sec missiles have been studied [RINEHART and WHITE, 1952]. These studies have demonstrated conclusively that the diameter of the meteorite crater can be many times the diameter of the meteorite. A half-inch diameter steel pellet that strikes a plaster of Paris target at 8,000 ft/sec produces a crater whose diameter is roughly four inches or eight times that of the pellet. The area of the mouth of the crater seems to depend primarily upon the kinetic energy of the missile. A 15,000 ft/sec, 0.6-gm aluminum pellet and a 8,000 ft/sec, 2-gm steel pellet, which are equivalent energywise, make about the same size openings.

Perhaps of more significance as regards the present discussion are the observations that have been made on the shapes of the mouths of the craters. In these studies the angle of impact was varied from normal incidence to 75° from the normal. For angles between normal and 45° from normal the opening is nearly circular. Between 45° and 60° from normal the opening has roughly the shape of an arrowhead with the tip of the head directed toward the point from which the missile was fired. Beyond 60° the projectile usually ricochets and produces a trench that is parallel to the direction of fire. The trench closely approximates a true ellipse. The ratio of major axis to minor axis is about 2 to 1. It appears from the foregoing observations that, insofar as data obtained from small-scale testing can be extrapolated to the impact of large meteorites, there appears to be no single shape that can be used to identify unambiguously a meteorite impact. Both Johnson's ovoids and Schriever's ellipses might have been caused by meteorites.

The contour of the Barringer Meteorite Crater in Arizona is, contrary to Schriever's statement, approximately square although the corners are somewhat rounded [ZIMMERMAN, 1948]. The shape of the Odessa Crater is roughly that of an arrowhead [MONNIG and BROWN, 1935].

The indisputable evidence for the meteoritic origin of a crater is the discovery of meteorite fragments in or near the crater. The fact that none has been found near the Carolina Bays is strong evidence against their meteoritic origin. NINNINGER [1951] has recently successfully extracted large quantities of minute spherules of meteoritic material from the soil around the Arizona Crater. Perhaps a similar sampling of soil around the Carolina Bays area might reveal the presence of meteoritic material.

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William Schriever (University of Oklahoma, Norman, Okla., Author's closure)--Rinehart's interesting and pertinent discussion is very much appreciated. Extrapolating the data obtained with high-velocity missiles to the impacts of large meteorites cannot lead to conclusions that are not subject to considerable doubt. Regardless of the origin of the Carolina Bays, aerial photographs of Johnson's "ovoid" bays indicate clearly that they were elliptical before the sand filling took place.

I have before me an excellent vertical aerial photograph of Meteor Crater, Arizona, which I obtained from the Fairchild Aerial Surveys. I believe "round" would be a better description of the shape of the rim of the crater at the present time than is the term "square." Actually the crater is slightly elliptical. Since the walls of the crater have been severely eroded, the present shape is not necessarily the original shape.

If the Carolina Bays had an origin similar to that indicated in the original paper, then one would not expect to find either fragments of the meteorites or Ninninger's little ferromagnetic pellets in the neighborhood of these bays. Failure to find fragments would not be indisputable evidence that the craters were not formed by meteorites.

Some day core-drilling should help solve this intriguing problem of the origin of the Carolina Bays.

DISCUSSION OF

"THE FORMATION OF FRAZIL AND ANCHOR ICE IN COLD WATER"

by VINCENT J. SCHAEFER

[Trans., v. 31, pp. 885-893, 1950]

R. W. Gerdel (Snow, Ice and Permafrost Research Establishment, Wilmette, Illinois)--Any technical paper on the phenomena of anchor- and frazil-ice formation is incomplete without a reference to, and discussion of, the research and reports in this field by Altberg, Byden [see ELGES, 1940], and DEVIK [1949], and a reference to the exceptionally complete bibliography on underwater ice prepared by ELGES [1940]. This excellent bibliography contains 91 references to publications on underwater and anchor ice of which 16 are by Altberg and 16 by Bydin. It is probable that the report by ALTBERG [1936] constitutes one of the greatest contributions to be found in the literature on both fundamental and applied research in anchor- and frazil-ice formation.

Schaefer has reopened the question of anchor-ice formation by long-wave radiation loss from submerged objects. He, at least, implies that BARNES [1906] may have been right in attributing the formation of anchor ice on the stream bed to cooling by long-wave radiational heat losses through the water. This implication, added to the definite statements by PARSONS [1942] and by LINSLEY and Others [1949, p. 147] that anchor ice occurs only in streams with dark rock bottoms and never on cloudy nights or under bridges, has been thoroughly refuted by Altberg. It is rather astonishing to note that the major works of Barnes and Altberg are cited, side by side, in the references on page 148 in Meinzer's "Hydrology," but that credence is given in the text only to Barnes' theory which even at that time (1940) was in disrepute.

Altberg conducted experiments in his laboratory with large tanks of water in which he could control the temperature and completely eliminate any radiation from the bottom of the tank. He was able to produce frazil and subsequently anchor ice without, as he says, "any importation of elements from the surface and without any participation of radiation."

Computations based upon data for the monochromatic absorptivity of water presented by DORSEY [1940] will show that all of the long-wave radiation (greater than $1.5\,\mu$) which can be lost from the bed of a stream at 0°C would be absorbed in less than one centimeter of water. Since anchor ice is produced at depths of several feet, radiational losses of heat must be ruled out. That Barnes himself realized the error in his hypothesis is shown by ALTBERG's [1936, p. 377] statement that Koblenz finally succeeded in persuading Barnes that his point of view had to be revised. It would be difficult also to refute the conclusion by Altberg that anchor and frazil ice are identical although Schaefer implies there is a difference.

The accumulation of anchor ice on submerged weeds, as reported by Schaefer, is a partial refutation of the theory of radiational heat losses from dark rocks on the stream bottom as a cause of anchor-ice formation. At temperatures of 0°C under the cloudless night-sky conditions cited as essential for the formation of anchor ice, all rocks, regardless of color, are about equally efficient black bodies and water is practically opaque to such radiation. The submerged weeds, like all vegetation, may be considered as composed of approximately 20 pct cellulose and allied organic materials and 80 pct water. Long-wave radiational heat losses from such a negligible mass of solid material could not support the growth of a volume of ice shown in the illustrations in Schaefer's paper.

Devik's use of seeding techniques to induce both frazil- and surface-ice formation to protect hydroelectric installations precedes the work of Schaefer by 20 years. Results of his more recent investigations in this field are also available [DEVIK, 1949]. Neither Devik, Altberg, nor the host of others who have worked on this problem used dry ice as the seeding agent to induce either frazil- or sheet-ice formation. However, most of them obtained identical results when cold sand, ice shavings, or snow pellets were introduced into supercooled water.

In view of the increasing economic importance of hydroelectric installations in northern United States and Canada, and the operational problems created by both anchor ice and frazil ice, this discussion has been prepared in the hope that further research, which is sorely needed, will not ignore the sound foundation prepared by many previous investigators in this field.

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Vincent J. Schaefer (General Electric Research Laboratory, Schenectady, N. Y., Author's closure)--The comments of Gerdel on the state of the formal literature concerning frazil and anchor ice are quite pertinent and are appreciated. I am sorry if I gave the impression in my paper that I accepted radiation cooling as the explanation of anchor-ice formation. Without any pertinent experimental data either for or against the conclusions of Barnes, I meant rather to infer that there was indeed a question about the validity of this explanation in terms of his experimental evidence. I am glad to have a reference to the important work of Altberg.

The main purpose of my paper was to report on what to me were interesting field experiments and observations and to indicate an interesting new approach to the seeding of supercooled water. The use of dry ice for seeding provides quantities of nuclei of a much higher order than is possible by throwing snow, sand, or any particulate matter into the water. Under proper conditions it is possible to produce as many as 10^{14} to 10^{16} ice crystals per gram of dry ice in air which is supersaturated with respect to ice. On a water surface this number will be considerably less and yet much greater than is possible by other methods.

If anchor ice is to be used to describe underwater accumulations of frazil ice, then a new term should be devised to describe the formations which I illustrated, such as are found firmly attached to underwater objects which have provided the nucleation sites. I would prefer to call the underwater accumulations of tiny, plate-like crystals frazil ice since it is my understanding that this word was originally coined to describe the flocs of underwater frazil which look like cinder.

I am quite in agreement with Gerdel that more basic research in this interesting field is needed since a number of fascinating problems remain to be answered. With the increase of interest in using the St. Lawrence River and other northern streams for power purposes, it is important in my opinion that scientists interested in the various states of H_2O become interested in solving these problems.

Transactions, American Geophysical Union   Volume 33, Number 1   February 1952

REVIEWS, ABSTRACTS, AND BIBLIOGRAPHY

REVIEWS AND ABSTRACTS

TARDI, PIERRE, and GEORGES LACLAVERE, Traite de Géodésie, Volume I, Book I, Gauthier-Villars, Paris, French text, xvii + 387 pp., 1951.

This is the first portion of a complete revision of a similar work by these authors which first appeared in 1933. The present book bears the sub-title "Mathematical foundations of geodesy and field operations." Book II of Volume I, devoted to triangulation computations, will appear, according to the prospectus, in 1952. Volume II entitled "Geodetic astronomy" and Volume III dealing with terrestrial potential fields and figure of the Earth will complete this treatise.

The authors state that their aim is to achieve theoretical comprehensiveness together with practical appeal, not seeking, however, to compete with purely theoretical works nor pretending to produce a field manual. This they achieve with typical French clarity and the pedagogical skill acquired in their dual roles of engineering officials in the French government and long-time professors of geodesy at outstanding technical schools.

The notation used in the text is the one sponsored and officially adopted by the International Association of Geodesy and it is to be hoped that its use is a stride toward the elimination of confusion and time-consuming annoyance caused by stubbornly persisting instances of divergence in other publications. Other notable features of the work are the up-to-date and readable sections on new subjects in geodesy such as radar surveying and Bergstrand's geodimeter innovations which may well revolutionize geodetic techniques. The omission of an alphabetical index in a work of this type seems rather unfortunate.

ERWIN SCHMID

BIBLIOGRAPHY

Scope of this Bibliography

This statement relates to the bibliographies published in the immediate past, and will be the guide for the selection of material for the current volume. The bibliography does not include papers appearing in the Transactions or in the usually available journals devoted in large part to any aspect of geophysics, except as the following statements may indicate. Emphasis in general is placed on the fundamental aspects of the several fields rather than their applications, and on out-of-the-way sources likely to escape general notice. Except in Oceanography, little Russian literature is covered.

General--The items include those of broad interest in geophysics, and those of specific interest to two or more Sections, garnered in the search of the literature of particular Sections, especially from foreign and out-of-the-way domestic publications.

Geodesy--An attempt is made to include all papers on theoretical geodesy. In general, papers on cartography and photogrammetry and those in such journals as the Journal of the American Congress of Surveying and Mapping, and the Empire Survey Review, are excluded.

Seismology--All aspects are covered except applications to prospecting. Papers that are primarily geological, or that are more properly assigned to Tectonophysics, are omitted.

Meteorology--Emphasis is placed on items gleaned from the less well-known foreign publications and from publications generally of a non-meteorologic nature. Meteorological Abstracts and Bibliography, published monthly by the American Meteorological Society, largely supplies the needs in this field.

Terrestrial Magnetism and Electricity--Some items which concern a particular segment of this field, but which appear in journals not devoted largely to that segment, are included. Thus a paper on magnetic storms appearing in Proceedings of the Institute of Radio Engineers, or one on ionospheric radio propagation in the Journal of Atmospheric and Terrestrial Physics, might be included, whereas neither would be entered if their sources were reversed. Items previously listed by W. E. Scott in the List of Recent Publications in the Journal of Geophysical Research are not ordinarily repeated here.

Oceanography--All aspects of Oceanography are covered save the purely biological. Readily available journals devoted in substantial measure to Oceanography are excluded, notably Journal of Marine Research, Papers in Physical Oceanography and Meteorology, Bulletin of the Beach Erosion Board, Journal du Conseil Permanent International pour l'Exploration de la Mer, Deutsche Hydrographische Zeitschrift, Bulletin d'Information de la Comite Central d'Oceanographie et d'Etude des Côtes, International Hydrographic Review, and the Oceanographical Magazine (Tokyo).

Volcanology--All aspects of the broad field of Volcanology are included, including petrology, heat processes, radioactivity, and geochemistry as they relate to geophysics, as well as classical Volcanology, except items from publications usually available in the libraries of schools where the Earth sciences are taught (such as the Bulletin of the Geological Society of America, American Mineralogist, American Journal of Science, and the Journal of Geology). Out-of-the-way publications and journals, and other foreign publications, are emphasized.

Hydrology--Includes most of the current literature developing hydrologic principles. It is selective only to the extent that regular serial reports on stream flow, ground water, and other basic data are excluded. Also excluded are engineering reports based on hydrologic applications, unless it is found that some new principle is set forth. It is not complete, especially in some of the peripheral aspects, such as glaciology and sedimentation.

Tectonophysics--All aspects of Tectonophysics are included, but the journals readily available in the United States are not covered (see under Volcanology). It is the intent to include general geologic papers having detailed discussion of structure.

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NOTES AND PERSONALIA

NOTES

Corrigenda--In the issue of December 1951, page 815 of the paper by Carl I. Aslakson, the right-hand members of Eq. (3) should read

$$\left\{[(V_o - 2f)/2f] \times 10^6 - \sum_{h=k}^{h=H} \Delta\mu\right\} \bar{S}_a \times 10^{-6}$$

On page 819, at the end of the third paragraph, the statement should read $\beta - 180° = 37.0°$.

On page 929, Table 1, in the note by B. C. Goodell, the last vertical line in the table headings should extend to the bottom of the table, as the material to the right has no line-for-line relationship to the material at the left.

New publication on volcanoes--A Catalogue of the Active Volcanoes of the World including Solfatara Fields, Part I, covering Indonesia, written by M. Neumann van Padang, has been issued by the Association of Volcanology, IUGG. This is, in fact, more than a "catalogue" as it contains material on form and structure of the volcanoes, volcanic activity and petrography, and has an extensive bibliography. It contains almost 200 pages and lists 128 volcanoes and solfatara fields. A limited edition of 1000 has been published. Inquiries and orders should be sent to Francesco Signore, Secretary of the Association, Via Tasso N. 199, Naples, Italy.

Fulbright awards for 1951-52--The Committee on International Exchange of Persons of the Conference Board of Associated Research Councils has announced a total of 232 Fulbright travel grants to scholars in the natural sciences from 17 countries for 1951-52. These are awards for study and work in the United States. In addition, many of these were given aid to cover their expenses while in the United States under the Smith-Mundt Act. The Committee has advised that many of these visiting scholars would welcome invitations to lecture and visit from colleges and universities while they are here, providing expense of travel is covered by the inviting institutions. Details may be secured by writing to the Committee cited above at 2101 Constitution Avenue, Washington 25, D. C.

New edition of Internal Constitution of the Earth--A new revised edition of Internal Constitution of the Earth, prepared by 11 authors and edited by Beno Gutenberg, has been released by Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. This is Volume VII of the Physics of the Earth Series which was originally published in 1939. Much has been largely rewritten, however, and considerable new material has been added.

New chair of geology at Rice Institute--The Harry Carothers Wiess Chair of Geology has been created at Rice Institute by a gift of $1,570,000 by the widow of Mr. Wiess who was a Trustee of Rice Institute, a member of the Corporation of the Massachusetts Institute of Technology, a Charter Trustee and permanent member of the Board of Princeton University. It is anticipated that the Chair will consist of a full professor of geology and two associates, with particular attention to be given to marine geology.

Graduate assistantships in meteorology at Florida State University--Several graduate assistantships in meteorology, carrying stipends of $125 and $150 per month, are available for graduate study in meteorology and climatology at Florida State University Interested persons should write promptly to the Dean of the Graduate School at the University, Tallahassee, Florida.

Civil Service positions for geophysicists--Positions in various grades with initial salary varying from $4200 to $10,800 per year are available in the Geophysics Research Division of the Air Force Cambridge Research Center, Cambridge 39, Mass. Interested persons should write to the Board of U. S. Civil Service Examiners, Air Force Cambridge Research Center, 415 Summer St., Boston 10, Mass., requesting Announcement 1-12-1 (52) dated January 8, 1952.

Intelligence research specialists needed--The Civil Service Commission has announced (Announcement No. 258) the availability of positions of various grades of intelligence research specialists for service in various agencies, chiefly in the Washington, D. C., area. Grades range from GS7 to GS12. Interested persons should write to the Commission in Washington, D. C.

Seminars of the Geophysics Research Division of the Air Force Cambridge Research Center--These seminars are currently being held in Room 10-275 of Massachusetts Institute of Technology at 15h (15h 30m on February 29). The following subjects have been announced: February 29, 1952, Some experiments bearing on the elasticity of the Earth's crust, by Francis Birch; March 7, 1952, Atmospheric electricity, by W. F. G. Swann; and March 28, 1952, A review of recent work on "astronomical seeing," by Gordon W. Wares.

Proceedings of Second Glaciological Conference--A 48-page mimeographed report of the Second Glaciological Conference held under the joint sponsorship of the American Geographical Society, the Arctic Institute of North America, and the Research Committee on Glaciers of the American Geophysical Union, in January 1951, is now available through the American Geographical Society, 156th St. and Broadway, New York 32, N. Y.

Geological meeting in Tucson, Arizona--The Cordilleran Section of the Geological Society of America, the Seismological Society of America, the Mineralogical Society of America, and the Pacific Coast Branch of the Paleontological Society will meet at the University of Arizona in Tucson, April 11 and 12, 1952 under the sponsorship of the Arizona Geological Society. Included will be a panel discussion on orogeny. Pre- and post-meeting field trips are planned. Interested persons should write to L. A. Heindl, P. O. Box 2270, Tucson, Arizona.

New organization of geology teachers--On November 10, 1951, geology teachers from ten states met in Detroit for the purpose of organizing an Association of Geology Teachers, joining two regional organizations that have been in existence for a number of years and expanding to a nation-wide basis. The aim will be to foster improvement of teaching sciences of the Earth at all levels and to emphasize the cultural significance of these sciences. The organization will publish its Transactions and the Journal of Geologic Education. The Pr sident elected for 1951-52 is Kurt E. Lowe of the City College of New York, and the Secretary, Ralph Digman, Harpur College, Endicott, N. Y.

Second Alaskan Science Conference--The Second Alaskan Science Conference was held at Mt. McKinley National Park, Alaska, September 4-8, 1951. This was a follow-up to the First Conference held at Washington, D. C., in 1950, and marks the beginning of formal organization of science on a territory-wide basis in Alaska. About 200 attended the conference and many scientific papers were presented, at least 15 bearing heavily on various aspects of geophysics, which is exceedingly important in Alaska. The occasion was marked by the formal installation of the Alaskan Division of the American Association for the Advancement of Science.

New laboratory for oceanographic studies--Plans are under way for the construction by the Navy of a $600,000 laboratory building at Woods Hole in order to facilitate the cooperative and contract work being undertaken by the Woods Hole Oceanographic Institution and the Navy.

Geology and mineralogy meeting at Roanoke, Va.--The Southeastern Section of the Geological Society of America and the Southeastern Mineral Symposium will have a meeting in Roanoke, Va., May 1-3, 1952. Inquiries should be addressed to the Chairman of the Program Committee, Byron N. Cooper, P. O. Box 634, Blacksburg, Va.

Meeting of the U. S. National Committee of the International Scientific Radio Union (URSI)--The annual spring meeting of URSI jointly with IRE Professional Group on Antennas and Propagation will be held at the National Bureau of Standards in Washington, D. C., April 21-24, 1952. Interested persons should write to the Secretary of the U. S. National Committee, A. H. Waynick, Pennsylvania State College, State College, Pa.

Organization of a Council on Wave Research--A Council on Wave Research has been organized under the auspices of the Engineering Foundation to foster and coordinate studies of wind-generated waves and their resulting problems such as beach erosion and coastal protection. The first conference of the Council was held in Long Beach, California, in 1950, and the papers have now been published as the Proceedings of the First Conference on Coastal Engineering. Interested persons should write to the Council, 244 Hesse Hall, University of California, Berkeley 4, Calif. The second conference was held in Houston, Texas, in November 1951.

Committee on Arid Zone Research--The second session of the Advisory Committee on Arid Zone Research was held at UNESCO House, Paris, September 3-5, 1951. About 40 specialists took part. A five-point program of recommendations was adopted. The Committee will meet again in the United Kingdom just preceding or following a meeting of the Conference on the Biology and Productivity of Hot and Cold Climates being organized by the British Institute of Biology.

Study of wind stress on the sea surface--The schooner Reverie of the Scripps Institution of Oceanography and a bomber of the U. S. Air Force are engaged in gathering coordinated data and observations in connection with a study of wind stress on the sea surface. The study is under the direction of Walter H. Munk of Scripps.

Study of typhoons--Plans have been made by the U. S. Weather Bureau and the Air Weather Service for typhoon research in the Pacific during the typhoon season of 1952. The structure of the typhoon will be a particular phase of the study.

PERSONALIA

Fred M. Bullard, Professor of Geology at the University of Texas, will be on leave for a year beginning February 1, 1952. He will study Italian volcanoes, including Vesuvius, Etna, and Stromboli, under a Fulbright grant.

William S. Dawson, Major, U.S.A.F , has been designated Director of Plans and Requirements for Headquarters 1808th Airways and Air Communications Service Wing, and is responsible for this work throughout the entire Pacific and Asia, including Korea.

Jack W. E. Edmonson has left the Shell Oil Company and has joined P. E. Navarte in consulting geophysical work with headquarters in San Antonio, Texas.

Pentti Eskola, petrologist of the Geological Institute of Finland, has been named the recipient of the Penrose Medal of the Geological Society of America for his original contributions to science.

A. W. Friend, recently of the staff of RCA, has been appointed Director of Engineering and Development of Magnetic Metals Co. of Camden, N. J.

Kenneth C. Heald, Vice President of the Gulf Oil Corporation of Pittsburgh, has been named as the 1952 recipient of the Sidney Powers Memorial Medal of the American Association of Petroleum Geologists. The presentation will be made at the meeting of the Association at Los Angeles in March.

J. Laurence Kulp, geochemist of the Lamont Observatory of Columbia University, has been awarded the Newcomb Cleveland Prize of the American Association for the Advancement of Science for his work in carbon-14 studies, notably those relating to studies pertaining to carbon dioxide in deep-sea water.

J. J. Levallois has been appointed as an assistant to P. Tardi, General Secretary of the Association of Geodesy of the IUGG, and will be in charge of preparation of material for the *Bulletin Géodesique*.

J. L. Rannie, Dominion Geodesist, Canada, has retired. J. E. R. Ross has been appointed to succeed him.

William W. Rubey, Research Geologist of the U. S. Geological Survey, has been appointed Chairman of the National Research Council of the National Academy of Sciences.

Orville F. Tuttle of the Geophysical Laboratory, Carnegie Institution of Washington, has been named the first recipient of a new award of the Mineralogical Society of America to be given to young scientists under 35 who make outstanding contributions to the mineralogical sciences.

Theodore von Karman, Director of the Scientific Advisory Board of the U. S. Air Force, has been given a Fulbright grant to lecture on fluid mechanics at the University of Paris.

Theron Wasson, Chief Geologist of The Pure Oil Company, has been given the Award of Merit by the Alumni Federation of the Carnegie Institute of Technology in recognition of his achievements in the field of petroleum geology.

Erwin J. Hoeck, Director of the Research Laboratory in Hydrology at the Federal Technical College at Zürich, Switzerland, and in that work successor to the late Otto Lütschg-Lötscher, died July 27, 1951.

James Otis Letts, Engineer with the Corps of Engineers on river and flood-control work at Rock Island, Illinois, died March 6, 1950 at the age of 45. He was born at Sedalia, Missouri, and received his baccalaureate degree from Missouri School of Mines in 1930. He entered the services of the Corps of Engineers in 1934, and devoted his professional years to hydraulic and hydrologic studies. He had been a member of AGU since 1941.

Jacob Menno Tienstra, Professor of Geodesy at the Technical University of Delft, President of The Netherlands Geodetic Commission, and internationally known geodesist, died September 15, 1951 at the age of 56. He was born at Sneek in the Province of Friesland, The Netherlands. He was trained as a land surveyor, later taking a degree in mathematics. His early professional work was in Zutphen and, in 1925, was engaged in land-surveying work in the Amsterdam area. In 1931, he became a professor of land surveying at the Agricultural Institute of Wageningen and, in 1935, took up his duties at Delft. He was a knight in the Order of Orange Nassau. He was considered an outstanding authority on problems of triangulation, was the author of extensive writings bearing on this subject, and had only recently completed a book on least squares which is to be published posthumously.

Thomas Wayland Vaughan, Director Emeritus of Scripps Institution of Oceanography, currently engaged on paleontological work with the National Museum, died January 16, 1952 at the age of 81. He was born at Jonesville, Texas, graduated from Tulane University, and in 1903 took the degree of Doctor of Philosophy at Harvard University. He worked with the U. S. Geological Survey from 1894 to 1924, and from 1936 to 1939. From 1924 to 1936 he was Professor of Oceanography at Scripps, serving also as Director. He was the recipient of many awards, including the Penrose Medal of the Geological Society of America, and served as President of the Section of Oceanography, AGU, from 1926 to 1928. He had been a member of AGU since 1921.

Walter Byron Wilson, Consulting Geologist with the Tulsa Division of the Gulf Oil Co., died May 21, 1951 at the age of 65. He was born at Fort Madison, Iowa, took his baccalaureate and master's degrees from the University of Missouri in 1913 and 1914, and subsequently, for two years, was a Fellow at the University of Chicago. In 1916, he entered the service of the Gulf Oil Co. which he served in diverse capacities until his death. He became a member of AGU in 1947.

SPECIAL ANNOUNCEMENTS

Thirty-Third Annual Meeting, Washington, D. C , May 5-7, 1952--Your attention is called to the previous announcements in the Transactions [Oct. 1951, p. 807; Dec. 1951, p. 954] regarding the Thirty-Third Annual Meeting of AGU to be held in Washington, D. C., May 5-7, 1952. Note that titles of papers, with three copies of the abstracts, are to be in not later than March 10, 1952, with advice concerning time required (ten to 20 minutes usually allowed) and projection equipment needed. Word may be sent to the office of AGU or to the Section Secretary or Program Committee Chairman as shown in the October announcement. Note that programs may be closed in advance of March 10 if they are filled. The December announcement gives information concerning hotel reservations.

Regional Meeting in Chicago, Illinois, September 11-12, 1952--In connection with the celebration of the Centennial of Engineering to be held in Chicago in September 1952, AGU will hold a Mid-West Meeting (Twenty-Eighth Regional Meeting) in which the Sections of Meteorology and Hydrology are joining. While the scientific sessions will be held at the University of Chicago, the hotel headquarters will be The Del Prado. A Committee has been appointed consisting of Horace R. Byers (Chairman), Department of Meteorology, University of Chicago, Chicago, Illinois; W. S. Hamilton, Department of Civil Engineering, Northwestern University, Evanston, Illinois; William C. Krumbein, Department of Geology, Northwestern University, Evanston, Illinois; Arno T. Lenz, Hydraulic Laboratory, University of Wisconsin, Madison 6, Wisconsin; Mathew L. Rorabaugh, U. S. Geological Survey, 531 Federal Building, Louisville, Kentucky; Glenn E. Stout, 708 N. Lincoln Street, Urbana, Illinois; and Chester O. Wisler, Department of Civil Engineering, University of Michigan, Ann Arbor, Michigan. Interested persons should get in touch with the Committee.

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AMERICAN GEOPHYSICAL UNION

Volume 33 APRIL 1952 Number 2

SYMPOSIUM ON THE MEASUREMENT OF GEOLOGIC TIME

INTRODUCTION TO THE SYMPOSIUM

John Putnam Marble

The last time that a symposium on the topic of the measurement of geologic time took place was at the 1933 Meeting of the American Association for the Advancement of Science in Chicago. Those who may be interested in the progress made in this field may care to compare the topics covered by the papers on these two occasions. In 1933 there were no talks dealing with the age of the Earth's crust, with the measurement of lead isotope ratios, with "carbon-14," or with the rubidium-strontium ratio method, for the simple reason that these techniques had not then been thought of, much less developed. We feel that the present general audience of active workers in all fields of geophysics and geochemistry will be interested to hear from the experts of these new developments in the application of physical and chemical techniques to a very fundamental series of problems in Earth science, and also to learn of recent progress in fields that may now be termed "classical."

We cannot, of course, hope to cover all of the interesting work that is now being carried on, in the limited time at our disposal. We have deliberately omitted from this program papers dealing with the "lead method." This, which utilizes the ratio of the lead produced by processes of natural radioactive decay to the undecayed residues of the mother elements uranium and thorium as its "clock," is well worked out as to its principles, and its limitations are reasonably well understood. Improvements in existing analytical techniques and the introduction of new ones go on apace; the fundamental radioactive constants involved are becoming known with greater precision; and data for the time scale by this method are being accumulated in increasing volume and of greater accuracy. It is to be hoped that in the near future some student of this particular field will provide an up-to-date synthesis of present knowledged therein. This has not been done at all since the publication of Holmes' little book, The Age of the Earth (Nelson), in 1937, and not with real thoroughness since the work of Holmes, Knopf, Kovarik, and others in Bulletin 80 of the National Research Council in 1931.

Limitations of time have made it also impossible to discuss the most recent work involving the Ra/Io/U ratios of deep-sea cores with the implications for sedimentation rates and for physical oceanography in general therein involved. No discussion will be presented, unfortunately, of the very recently developed application of the so-called "nuclear" photographic emulsion to the distribution of radioactive material in rocks and the semi-quantitative measurement of the content of such elements by this means. Further, we are not presenting any material on the possibilities, interesting to the Chairman, of applying any of the recently discovered weak natural radioactivities, such as those of samarium, lutecium, and rhenium, to the absolute measurement of geologic time.

We have been forced to pass over many actively pursued studies concerned with limited portions of the geologic time scale, including varve counting, pollen profiles, the relative fluorine content of recently fossilized bone, tree-ring counts, and possible cyclic variations in the Earth's magnetic field. All of these are interesting and valuable, but "you can't have everything."

Finally, we have had to omit papers dealing with the recently proved alternative radioactive decay of potassium, $K^{40} \rightarrow A^{40}$. This is being actively studied in several laboratories. Its implications to problems of the composition and age of the atmosphere will occur to meteorologists.

However, we have a full program, in spite of all that has been left out. Those who desire to read recently published work in any of these topics are referred to the Annotated Bibliographies which accompany the Reports of the Committee on Measurement of Geologic Time of the Division of Geology and Geography of the National Research Council.

3221 Macomb Street,
Washington 8, D. C.

(Manuscript received October 5, 1952; presented at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1952.)

STRATIGRAPHICAL VIEWPOINTS IN MEASUREMENT OF GEOLOGIC TIME

Raymond C. Moore

Abstract--Although geologists are dependent wholly on physical and chemical research dealing primarily with radioactive substances in the Earth's crust for reliable long-range quantitative measurement of geologic time, stratigraphical information is indispensable for interpretation of findings if they are to be useful in geochronology. A chief problem is placement of time-measured mineral or rock samples in proper position in the geologic succession, and unless such accurate placement can be made, the age determinations have relatively little value. Stratigraphical methods for establishing age relationships of different rock bodies are applicable to Precambrian, as well as to Paleozoic and younger rocks, but only in fossil-bearing strata can correlation of rock units in widely separated regions be fixed with reasonable accuracy. Given geologic-time bench marks distributed through the rock column, stratigraphical techniques based on studies of sedimentation, paleontology, paleogeography, and diastrophism may be employed in constructing reasonably trustworthy interpolations in the time scale. Progress in the direction of precise detailed geochronology depends on coordination of diverse skills, coupled with improvement and greatest possible extension of researches serving to measure geologic events in terms of time.

Introduction

As an introduction to the discussion which I have been asked to prepare for this symposium, I quote remarks by KNOPF [1949, p. 4] in a recent lecture on The Geologic Records of Time. He said:

> A few weeks ago Time gave an account of the finding by Dr. J. B. Reeside of a fossil wasp nest in rocks of late Cretaceous age in Utah, adding that the nest is 80 million years old. I wrote to my former colleague in the United States Geological Survey, congratulating him on his remarkable discovery and then in lighter vein asked him, "Where did you get the 80 million years?" and suggested that 40 million years is as likely to be right from anything we now know as 80 million years. To which he replied: "I have lots of sins to answer for, but stating the age of a fossil in years is not among them. All I have vouched for is that the specimen came out of the Upper Cretaceous rocks. It will be perfectly satisfactory to me for you men who know all about geologic ages in years to call it 40 million rather than 80--I take either with two grains of salt." (And Professor Knopf adds:) The wasp's nest I stirred up is most probably 80 million years old.

This incident and the borrowed words describing it serve very well, I think, to set the stage for what I now undertake to present for consideration of "you men who know all about geologic ages in years."

I share the open-mindedness and moderate scepticism of my friend, John Reeside, on statements about geologic age of fossils in terms of years. Like nearly all geologists, we are eager to learn about reliable, provable time-datum points located in any part of the geologic column, and we are glad to contribute in every possible way to obtaining them. When we learn that one of the very few now known fixed points which can be applied to calibration of the time scale has the value of 58 million years and that this defines the age of one of the Laramide mountain-making movements, it becomes patent that fossil-bearing beds some distance below the top of the Creataceous must be appreciably older than 58 million years. And we are much more assured of this when told that present laboratory techniques provide three independent means of age computation which yield, in this case, mutually confirming results. The antiquity of our fossil wasp nest, however, is indicated only by a limiting minimum figure. How much older than 58 million years shall we say? In what manner, if at all, can this be determined? Also, we must not overlook question amounting to perhaps as much as ten million years in the stratigraphic placement of our starting point, which is the 58-million-year bench mark. We do not question the age figure but need to know whether it belongs approximately at the beginning of Paleocene time (equivalent to close of the Cretaceous) or at the end of Paleocene time. On this there is doubt. The question can be answered only by geological study, and answer is not yet available. Then comes the problem of finding the geologic time span represented by beds between the horizon of the wasp nest and summit of the Cretaceous or Paleocene. With whatever help he can get, the geologist must take responsibility for this task.

These considerations define my part in the present program. Analyses and computations by specially skilled chemists and physicists working on suitable materials in the laboratory provide acceptable measurements of geologic time. Usefulness of these measurements in building a geologic time scale expressed in years depends on the work of geologists, first in determining stratigraphic placement of measured points, and second in devising acceptable interpolations between the geologic-time bench marks. The purpose of my paper is to outline some of these geological aspects in the coordinated attack by chemists, physicists, and geologists on measurement of geologic time. The discussion logically divides itself in two parts: (1) How and with what precision can absolute time-computed points be assigned position in the geologic column? (2) How and with what definiteness can geologic age be fixed for many stratigraphic horizons located between absolute time-computed bench marks?

Geologic placement of absolute-time bench marks

Igneous source materials--The fact that best materials for long-range geologic-age determinations occur almost exclusively in minerals and rocks of igneous origin, especially pegmatites, granitic rocks, and some metalliferous veins, constitutes a major difficulty when age of these source materials is sought in terms of the standard geologic column. Everyone concerned with geologic-time studies knows this well. Placement of the pegmatite or other source rocks cannot be fixed by identifying fossils in rocks just below and above them, even if the igneous source is Paleozoic or younger. Such conditions do not exist, and if they did, we would merely know that the igneous rock is younger than any adjoining materials penetrated by it. Depositional contact of fossil-bearing strata on the igneous rock is possible and, if found, it allows determination of limiting post-intrusion age. Such occurrence, because of rarity, lacks practical significance. Accordingly, geologic age in terms of the rock column must be established by indirection. It follows that almost all degrees of uncertainty, depending on local setting, may remain even after painstaking geological field studies. The rule that an intrusive igneous rock or vein must be younger than country rock which it intersects, may help little or not at all. If the rock which bears uranium, thorium, helium, or other time-measuring substances occurs as veins in igneous rock far removed from datable sedimentary formations and having indecipherable structural relations to such nearest formations of known age, placement of the age measurement in terms of recognized geologic periods and epochs may well seem hopeless. This is not necessarily true, however, because advances in understanding the characteristic association of plutonic and volcanic activity with geologic evolution of geosynclines and mountain-building deformations indicate promise of obtaining acceptable age determinations of many igneous rock bodies which heretofore have been excluded from assignment to a specific geologic period or part of a period. Analysis of the relations of magmatic activity to mountain building and age of both has been undertaken especially in papers of **STILLE** [1924, 1940, 1944, 1950b], and in this connection special attention of geologists may be called to papers by **KENNEDY** [1933] and **KENNEDY** and **ANDERSON** [1938], which **READ** [1944] appraises as belonging among foremost of all contributions to petrogenesis.

GILLULY [1949] recently has raised question as to the periodicity of mountain building and the accuracy of geological dating of such deformations in various quarters of the world. In particular, he has challenged the validity of concepts advanced by Stille of world-wide mountain building at specified moments in geologic history. Because igneous activity, with which emplacement of radioactive minerals is concerned, accompanies most of the mountain-building episodes, questions on the age of orogenic diastrophism has importance for us in considering geologic placement of absolute-time bench marks derived from study of these minerals. Gilluly is quite right in scrutinizing closely the premises employed by Stille in his thesis that mountain building has occurred coincidently in widely separated parts of the world at some 40 specified times in Earth history, and that each such mountain-making deformation has lasted on the average about 300,000 years.

Although **BUCHER** [1933], **UMBGROVE** [1947], and very many other geologists accept Stille's orogenic chronology, myself included as regards many features, I join with Gilluly in judgment that precision of geologic dating in numerous regions and parts of the record is far from sufficient to fix the assigned placements. In support of Stille is ample evidence almost everywhere that the crust has remained quiescent for long periods. Then may come disturbance that folds and faults the crust, and such deformation is surely of short duration compared to preceding calm. If, as I have just stated in agreeing with Gilluly, geologic dating of these deformations is imprecise, so that various ones come to be recorded a notch or two or three out of their actual time placement, it is easy to class nonequivalent things as equivalent. In this way, Stille can report orogenies of somewhat different dates in various continents as parts of a single world-wide mountain-making movement. The filing of the diastrophic records in wrong pigeonholes, probably because too few of them are yet made, can give rise to a spurious simplicity, and lead to false generalizations concerning coincident movements throughout the globe. Discussions by **STILLE** [1950a] and **GILLULY** [1950] on distribution of mountain building in geologic time, published subsequently to **GILLULY's** [1949] address on this subject, have clarified distinction between date of origin of mountain structures and that of mountain topography and have served reasonably to settle other points, but they have not disposed of essential questions of the contemporaneity of origin of mountain structures in widely separated regions nor distinction between structures defined as orogenic and those defined as epirogenic. Plainly, however, Gilluly holds to judgment, as I do, that Gleichzeitigkeit -- time identity -- of deformative crustal movements in distantly separated regions is far from proved.

The bearing of all this on geologic placement of absolute-time bench marks based on study of igneous source materials should be evident. It simply means that if given radioactive minerals are known to represent the time of magmatic activity associated with mountain-making deformation in a part of Mexico, for example, definition of geologic date cannot be fixed reliably by reference simply to one of the Stille-named orogenic epochs. It may be significantly older or younger.

Sedimentary source materials--Scant notice need be given to this part of my subject, because answers to questions of geologic placement are mostly obvious. Clastic mineral grains, some of which may bear occluded helium, and pebbles consisting of abrasion-rounded uraninite crystals, which are known in some sedimentary formations, come from sources older than the rock layer where they now lodge. Naturally, age of the sedimentary rock and its place in the geologic column are not usable as a guide in establishing position of an absolute-time point based on analysis of its constituent clastic materials.

On the other hand, chemical precipitates and biogenic materials of sedimentary rocks may yield important absolute-time bench marks, provided they are susceptible to time-measurement techniques. The Upper Cambrian kolm shale from Sweden, which contains trilobites and other fossils, is a source of one of the most important absolute-time bench marks, partly because of its antiquity in terms of the fossil-bearing segment of the rock column and partly because it can be entered in proper position without question. Many middle and late Paleozoic black shales, some containing much phosphatic material, are known to be high in radioactive content, at least as regards emanation of gamma rays. If means for measurement of absolute time of these sedimentary rocks can be found, closely spaced time bench marks will furnish extremely useful, not to say exciting, approach to interpretation of cyclic sedimentation and other features of the geologic record. Let some one please achieve these time measurements.

Placement of stratigraphic horizons between absolute-time bench marks

Four methods of determining positions of points on the geologic time scale between bench marks established by measurements of radiogenic materials are available. As now viewed, some are decidedly superior to others, but each should be utilized as far as possible in such manner that one supplements another. Consensus of judgments by groups of geologists who are best acquainted with stratigraphy of respective segments of the column between the absolute-time datum points is likely to eliminate undesired subjectivity and give best-balanced evaluation of available data. Of course, the more closely we can space positions of bench marks, the easier is the task of making interpolations between them. The methods for filling in calibration of the time scale comprise application of techniques in the study of (1) sedimentation, (2) paleontology, (3) paleogeography, and (4) diastrophism. Not included here are special techniques, such as measurements based on carbon-14, tree rings, archeological data, and the like, which are applicable only to a tiny part of most recent geologic time. Likewise, I omit astronomic, paleoclimatologic, and some other possible tools.

Computations based on sedimentation--Time measurements based on studies of sedimentation are chiefly of two sorts. One consists in making counts of superposed laminae which, like varves, are presumed to denote annual accretions of sedimentary materials. Thus, laminated Eocene oil shale (Green River) in Colorado and adjacent states studied by BRADLEY [1929, 1936] is found to contain varve-type layers averaging 0.175 mm in thickness, signifying that accumulation of the whole deposit, 2600 ft thick, required approximately 6.5 million years. KNOPF [1949] cites Upper Devonian laminated shale in Germany representing approximately a million years of annual deposits arranged in clearly discernible sunspot cycles of 11.4 years, which correspond precisely to the present-day sunspot period.

Another method of measuring time in terms of sedimentation is based on ratio of thickness of sedimentary deposits to the time required for their accumulation. This is a very crude way indeed to seek determination of intermediate points on the geologic time scale, because (1) sedimentation rates of various sorts of clastic and nonclastic deposits surely embrace an extremely wide range between minima and maxima, (2) no trustworthy approximations of an average rate of sedimentary accumulation are available, (3) time evaluation of effects of diagenetic changes and of breaks in sedimentation cannot be more than a vague sort of guess, and (4) errors or inadequacies of several kinds enter into selection and compilation of stratigraphic data used in making sedimentation rate computations. It is very doubtful that efforts of SCHUCHERT [1931] and others along this line even indicate correct order of relative magnitudes. Yet in lieu of something better, and in spite of fallacies of interpretation partly noted by GILLULY [1949, p. 578], boundaries between geologic systems have been placed in the geologic time scale, based on comparison of maximum known thicknesses of sedimentary rocks belonging to major stratigraphic divisions. All columns shown in Figure 1 reflect this approach. Columns 1a and 1b represent alternative judgments of HOLMES [1948] in which absolute-time bench marks of post-Silurian date are differently interpreted as to stratigraphic placement, shaded spaces marking the degree of divergence between the two columns. Column 2 is compilation by MARBLE [1950] in which the shaded spaces denote latitude in fixation of boundaries; they become progressively "fuzzier" proceeding toward the Precambrian. Column 3 is based on thickness of sediments according to SCHUCHERT [1931], weighted by increasing time value of carbonate rocks in the ratio of 6 to 1, as by GILLULY [1949, pp. 580-581], and based on arbitrary assignment of 520 million years to the beginning of Cambrian but without adjustment to any intervening absolute-time bench marks. Column 4 depicts maximum thicknesses of sedimentary rocks according to Schuchert, Sollas, and Woodward [SCHUCHERT, 1931], without adjustment for carbonates but utilizing all accepted absolute-time bench marks. Not included in making columns 3 and 4 are 65 million years estimated by SCHUCHERT [1931, p. 49] as allotment for diastems and unconformities. The discrepancies in the five columns speak for themselves.

Computations based on paleontology--The whole framework of interregional correlations and determination of comparative geologic dates in rocks younger than Precambrian depends on fossils. As GILLULY [1949, p. 582] remarks, "Geology could hardly be considered a science before the recognition of the stratigraphic significance of fossils." Yet precision of dating sedimentary deposits in terms of fossils has been challenged repeatedly. To some extent and with reference to particular cited examples, such challenge is well warranted. Mostly, however, the opposite is true. Despite manner in which plants and animals, both of the present and past, are influenced by environment and controlled in distribution by modes of dispersal and such things as geographic barriers; despite marked differences in evolutionary differentiation and longevity of individual life forms; and despite many other modifiers which the paleontologist must evaluate; the sum total of interwoven stratigraphic equivalences framed in terms of fossils is unshakably firm, amazingly detailed, and in varying degree, very satisfactorily precise. Examples without number can be cited readily, although I have time for not even one of them here. Let me simply call attention to the facts of identical successions of distinctive faunal zones in widely separated places, as in marine Cretaceous rocks of Europe, North and South America, and other continents [GIGNOUX, 1943, pp. 377-447; STEPHENSON and Others, 1942; IMLAY, 1944] as demonstration both of time-equivalent sedimentation and long-persistent, quiet interorogenic epochs in Earth history. Also, one may see many such examples of contemporaneous and near-contemporaneous fossil assemblages throughout the world by consulting the 99 paleobiogeographic maps by JOLEAUD [1939].

The foregoing remarks briefly assert the importance of paleontological work in arranging fossil-bearing strata in proper sequence. Fossils, however, furnish no satisfactory quantitative means for recording lapse of time. Evolutionary rates obviously have varied so enormously, both among different organic stocks and at different times in the same stock, that only subjective judgment can translate these things roughly into time. Even for this, some sort of time-measuring yardstick is needed to give scale. Evolutionary rates can be correlated with thickness of sediments through which a group of related fossils is distributed, but this too means little. Hence, we cannot look to paleontology for significant help along this line.

Computations based on paleogeography--SCHUCHERT [1916 and several later papers] has emphasized especially the role of paleogeography in determining significant features of geochronology. Synthesis of many sorts of stratigraphical data, including information from paleontology, expresses in map form inferred conditions prevailing on the Earth at a selected point on the geologic time scale. Many kinds of difficulties are encountered in paleogeographic studies, but because the end product is a series of graphic representations of significant changing conditions from epoch to epoch, they furnish useful background for appraising partition of the geologic column and for estimating the value of specified segments of the geologic time scale. Like paleontology, however, paleogeography can furnish no independent quantitative approach to determining the position of intermediate points on the scale between absolute-time bench marks. It can merely suggest what points have importance.

Computations based on diastrophism--Major crustal movements of the Earth, particularly mountain-building deformations, provide chief punctuation points in the geological record. Wide-

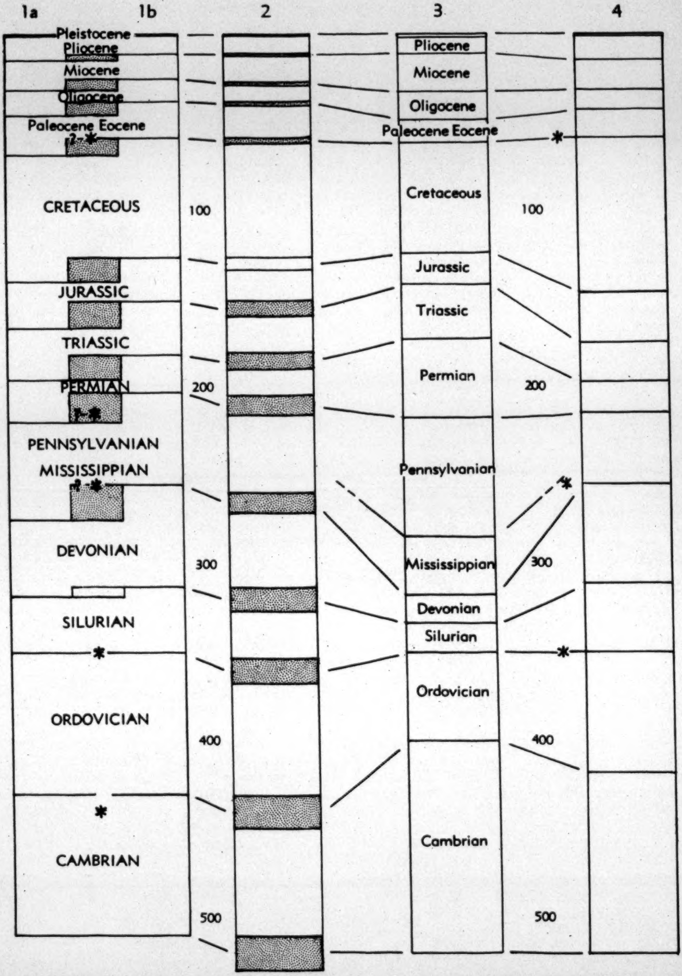

Fig. 1--Geologic time scales, based on absolute time measurements and thickness of sedimentary deposits; (1a-b) scales by HOLMES [1945], based on alternative postulates as to placement of post-Silurian radiogenic measurements, shaded areas indicating divergences of resulting period boundaries; (2) scale by MARBLE [1950], progressively widening shaded bands toward the base denoting uncertainty in placement of boundaries; (3) scale based on maximum thicknesses of sedimentary deposits in each division according to SCHUCHERT [1931], weighted by increase of carbonate rocks in ratio of 6:1 and plotted against assumed beginning of Cambrian at 520 million years but no other adjustment for absolute-time measurements; (4) scale based on maximum thicknesses of sedimentary deposits in each division given by Schuchert, Sollas, and Woodward [SCHUCHERT, 1931], not weighted for carbonate rocks, plotted against 520 million years at beginning of Cambrian and adjusted to each absolute-time measurement shown spread interruptions in sedimentation and accompanying erosion of earlier-formed rocks are associated with such diastrophism, as is readily demonstrable on all continents and in nearly all parts of the rock column. Formerly, these movements, classed as disturbances or revolutions, prevailingly were identified as belonging at the boundaries between geologic periods, or, by some, as belonging in a sort of no-man's land between the periods, a twilight zone included in neither of the periods. Although we have learned that many mountain-making formations surely begin and end within the span of time defined as constituting a single geologic period, many geologists continue to interpret long-known orogenies as inter-period or boundary-of-period affairs. Thus, Caledonian mountain building is post-Silurian pre-Devonian, and the Appalachian mountain building belongs at the close of the Permian, partly defining the abyss between Paleozoic and Mesozoic. The dating of these crustal movements is not fixed, of course, by misconceptions, even if they appear in textbooks, and it seems clear that critical review of all aspects of the time-placement problem as applied to mountain building has prime importance, both in establishing stratigraphic positions of the absolute-time bench marks and in making interpolations between them.

Although mountain-making diastrophism in any specified area consisting of half, or even all of a continent clearly is not regularly periodic, it may be so for the Earth as a whole. I do not mean by this that each deformation has world-wide expression in major or minor crustal movements, as Stille's thesis suggests, but rather that periodic movements are definitely localized in varying degree. If, as I have pointed out elsewhere [MOORE, 1935, 1936], such deformations are accompanied by eustatic changes in sea level (and this seems reasonably demonstrable by stratigraphic comparisons of late Paleozoic rock successions in Europe and North America), mountain building in any quarter of the globe may be reflected by stratigraphic consequences at opposite ends of the Earth. Coincident rhythmic submergences and emergences of continental areas which are recorded throughout the column find explanation in such a mechanism. GRIGGS [1939] has suggested a convective mechanism for periodic mountain building which conforms to known gravity and structural conditions, as well as to inferred mobility of crustal and subcrustal zones in the Earth. JOLY [1925] and VENING MEINESZ [1934, 1948] have also outlined conditions leading to periodic diastrophism. If such periodicity is firmly demonstrated for any portion of the geologic time scale, and I think that it can, periodicity of like sort may be assumed to characterize other parts, even if records of it are incomplete and ambiguous. Still to be determined is the question of regularity in time spacing of the diastrophic periods. At all events, I judge this to be one of the most promising fields of geologic study concerned with measurement of geologic time.

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(Manuscript received April 19, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1951; open for formal discussion until September 1, 1952.)

THE USE OF LEAD ISOTOPE RATIOS IN ESTIMATING THE AGE OF THE EARTH

Edward McCrady

Abstract--Since the publication of Nier's determinations of the lead isotope ratios in lead ores of different geologic horizons from Precambrian to Tertiary, four more or less independent efforts have been made to use these data as a basis for calculating the age of the Earth. In addition to the fact that more data are required to establish statistical validity for any of these methods, they all share a common fault of assuming that we already know with reasonable accuracy the age in years of one or more of these ores from their field associations. The author contends that the age of the enclosing rocks need have no correlation with the age of the ore if, by the latter, we mean the time of its extraction from source crustal rocks or magmas (not the time of its emplacement). A method is proposed for determining the age of granites in years measured backward from now instead of from the assumed age of any given rock body; and suggestions are made as to how the same data may provide a basis for determining the age of the original crust of the Earth. A least-squares analysis of the data from isotope ratios in lead ores is used to show that the maximal possible age of matter is 5.07×10^9 years. Reasons are given for thinking the real age less than that. The fact that least-squares equations, assuming alpha decay as the sole source of changes in the isotope ratios of uranium and lead in the past, can be extrapolated to the present ratio of U^{235}/U^{238} with a Lagrange multiplier not significantly different from zero, is interpreted to mean that other processes than alpha decay (such as natural fission, isotopic selectivity in crystal formation, and radon loss) have played no significant role in changing the composition of the ores.

The lead in all lead ores is composed of four stable isotopes of weights 204, 206, 207, and 208, in various relative abundances. Of these four, only one (Pb^{204}) may be considered to have remained essentially unchanged in total abundance throughout the history of the Earth, since it is not a product of any known process of natural radioactive decay, nor is it thought to be produced in appreciable quantity by any other kind of nuclear conversion. Pb^{206} and Pb^{207}, on the other hand, are being continually generated by the breakdown of U^{238} and U^{235}, respectively; and Pb^{208}

similarly increases at the expense of Th^{232}. If all four isotopes are treated impartially in chemical processes (some of the evidence that they are treated impartially will appear below), the conclusion seems inescapable that the great differences actually found in their relative abundances in the different ores must represent variations among the radiogenic additions resulting from one or more of the following circumstances. (1) The ores may have been extracted from their source magmas or crustal rocks at different times; in which case, those more recently extracted will have had greater lengths of time in which to accumulate radiogenic lead. (2) The source rocks or magmas may have varied in content of uranium or thorium or both. (3) The intermediate daughter elements may have been differently dispersed before final conversion to lead. (4) Ores differing as a result of any of the above causes may by geologic changes come to be combined in any proportions to form new ore bodies.

The actual data upon which all modern studies of this subject are based were obtained by A. O. Nier and his collaborators [NIER, 1938, 1939ab; NIER, THOMPSON, and MURPHEY, 1941]. Their examination of the isotopic constitution of lead from 25 lead ores collected from all over the world, and from horizons ranging from Precambrian to late Tertiary, showed that those specimens which contain relatively more of any one radiogenic isotope as compared with Pb^{204} also contain relatively more of each of the other radiogenic isotopes. As it is difficult to imagine any other factor which could possibly account for such a result, Nier suggested that the differences between the ores are, indeed, attributable to radiogenic contributions; but his work did not go far enough to evaluate the extent to which the four possibilities mentioned above are realized in nature, or to make any estimate of the age of the Earth.

The first effort to utilize these data as the basis for a calculation of the age of the Earth was that of GERLING [1942]. As his fundamental equation he gives

$$\frac{AcD}{RaG} = \frac{e^{(U^{238}/U^{235})R\lambda_U T} - 1}{(U^{238}/U^{235})(e^{\lambda_U T} - 1)} \qquad (1)$$

which merely asserts that the ratio of the lead isotopes generated during any period of time T is equal to the difference between the ratios of the corresponding uranium parent isotopes at the beginning and at the end of the same period. This will, perhaps, be more obvious if we write his equation in the form

$$Pb^{207}_{\Delta t}/Pb^{206}_{\Delta t} = U^{235}(e^{\lambda'\Delta t} - 1)/U^{238}(e^{\lambda\Delta t} - 1) \qquad (2)$$

where $Pb^{207}_{\Delta t}$ is the amount of radiogenic Pb^{207} produced in the time interval Δt, $U^{235}\,e^{\lambda'\Delta t}$ is the amount of the parent uranium isotope present at the beginning of the interval and U^{235} is the amount at the end, λ' is the decay constant for U^{235}, and the terms in the denominators are analogous. (Perhaps it should be mentioned that the form of Gerling's equation as he wrote it, though not incorrect, is unaccountable. The term $e^{(U^{238}/U^{235})R\lambda_U T}$ is used as the equivalent of $e^{\lambda'\Delta t}$, where λ' is the decay constant for U^{235}. It is true that the product of the ratio of the parents times the ratio of their activities times the decay constant of the one in the numerator is equal to the decay constant of the one in the denominator. This product has a specious appearance of avoiding reliance upon the less well-determined value of λ' by expressing it as a function of λ, which is fairly accurately known. However, the ratio of the parents actually used in the calculation is not the accurately measurable one for today but the one for 130 million years ago. This latter value of the ratio of the parents cannot be calculated unless we already know the values of both λ and λ'. Hence nothing is gained by this obscure formulation, so I have rewritten (1) in the more obvious form (2).)

Gerling next observed that the radiogenic isotopes of lead are least abundant in the galena from Ivigtut, Greenland, and correspondingly he takes this ore to be the oldest known sample of the Earth's crust. The difference between the isotope ratios in it and those in any other ore must then represent the radiogenic contribution in the interval of time between the extractions of the respective ores. Values for this radiogenic component for each isotope in each ore are then obtained from Nier's data by subtracting the Ivigtut value from the corresponding values in eight other ores which were all of Nier's analyses that Gerling had seen. The average of all of these remainders he takes as the radiogenic contribution between the time of origin of the Ivigtut ore and a time representing the average age of all the other ores. The numbers thus obtained are the values used on the left side of (1) or (2). He then averages the recorded geologic ages of all the ores except the Ivigtut, which average he finds to be 130×10^6 years, and calculates the ratio

U^{235}/U^{238} for the time from the known present ratio and the known decay constants. This gives the ratio U^{235}/U^{238} required on the right side of (1) or (2), and leaves as the only unknown, Δt, the time required to convert lead of the Ivigtut type to a typical lead ore of age 130×10^6 years. It is then possible to calculate Δt, and the value he obtains is 3.1×10^9 years. The real age of the Ivigtut ore would accordingly be the sum of 3.1×10^9 and 130×10^6, which is 3.23×10^9 years.

My criticisms of this approach are: (1) that, at best, it is designed to determine the age of the Ivigtut ore, not the age of the Earth; and (2) that to obtain this value we are required to assume that we already know the ages of all the other ores rather well, at least well enough to allow the whole weight of the argument to rest upon our figure for their average age.

The "age" of the Ivigtut ore as given in Nier's source paper is 600 million years. The age obtained by Gerling differs from this by 2630 million years. Why should we accept without qualm so great an "error" in the recorded age of this ore, and yet assume that we can trust the recorded ages of the others? How many of the others may involve errors of 500 pct? No solution of such doubts is offered in Gerling's paper, nor are the questions raised.

He proposed one other method which consists of accepting the age of the galena from Great Bear Lake as 1.25×10^9 years, and then calculating the time required to convert lead of the Ivigtut constitution to lead like that of Great Bear Lake. The time required turns out to be 2.7×10^9 years. Adding this figure to the time assumed to have elapsed since the formation of the galena gives the age of the Ivigtut ore as 3.95×10^9 years. He concludes that the Earth is not less than three to four billion years old.

Gerling's second method differs from his first only in relying upon the presumed age of one ore rather than the average of eight. I doubt that we are justified in placing much confidence in the figure 1.25×10^9 years as the age of the Great Bear Lake lead. I shall show later on that the "ages" of lead ores which are compatible with the theory that their isotopic constitution is a criterion of age differ inordinately from those assigned on the basis of regional stratigraphy.

Why Gerling considers the ages which he has computed as minimal is not explained, but presumably he means simply that there may be parts of the Earth's crust older than the Ivigtut ore.

Four years later and without knowledge of Gerling's work, HOUTERMANS [1947] in Germany and HOLMES [1947] in Scotland independently undertook to solve the same problem with the same data, but by different methods. For convenience I shall discuss Houtermans' method first.

Houtermans begins with the same equation in the form

$$(AcD)_t/(RaG)_t = [(AcU)_t/(UI)_t](e^{\lambda' t} - 1)/(e^{\lambda t} - 1) \quad \ldots\ldots\ldots\ldots\ldots (3)$$

but his use of it is quite different. He immediately modifies it to a slightly more general form by pointing out that the radiogenic Pb^{207} is the total amount measured in any ore minus whatever was there to begin with $(\beta - \beta_0)$. He does not commit himself definitely as to what he means by "to begin with." He admits that it might mean the time of creation of uranium itself, that is, the time at which uranium began decaying to lead. But he also admits that if in the early history of our planet there were, for some time after the origin of matter, gaseous or liquid stages which permitted fairly uniform mixing of primordial and radiogenic lead, then the ratio β_0/α_0 would represent the ratio of Pb^{207}/Pb^{206} at the time of formation of a solid crust, because no sample of the isotopic constitution of lead could have been extracted and preserved until after it was possible for atoms to be locked up in the rigid structure of solids. Once a crust had been formed, however, solid ore bodies, resulting from chemical separation of lead, would differ in isotopic constitution in accordance with their different times of extraction. Those extracted at the time of crust formation would be identical or very nearly so in isotopic constitution, whereas those more recently segregated from their magmatic or crustal sources would contain proportionately greater amounts of the radiogenic isotopes.

These considerations (not fully explicit in Houtermans' paper) provide the necessary theoretical justification for his converting (3) to the form

$$(\beta - \beta_0)/(\alpha - \alpha_0) = (1/139)\,(e^{\lambda' w} - e^{\lambda' p})/(e^{\lambda w} - e^{\lambda p}) \quad \ldots\ldots\ldots\ldots\ldots (4)$$

in which α and β represent the content of Pb^{206} and Pb^{207} in a given ore (expressed as a multiple of Pb^{204} in the same ore); α_0 and β_0, the corresponding proportions at the time of formation of the Earth's crust; 1/139, the present ratio of U^{235} to U^{238}; w, the number of years between the origin of the crust and now; and p, the years elapsed since the extraction of the ore.

If the mixing was essentially complete prior to crustal formation, α_0 and β_0 are approximately constants. W is assumed constant, and for ores of one age p is constant. Therefore, on an α, β graph showing the relation of α to β, (4) is a straight line for all ores of one age. The different straight lines representing different ages are called isochrons. Their slopes are $(\beta - \beta_0)/(\alpha - \alpha_0)$, and they all pass through the point α_0, β_0.

Since the ratio of uranium to lead is different in different parts of the crust, the values of α and β may differ in ores of the same age if they are from different localities; but their values will lie on the same isochron. Any two different points on an isochron will determine its slope. Therefore, if we measure α and β accurately in any two ores of the same age p (which age also must be known accurately), we can determine the slope $(\beta - \beta_0)/(\alpha - \alpha_0)$ and thus solve for w. The value found is $w = (2.9 \pm 0.34) \times 10^9$ years. This gives us the age of the crust but not its original isotopic constitution β_0/α_0. The latter, however, may be obtained if we know the isotopic constitution of one other ore of a known and different age. For then, with both w and p known, the slope of the second isochron is fixed, and the intersection of the two isochrons is the point α_0, β_0.

This method has considerable mathematical elegance and two clear advantages over that of Gerling. First, it is concerned with the age of the crust of the Earth itself, instead of the age of the Ivigtut ore; and it does not require us to find or identify the oldest portion of the crust in order to estimate its age. Second, by way of prerequisite information, it requires that we have independent knowledge of the age of only three ores, instead of the much larger number utilized by Gerling in his first method.

On the other hand, it must be admitted that until we have more certain knowledge than we now have of the ages of the individual ores, this method cannot be applied with precision; and in any case, it is based upon what must be oversimplified geological assumptions, such as, that the whole crust formed at one time, that all lead ores are derived from the crust, and that no mixing or recycling of ore lead has occurred.

Holmes utilizes the same data to estimate the time required to convert lead of the Ivigtut type to lead of the Joplin type, that is, lead with the least proportion of radiogenic isotopes to lead with the greatest. Thus far, except for his choice of a reference ore, Holmes is employing Gerling's second method, though he extends it in a new way. For his purposes (2) should be written

$$(Pb_J^{207} - Pb_I^{207})/(Pb_J^{206} - Pb_I^{206}) = U_t^{235}\ (1 - e^{-\lambda' \Delta t})/U_t^{238}\ (1 - e^{-\lambda \Delta t}) \ldots\ldots\ldots \quad (5)$$

That is to say, the amount of U^{235} in the source at the time t when the Ivigtut ore was extracted was such as to require a certain time interval (Δt) in which to produce the additional Pb^{207} found in the Joplin III ore ($Pb_J^{207} - Pb_I^{207}$); and that there was also in the source at the same time exactly the right amount of U^{238} to produce the excess Pb^{206} of the Joplin III ore in the same number of years, Δt. Holmes and Houtermans also made use of the data from Pb^{208} and Th^{232}, but since the entire argument can be illustrated by reference to the uranogenic leads alone, I shall omit Pb^{208} from most of the present account.

In order to establish the time scale correlated with lead isotope ratios we must solve this equation for Δt. All of the terms on the left are known from direct measurement, so we may substitute for them their numerical equivalent, which is 0.194. We know the ratio U^{235}/U^{238} which exists today, and hence may calculate it for any other time by means of

$$U_t^{235}/U_t^{238} = (1/139)\ e^{\lambda' t}/e^{\lambda t} \ \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where λ' and λ are the decay constants for U^{235} and U^{238}, respectively, and 1/139 is the ratio used by Holmes for U^{235}/U^{238} today. This calculation is valid provided that the only factors which have influenced the relative abundance of the uranium isotopes at different times in the past are their different rates of radioactive decay.

It is clear from these equations that any value whatsoever of the time interval Δt (provided that it is compatible with the deposition of the Joplin ores in the past, not the present or future, that is, $\Delta t < t$) can be made to satisfy the equation as well as any other by a suitable adjustment of the value of t. The older the ores, the shorter the interval required to account for a given disparity in isotope ratios, because the greater the amount of uranium generating lead. Hence it is necessary to know the age of at least one of the ores independently before we can proceed any further. Or, to express it mathematically, we have an equation in two unknowns (t and Δt) which cannot be solved without an additional independent equation relating the two unknowns. The age of the older ore is t. If this were known, there would remain only one unknown, Δt, and we could solve for it directly. The age of the younger ore is $t - \Delta t$. If this were known, we should have the required second equation relating the unknowns, which would enable us to solve for both. But unless we know the age of either the older or the younger ore, we can proceed no further.

We thus find ourselves faced with the same kind of difficulty encountered by Gerling and Houtermans, but with these advantages: (1) Instead of requiring previous knowledge of the age of three or more ores, we reduce the number to one as in Gerling's second method. (2) The ore whose age we require is, by isotopic evidence, the youngest of them all, so that any numerical error in years will presumably be minimal. (3) If we construct decay curves through points separated by a maximal time interval, any angular error involved should be less than in curves fixed by points closer together on the time scale. Nonetheless, we should admit that this is in principle still a weak point in the argument. The apparent rigor of the mathematical approach is vitiated by its dependence upon information not derived by the same or equally rigorous means.

At this point Holmes assumes that we know the age of the Joplin ores to be about 100 million years, and cites BASTIN [1939, p. 131] for his authority. In that report there seems to be no very confident statement about age anywhere, and certainly no commitment in terms of years. The nearest approach is the following sentence [p. 132]: "If the lead and zinc mineralization of the Mississippi Valley region is to be attributed to magmatic sources, a mid-Cretaceous age is the most probable on the basis of present knowledge." There is not, at present, unanimity about the magmatic origin of these ores, and the assignment of an age of 100 million years to mid-Cretaceous strata is not the result of any evidence in that paper. I do not mean to say that this is a bad figure. There is considerable reason to think it a good one, though on the basis of helium ratios from magnetites and strontium-rubidium ratios a reasonable case can be made for assigning that age to late Jurassic instead of mid-Cretaceous. However, the real point I want to make is that, excluding isotope ratios, whatever geologic evidence there is at present for dating the Joplin ores applies with as much validity to one of them as to another; whereas, Holmes' argument, if followed through, leads to the conclusion that there is an age difference of 400 or 500 million years between Galena II from Joplin and Galena III from Joplin. The argument that two ores have ages differing by this much (which in this case means 400-500 pct) should not be allowed to rest upon the argument that they are both approximately the same age, namely, 100 million years. Logically this is bad, but it will be shown below that in practice it makes surprisingly little difference.

If we accept the figure, 1×10^8 years, as the probable age of Joplin Galena III, then, using the values for the decay constants which Holmes used, it is easy to show from (5) and (6) that the Ivigtut ore must have been extracted from its source some 2.76×10^9 years ago (that is, $t = 2.76 \times 10^9$ and $\Delta t = 2.66 \times 10^9$). But we should not accept this conclusion without recognizing that it means jettisoning the geologic age given in Table 1 of the same paper by Holmes, where the Ivigtut ore is recorded as "Late Precambrian, 600 million years," which is less than one-fourth of the new figure. If we can discard this age so lightly, were we justified in laying so much store by the age of the Joplin ore which we needed to know in order to make this calculation?

Holmes' next step is to obtain dates for all the other ores by converting a scale of isotopic abundances to a time scale. To do this he simply graphs Nier's data, plotting the individual isotopic abundances on the ordinate and the sums of the isotopic abundances for each ore on the abscissa, and then converting the latter scale to time by taking the interval between the Ivigtut ore and the Joplin III as 2.66×10^9 years. Radiogenic curves constructed through the points for these two extreme cases (that is, the ore with the least and the ore with the greatest sum of the isotopic abundances) then turn out to fit the data from the other ores with remarkable closeness. In spite of some small scatter, the fit is so good as to leave little room for doubt that the differences in isotopic constitution are, indeed, primarily the result of radiogenic additions, and that the parents were distributed approximately uniformly in the source magmas or rocks at all times during the billions of years involved in the history of these ores, and at all of the places around the globe from which those ores were extracted.

These conclusions in themselves are impressive, but we should not allow them to give us an unjustified confidence in the "ages" indicated for the individual ores. In the first place, Holmes was not justified in using a linear conversion of the isotopic-abundance scale to a time scale. The relation is really exponential, and if the proper conversion is made, the "ages" of all the ores, except the Ivigtut and Joplin III which provide the fixed points, will be pushed farther back

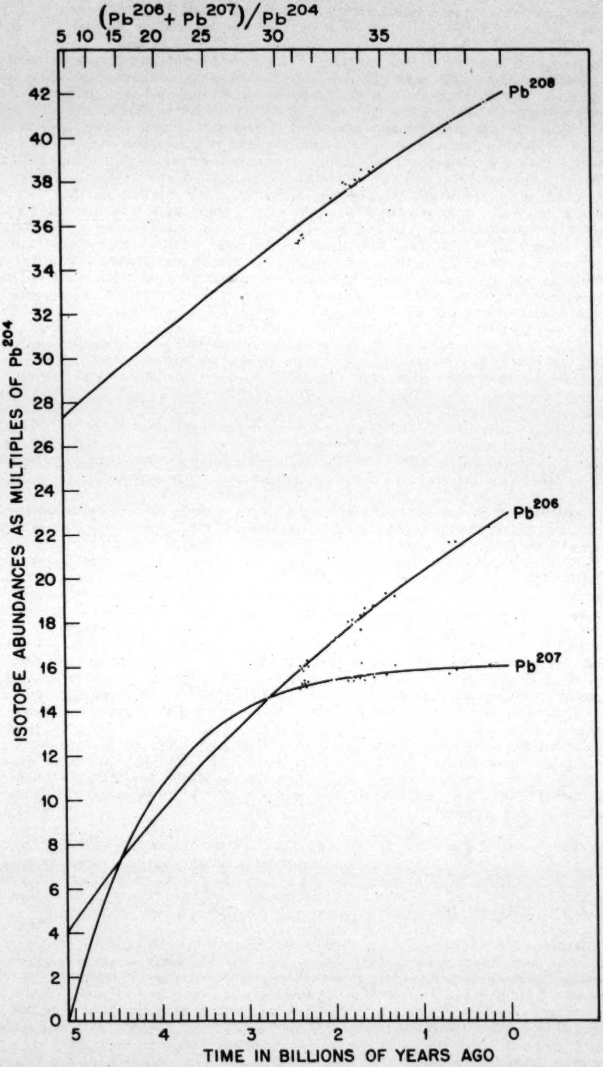

Fig. 1--Relationships of isotopic abundance in time than is indicated in Holmes' Figure 1; but they will still fit the decay curves, since the latter will be appropriately altered in shape by the new scale (see my Fig. 1).

Furthermore, the "ages" thus indicated need not bear any fixed relation to the ages of the rocks in which the ores are found. The familiar statement that ores cannot be older than the rocks in which they occur means only that they almost certainly had to be deposited in their present locations after the origin of the rocks which contain them. But we cannot be certain that the times of their final emplacement correspond to the times of their extraction from their original sources. We know nothing about how often they may have been redissolved and transported and redeposited, or to what extent they may be mixtures of extracts of different ages. Any two or more of these ores could be combined in any proportions one might arbitrarily choose and the isotopic assay of the resulting mixture would fall rather neatly into position on or near the decay curves at a time value intermediate between the ages of the original components. This is because throughout the interval of time sampled by these ores the decay curves are very nearly linear. The apparent age of the composite specimen could be made to fall anywhere between the ages of the oldest and the youngest constituents by varying the proportions of the mixture; but it would, of course, be impossible to make a mixture the isotopic composition of which would resemble at all closely that corresponding to any age greater or less than the extremes of its components. Accordingly, when one says that the Ivigtut ore is 2.76×10^9 years old, one means that it is either an unmodified primary sample of the Earth's lead extracted that many years ago, or that it is a mixture of younger and older samples. In any case, it is as certain as our knowledge of the decay constants and the age of the Joplin III ore that the Ivigtut ore includes some ingredients at least that old, unless some other factors than alpha decay have significantly altered the abundances of the parents in the past.

This consideration provides one kind of limiting value for the age of the crust of the Earth. The Earth cannot be younger than the oldest ore extracted from it, and the oldest ore thus far found cannot be younger than 2.76×10^9 years. Of course, the Earth may be older than that, for we have no assurance that we have yet found the oldest portion of its crust.

A maximal limit for the age of the Earth is also proposed by Holmes on the basis of extension of the radiogenic curves backwards beyond the point where they cross to the point where they reconverge at 4.46×10^9 years on his scale. This point, he says, corresponds to the time at which U^{238} and U^{235} "first began to disintegrate." I am unable to find any justification for that conclusion. Equations of the form $y = a + b\,(1 - e^{-\lambda x})$ may, by varying the values of a, b, and λ, be made to intersect: (1) at no point at all; (2) at any one point between their limits; or (3) at any two points between their limits. Mathematically there are 27 possible combinations of conditions (combinations of $a =, >, <, a'$; $b =, >, <, b'$; $\lambda =, >, <, \lambda'$); but since we know that $\lambda < \lambda'$, the number that we need to consider in this case is reduced to nine. Among these nine only one would give the result postulated by Holmes. This condition is that $a = a'$ and $b > b'$; that is to say, the amounts of Pb^{206} and Pb^{207} were equal to begin with, and U^{238} was more abundant than U^{235}. There is not at present any compelling evidence in support of these assumptions. It is possible to extend his curves backward in time even farther than he did without any inconsistencies or unreasonable assumptions. The only question involved is, what fraction of the Pb^{207} present in the Earth today has been contributed by the decay of U^{235} since the formation of the crust? We do not know the answer at present, but if we are interested in a maximal limit for the age of the Earth, we can be sure of one thing - not more than all of its Pb^{207} came from U^{235}. So if we extend the curves back to the point where Pb^{207} goes to zero, we can be confident that we have reached a real limit in time.

I have made such a graph and will discuss it in more detail below, but for the moment suffice it to say that the maximum possible age thus determined turns out to be approximately 5×10^9 years. The actual figure may be somewhat in error, due to inaccuracies in our data and constants, but in principle such a limit is absolute. It is impossible for the Earth to be older than the limit thus indicated and to contain lead and uranium with their present isotopic ratios.

Holmes went further than the suggestion of limits. He proposed a method for fixing an actual value between them. He pointed out that instead of passing his decay curves through the points representing the Joplin III and the Ivigtut ores he might have chosen any other pair of ores as the fixed points. All such curves for Pb^{206} he claimed should ideally converge to pass through one point representing the original amount of Pb^{206} in the crust of the Earth, and a similar pencil of curves for Pb^{207} should determine its original amount. He then constructed such curves through all possible pairs of pairs which share one ore in common, discarded those combinations which resulted in coincident, or parallel, or divergent lines, and found the mode of the time values of the points of intersection of the remaining Pb^{206} and Pb^{207} curves. He thus obtained a figure of approximately 3×10^9 years for the age of the crust of the Earth, which in a later paper [HOLMES, 1947] he modified to 3.35×10^9 years.

Holmes offered no explanation of the theory behind this method, and it is not easy to be sure what he had in mind. Presumably he means that the failure of the dots on the graph to fall exclusively upon a single decay curve is due to inequalities in the distribution of the parent elements. It has already been mentioned that the extent to which the theoretical decay curves fit the data is strong evidence of the approximately uniform distribution of the parents in the source rocks or magmas, but if the distribution had been completely uniform, no pencils of curves such as Holmes postulates would occur. All of the curves would be coincident throughout their lengths, and no solution such as he proposes would be possible. On the other hand, if at the time of formation of the crust the ratio of uranium to lead were greater in one locality than in another, the ratios of Pb^{206} and Pb^{207} to Pb^{204} would increase more rapidly where uranium was more abundant. Such inequalities in the concentration of the parent would produce a scattering of the dots on such a graph as Figure 1, which scattering might increase with the age of the Earth, and conversely, reduce to zero at the time of formation of the crust. A statistical study of the standard deviations of the isotopic abundances in differing age groups might thus offer a means of calculating the time of formation of the crust, unless recycling and mixing have occurred frequently enough to obscure the trend or even establish an equilibrium.

Holmes did not make such a study of standard deviations. Instead, he constructed pencils of radiogenic curves utilizing different combinations of ores and he took the mode of their intersections as the age of the crust. This method would require a very much larger number of ores than he employed to make a statistically valid determination. JEFFREYS [1948] criticized Holmes' method and then proceeded to make the study mentioned in the preceding paragraph. He classified the ores analyzed by Nier into four age groups averaging 41, 214, 700, and 1243 million years, respectively. Next he found the mean and standard deviation for each isotope in each age group. He then set up his equations of condition and solved them by the method of least squares. The result gave values of t_0 ranging from 1.25×10^9 years to 1.43×10^9 years, but with X^2 values so high that he concluded, "Plainly there is some source of variation besides those contemplated in the hypothesis."

My criticism of this approach is that the "ages" used in establishing his groups of ores are the "geological ages" given in Nier's papers, and do not correspond at all to the "ages" obtained by converting the scale of isotopic abundances to a time scale. A study of the standard deviations of the isotopic constitution of ores of different age groups is only as dependable as our knowledge of the ages of the groups.

The most recent attempt to use Nier's data in determining the age of the Earth is that of BULLARD and STANLEY [1949]. Like Jeffreys, they used a least-squares method, but instead of classifying the ores into four age groups they combined all specimens of a single "age" and locality and treated the mean of each such combination as a single specimen. This reduces the number of "specimens" from 25 to 13. But they found that 96 pct of the sum of the squares was contributed by two of the "specimens," which represent five of Nier's samples, so they judged these to be anomalous in some way and discarded them. From the remaining 11 they obtained an age for the original crust of 3.29×10^9 years. They point out that it is "unsatisfactory that this result can only be obtained after rejecting two sets of data out of 13," and add that "it is most desirable that further lead ores be examined to see how common such exceptions are. Until they have been shown to be rare, or to occur only in some well-defined class of ores, it will remain possible that the apparent agreement of the 11 concordant ores is the exception, and that the deviation of the remaining two is the rule." Aside from this objection, I want to point out that their method, like all the prior ones, rests upon the assumption that the "ages" assigned to the ores in Nier's tables are dependable. It is permissible to question which of the two criteria of age is the more dependable, but as long as one uses equations implying isotopic criteria of age it is simply inconsistent to employ age data incompatible with the equations.

As this seems to be the most fundamental fault in all previous work, it deserves some emphasis. The ages assigned to the ores in Nier's tables differ from those calculated on the basis of isotope ratios in many cases by a whole order of magnitude and at one extreme by nearly two orders of magnitude (that is, nearly a factor of 100). Furthermore, even the chronological sequence becomes completely scrambled when the ores are arranged by the sum of the isotopic abundances. To illustrate this, if the ores are numbered from 1 to 25 in chronological succession according to geologic field evidence, and then rearranged into chronological sequence as indicated by the sums of the isotopic abundances, the series becomes 11, 9, 10, 4, 7, 1, 6, 2, 16, 18, 12, 3, 13, 14, 17, 5, 15, (20), 22, 21, (24), 25, 23, 8, 19. (Those in parentheses had a Precambrian geologic assignment but no year age.) This is almost as complete a scuttling of field evidence as one could arrange, but it does not, in itself, necessarily mean that the isotopic "ages" are wrong. I have already mentioned that ores may be older or younger than the rocks with which they are associated, and, indeed, may be of mixed age. Lead might be extracted today from rocks or magmas, and might percolate through strata of various ages and be deposited in some very ancient formation. Or conversely a very ancient ore body might be eroded away today and redeposited in a young rock, or again it might be mixed with lead from other sources and ages before being redeposited.

If we could date any one ore accurately, then we could establish time intervals between it and the others by means of radiogenic curves. What we need, therefore, is at least one accurately dated ore upon which to hang our time scale. Perhaps the surest way to avoid any uncertainty about the time of extraction of an ore is to extract one ourselves and record the time; in other words, dissolve the lead from a primary igneous rock now and determine the isotope ratios characteristic of an ore extracted today, then measure time backward from now. Fortunately, if we use granite as the source we should be able to fix two points upon the time scale--now, and the time of solidification of the granite from a magma. The reason for this is that as a magma cools its different ingredients solidify at different times and into different mineral components of the final rock. It is well known that the substitution of potassium by lead takes place in the feldspars as noted by **RANKAMA** and **SAHAMA** [1950, p. 104]. "Both potash feldspar and lead feldspar are known as natural or artificial compounds. They have similar crystal structures, in which the K and the Pb^2 ions occupy identical positions. Therefore, natural potash feldspars, which contain only minute quantities of lead, may also be considered to be isomorphic mixtures of the compounds $KAlSi_3O_8$ and $PbAl_2Si_2O_8$, with lead replacing potassium in random distribution." Accordingly, if we segregate the orthoclase from a granite and determine the isotope ratios of the lead in it, we should have a sample showing the constitution of the lead at the time of solidification of the granite. As uranium and thorium occur in other parts of the rock and contribute radiogenic lead we should extract the lead from the rock as a whole and determine its isotopic constitution also. The difference between the isotope ratios in the rock as a whole and in the orthoclase alone should represent the radiogenic additions between the time when the granite solidified and the time when we dissolve it and separate the lead. This information would make it possible to calculate the age of the granite in years measured backward from now, and at the same time would give us the isotopic constitution of two accurately dated ores--the contemporary one from the whole rock, and the ancient one from the orthoclase alone.

We are very much indebted to E. S. Larsen, Jr., of the U. S. Geological Survey, who has separated the orthoclase from several granites of known geologic horizons and presented the material to us. John W. Ruch, Allen J. Vander Weyden, and I are now engaged in dissolving the rock, extracting the lead, and analyzing its isotopic constitution. We expect to have data within the next few months which will be published separately.

Meanwhile some important results have been obtained from least-square analyses. There is at present no reason for requiring radiogenic curves to pass through the points representing the isotope ratios in the Ivigtut and Joplin ores, as was done in Holmes' work. As these two ores, like any others, may be in some degree atypical, it is much better to employ, for lines of reference, curves which make the best possible fit to all of the data collectively. For this purpose it is convenient to write the fundamental equations in the form

$$Y' = a - b\,(e^{\lambda t} - 1) \quad \ldots\ldots (7)$$

$$X' = a' - b'\,(e^{\lambda' t} - 1) \quad \ldots\ldots (8)$$

where $Y' = Pb^{206}$ at time t measured in years backward from now; $a = Pb^{206}/Pb^{204}$ in source rocks or magmas now; $b = U^{238}/Pb^{204}$ now; and $\lambda = 1.543 \times 10^{-10}$. The second equation is analogous for Pb^{207}, and U^{235}, and $\lambda' = 9.76 \times 10^{-10}$. In this form it will be possible to solve for the present amounts of the lead isotopes and their uranium parents in source rocks or magmas today, which may be helpful in identifying the sources. Since we know that the present ratio of the uranium parents is

$$b/b' = 137.8 \quad \ldots\ldots (9)$$

we may make the curves conform to this condition by inserting a Lagrange multiplier. The function to be minimized, therefore, is

$$V = \Sigma(Y - Y')^2 + \Sigma(X - X')^2 + L(b - b'\,137.8) \quad \ldots\ldots (10)$$

To obtain the values of a, b, a′, b′, and L, which make the sum of the squares of the errors under this limitation a minimum, we must make

$$(\partial V/\partial a)\, da + (\partial V/\partial b)\, db + (\partial V/\partial a')\, da' + (\partial V/\partial b')\, db' + (\partial V/\partial L)\, dL = 0 \quad \ldots\ldots (11)$$

Solution of this equation gives

$$a = (\Sigma Y + b\Sigma e^{\lambda t} - nb)/n \quad \ldots\ldots (12)$$

$$b = [\Sigma e^{\lambda t} \times \Sigma Y - n\Sigma(e^{\lambda t} Y) - nL]/[n\Sigma e^{2\lambda t} - (\Sigma e^{\lambda t})^2] \quad \ldots\ldots (13)$$

$$b' = [\Sigma e^{\lambda' t} \times \Sigma X - n\Sigma(e^{\lambda' t} X) - 137.8\, nL]/[n\Sigma e^{2\lambda' t} - (\Sigma e^{\lambda' t})^2] \quad \ldots\ldots (14)$$

$$a' = [\Sigma X + b'\Sigma e^{\lambda' t} - nb]/n \quad \ldots\ldots (15)$$

$$L = \left\{c\, [\Sigma e^{\lambda' t}\Sigma X - n\Sigma(e^{\lambda' t} X)] - c'\, [\Sigma e^{\lambda t}\, \Sigma Y - n\,\Sigma\,(e^{\lambda t}\, Y)]\right\}/N\,(137.8\, c + c') \quad \ldots\ldots (16)$$

where $c = 137.8\, [n\, \Sigma e^{2\lambda t} - (\Sigma e^{\lambda t})^2]$ and $c' = n\,\Sigma\, e^{2\lambda' t} - (\Sigma e^{\lambda' t})^2$.

In order to apply these equations to the data we must locate the ores on a time scale consistent with the hypotheses on which the equations are based, that is, a time scale based upon isotopic abundances. To approximate this I have computed the age of the Ivigtut ore by Holmes' method, then passed radiogenic curves through the points representing the Ivigtut and Joplin ore data on the time scale thus derived, and calculated the sum of the isotopic abundances as a function of time by the relation

$$Pb_t^{206} + Pb_t^{207} = a - b\,(e^{\lambda t} - 1) + a' - b'\,(e^{\lambda' t} - 1) \quad \ldots\ldots (17)$$

in which a and a′ are the amounts of Pb^{206} and Pb^{207}, respectively, in contemporary sources (obtained by projecting the curves to present time), and b and b′ are the amounts of U^{238} and U^{235} in contemporary sources (the only values which will allow the curves to pass through the Joplin and Ivogtut data at the points assigned them on the time scale), all expressed as multiples of Pb^{204}. The results are given in Table 1. Ages strictly consistent with the sum of the isotopic abundances

Table 1--Computation of rock ages

Ore no.	Pb^{206}	Pb^{207}	$Pb^{206} + Pb^{207}$	Isotopic age	Age of enclosing rocks[a]
				Years × 10^9	Years × 10^9
11	22.38	16.15	38.53	0.10	0.100
9	21.65	15.88	37.53	0.42	0.100
10	21.65	15.74	37.39	0.53	0.100
4	19.22	16.17	35.39	1.27	0.025
7	19.30	15.73	35.03	1.35	0.060
1	18.85	15.66	34.51	1.50	0.025
6	18.71	15.70	34.41	1.55	0.025
2	18.67	15.45	34.12	1.65	0.025
16	18.46	15.66	34.12	1.65	0.220
18	18.43	15.61	34.04	1.66	0.220
12	17.75	16.21	33.96	1.69	0.175
3	18.40	15.53	33.93	1.70	0.025
13	18.10	15.57	33.67	1.77	0.220
14	18.20	15.46	33.66	1.78	0.220
17	17.95	15.57	33.52	1.80	0.220
5	18.07	15.40	33.47	1.84	0.025
15	17.36	15.46	32.82	2.00	0.220
20	17.15	15.45	32.60	2.05	(Precambrian)
22	16.07	15.40	31.47	2.35	1.200
21	16.27	15.16	31.43	2.36	0.800
24	15.83	15.45	31.28	2.39	(Precambrian)
25	15.93	15.30	31.23	2.40	1.330
23	15.93	15.29	31.22	2.41	1.200
8	16.04	15.11	31.15	2.43	0.060
19	14.65	14.65	29.30	2.78	0.600

[a] **Based on Holmes' time scale.**

on least-square curves for all the data would be expected to be somewhat different from these which are based upon curves passing through the Ivigtut and Joplin III data (used here as a first approximation), but the differences turn out to be negligible.

The scrambling of the geologic sequence should be noted, and it should be admitted that it is surprising to find only three of the 25 ores (the three from Joplin) indicated as younger than 550 million years, though 18 of them occur in post-Cambrian strata. All of the ores are indicated as considerably older than the presumed ages of the rocks which contain them except the single specimen upon which the time scale hangs, and which remains fixed arbitrarily. What if we are considerably in error in supposing the Joplin III to be 100 million years of age? R. L. Macklin and G. B. Knight (unpublished) pointed out that varying the assumed age of the Joplin ores throughout the entire range of post-Cambrian time (that is, from 500 million years to zero) makes the calculated age of the Ivigtut ore change only from 2.8×10^9 to 2.6×10^9 years, some seven per cent. Thus a very wide range of error in this datum would have so slight an effect upon the time scale that it need give us little concern. A much more sensitive factor in the equations is the value of λ' in which a change of ± 2 per cent makes a change of ± 3.6 per cent in the age of the Ivigtut. This kind of error is thus amplified rather than diminished in the age calculations; but even so, as the uncertainty in the value 9.76×10^{-10} yr^{-1} is considered to be only about two per cent, we still are not dealing with any great source of error.

For the purpose of testing the theory further we therefore seem justified in accepting the ages thus determined as having no serious sources of error in them. Accordingly, I have used the data of Table 1 in solving (12) to (16) inclusive, and obtained the following numerical values for the constants

$L = 0.041466$
$a = Pb^{206}$ in contemporary sources = 23.019
$a' = Pb^{207}$ in contemporary sources = 16.174
$b = U^{238}$ in contemporary sources = 15.874
$b' = U^{235}$ in contemporary sources = 0.11519

It is interesting to compare these figures with data derived from analysis of crustal rocks. According to EVANS and GOODMAN [1941] and KEEVIL [1944], the ratio Th/U in granitic rocks averages 3.4, and in basaltic rocks 3.3. According to my least-squares solutions this ratio in the sources of these ores should average $52.043/15.989 = 3.26$, which differs from the above granite average by 4.12 pct and from the basalt average by 1.2 pct.

Since the time scale was based upon the decay constants of U^{238} and U^{235}, both of which are very different from the corresponding factor for Th^{232}, and since one of the Th^{232} daughters, Rn^{220}, has, on account of its short half-life, much less chance to escape while gaseous, than has the corresponding daughter for U^{238}, it is conceivable that the time scale based on the uranium data would not be confirmed by the thorium data. To test this, I have used the uranium time scale and calculated what amount of Pb^{208} should have been present in the Ivigtut ore if the present amounts of Pb^{208} and Th^{232} in the source correspond to my least-squares solutions. The result gives 34.68. The actual amount found was 34.48. The error is 0.527 pct.

Similar tests were applied to the uranium lead data. The lead in granitic rocks is said to average 20×10^{-6} gm/gm. The sum of the isotopic abundances in sources today according to my least-squares solutions is 82.62 (in multiples of Pb^{204}). Therefore, the amount of Pb^{204} in sources today should be $(20/82.62) \times 10^{-6} = 0.242 \times 10^{-6}$ gm/gm. The same equations indicate that the amount of uranium in sources today should be 15.98 times the amount of Pb^{204} = 3.87×10^{-6} gm/gm. The average amount found in granites is about 3.31×10^{-6} gm/gm.

In general, the results of such tests seem very favorable, and the predicted values for the constants appear quite reasonable, though this need not necessarily mean that granite or basalt should be regarded as the source of all the ores, or even of most of them. We do not yet have comparable data from effusive rocks, and such data may agree with the calculated values just as well.

We may now find the maximum possible age of the Earth by solving (8) for the time when there was no Pb^{207} in it at all. This makes $a' = b' (e^{\lambda' t} - 1)$, from which we obtain

$$t = (1/\lambda') \log_e [(a' + b')/b'] = 5.07 \times 10^9 \text{ years}$$

If the Earth, or any assemblage of matter which was later to be incorporated into the Earth, existed prior to that time, either there was no U^{235} in it to produce Pb^{207} (which seems improbable, because the amount of U^{235} increases as we go backward in time) or some process destroyed all of the Pb^{207} produced prior to that (for which latter possibility we can find no physical basis). If matter existed as far back as this limiting time, the original amounts of the lead isotopes were

$$Pb^{207} = 0$$
$$Pb^{204} = 1$$
$$Pb^{206} = 4.17$$
$$Pb^{208} = 27.41$$

However, it is unlikely that no Pb^{207} at all was formed to begin with, since the theory of nucleogenesis accounts for the origin of the heavier atoms from the lighter ones by means of neutron capture with or without subsequent radioactive decay. There would seem to be no way of producing the original Pb^{208} without passing through a Pb^{207} stage or a Tl^{207}, which would decay to Pb^{207}. So the origin of matter must have come sometime after 5.07 × 10^9 years. Figure 1 shows that the starting ratios could have had any of the values indicated by time intercepts on the curves between 5.07 × 10^9 years and the age of the oldest ore yet found. The initial amount of Pb^{207} could have been less, equal to, or greater than the initial amount of Pb^{206} depending on when the "atom cooking" began. In any case, the original abundance of Pb^{208} was the highest of the three. If the cross-section for neutron capture can be measured for each of these isotopes with sufficient accuracy to determine the relative abundances in which they were first created, the ratios thus determined should theoretically correspond to one of the time intercepts on these least-squares curves, and correspondingly should fix the time of creation of matter.

The fact that the Lagrange multiplier has a value other than zero is a point of great theoretical interest, provided, of course, that the difference from zero is statistically valid. If the differences between the lead ores are due solely to the alpha decay of the parents, and if the relative abundances of the parents in the past have changed solely as a consequence of their different rates of alpha decay, then the result of the calculations should be the same whether a Lagrange multiplier were included in the equations or not. In other words, the value of L should be zero. A significant departure from zero would indicate that either the relative abundances of the parents, or of the daughters, or of both, have been changed in the past by some factor or factors not included in our equations. Accordingly, an evaluation of L constitutes a test of the extent to which such possible factors have influenced the data.

It is easy to show that omission of L from the equations will change the indicated value of U^{238}/U^{235} in contemporary sources by 18.5 pct, and that an error of that magnitude in the accepted value, 137.8, is out of the question. But it is also easy to show that such an error would result from an error of only 1.45 pct in the least-square value of Pb^{206} today and of only 0.69 pct of the Pb^{207} today, both of which are well within the probable error of the data. Accordingly, we must conclude that there is not at present any evidence that factors other than alpha decay have appreciably affected the abundances of the uranium or lead isotopes in the past two or three billion years.

This is important, because it throws light on a number of questions which might not be easy to evaluate directly. A good many factors are known to be operative in nature which might conceivably make significant alterations in the isotope ratios of uranium and lead. For example, spontaneous fission of U^{238} reduces the amount of U^{238} without adding any Pb^{206}; neutron capture by U^{238} with subsequent beta decay produces Pu^{239}, which by alpha decay becomes U^{235} and finally Pb^{207}; spontaneous fission of U^{235} reduces U^{235} and produces no Pb^{207}; neutron capture by U^{235} produces U^{236} with immediate splitting which also leaves no lead; and (n, 2n) reactions in Th^{232} produce Th^{231} which decays to Pb^{207} instead of Pb^{208}. For these and several other such reactions we do not at present have enough data on cross-sections and energy requirements to permit calculation of the net effect. What information we have suggests that the effect would not be of sufficient magnitude to alter our curves appreciably; but a more convincing argument is the fact that least-squares equations, assuming no significant factor besides alpha decay, predict the contemporary relative abundances of all the parent and daughter isotopes with a Lagrange multiplier not significantly different from zero. Without such evidence one might wonder whether in granites the presence of large numbers of atoms of Si, O, and Mg with high cross-sections for (α, n) reactions might not increase the local neutron flux enough to make the neutron capture reactions appreciable.

Similarly, there are factors which might conceivably alter the isotope ratios in lead ores independently of factors affecting the uranium and thorium parents. Isotopic selectivity during crystal formation, due to the effect of the different weights on solution kinetics, might play such a role. The fact that the percentage weight difference is so slight (about 1.92 pct between the extremes) argues against it, but the most convincing evidence is again from the least-squares curves--the fact that contemporary average ratios of Th/U in the sources, predicted from the lead isotope ratios in the ores, agree with the actual Th/U ratios measured in crustal rocks with such a high degree of accuracy (within four per cent).

The last possible factor to be considered is the loss of intermediate daughters. All three of the decay chains which lead to lead pass through a radon stage which is gaseous, chemically inert, and capable of escaping from the source by diffusion. That this actually occurs is easily demonstrable. Radon in the air can be measured with sufficient accuracy to show that it is greater over granitic rocks than over less uraniferous shales, and that it accumulates during temperature inversions and is dispersed when the normal lapse is restored. If all three radon daughters escaped in equal proportion the result would be merely to retard the increase of radiogenic lead in proportion to Pb^{204}, but would not alter the rate of change of the proportions among the radiogenic isotopes, which latter could still provide a basis for a time scale. However, the half lives, and correspondingly the opportunity for escape, of the three radons are very different. Rn^{222} (called radon), which comes from U^{238}, has a half life of about 3.8 days; whereas Rn^{220} (called thoron) from Th^{232} has a half life of 54.5 sec; and Rn^{219} (called actinon) from U^{235} has a half life of only 3.9 sec. This means that the chain leading toward Pb^{206} has more than 86,000 times as great a probability of being interupted at any one spot on the Earth and completed elsewhere as has the chain leading toward Pb^{207}; and more than 6000 times as great a chance as the chain leading toward Pb^{208}. If such processes have resulted in alteration of the isotopic ratios of lead locally, the result would be the production of lead deficient in or oversupplied with Pb^{206} as compared with the other three isotopes. There is no evidence that the Pb^{206} data are any more variable than the others, and therefore no evidence that this process has altered the composition of the ores appreciably, but the best test will come when we measure the isotopic abundances in rock lead and compare them with our least-squares predictions.

It seems likely that radon atoms do not migrate far through dense granites before transmutation to solids. Escape is possible only from a thin layer very near the surface. If there is a great surface exposure in proportion to volume of the rock, a significant fraction of the Rn^{222} might escape into the air, but even this will in a few days change to solid matter which will eventually settle down as Pb^{206}. So what Pb^{206} is transported away from one spot as Rn^{222} may quite possibly be replaced by what settles down after transportation from other spots. The result may, indeed, be a mixing process which keeps Pb^{206} more uniformly distributed around the world than it would otherwise have been.

What then, in summary, can be said about the extent to which we can be justified in attaching any chronological significance to isotope ratios in lead ores? Modern cosmogony [see GAMOW, 1951] pictures the material universe as arising in an unparalleled explosion some three to six billion years ago with subsequent expansion during the first few hours of which the different nuclear species were built up from the lightest to the heaviest by neutron capture. The process of expansion very quickly reduced the temperature from billions of degrees Centigrade to a level below which nuclear synthesis could not occur and the origin of matter ceased abruptly. For some 30 million years thereafter all matter existed in the form of a rapidly expanding and rapidly cooling gas, and during that time there must have been rather thorough mixing of all the ingredients. After some such period the combined influence of radiation pressure and gravitation produced a clotting of matter into galaxies and these in turn into suns where the process of condensation again provided temperatures of thousands and even millions of degrees Centigrade. We do not know enough about the manner of origin of planets to settle the question whether our Earth in its early stages shared the high temperatures of the Sun, or whether it arose as an aggregation of cooler dust particles. But in any case, the formation of the ores with which the present study is involved could not have taken place until after the gas, or liquid, or cloud of dust particles had condensed into a sphere with at least its surface cool enough to be solid. For in a gas, or a liquid, or a cloud of dust particles, there is no way by which a sample of the lead atoms can be segregated from the remaining matter and preserved.

If, prior to the formation of a solid crust, mixing of uranium, thorium, and lead was rather thorough, then the isotopic composition of all ores formed at first was approximately the same, and this composition represented the original proportions of the isotopes at the end of the first hour or so of nucleogenesis, plus the radiogenic additions up to the time of formation of a solid crust. Thereafter conditions arose which necessarily produced ores of different composition at different times and at different places. Since the chemistry of each of the elements, lead, uranium, and thorium, is different from that of the others, it is to be expected that they would become to some extent unequally distributed throughout the solid part of the Earth. Where the concentrations of the parents and of primeval lead were average in amount the accumulation of radiogenic lead would proceed in a simple and regular way which would make the sum of the isotopic abundances an accurate criterion of time. Where there happened to be more or less than average abundance of the parents in proportion to original lead, the accumulation of radiogenic isotopes would be more or less rapid than average, though the ratio of additions of Pb^{207} to additions of Pb^{206} would continue to be the same function of time as in the average case. However, in these cases the sum of the isotopic abundances (with Pb^{204} as unity) would suggest a younger or older age than the real one, and the data for the radiogenic isotopes would disclose the error by not falling accurately upon the curves representing average conditions for any time value, as long as the time scale is calculated as a function of the sum of the isotopic abundances. This would produce just such a scatter as we actually find in a graph like Figure 1. If there is any wonder, it is that the scatter is as small as we find it; and the consequent errors in the indicated ages must be less than one might have anticipated.

The facts that different portions of the Earth's outermost crust have congealed from magmas at different times, and that parts of this crust may have been locally remelted and reincorporated into underlying magmas perhaps later to be solidified again, do not affect the general account given above. The accumulation of radiogenic lead would go on at presumably the same pace in the molten and in the solid parts. An ore extracted at a given time and place would be likely to have essentially the same composition whether its source had gone through one or several such cycles. The only circumstance which could prevent this would be the segregation of a large percentage of the total lead, or the total parent elements of the area into ore bodies during the solid stage which then escape remelting and remixing. It seems unlikely that the amount of lead, uranium, or thorium segregated into ore bodies is ever a large fraction of the total amount of the element present in such areas of the outermost crust or the underlying magmas as are involved in such processes, or that the segregated elements would in large proportion escape re-cycling. If the underlying magmas are themselves segregated into local pools not sharing in any general mixing, then there is no reason to expect the various pools to differ from one another in isotopic composition to any greater extent than the various areas of the solid crust. They cannot contain anything analogous to ores unless there are certain pools which have at some time in the past exhausted or lost their uranium and thorium, in which case the lead isotope ratios in these should have remained fixed since such time, just as the isotopic composition of an ore remains fixed. Otherwise they should show contemporary ratios with an average like that in contemporary rock lead. It will be interesting to examine contemporary lavas with this in mind, and to compare them with the least-square solutions and with what we find in granite.

In conclusion, the unique contribution from the granite studies should be that for the first time we shall obtain artificially pairs of "ores" which are known to belong to the same genetic sequence, and one of which in each pair is rigorously located on the time scale as now. Accordingly, we may be able not only to determine the date of origin of the granites themselves accurately, but also to obtain evidence concerning many of the questions raised in the above discussion. If the granite was crystallized from a magma which had been part of a common well-mixed pool, the orthoclase lead data should fall upon the least-square curves, and the decay curves from different granites of the same age should intersect on the least-square curves at the orthoclase points. If we are dealing with secondarily granitized sedimentary material, the orthoclase points should in general not be expected to fall upon the least-square curves, and the decay curves from several granites of the same or different ages might meet the least-square curves only at a common and remote time point, as Holmes anticipated for his common ores, but without his statistical difficulties, since we would construct decay curves only through pairs of ores known to belong to the same genetic sequence. Finally, the decay curves may turn out for the most part to intersect the least-square curves in a random fashion, indicating frequent re-cycling and mixing in the past, and our only hope of finding an age of the first crust would then be that the decay curves of the very oldest granites might all converge upon the least-square curves at a common time point. We hope to have new data bearing upon these questions soon.

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(Manuscript received July 20, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1951; open for formal discussion until September 1, 1952.)

CARBON-14 DATES AND ARCHEOLOGY

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Abstract--The development of a method based on the half-life of radio-carbon whereby the age of certain objects can be determined through a series of laboratory tests has provided the archeologist with a time scale which is a valuable aid in placing various types of remains in their proper chronological position. Although the standard deviations in some cases are relatively large, the approximate age indicated is very helpful. Thus far, it has been found that the method is applicable for periods of from 1000 to 25,000 years before the present.

Materials from various archeological sites tested by the carbon-14 method have shown human occupation in the New World to go back as far as approximately 10,000 years before the present. There are intervening stages between 8000 and 8500 years, others centering around 7000 years, with the largest group of dates falling in the 4000- to 5000-year category. The intervals of several hundreds of years in some cases may mean that sites which belong there have not been found or they may indicate a break in occupation with recurring waves of migration. The carbon-14 dates definitely show that beginning about 1500 B.C. there was a marked increase in the population and that it continued to expand until the arrival of the white man.

The development of a method based on the half-life of natural radio-carbon whereby the age of certain objects can be determined through a series of laboratory tests has provided archeologists with a time scale which is a valuable aid in placing various types of remains in their proper chronological position. It is not the purpose of this paper to explain and discuss the method whereby radio-carbon is made to serve such a purpose.

In the course of the various studies it had been determined that the half-life of carbon-14 is about 5568 ± 30 years, which means that one ounce of the material is reduced by decay to half an ounce during the 5568-year period and that half of the remainder decays during the next 5568 years, leaving a quarter of an ounce, and so on. Because of this, Libby was convinced that the amount of carbon-14 present in any particular object would be an indication of the age of that object and proceeded to develop a method for measuring it. Then he and his associate, James R. Arnold, ran a series of tests on samples whose ages had been fairly accurately established by other means but were unknown to them. Material from Egyptian tombs, from archeological sites in our own Southwest, and from redwood trees was provided by different museums and it was found that the carbon-14 dates obtained for them agreed with the known ages within the calculated error of the method. In making the preliminary tests it was found that the most useful materials were plant fibers and wood, charcoal, antler, burned bone, shell, dung, and peat.

Arrangements were then made with a committee representing the American Anthropological Association and the Geological Society of America to obtain samples for testing, and a grant was made by The Viking Fund to assist in the support of the program. Archeologists and geologists began sending in the necessary materials and the series of tests got under way in the spring of 1949. Announcements of the dates obtained were made informally from time to time but it was not until October 1950 that a lengthy series was made public [ARNOLD and LIBBY, 1951]. Libby has sent out informal mimeographed announcements of additional dates in recent weeks, some of which have been published [LIBBY, 1951]. Others will be published in due course.

From the standpoint of archeology, most of the dates reported thus far have been fairly satisfactory, but in a number of instances there appears to be a contradiction between the archeological evidence and the age obtained from the carbon-14 tests. One factor to be considered in this connection is that the older material appears to be more consistent than that of relatively recent times, and even though the error in the method may make a difference of several hundred years in the actual chronology, the results are very helpful and will aid materially in making the syntheses of cultural relationships and developments that are essential to an understanding of past history. For geophysicists and geologists, greatest interest probably attaches to the archeological remains which fall within what may be called geologic time. The results in this Early Man category are in some respects as surprising as a few of those in other fields, but on the whole they are reasonably satisfactory.

The age of the well-known Folsom complex caused considerable comment when the figure pertaining to the type site was released. As a matter of fact, that discussion has continued actively to the present. Unfortunately many of the arguments which it produced were not necessary because the announced date was not that of the culture-bearing horizon of the Folsom complex but that of a fire pit in the fill of a secondary channel which had cut through the original deposit of bison bones and artifacts. Such was known at the time when the first announcement was made, but unfortunately the explanation accompanying the date was not clear. If, as the geologists who have examined this site maintained, the cultural stratum was of very late Pleistocene or early Recent age [BROWN, 1929; BRYAN, 1937], the date of 4283 ± 250 years obviously was wrong or else the geologists were greatly mistaken in their identification of the deposits. Subsequently, material from a Folsom horizon at Lubbock, Texas, was tested and a carbon-14 date of 9883 ± 350 years was obtained. The latter more closely approximates the value estimated for Folsom on geologic evidence. The deposits at Lubbock, in the opinion of E. H. Sellards of the Texas Memorial Museum and his associates, correlate closely with those at the site in the Black Water Draw near Clovis, New Mexico, which ANTEVS [1949] has identified as belonging to a pluvial period which he believes corresponds to the end of the Pleistocene and has estimated the age as being from 10,000 to 13,000 years. It is possible, of course, that some of the material at Clovis is older than the Lubbock site because the Folsom culture undoubtedly lasted over a reasonably long period. Also, certain faunal differences suggest a greater age for some of the Clovis manifestations. The difference between the carbon-14 date and Antevs' estimate is not too great, however, and it seems a fair assumption that the general idea that the Folsom complex is about 10,000 years old is not far out of line.

BRYAN and RAY [1940], after completing their studies at the Lindenmeier site in northern Colorado, another Folsom location, estimated its age as from 10,000 to 25,000 years, with the statement that they believed it nearer the latter than the former. Thus far, it has not been possible to obtain charcoal or other material suitable for carbon-14 tests from that location and we do not know if the estimate is too great or if perhaps the occupation there was earlier than at the similar sites farther south. Geologically the age appears to be somewhat older. Bryan and Ray concluded that the occupation at the Lindenmeier site was in Wisconsin III times or late Mankato. Of interest in this connection is the fact that some early sites on Lime Creek in western Nebraska have been correlated with the Mankato [SCHULTZ and Others, 1949] and carbon-14 dates of 9524 ± 450 years, and 10,493 ± 1500 years have been announced for them. The cultural material is not Folsom, but projectile points of the types found there have been collected elsewhere at sites where the characteristically fluted Folsom type occurs in a lower and older stratum. What this dating means with respect to the Mankato is something the geologist can pass on better than I. In that connection, however, I understand from Libby that the dates for the Mankato, based on other materials, have consistently run between 11,000 and 12,000, with an average age of 11,400 years.

Other archeological remains of about the same age on the basis of carbon-14 dates are

Gypsum Cave in Nevada, with an average of 10,455 ± 340 years for the 6-foot 4-inch level and an average of 8527 ± 250 years for the 2-foot 6-inch level; the Fort Rock Cave in Oregon, 9053± 350 years; and Palliaike Cave at the tip of South America, with 8639 ± 450 years. The deposits in Gypsum Cave where the oldest artifacts were found have been correlated with a dry period immediately following the Provo Pluvial and the age estimate was placed at from 7000 to 9000 years ago [ANTEVS, 1948]. In that case it appears that the estimated and the carbon-14 dates are in fairly close agreement. Archeologically such an age would not be unreasonable, although there is still some question as to the contemporaneity of the associated wooden objects and the sloth dung. In connection with the Gypsum Cave dates it might be noted that the accumulation of material between the 6-foot 4-inch level and the 2-foot 6-inch level, a depth of a little less than four feet, represents approximately 2300 years. The Fort Rock Cave in Oregon is particularly interesting as well as somewhat puzzling. At that location a large number of fiber sandals and some basketry were recovered from beneath a layer of ash which has been identified with the Newberry eruption which presumably followed that of the Mount Mazama eruption which produced Crater Lake [CRESSMAN and WILLIAMS, 1940]. The Mount Mazama eruption has been dated by a carbon-14 test of charcoal from a tree killed by the eruption and a date of 6453 ± 250 years was obtained. WILLIAMS [1942] had previously estimated it to be between 4000 and 7000 years, ALLISON [1946] had placed it at 12,000 to 14,000 years, and HANSEN [1946] on the basis of pollen stratigraphy, suggested 12,500 years or somewhat later. The sandals from Fort Rock Cave were found to have an average age of 9053 ± 350 years, the oldest actual artifacts thus far dated. If the assumption that the Newberry eruption followed that of Mount Mazama is correct, it is rather difficult to explain how sandals remained on the surface of a cave floor for 2600 years without alteration and were then charred by the heat from the Newberry pumice. However, Hansen estimates the Newberry eruption at between 7500 and 9500 years, and Allison places it at from 11,000 to 12,000 years. If the date for the sandals is correct, it is extremely significant because it shows that the inhabitants of North America at that time had not only developed a fine technique in the manufacture of protection for the feet but that they had also developed a highly artistic form of basketry decorated with a false embroidery. It is possible, of course, that when perishable materials belonging in the complex of some of the other cultures of this period are found an equally high kind of industry will be revealed. For the present, however, the critical factor is the apparent reverse order of the dates of the volcanic eruptions. Archeologically, the Mount Mazama pumice is a useful feature in Oregon as at many sites artifacts are found beneath it. The date for the Palliaike Cave in South America is important in indicating that man had reached that area at a reasonably early time. Another cave in the same region, while it did not contain archeological remains, is particularly significant because of the possibility of correlating the deposits in it with the last ice advance in Patagonia. It produced a date of 10,832 ± 400 years. That particular date was obtained from sloth dung and agrees very closely with the 10,455 ± 340 years obtained from similar material from Gypsum Cave.

There is a somewhat younger group of cultures which are represented at various places in the West and Southwest, which may represent developments out of the older remains previously mentioned, but which, on the other hand, may indicate subsequent migrations. That is a problem which still needs to be solved by the archeologists. Nevertheless, there is some significance in the carbon-14 indications.

Associated with Folsom materials at sites where artifacts were picked up from the surface were types of points which were given the name Yuma. The latter became somewhat of a catchall designation for points which could not otherwise be identified and has more or less been replaced by specific site names. The first points of this type were believed to be contemporaneous with Folsom points, but subsequent work showed that their contemporaneity at best was a very late one and that they more likely actually represented a subsequent horizon. In this category are specimens such as those found by G. L. Jepsen of Princeton University at Sage Creek in Wyoming. The average date for that material, derived from partially burned bison bones, is 6876 ± 250 years. Other specimens falling in the same category were found in a site at the Angostura Reservoir in South Dakota. Their age was determined from charcoal found at the occupation level and is 7715 ± 740 years. Geologic studies at both of those locations have not yet been completed so we do not know what the conclusions are with respect to their geologic age.

Caves in the Humboldt Valley in Nevada have furnished specimens which have been dated 7038 ± 350 years, 5737 ± 250 years, and 2482 ± 260 years. These dates were obtained from bat guano and archeological artifacts. In the cave containing the oldest objects, the guano layer below the artifact-bearing stratum rested on Pleistocene gravels and material from the contact tested 11,199 ± 570 years. Briefly, it may be said that the evidence there shows that man was present in that district by 5000 B.C. and that after approximately 1500 years' occupation the region was then abandoned until about 500 B.C. from which time there is an unbroken archeological record to the present day. The oldest date thus far for California is 4052 ± 160 years, but since Olivella biplicata shell beads were found at the 7000-year-old level in one of the caves in Nevada, it seems evident that there must have been people along the Pacific coast at that time.

In southeastern Arizona a series of cultural horizons designated the Cochise yielded carbon-14 dates of 7756 ± 370 and 6210 ± 450 years for the oldest stage. The next or second stage yielded dates from 4508 ± 680 to 4006 ± 270 years, while the third stage gave 2463 ± 310 years. From these dates it is obvious that the sequence of the three stages which was established on geologic and typological evidence was sound. The dates themselves, however, are somewhat lower than previous estimates based on climatological studies, particularly in the case of the first period which is about 2000 years younger [SAYLES and ANTEVS, 1941]. The carbon-14 figures indicate that, contrary to the opinion of many, the Cochise peoples had not moved into that area prior to the appearance of hunting peoples of the Folsom type in the region immediately to the east. In that connection it may be pointed out that the fire pit with a date of 4283 ± 250 years at Folsom was roughly contemporaneous with the second stage of the Cochise in Arizona and the Early Horizon culture in California. Furthermore, the date falls in the same general horizon as some of the Archaic remains in the eastern part of the United States, as that of the site of the much-debated Tepexpan Man in Mexico with its 4118 ± 300 years, and of the Huaca Prieta Mound No. 3 in Peru with 4044 ± 300 years.

In the eastern United States the Archaic at Frontenac Island, New York, gave a date of 4930 ± 260 years. A site at Lamoka Lake, New York, produced charcoal which tested 5383 ± 250 years, while shell mounds in Kentucky yielded dates from 4900 ± 250 to 5149 ± 300 years. Geological determinations have not played a particularly important part in the studies of those sites, although such investigations as were made there would indicate that there was some expectation of reasonable antiquity. Probably somewhat younger but still falling within that period is the fish-weir at Boston where peat from the Boylston Street site gave a carbon-14 date of 5717 ± 500 years for the lower peat underlying the weir. A second date was obtained from a fragment of coniferous wood which was taken from marine silt overlying the lower peat and the weir. It was 3851 ± 390 years old. The weir itself presumably should be younger than the oldest date but older than the later one. On the basis of climatological evidence it had been estimated that the weir was in use just prior to 2000 B.C., and that appears to have been a very good guess [ANTEVS, 1943]. At any rate, it appears that the people who built and used the weir may have been contemporaneous with those who lived at the Frontenac Lake, Lamoka, and Kentucky sites. These dates from the eastern United States are somewhat older than had been anticipated and show that the migration to this area was relatively early.

Curiously enough Alaska, which should give the oldest dates if the migration theory for the populating of the New World is correct, thus far has shown nothing older than 5993 ± 280 years, and it is not clear if the charcoal and willows which gave that result actually were in association with human material. Most of the sites thus far dated for that area fall within the Christian Era. The explanation for that situation probably is that owing to the small amount of work thus far done the older places have not yet been found. Much older dates have been obtained for Alaska, however, from materials not related to archeological sites. There is 12,622 ± 750 years for wood associated with the bones of extinct mammals at a depth of from 30 to 60 ft in the banks of Fairbanks Creek and more than 20,000 years for wood beneath frozen muck in the gold diggings near Eva Creek.

There are various other dates which are now available but time will not permit mentioning or discussing them. A large percentage of those for the New World fall within the period beginning about 1500 B.C., and extending to late pre-Columbian times. Some of the Old World remains show more antiquity than anything thus far found on this side of the water. Charcoal from the Lascaux Cave, near Montignac in the Dordogne, which contains some of the finest known stone-age paintings, tested 15,516 ± 900 years; a Magdalenian site, also in France, gave 11,109 ± 180 years; and carbonized wood from a late stone-age site in West Africa shows 11,189 ± 490 years. The latter was in a deposit identified as late Upper Pleistocene. Dates approximating 9000 years have been obtained from houses in Denmark, and materials from a Neolithic cave in Iran range from 8000 to 10,000 years. There are others, but for now these must suffice.

Whether it is pure coincidence, the result of the choice of the samples selected for testing, or a matter of some significance remains to be seen, but it is worthy of note that the remains of the so-called Paleo-Indian in the New World tend to fall into certain date patterns. The oldest approximate 10,000 years old, then there is a group between 8000 and 8500 years old, others center around 7000 years old, while the largest number occur in the 4000 to 5000-year category. The intervals of several hundreds of years in some cases may mean that we merely have not found the sites that belong there. On the other hand, they may indicate a break in occupation with recurring waves of migration. It is certain that beginning about 1500 B.C. there was a marked increase in the population of North America and that it continued to expand until the arrival of the white man. As previously stated, carbon-14 dating has been found to agree in the main with earlier archeological concepts, particularly in the matter of cultural sequences if not in their estimated ages, and is proving to be a very useful tool. There are a few instances where its results run contrary to archeological interpretations, especially in remains falling within the period of the Christian Era. However, this may indicate the necessity for a review and reconsideration of the archeological data rather than a shortcoming in the carbon-14 method.

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(Manuscript received June 21, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until September 1, 1952.)

ALPHA IONIZATION DAMAGE AS A CAUSE OF LOW HELIUM RATIOS

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Abstract--The possibility has been investigated that helium atoms escape from the crystal structures of minerals only as these structures become damaged by intense alpha-particle ionization. The escape of helium from zircon and sphene has been found to be almost zero for cases of low alpha bombardment, and increases progressively until almost all of the helium is lost in cases where the samples have suffered a bombardment of $10^{16}\ \alpha/mg$. The zircons in granites are well up in the range of activity where this damage will be important so that it is concluded that one of the principal causes of the high loss of helium from granitic rocks is the concentration of the radioactive elements and consequently helium into centers in which the damage will be great enough for the helium to escape.

The relationship between helium loss and the number of alpha particles per milligram has been established for zircon and sphene for use in approximate age measurements on these minerals. Theoretical curves have been derived that appear to represent the process fairly well, and permit age estimates to be made even on material that is severely damaged.

Introduction--A considerable number of helium ratios have been measured on rocks and minerals by various workers in the past. A summary of all findings may be stated simply as follows. Measurements on whole samples of granitic rocks have yielded widely scattered helium ratios, always lower than the geologic age of the rock, and with a distribution of which the mean is about one-quarter of the age of the rock. Measurements on fine- to medium-grained basic rocks have shown less scatter, with a mean closer to half of the geologic age of the rock, and with very occasional ratios exceeding the age of the rock to such an extent that primary helium is inferred. Measurements on separated minerals have indicated that the helium content bears a close relationship to the content of radioelements and that in the same rock the helium will be high in minerals of high activity and low in minerals of low activity, and in general that the lowest age ratios are found in the minerals of lowest activity. This indicates that there is no tendency for helium to diffuse into the minerals of lowest helium content.

Some attempt has been made to assign values of "helium retentivity" to various common rock minerals as a result of these findings, but it is believed that these values are meaningless, and the idea of varying helium retentivity between minerals has tended to direct thought away from the probable true causes of the low ratios. For instance, quartz and feldspar have always been stated to have low retentivities, whereas the very low amounts of the radioelements found in these minerals make much more probable their contamination by submicroscopic radioactive mineral impurities. Acid treatments [HURLEY, 1950] have indicated that the removal of impurities tends to bring the age ratios up to levels in accord with the age of the minerals. The program of work described below was initiated to test the possibility that helium escaped only when the crystal structures of the minerals became damaged by intense localizations of alpha-particle ionization.

The suggestion that this cause of helium escape might be an important one resulted from the observation that in a number of measurements on magnetite of hydrothermal or metamorphic origin the helium ratios agreed well with the true age of the samples, whereas the same mineral yielded low helium ratios when it occurred in granitic types of rock, or rocks in which concentrations of radioactivity into certain accessory minerals suggested that much of the helium would occur in regions of high ionization damage [HURLEY and GOODMAN, 1943].

In order to evaluate the effect of alpha-particle damage on helium retention it was necessary to select minerals that had a tolerance for uranium or thorium in their crystal structures at the time of crystallization, so that the material worked with would have a uniform concentration of radioelements throughout, and at the same time have a level of alpha activity high enough to damage the structures within the lifetime of the mineral. It was also necessary that the crystals investigated be large enough to have much greater dimensions than the range of the alpha particles themselves. Also a number of the samples had to be of known age as determined by lead isotope ratios in the same area. Zircon and sphene were chosen as best fulfilling these criteria. A number of specimens of these minerals were generously supplied by the Harvard Mineralogical Museum and the Ontario Bureau of Mines.

Measurements on zircon and sphene from Ontario--The well-known occurrences of zircon and sphene in Ontario are probably of the same general age as the uranium minerals on which lead isotope measurements were made by NIER [1939]. These are listed in Table 1 and the average of the results (920 million years) has been used as the best single age for the zircons and sphenes, without attempting to differentiate between the various pegmatite age groups with which the zircons and sphenes might be associated.

In Figure 1 is shown the relation of helium age ratios to alpha activity and total input of energy for the Ontario zircons, and in Figure 2 is shown a similar plot for the Ontario sphenes. The ratio of thorium to uranium was not measured in these samples but variations of this ratio do not change the age ratios sufficiently to be significant in this investigation.

Interpretation of results--It is clear that essentially all of the helium is retained in the zircon and sphene samples of low activity. It is thus evident that the crystal structures of zircon and sphene will retain all of their helium, regardless of time, in their normal state and that helium loss occurs progressively under the effects of alpha-particle damage.

Table 1--Age of Ontario pegmatites from lead isotopes
[NIER, 1939]

Locality	Mineral	Ratio	Age m y[a]
Wilberforce, Ont.	Uraninite	Pb^{206}/U^{238}	1080
Wilberforce, Ont.	Uraninite	Pb^{208}/Th^{232}	980
Wilberforce, Ont.	Uraninite	Pb^{207}/Pb^{206}	1040
Pieds des Monts, Que.	Cleveite	Pb^{206}/U^{238}	880
Pieds des Monts, Que.	Cleveite	Pb^{207}/Pb^{206}	910
Besner, Ont.	Uraninite	Pb^{206}/U^{238}	770
Besner, Ont.	Uraninite	Pb^{208}/Th^{232}	790
Besner, Ont.	Uraninite	Pb^{207}/Pb^{206}	820

[a] m y = million years.

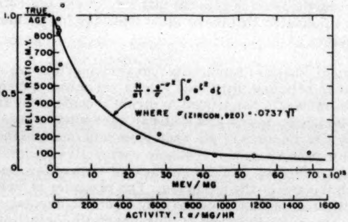

Fig. 1--Ontario zircons: helium ratio versus alpha activity

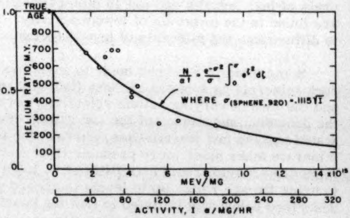

Fig. 2--Ontario sphenes: helium ratio versus alpha activity

The exact nature of the structural disturbance caused by heavy ionizing particles is not known. It is well known that x-ray diffraction patterns become obliterated under heavy dosages of alpha particles. If the process in solids is at all similar to that in gases, an alpha particle will lose energy chiefly by interaction with electrons, and not by nuclear reactions or scattering. According to classical theory, each alpha particle would initiate in the order of 10^5 secondary electrons of mean energy 100-200 ev before coming to rest. This would leave a cylindrical volume along the track intensely irradiated by these electrons and those which they in turn displaced. The electrons presumably remain out of place in traps, and the presence of the extra electrons as well as the disrupted bonds cause displacement of the atoms, leaving them in a weakly bound condition.

In these results we have knowledge of the number of alpha particles of known energy that have traversed unit volumes of the zircon and sphene, and a quantitative measure of the degree of disturbance in terms of helium leakage. The number of helium atoms at the time when leakage becomes important is only about one for every 10^4 atoms in the structure. Thus it seems probable that the helium atoms themselves (or the one or two nuclear displacements that they may have caused directly as alpha particles) would not be sufficient to produce the disturbance observed. In comparison, the number of electron displacements would be about equal to the total number of atoms in any volume, which would seem to present sufficient cause for observed structural distortions. It is not known whether the helium migrates through the structure in a random path as a general condition of weakness is developed, or whether a helium atom will migrate only along alpha tracks.

As a basis for a theoretical curve expressing the relationship between helium loss and total alpha bombardment it is therefore assumed that helium will escape progressively as a network of tracks develops throughout the mineral, and that the chance of escape will be proportional to the total number of tracks, at least in the early stages. Furthermore, it is assumed that lineage imperfections and other planes of weakness will occur throughout the structure at sufficiently close spacing so that any helium atom will not have to travel far before finding an easy avenue of exit from the mineral grain, and the time of establishing near equilibrium in the diffusion of helium atoms out of the structure at any level of irradiation damage is short compared to the total times involved. Thus as a first approximation

$$dN/dt = \alpha - K_1 N K_2 \alpha t \ldots\ldots (1)$$

where N = number of alpha particles (helium atoms) per mg
t = time in millions of years after the formation of the crystal
α = number of alpha particles added per mg per million years (assumed to be constant for $t < 1000$ m y)
K_1, K_2 are factors of proportionality in the rate of helium loss: number of helium atoms present (K_1 N) and the openness of the structure (K_2 αt).

The alpha activity of minerals is more commonly reported in terms of the activity index, I, in alphas/mg hr.

$$I = 1.04 \times 10^{12}\ \mathrm{Ra} + 88600\ \mathrm{Th} \ldots\ldots (2)$$

where Ra = radium content in gm/gm (in equilibrium in uranium series) and Th = thorium content in gm/gm, so that $\alpha = 8.77 \times 10^9$ I alphas/mg m y.

The solution of the above differential equation in its general form is

$$N = K_3\, e^{-K_1K_2\alpha t^2/2} + e^{-K_1K_2\alpha t^2/2} \int_0^t \alpha e^{K_1K_2\alpha t^2/2}\, dt \ldots\ldots (3)$$

Under these boundary conditions (no primary helium) N = 0 at t = 0, so that $K_3 = 0$

$$N/\alpha t = (e^{-K_1K_2\alpha t^2/2}/t) \int_0^t e^{K_1K_2\alpha t^2/2}\, dt \ldots\ldots (4)$$

Let $\sigma = t/\sqrt{2/K_1K_2\alpha}$

$$N/\alpha t = (e^{-\sigma^2}/\sigma) \int_0^\sigma e^{\rho^2}\, d\rho \ldots\ldots (5)$$

The natural time unit is $\sqrt{2/K_1K_2\alpha}$ so that the time of maximum helium is proportional to $1/\sqrt{\alpha}$. The theoretical curve may be fitted to the experimental points for zircon in Figure 1 by taking the point where helium retention is one-half, or $N/\alpha t = 0.5$, at a value for activity I = 210 alphas/mg hr.

In Figure 3 is shown a plot of $N/\alpha t = (e^{-\sigma^2}/\sigma) \int_0^\sigma e^{\rho^2}\, d\rho$ as a function of σ. From this curve $N/\alpha t = 0.5$ when $\sigma = 1.07$. In the zircon case at t = 920, $\alpha = 1.84 \times 10^{12}$ and $\alpha = 1.07$, the value of K_1K_2 is found to be 1.472×10^{-18}. Thus σ (zircon) = $.0737 \sqrt{I}$ for crystals of 920 million years of age. Similarly from the sphene points in Figure 2 σ (sphene) = $0.1115 \sqrt{I}$.

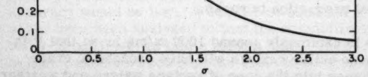

Fig. 3--The relationship of σ to helium retention

From these values and the function in Figure 3 the theoretical curves are plotted as shown in Figures 1 and 2. In the zircon case it is seen that the curve fits the points above and below the point of fit about as well as the data permit. This suggests that the permeability of the structure remains proportional to the increase of total alpha ionization over the range of the samples tested.

In Figure 2, the higher sphene ratios do not fit the curve too well, but the number of measurements are deemed insufficient to warrant a modification of the hypothesis that seems to fit the zircon case satisfactorily.

From the above interpretation it appears that the total amount of helium in a gram of zircon will increase with time to a maximum and then decrease again and approach zero as the crystal

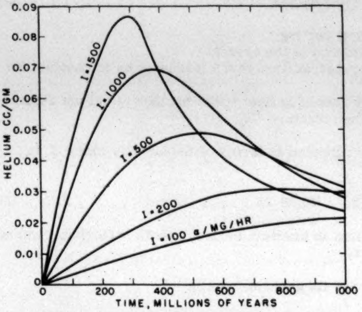

Fig. 4--Zircon: helium versus time for several activity values

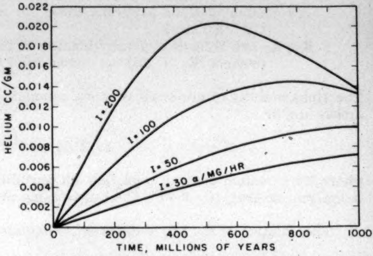

Fig. 5--Sphene: helium versus time for several activity values

structure deteriorates completely. In the true case this condition will not be completely met since there will be some lag in the escape of helium, but within the range of activities and times involved this lag is apparently small and the inflow and outflow of helium appear to be almost in equilibrium for the degree of leakiness at any one time.

In Figure 4, the amount of helium to be found in a zircon crystal of any age is plotted against the age of the crystal for cases covering an activity range from 100 alphas/mg hr to 1500 alphas/mg hr. These curves are obtained from

$$\text{Helium } (10^{-5}\ \text{cc/gm}) = tI/30.8 \times N/\alpha t = (tI/30.8)\,(e^{-\sigma^2}/\sigma)\int_0^{\sigma} e^{\rho^2}\,d\rho \quad \ldots\ldots (6)$$

where $\sigma = 0.859 \times 10^{-9}\, t\sqrt{\alpha}$, using the constants derived from the fitted curve in Figure 1. From this set of curves it is seen that zircons in this range of activity will never contain more helium than 8650×10^{-5} cc/gm; and this is found to be the case. Furthermore, it is seen that there are two age solutions for any measurement of helium and activity, one being in the period of increasing helium and the other lying in the period of decreasing helium. This ambiguity can normally be resolved by geological knowledge or the analysis of more than one sample from any one area. In Figure 5 a similar set of curves is plotted for sphene.

X-ray powder photographs were taken of samples of zircon over the range of activity to see if there was any visible effect on the structure. These are reproduced in Figure 6 with explanatory information in Table 2, and it is apparent that the clearness of the powder pictures decreases with increasing alpha ionization. A quantitative measurement of the breadth and intensity of the x-ray lines, as determined by x-ray spectrometer, will be reported at a later date. Differential thermographic measurements are being made to determine the amount of heat given off as the zircon becomes ordered again at elevated temperature, as part of a study of the proportion of alpha-particle energy that goes into permanent disturbance of the crystal structure. This knowledge is particularly needed for the calculation of radiogenic heat production in rocks.

The activity of accessory zircons in granite is commonly around 1000 α/mg hr so that it is apparent that most of the helium generated by these active centers will have escaped to grain boundary openings and be removed as the rock comes into the zone of surface waters and weathering. This process of helium escape could account for most of the helium loss observed in granites since more than half of the radioactive elements are known to be concentrated in active accessory minerals.

From the effect of acid treatment, it appears that most of the radioelements that do not go into visible accessory minerals form acid-soluble compounds along grain boundaries or other planes of weakness. This fact is also borne out by the observations of PICCIOTTO [1950] and others who have worked with nuclear track emulsions on granite sections. In this case the helium atoms might

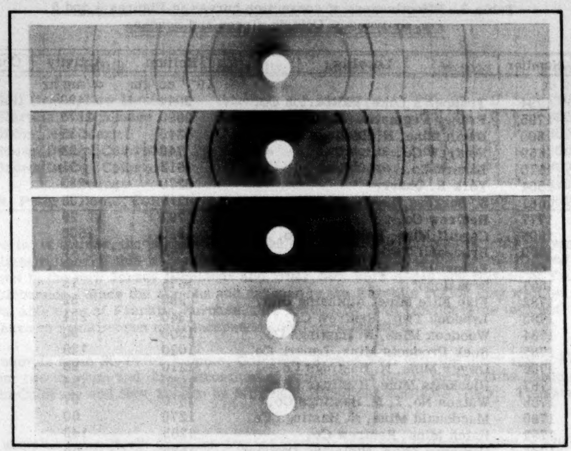

Fig. 6--Decrease in intensity of x-ray lines with increase of total alpha irradiation for zircon samples of Table 2

Table 2--Data on zircon samples of Figure 6, top to bottom

Zircon no.	Probable age	Activity	Total α ionization
	m y	α/mg hr	Mev/mg
1659	920	28	1.3×10^{15}
1670	920	51	2.3×10^{15}
1678	1050	490	28×10^{15}
1800	920	615	28×10^{15}
1805	920	1500	69×10^{15}

simply escape by returning along their own tracks to the boundary from which they originated without the necessity of the extreme concentrations.

Therefore, in predicting what geological material might be used for helium age measurement it appears that minerals which have a uniform low distribution of radioelements as substitution solid solutions should be the best, or at least with no strongly active mineralization superimposed. This is in keeping with the previous findings of good helium ratios on hydrothermal magnetite, pyrite, chalcopyrite, and galena.

Measurements on other macroscopic zircons and sphenes--From Figures 4 and 5 it should be possible to determine the age of unknown zircon or sphene, although it is to be expected that the accuracy would be low. A number of samples of zircon and sphene of ages other than 920 million years have been analyzed to test the possibility of age determination on samples in which the activity is sufficient to cause helium loss. These measurements are listed in Tables 3 and 4, with helium ratios corrected on the basis of the curves in Figures 4 and 5. Although the curves in Figures 4 and 5 are based on the zircon and sphene measurements from Ontario, these same specimens are again listed with their corrected helium age values in order to show the effect of the correction over the complete range of time and activities that has been studied.

Discussion of results--The expected accuracy of this method of age measurement is still sufficient to give rise to some speculation regarding the results on the zircons and sphenes from the regions near Franklin, New Jersey, and in lower New York State. These appear to be grouped about an age of 310 million years which is sufficiently younger than the expected Precambrian age to raise the question of whether there is an additive error in the helium ratios that is caused by

Table 3--Effectiveness of correction curves in Figures 4 and 5 on analyses of Ontario zircon and sphene

Mineral	Number	Locality	Helium	Activity	Corrected age
			10^{-5} cc/gm	α/mg hr	m y
Zircon	1794	Trapper's Lake, Haliburton Co.	3044[a]	490[a]	1000
Zircon	1795	Fraser Pegmatite, Haliburton Co.	2980	1175	950
Zircon	1800	Davis Mine, N. Hastings Co.	4240	615	800
Zircon	1659	Renfrew Co.	740	28	940
Zircon	1670	Lanark Co.	1618	51	1200
Zircon	1804	York R. Quarry	3270	235	800
Zircon	1681	S. Burgess, Leeds Co.	715	35	740
Zircon	1777	Renfrew Co.	797	29	960
Zircon	1805	Cardiff Mine, Haliburton Co.	4950	1500	530
Zircon	1660	Brudenell Twp.	742	28	940
Sphene	1668	Eganville, Renfrew Co.	830	38	1000
Sphene	1667	Sebastopol	1675	75	950[b]
Sphene	1782	Five Mile Mine, Nipissing Dist.	996	52	900
Sphene	1783	Lyndoch Twp., Renfrew Co.	1340	254	1000
Sphene	1784	Woodcox Mine, N. Hastings Co.	1300	90	960
Sphene	1785	Rock Products Mine, Lanark Co.	1020	120	1200
Sphene	1786	Dwyer Mine, N. Hastings Co.	1210	90	1050
Sphene	1787	Plunketts Mine, Hastings Co.	1500	185	910
Sphene	1788	Watson No. 1, N. Hastings Co.	1510	97	800[b]
Sphene	1789	Macdonald Mine, N. Hastings Co.	1270	60	1050[b]
Sphene	1792	Hatch Mine, Renfrew Co.	1285	143	1030
Sphene	1781	Cameron Mine, Nipissing District	1525	68	1000[b]

[a] Standard deviation of repeat helium measurements about seven per cent; activity measurements about six per cent.

[b] Helium too high for age solution, closest point chosen.

Table 4--Corrected age ratios on zircon and sphene from other localities

Mineral	Number	Locality	Helium	Activity	Corrected age
			10^{-5} cc/gm	α/mg hr	m y
Zircon	1678	Hammond, Adirondacks	2340	490	1170
Zircon	1685	St. Peter's Dome, Colorado	271	9.7	860
Sphene	1679	Bolton, Massachusetts	1860	179	Two solutions, 460 or 580
Zircon	1642	Fredriksvarn, Norway	783	78	320
Zircon	1674	Ball's Hill, New Jersey	3220	325	380 (or 860)
Zircon	1683	Franklin, New Jersey	1908	180	370
Sphene	1675	Franklin, New Jersey	310	36	300
Sphene	1684	Franklin, New Jersey	705	87	275
Sphene	1686	Franklin, New Jersey	1340	179	285
Sphene	1682	Tilly Foster, New York	24	2.6	300
Sphene	1715	Pitcairn, New York	238	36	230
Sphene	1676	Williamsbridge, New York	44	4.0	335
Zircon	1680	Edenville, New York	1775	183	330
Zircon	1644A	Beach Sands, North Carolina	475	97	160
Zircon	1661	Iredell Co., North Carolina	1380	202	225
Zircon	1662	Henderson Co., North Carolina	1500	275	180
Zircon	1669	Buncombe Co., North Carolina	2058	592	120
Zircon	1776	Statesville, North Carolina	1352	210	220
Zircon	1819	Max Patch, North Carolina	486	97	160

helium loss during the time the mineral is still buried in an environment of elevated temperatures. The erosion and isostatic rise of a mountain belt may well take several hundred million years in some cases, but the drop in temperature at depth may precede the rise of the mineral to the surface so that it is not possible to make much of an estimate of the magnitude of this correction by geological reasoning. However, the age ratios on the two samples of zircon from the young Nova Scotia granites described below show that this correction probably does not exceed a few tens of millions of years.

Table 5--Comparison of St. Peter's Dome with other age measurements in Colorado and New Mexico

Locality	Authority	Mineral	Method	Age
				m y
Bull Creek, New Mexico	Muench	Monazite	Pb/Th + U	770
Glorieta, New Mexico	Muench	Monazite	Pb/Th + U	860
Dixon, New Mexico	Ahrens	Lepidolite	Sr^{87}/Rb^{87}	800
Brown Derby, Colorado	Ahrens	Lepidolite	Sr^{87}/Rb^{87}	900
Brown Derby, Colorado	Ahrens	Lepidolite	Sr^{87}/Rb^{87}	800
Average				820
St. Peter's Dome, Colorado	....	Zircon	He/Activity	860

There is, of course, the possibility that the age of these specimens is truly the age suggested by the helium ratios. If this is the case it would mean that some of the pegmatitic and other mineralizations in the New Jersey Highlands and lower New York areas are related to Taconic or later disturbances. Since the zircons and sphenes in the Franklin region may well be related in time to the zinc ores of Franklin Furnace, this possibility is of considerable interest. The question will have to remain open until independent evidence appears.

The sample from St. Peter's Dome, Colorado, agrees well with the mean age of 830 million years from two thorium and three strontium determinations on the surrounding basement exposures in Colorado and New Mexico by MUENCH [1950] and AHRENS [1949]. These are listed in Table 5.

The samples from North Carolina are not younger than late Paleozoic (say > 220 million years) on geological evidence so that the helium measurements appear definitely low. This indicates that other factors must enter the picture. For instance, it is expected that compositional variations of the zircon and sphene would be important.

Measurements on accessory zircon separated from granites--The accessory zircon was separated from two samples of granite of presumably Devonian age from Nova Scotia to test the possibility of helium age measurements on granite by use of corrected zircon ratios. The rock was granulated to pass 20-mesh and put through bromoform to eliminate feldspar and quartz. The residue was then put through methylene iodide and magnetite was separated from the final residue by magnet. Almost pure zircon was obtained from this residue in repeated passes through a Franz magnetic separator, it being the mineral of the lowest magnetic susceptibility. The zircon from granite No. 1712A, St. Margaret Bay, Nova Scotia, was screened into a number of sized fractions and a count made under a microscope of the diameter of the grains in each fraction. Histograms of these counts are shown in Figure 7. In Figure 7a the cut-off of zircon below 20 microns probably means that crystals of these sizes were not released from the other minerals with this coarseness of grinding.

Since the diameters of these accessory zircon crystals are not much greater than the ranges of the alpha particles themselves, it was necessary to determine the proportion of alpha particles that left the boundaries of the zircon crystals for each sized fraction. This determination was made by a count of the alpha emission from a surface layer one crystal deep, of known weight, and by a thick source measurement of the radioelement content of the zircon. There is a distribution of diameter sizes for each sized fraction so it was not possible to obtain an emission value directly for each diameter interval. By assuming that the weight of a single crystal varied as the cube of its diameter and its emission varied as the square of its diameter, the emission for each diameter interval was calculated for each of the sized fractions. The last assumption is not strictly correct and tends to weight the alpha emission too heavily in favor of the finer sizes. However, since the data were not available for a more realistic assumption in which the variation of the length of the crystals is known in relation to their diameter, the approximate assumption has been used and the variation of whole space emission is plotted against crystal diameter for each of the sized fractions in Figure 8. An improved assumption would bring the curves together into a single curve for all the fractions. The total radioelement content of this zircon sample is equivalent to 420 alphas/mg hr. Figure 8 is, therefore, of general use in showing the proportion of alpha particles that leave the boundaries of accessory zircon crystals for various diameters or screen sizes, regardless of the activity of the zircon. This knowledge is necessary if accessory zircon is to be used for helium age measurements.

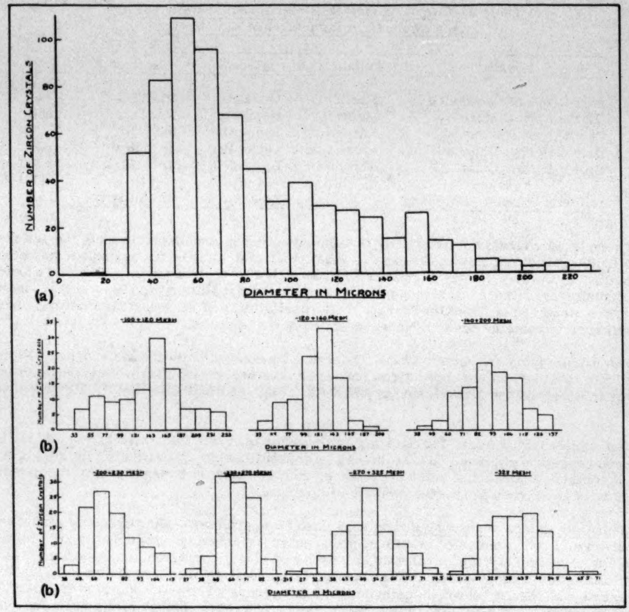

Fig. 7--Distribution of crystal diameters in various screen-size fractions of accessory zircon separated from R-1712A granite; (a) diameters of zircon crystals; (b) size distribution in screen fractions of zircon crystals (diameters)

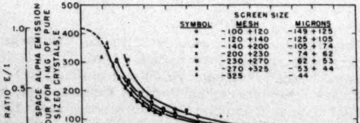

Fig. 8--Ratio of number of alpha particles driven out to total number produced in crystals of accessory zircon, plotted against crystal diameter

If I alphas/mg hr is the total activity of a well-sized sample of accessory zircon and E is the whole space emission as obtained from the curve in Figure 8, then I - E gives the rate at which helium atoms are built up within the boundaries of the zircon crystals. The helium ratio of the accessory zircon would therefore be found from the equation

$$\text{Age } t = [30.8\, H/(I - E)] \times \sigma \left\{ e^{-\sigma^2} \int_0^{\sigma} e^{\rho^2}\, d\rho \right\}^{-1} \ldots\ldots\ (7)$$

where H is the helium content in 10^{-5} cc/gm, and $\sigma = t/\sqrt{2/K_1K_2\alpha'}$. The value of α' should be somewhere between 8.76×10^9 I and 8.76×10^9 (I - E), since the alphas that leave the crystals do not contribute to helium but do contribute to damage. For the case of the Nova Scotia zircon α' was chosen to be 8.76×10^9 (I - E/2) and the ages taken directly from the curves in Figure 4.

The age ratios on the different sized fractions of zircon from the Nova Scotia granites are shown in Table 6. The granite is believed to be Devonian in age, or between 250 and 300 million years from scant lead data on similar granitic emplacements in New England. The helium results are not inconsistent with this. The scatter is large and the age determination is of reconnaissance value only in this case, but the results indicate that helium ratios by this method may

Table 6--Helium ratios on accessory zircon from Nova Scotia granites

Sample no.	Screen size	Helium	Activity, I	Whole crystal emission, E	I - E	Corrected age ratio
	microns	10^{-5} cc/gm	α/mg hr	α/mg hr		m y
1710[a]	-44	1465	466	257	209	245
	-53+44	1820	466	232	234	270
	-105+74	1790	466	127	339	185
Average	....	......	......	......	......	235
1712A[b]	-53+44	2350	420	154	266	310
	-74+62	2020	420	120	300	235
	-105+74	1785	420	106	314	200
Average	....	......	......	......	......	250

[a] Sample 1710 is from Route 3, five miles from Chester, N. S., toward Halifax.

[b] Sample 1712A is from road to Peggy's Cove, seven miles from Route 3, N. S.

be of use in the correlation of young granitic rocks where little is known of their age at present. Part of the scatter is undoubtedly due to the fact that the samples of zircon were seen under the microscope to have not been completely pure.

Acknowledgments--This research was supported by the Office of Naval Research in a laboratory established at the outset by a grant from the Geological Society of America. This paper has been filed as a Technical Report under Contract N5ori-07829 of the Office of Naval Research. The writer thanks Fanny M. Beasley for her very able assistance in the measurements. The samples of zircon and sphene were generously supplied by Clifford Frondel and C. S. Hurlbut of Harvard University, and by D. F. Hewitt of the Ontario Bureau of Mines. The writer is very grateful to R. W. Williams for his ideas and help in the interpretation of the results and for reading the manuscript. L. V. Azaroff made the x-ray powder photographs.

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(Manuscript received June 14, 1951; presented in part at the Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until September 1, 1952.)

TECHNIQUES OF NATURAL CARBON-14 DETERMINATION

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(Contribution from the Lamont Geological Observatory, No. 53)

Abstract--The techniques which are in use at the Lamont Geological Observatory for the preparation and counting of the carbon-14 in natural carbon-bearing samples are described in detail. The first phase of the process includes preparation of pure carbon dioxide by burning or hydrolysis, the reduction to carbon with hot magnesium, and the extraction of the product to remove all substances except the carbon. The yield is generally 90 pct of the carbon in the original sample with an ash content of about one per cent.

The carbon samples are mounted in a stainless-steel cylinder and counted against a background cylinder of stainless steel in a modified Libby screen-wall-type counter with an active counting length of eight inches and a diameter of three inches. This counter is surrounded with 30- by 2-inch cosmic-ray counters which are used in anticoincidence. The entire assembly is placed in a large shield with eight-inch thick iron walls. The electronic unit contains two regulated high-voltage supplies, drag-in voltage, monitor selector switch, anticoincidence analyzer, scale of four, and register.

This arrangement has provided background as low as 2.5 counts per minute with normal operation ranging from four to six counts per minute. The efficiency of the screen-wall counter is about five per cent. With three days of counting, this arrangement can measure samples up to 25,000 years old for wood and 30,000 years for shells. Application of this quantitative method of age determination to a wide variety of samples of geological interest is in process.

Introduction--One of the most important scientific discoveries of the past decade is the existance of measurable amounts of carbon-14 in the atmosphere, hydrosphere, and biosphere by W. F. Libby and his associates [LIBBY, ANDERSON, and ARNOLD, 1949; ARNOLD and LIBBY, 1949; ANDERSON and LIBBY, 1951] at the Institute for Nuclear Studies, University of Chicago. Since the half-life of carbon-14 is reasonably long (5568 ± 30 years) [ANDERSON and LIBBY, 1951], and since a constant concentration is present in living things, air, and surface water, the time of removal from the carbon dioxide-life cycle can be measured quantitatively if the carbon-14 concentration is determined. Thus a method was created which permits relatively precise age determination of archaeological and geological materials from the present back to about 30,000 years ago. Since the time dimension is uniquely important to the sciences of archaeology, anthropology, and geology, and since no quantitative method was previously available for this time interval, the application of the method may cause a major advance in these sciences.

During the past year a second laboratory for the measurement of natural radiocarbon has been completed at the Lamont Geological Observatory. The many problems to which this method may be applied and the large amount of time and effort required per sample make the development of numerous laboratories with such facilities desirable. The interchecking from such independent groups should establish the method even more firmly.

Since it is anticipated that a continuing series of carbon-14 determinations will be made at the Lamont Geological Observatory, it seems desirable at the outset to describe in one paper the techniques in use. In general aspect the procedure is similar to that of ANDERSON, ARNOLD, and LIBBY [1951]; however, numerous minor modifications have been made which appear to increase either the ease or efficiency of operation.

The process consists of preparing a sample of pure carbon from the natural carbon-bearing material, mounting this in a special screen-wall Geiger counter, and counting it against background for several days. The key to the success of the procedure is the reduction of the unshielded background of the counter (about 350 cpm) to about five counts per minute by the use of a large iron shield and anticoincidence counters and circuitry.

Carbon preparation

Sample pretreatment--Types of natural material which are subject to carbon-14 determination include plant and animal tissue or decomposition products, air, water, ice, shells, and carbonate sediments. The history of the sample is particularly important in the case of air, water, ice, and carbonates since isotopic exchange is possible under proper conditions. It appears that wood is not particularly susceptible to exchange. If a wood sample contains some adherent carbonate, the latter is very likely of a different age. Thus a pretreatment scheme of boiling for 30 minutes with 1N HCl has been adopted for wood, peat, charcoal, and dung samples. In addition, the samples which must be oxidized are pre-dried to decrease the water evolution during the oxidation. All samples are inspected, before treatment is started, for evidence of inhomogeneity and an estimate of the carbon content is made. If abundant material is available, enough is taken to yield about 12 grams of pure carbon.

Oxidation and acidification--Figure 1 is a diagram of the system used in preparing the carbon from the natural samples. The first step in the operation is the conversion of the carbon in the sample to carbon dioxide. If the sample consists of an organic material or carbon, it is burned in a stream of oxygen. The sample is placed in a Vycor ignition tube (30 inches long and one inch inside diameter), the system from N to H is evacuated, and the copper oxide in the following similar tube is heated to about 800°C by furnace O. Oxygen is then admitted to the sample, which is

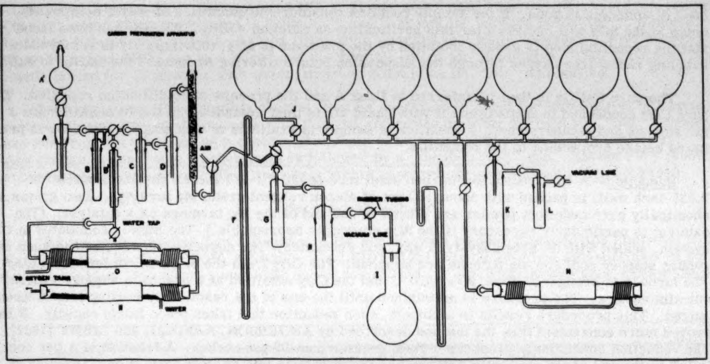

Fig. 1--Diagram of apparatus

ignited by furnace N. The pressure of the system at the manometer F is kept at about five cm. The gases formed are further oxidized by the copper oxide, dried by the condenser B', traps C and D and the drierite tube E. The carbon dioxide along with some of the oxides of nitrogen and sulfur, pyrolysis products which escaped oxidation, and radon are collected in traps G and H at liquid air temperatures.

The yield in the oxidation step depends largely on the rate of flow of oxygen. As long as the flow is slow and the net pressure less than five cm, quantitative experiments showed 98-100 pct recovery of the CO_2 released from the specimen. If a fairly fast rate of flow is used (about a liter per minute) at a total pressure of 30 cm, the yield ranges from 60 to 80 pct.

Specimens such as shells, carbonate sediments, or calcium carbonate precipitates from ocean water, are ground to a coarse powder, placed in two-liter flask A, and covered with one to two cm of distilled water. The system is evacuated from A to H until the water in A has boiled for several minutes at room temperature to remove dissolved air. The stopcock beyond H is closed and concentrated chemically pure. HCl is added slowly and continuously from A'. The CO_2 pressure is controlled so as to be less than ten cm by the rate of HCl addition. C, D, E, G, and H perform the same function as described above. The extra trap B is surrounded by ice, used to collect water and froth. With clear samples, the final pressure is below three cm. In certain samples where frothing is expected, five cm of air is admitted prior to acidification. If the sample contains considerable organic matter, it is generally pre-treated by a stream of oxygen at 400°C to burn off this material so that frothing is minimized in the reaction flask. Careful application of the procedure has consistently produced better than 99 pct yield.

The numerous ball joints in this system are necessary for ready cleaning and reassembly. For certain types of organic samples, the copper oxide does not decompose all the gaseous impurities, and hence the system may become dirty. This may also occur if excess frothing occurs.

Purification of CO_2--In order to eliminate the chemical impurities which would inhibit the reduction and remove the radon, it is necessary to purify the CO_2 produced by the oxidation or acidification reaction.

The gas phase above the condensed CO_2 in traps G and H is pumped off after closing the stopcock near E. The two-liter flask I is filled with hot solution of 500 cc of concentrated NH_4OH and 500 cc of distilled water. I is evacuated until it boils, then the stopcock is turned off. When the pressure in G and H is 75-80 cm as indicated on manometer F, the stopcock between H and I is opened and the CO_2 is rapidly absorbed in the solution with occasional shaking. When absorption is complete, the solution is heated to near boiling and added slowly to a hot solution of calcium chloride and shaken until precipitation of $CaCO_3$ is complete. This is stoppered and allowed to stand for several hours, filtered on a Buechner funnel, and washed with distilled water until the odor of ammonia is gone. If the sample contains considerable quantities of sulfur compounds, some of the SO_2 will remain after this purification as calcium sulfite. Since it has been found that the reduction step is greatly inhibited by the presence of SO_2, this impurity is removed by bubbling radon-free oxygen through the suspension before filtering to convert the sulfite to sulfate.

The pure $CaCO_3$ is then transferred to flask A and the process of acidification repeated. The pure CO_2 condensed in traps G and H with liquid air is then expanded into the five-liter bulbs J for storage and measurement. If insufficient sample is available at this stage, more CO_2 is prepared before proceeding to the reduction.

Reduction--A cold-rolled low carbon steel tube M (30 × 1 1/4 inches outside diameter × 0.035-inch wall) is packed with about 100 gm of Merck reagent grade Mg turnings. Two grams of chemically pure cadmium powder are placed at one end of the Mg turnings as a catalyst. (The catalyst is particularly necessary if the SO_2 content is appreciable.) The tube is connected to the system, sealed with de Khotinsky-type wax, and evacuated. The copper coils required to keep the rubber stopper cool and the furnace are attached. The CO_2 from the bulbs is condensed in trap K. The furnace is brought to about 900-1000°C and the CO_2 admitted at a pressure slightly exceeding one atmosphere. The pressure is maintained until the end of the reaction by heating trap K as required. This procedure results in a smooth, even reduction that takes place fairly rapidly. It has proved more convenient than the method described by ANDERSON, ARNOLD, and LIBBY [1951]. The reduction consistently produces a yield greater than 99 pct carbon. A fraction of a per cent is left in the gas phase unreacted and another fraction goes into carbide formation. Analysis of the acetylene evolved when water is added to the reduction product indicates that the carbon in the form of carbide is less than 0.5 pct.

Extraction--An improvement has been made in the extraction procedure described by Libby. The method discussed here requires only three to four hours, few operations, and consistently produces an ash content of about one per cent. After a thorough study of the extraction problem by using various chemical reagents and temperatures, it was found spectroscopically that the ash was pure MgO but that ten to 20 pct could not be removed by boiling HCl even though MgO is easily soluble in HCl. It appeared that this fraction of the MgO was coated firmly with graphite particles which prevented the HCl from making contact with the MgO. A reasonable solution appeared to be an oxidation treatment. Figure 2 is a diagram showing the general relationships obtained between ash content and carbon content due to oxidation. It is clear that a 15-minute boiling in concentrated HNO_3 results in a loss of only about eight per cent of the carbon while removing practically all of the ash. Libby's process of heating the leached dry carbon on "low" (hot plate) for four hours followed by subsequent leaching lowered the ash slowly by the same oxidation mechanism (in this case by air) but much less efficiently. The heating on "low" for four hours in air does not result in much less carbon loss than the more violent nitric acid treatment and the latter produces a much lower ash in less time. Table 1 gives the ash results on a number of representative specimens treated according to the following process. The products of the reduction are broken out of the tube into a

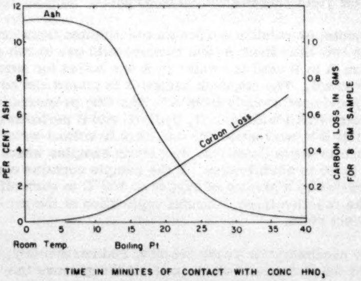

Fig. 2--General relationships between ash content and carbon content

Table 1--Representative set of carbon samples

Sample no.	Description	Ash pct
100 AB	Ocean water carbonate, 4000 m	0.7
101 B	Coaly Peat, Eagle River, Alaska, pre-Mankato	0.2
102 A	Wood under last major eruption, La Suffriére	0.8
103 B	Shells, Sagadahoc Bay	0.8
104 BC	Signal Butte Charcoal	1.2
105 C	Shell fragments, deep ocean cave off Bermuda	0.5
106 C	Wood under Mendenhall glacier outwash, Alaska	0.7
108 B	Sequoia tree-ring sample	0.9

clean three-liter beaker. Concentrated HCl is slowly added until vigorous effervescense ceases (30 min), after which the volume is made up to one liter and the solution boiled for 15 minutes. It is then filtered with a sintered glass filter stick attached to a water aspirator. Five hundred cc of concentrated HNO_3 are next added and the mixture boiled for 15 minutes. This is filtered and finally boiled for 15 minutes with water, dried on a hot plate on "low" and bottled.

Contamination--Considerable caution must be exercised to avoid the introduction of contamination in some part of the process. To this end anthracite or other geologically "old" samples are run from time to time as a check. Chemicals of highest purity are bought in large lots. Any change in apparatus or chemicals is followed by a "dead-carbon" run. Except for a piece of rubber tubing in the argon filling line early in this research, no contamination has appeared.

Screen-wall counter

Construction--Figure 3 is a diagram of the screen-wall counter modified and improved from that developed by ANDERSON, ARNOLD, and LIBBY [1951] for the efficient measurement of soft beta particles. This counter was built to our design by L. C. Eichner Instruments, Clifton, New Jersey.

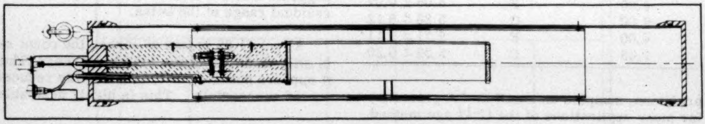

Fig. 3--Modified screen-wall counter

The major components are: (1) A sample cylinder consisting of two eight-inch long pieces is connected by a third piece by a friction-sleeve fit. The sample is mounted on one of these eight-inch pieces. The complete sample cylinder is slid from one end of the counter to the other for sample and background measurement. (2) The screen assembly is maintained at a "drag-in" potential of +100 volts. This feature considerably increases the efficiency of the large-diameter (6.7 cm) counter. The screen is insulated from the outside case of the counter with lucite. (3) The outside case, which is 24 inches long, has vacuum-tight end caps sealed with "o" rings and has a stopcock attached to one end cap for evacuation and filling procedures. (4) The center wire is brought in through the end cap in a vacuum seal, which is made by soldering the wire in a capillary Kovar tube, the Kovar being attached to glass to insulate the center wire from the case.

The inside of the counter including wires and screws, except the lucite insulators and the banana plug, is made of stainless steel. This was done to eliminate activity which is present in many metals including some lead-free brass stock.

The significant improvements over the original Libby model include: (1) Stopcock is placed on end cap instead of on the counter cylinder wall and is soldered in place via a Kovar metal-glass seal instead of being attached with de Khotinsky cement. This position of the stopcock permits use of fewer anticoincidence counters and more compact arrangement inside the shield, that is, a simple circular rack. (2) No de Khotinsky cement is required on the external wire glass seals which are permanently soldered in place. The entire glass-metal, solder-joint assembly is encased in a stainless steel shield to which are attached a banana-plug socket for "drag-in" voltage and an amphenol connector for high voltage for the center wire. This eliminates the frequent breakage or leakage due to mechanical stress on the fragile metal-glass seals and vacuum solder joints. (3) The entire screen-wall assembly, which is the only part of the counter that requires repeated repair, can be easily disconnected by means of a banana plug within the counter so that the cap assembly remains intact. (4) The end caps are kept vacuum tight with "o" rings rather than being waxed on. A minimum of apiezon N vacuum grease is used about the "o" rings. Thus the counter need not be heated excessively at any time and the end caps may be removed and replaced rapidly. (5) The screen wires are tightened at the base by means of "taper" screws. One wire starts at the base and returns as the next wire of the screen. The lucite piece at the far end is grooved to accept the wire so that no sharp corners are exposed. The replacement of these wires is very simple as is the tension adjustment.

Sample mounting--Eight grams of the carbon sample are ground in a mullite mortar and pestle, transferred to a 200-cc beaker, and about 50 cc of distilled water containing about 0.05 pct agar added and mixed until a smooth slurry of the desired viscosity is attained. The slurry is transferred to the one-half of the sample cylinder, spread evenly either by shaking or by rotation using a glass rod or spatula. The cylinder is then spun in the hand several times in each direction until a smooth even coat is attained. While constantly rotating the cylinder, warm air is passed through with a blower until the coat takes on a dull appearance. The cylinder is then mounted on a stand and warm, radon-free nitrogen allowed to flow slowly through for about two hours. At the end of this time the sample cylinder is assembled, placed into a screen-wall counter, and put on a rough mechanical vacuum pump with dry-ice trays for four hours or longer.

Sample size--A preliminary series of experiments were carried out to determine the relationship between sample weight and count rate. Table 2 shows the results of these studies. Providing an even coating of the carbon can be obtained, it is apparent that within the statistical error a sample of 4.0 grams gives the same count rate as one of 10.0 grams. A "thick source" requires 8.5 grams. That the count rate falls off more slowly than would be predicted from the absorption curve of C-14 betas suggests that the counter efficiency varies with residual range of the betas.

Table 2--Relation of sample size to count rate for modern wood sample

Weight of sample gm	Counter	Counts/min
9.80	C'	6.03 ± 0.14
8.00	F	6.05 ± 0.07
6.00	C	5.88 ± 0.12
4.00	F	5.71 ± 0.11
1.85	C	5.32 ± 0.20

At only 1.85 grams of carbon the count rate is still 85-90 pct of the "thick source." Thus it appears possible that, with slightly reduced precision, samples as small as 1.0 gram may be counted successfully. This is highly desirable for many applications of the C^{-14} age method.

Filling system--The counter is transferred to the filling system and a 90°C furnace placed around the center of the barrel. It is pumped down to one micron pressure or less, the furnace turned off, and the pumping continued for at least an hour. The pressure is now about 0.1 micron. The counter is then filled with 0.5 cm of ethylene and 9.5 cm of argon. The tank ethylene is previously purified by successively condensing into the trap and pumping on it, expanding, etc., and then stored in bulb D. The tank argon requires no further treatment. The starting potential is very sensitive to filling mixture, particularly the ethylene pressure. A clean counter gives a starting potential of about 1200 volts with this mixture.

Gamma-beta shield

The most critical problem in the soft-beta counting of samples of low activity is the reduction of the background to a value equal to or less than the activity of most samples. Thus for a thick source of a modern natural carbon sample in a screen-wall counter of the dimensions described above, the total sample count is six to seven counts per minute while the unshielded background count of the same counter would be on the order of 350 cpm. There are two techniques required to reduce this background; shielding, and anticoincidence counting. The former can eliminate all betas, most gammas, and possibly some soft cosmic-ray components. ANDERSON, ARNOLD, and LIBBY [1951] found that an eight-inch iron shield gave maximum shielding. This reduces the background to about 125 cpm.

The details of the shield built in this laboratory are given in Figures 4, 5, and 6. The advantages are: (1) Two compartments; (2) no single piece of iron weighs more than 125 lb. so that it can be easily put together by two men in restricted space without heavy equipment; (3) the very simple door movement which allows finger-tip control of the 600-lb doors without requiring any devices which can become defective, the doors rolling on solid two-inch cylinders of steel; (4) it is open underneath so that additional equipment, such as automatic sample-changing apparatus, can be added; (5) there is no machining except for drilling and bolting the doors.

The support is by six oak 8 × 8-inch blocks aided by four mechanical jacks, each of ten-ton capacity. The entire shield weighs about ten tons. The roof is made of 1/2-inch plates, the side of 1 × 8 × 40-inch plates; the bottom and back are 1 × 8 × 48-inch plates. The 1/2-inch roof plates alternately stacked are 26 and 30 inches long to give a total length of 56 inches. They are staggered to prevent a vertical crack. Figure 4 is a front view, Figure 5 a side view, and Figure 6 a top view omitting the roof. The dashed lines on the sides are openings for electrical connections and the squares in the floor of the counter space are for an automatic sample-changer eccentric rod.

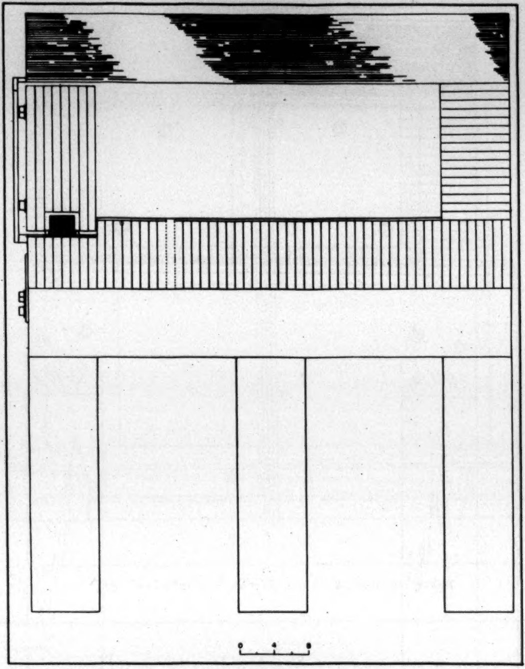

Fig. 4--Details of shield, front view

Anticoincidence counting equipment

Of the 125 cpm left in the background after the counter is placed inside the shield, 115-120 are due to cosmic radiation. This can be removed by the use of a ring of parallel Geiger counters 2 inches diameter by 30 inches long which are connected in anticoincidence with the screen-wall counter. Figure 7 is a diagram giving the essential elements of the anticoincidence circuit. Nine cosmic-ray counters are used in a tight ring around the screen-wall counter. Two regulated high-voltage supplies and amplifiers are required. The cosmic-ray tube pulses are stretched, the screen-wall pulses inverted, and then they are mixed in such a way that only pulses other than those penetrating the ring of counters are registered. A selector switch makes it possible to monitor the tube behavior with an oscilloscope by showing cosmic-ray pulses only, screen-wall pulses only, or the cancelled pulses plus those which are passed to the register. If the register is not at least as fast as a Cyclotron Specialty register, a scale of four is desirable between the mixer and the register. A counting loss of about eight per cent was observed with the Veeder-Root mechanical register in this application. The number of accidental coincidences using nine of these cosmic-ray counters is about 0.1 pct, which is well below the statistical counting error for a normal sample count.

The electronic unit was built according to our specifications by the Atomic Instrument Company, Cambridge, Massachusetts, and has been catalogued by them as the Model 503 B Anticoincidence Analyzer.

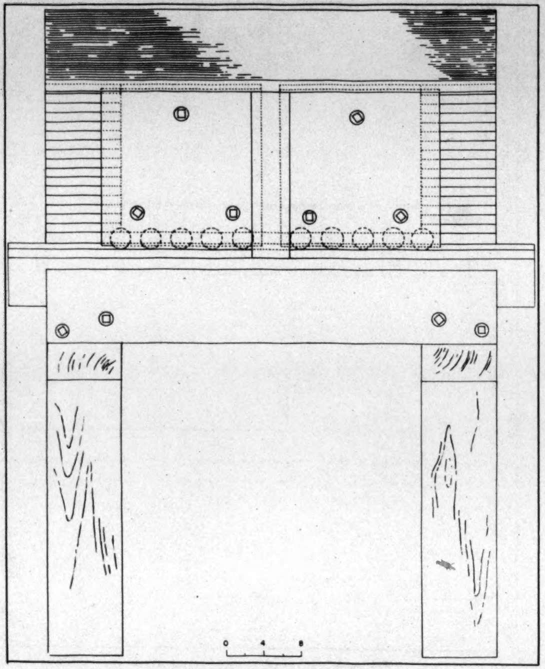

Fig. 5--Details of shield, side view

Numerous experiments have been made to study the source of the residual background. Six additional cosmic-ray tubes were placed on top of the ring of nine. No appreciable reduction in background occurred, indicating that the present apparatus is operating close to 100 pct efficiency as far as the cosmic-ray elimination is concerned. Preliminary work was carried out with 18-inch cosmic-ray counters. The use of 30-inch counters improved the background by about one count per minute. This is approximately the calculated value assuming a cosine-squared distribution in the intensity of cosmic rays as a function of angle from the vertical. The use of some additional counters arranged perpendicular to the axis of the nine cosmic-ray counters to eliminate any radiation coming in horizontally resulted in a further lowering of the background by about 0.3 cpm. Under these conditions, operating the stainless-steel screen-wall counter at 100 volts over the starting potential, the counters have backgrounds from 4.5 to 8.0 cpm depending on their history and cleanness. If the counter is operated at 30 volts over the starting potential, this can be reduced by 0.5 to 1.0 cpm.

Since the background is similar for stainless steel and copper wire, and since it is similar for stainless steel, pure copper, or special lead-free brass sample cylinders, it is probable that the residual background does not originate in the metal parts of the screen-wall counter. This leaves the shield itself, the rack, the cosmic-ray tubes, and the lucite as possible sources of activity. The lucite is made from dead carbon, according to Dupont officials. It was decided to check this. A sample of lucite was burned and the carbon prepared, counted, and found to yield 0.5 to 1.0 cpm over background. At the same time another counter was constructed using a

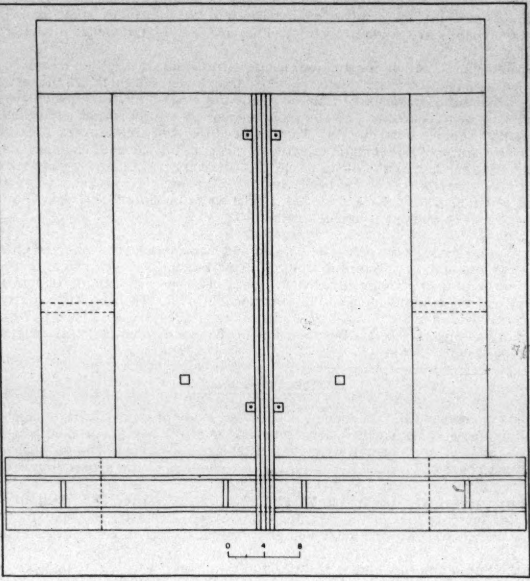

Fig. 6--Details of shield, top view, roof removed

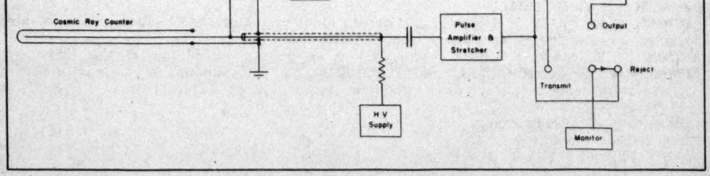
Cosmic Ray Counter
H V Supply
Screen Wall Counter
Pulse Amplifier & Inverter
Mixer
Register
Drag Supply
Output
Cosmic Ray Counter
Pulse Amplifier & Stretcher
Transmit
Reject
H V Supply
Monitor

Fig. 7--Anticoincidence circuit

different piece of lucite stock. This counter has a background about one count per minute lower than all the others. Contamination from the insulator used is being studied further. A counter is being constructed from polystyrene. The rack used by Libby was made of wood while ours was made of brass. While it appears that a test of the materials of which the cosmic-ray counters are made would be useful, the small gamma activity in the relatively small mass of the material of which these counters are made probably can not account for the residual background.

J. R. Arnold (personal communication) suggested the shield itself as the most likely source of residual background. To study this a mercury shield one inch thick, 12 inches long, was made to fit closely around the screen-wall counter inside the ring of anticoincidence counters. About 125 lb of mercury was required. The available mercury was of unknown purity but was processed with nitric acid to remove base metals. This reduced the background from 1.0 to 1.5 cpm. Since it is possible that some of the remaining activity (2.5 cpm for the best counter) may be coming from the mercury, another shield using triple-distilled mercury is under construction. Thus, at the present time, it is possible to get backgrounds as low as 2.5 cpm and conceivable to get in the neighborhood of 1.0 cpm. If this is achieved, it will add 10,000 to 20,000 years to the effective range of the carbon-14 method of age determination.

In the actual counting procedure, the sample and background positions are changed every four hours except for an eight-hour interval at night. This is continued for two to three days so that the probable error in the net count is about 0.1 cpm. The age of a sample is determined by obtaining the ratio of the sample count to that for modern wood. The specific activity of 14.7 dpm gm carefully determined for modern wood by W. F. Libby is accepted as a base to refer published results. It should be emphasized that the accuracy of the age determination is independent of the absolute age of the specific activity.

Acknowledgments

The research reported in this document has been made possible through support and sponsorship extended by the Geophysical Research Directorate of the Air Force Cambridge Research Laboratories, AMC, under Contract No. AF 19 (122)-214. It is published for technical information only and does not represent recommendations or conclusions of the sponsoring agency.

The authors are greatly indebted to W. F. Libby, who permitted J. L. Kulp to work in his laboratory at the Institute for Nuclear Studies, University of Chicago, in January 1950, and to W. F. Libby and J. R. Arnold, who made numerous helpful suggestions and criticisms.

Many of the minor improvements described in this paper were first suggested during discussions at Chicago. Without this generous cooperation it would have been impossible to accomplish the work herein described in so short a time.

This paper is submitted in partial fulfillment of the requirements for the Master of Science degree at Columbia University by L. E. Tryon. The authors acknowledge the considerable assistance of Hans J. Holland and Walter Eckelmann in the carbon preparation study, and Raymond Edwards in the design and construction of the shield.

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(Manuscript received August 24, 1951; part of presentation at Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until September 1, 1952.)

THE OLDEST ROCKS

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Abstract--The greatest geological ages thus far determined are from pegmatites from three widely separated areas--southeastern Manitoba (Canadian Shield), Southern Rhodesia (Southern African Shield), and Varuträsk, Sweden (Baltic Shield)--for each of which the age appears to be slightly over two billion years. Pegmatites from a fourth area, northern Karelia, may be of about the same age, or a little less. These great ages have been determined mainly by the strontium and lead methods.

The ages of the oldest rocks have long been a source of much speculation and investigation. Until fairly recently, radioactive age measurements were based on the crude lead method (see 1 below) and to some extent on the helium method. (The "lead method" includes several procedures which may be distinguished as follows: (1) a method based entirely on a chemical determination of the ratio Pb/U + Th as a function of age; (2) a combination of chemical analysis and a quantitative analysis of lead isotope composition, ages being calculated by various parent isotope/daughter isotope ratios and also Pb^{207}/Pb^{206} ratios; (3) quantitative isotope analysis alone in which only Pb^{207}/Pb^{206} ratios are determined as a function of age.)

Another method, the strontium method based on the natural radioactivity of rubidium, is relatively recent. Although this method is not well suited for the determination of relatively young ages, it is particularly valuable for determining the ages of some of the most ancient rocks. Many of the data discussed here, some of them new, are based on age measurements by the strontium method.

Southeastern Manitoba--It has been known for some time that pegmatites from Manitobe have yielded great ages, namely, about 2000 × 10^6 years. The earlier measurements were made by the lead method on uraninite and monazite (see discussion and calculations by HOLMES [1948], who recommends an age of 2000 × 10^6 years; a complete bibliography is given). (Later note: Recent precise measurements by FLEMING, GHIORSO, and CUNNINGHAM [1951] of the half-life of U^{235} (using > 99.95 pct pure U^{235}) give a value of 7.07 ± 0.11) × 10^8 years. This checks the value (7.13 × 10^8 years) determined by NIER [1939a] and used by NIER [1939b, 1941] for calculation of Pb^{207}/Pb^{206} ages of Manitoba uraninite (2200 × 10^6 years) and monazite (2600 × 10^6 years), and gives support to the validity of their ages. These two ages are in better agreement with the latest strontium age (see below) of 2400 × 10^6 years than the value of 2000 × 10^6 years.)

Strontium age measurements on lepidolite corroborate this magnitude. AHRENS [1947] gives strontium age measurements of 2100 to 2400 × 10^6 years (four separate specimens), and AHRENS and GORFINKLE [1950] give an age of 2400 × 10^6 years, using an entirely different analytical procedure. All these data, in addition to indirect support from helium age measurements [HURLEY, 1949], show an age magnitude of 2000 to 2400 × 10^6 years for southeastern Manitoba pegmatites.

Northern (Russian) Karelia--Pegmatites from a second area, namely, Russian Karelia, have also been known for some time to be of great age. Although their exact age, and the possibility that they should be the greatest age, has been a cause of some dispute, there seems little doubt that a magnitude of 1800 to 2000 × 10^6 years is reliable. Numerous lead age measurements (procedure (1), see above) have been made on uraninite and monazite from several pegmatites, and all agree in magnitude [KHLOPIN and VLADIMIROVNA, 1938]. According to a statement by these authors, the uraninite + monazite-bearing pegmatites and associated granites in northern Karelia represent an end phase of a complex cycle of orogenic events.

Southern Rhodesia--AHRENS and MACGREGOR [1951] have recently given nine strontium ages of pegmatites from different localities in the Precambrian Shield in Southern Rhodesia. With one exception, all ages are in good agreement (1900 to 2400 × 10^6 years) with an average of 2100 × 10^6 years. These ages have been calibrated by running a Manitoba lepidolite taken as 2200 × 10^6 years (mean of all lead and strontium ages) with the Rhodesian specimens.

Varuträsk, Sweden--A fourth area of like age appears to be that of Varuträsk, Sweden. Strontium ages on two lepidolite specimens (calibrated as above) made by Ahrens and Davidson (unpublished data) are given in Table 1. Strontium from the first specimen was concentrated and analyzed by W. T. Leland and A. O. Nier (Department of Physics, University of Minnesota), who found it essentially radiogenic. The first age (2150 × 10^6 years) is therefore more reliable than the second.

Table 1--Strontium ages of lepidolites from Varuträsk, Sweden; analysts, M. Davidson and L. H. Ahrens

Sample	Age
	10^6 yr
Coarse flakes	2200 1800 2800[a] 2000 1900
Average	2150
Medium-grained specimen	2300 2700 2500 2300 2500
Average	2400

[a] Average is 2000 if this value is omitted.

An earlier strontium age measurement on microcline from Varuträsk is reported by EKLUND [1946] as 1740×10^6 years. A half-life for Rb^{87} of 5.8×10^{10} years was used, whereas all strontium age determinations discussed heretofore in this paper are based on a value of 5.9×10^{10} years (see discussion on half-life of Rb^{87} by AHRENS [1950]). Using this value increases the age slightly to 1800×10^6 years. Eklund states that total strontium was determined by HYBBINETTE [1943], who employed a combined chemical enrichment--spectrochemical analysis procedure; 0.0069 ± 0.0006 pct Sr is reported. The tolerance is equivalent to about $\pm 200 \times 10^6$ years, and the age of 1800×10^6 years is not in actual disagreement with the above value of 2150×10^6 years which carries a similar tolerance. The latter value is, however, recommended for this discussion because it is based on the same method of analysis as employed for the other strontium age determinations and, moreover, has been calibrated as mentioned above.

The age measurements on all four areas discussed thus far are given in Table 2. These four areas are the only known ones which have yielded ages of such extreme antiquity.

Because all these great ages, with the possible exception of northern Karelia which may be a little younger, are very similar, they may represent a single period of major (world-wide) orogenic disturbance. Moreover, as no rocks of greater age have been reported, this period of crustal disturbance may have been the first major cycle of orogenic events in Earth history. If so, it forms a basis for division of Precambrian time into two major subdivisions, namely, pre- and post-2200×10^6 years (name ?) orogeny.

It is true that field evidence in Precambrian exposures shows that some of the dated pegmatites and associated granites intrude formations containing pebbles of a presumably older granite.

Table 2--Ages of greatest antiquity

Source of sample	Method of analysis	Authority	Age
			10^6 yr
Southeastern Manitoba along Winnipeg River	Pb (uraninite and monazite)	HOLMES [1948] recalculations	2000
	Sr	AHRENS [1947]	2100-2400
	Sr	AHRENS and GORFINKLE [1950]	2400
		Approximate mean	2200
Northern Karelia, Russia	Pb (many specimens of uraninite and monazite)	KHLOPIN and VLADIMIROVNA [1938]	1800-2000
New Salisbury, S. Rhodesia	Sr	AHRENS and MACGREGOR [1951]	2150 2200
Wankie Dist., S. Rhodesia	Sr	AHRENS and MACGREGOR [1951]	2100
Odzi Dist., S. Rhodesia	Sr	AHRENS and MACGREGOR [1951]	2100
Antelope Mine, S. Rhodesia	Sr	AHRENS and MACGREGOR [1951]	1900
Bikita Dist., S. Rhodesia	Sr	AHRENS and MACGREGOR [1951]	2150 2950[a] 2050 2300
		Approximate mean	2100
Varuträsk, Sweden	Sr	Table 1	2400 2150

[a] Rejected.

Such a granite pebble may, however, be representative of a granite belonging to an earlier igneous emplacement phase within the same major orogenic cycle, perhaps 100 to 200 × 10^6 years older, and not to a distinctly older cycle.

It should be stressed that the absolute value of about 2200 × 10^6 years as the age of these most ancient rocks should not be taken too rigorously. Some systematic error (bias), due either to error in the assumed values of decay constants, analytical error, or both, may be present. Such an error should not be large, however, and does not in any way disturb the conclusion about which are the oldest rocks dated thus far.

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(Manuscript received August 9, 1951; part of a paper presented at the Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until September 1, 1952.)

SOME CONSIDERATIONS REGARDING GEOCHRONOLOGY WITH SPECIAL REFERENCE TO PRECAMBRIAN TIME

J. Tuzo Wilson

Abstract--The recent dating of some Precambrian rocks in Canada by radioactive methods suggests that certain rocks of Proterozoic type are much older than others of Archean type. C. K. Leith had already observed that this might be the case. It has recently been noted that the Shield appears to be divided into provinces of Archean rocks, structurally and petrologically distinct, and that these may well be the roots of former mountains of different ages. It is suggested that each such province was active during a definite period, which can be approximately dated, during which time it was being built into a primary mountain belt. It is also suggested that the patches of Proterozoic rocks are remnants of secondary deposits each associated with an Archean belt. This classification is proposed as a means of subdividing geological time as well as for distinguishing provinces of the Shield.

The oldest provinces in at least three continents contain rocks of a greenstone facies, in which limestone is lacking but in which rich gold mineralization is widespread. It is thought that these greenstone belts may be contemporaneous and thus represent a distinct division of early Archean time not previously distinguished.

Introduction--Although I am only familiar in detail with work on chronology now in progress at the University of Toronto, I should first like to refer briefly to some of the work which is in progress elsewhere in Canada and then mention the work at Toronto. I propose next to discuss some difficulties about the present method of subdividing Precambrian time and rocks, and describe the progress being made in dividing the Canadian Shield into provinces and in dating them. Finally, I would like to suggest the existence of a new major division within the Archean and propose a general basis upon which time scales for the Precambrian may perhaps be established. Experimental details will shortly be published elsewhere.

Studies upon the ages of Canadian rocks--Age work in Canada may be reviewed from east to west by mentioning a few of the workers and one or two of the recent papers of each. Other references are contained in the Canadian Geophysical Bulletin of 1948. At Dalhousie University in Halifax, G. H. HENDERSON [1934, and HENDERSON and SPARKS, 1939] was probably the leading authority upon pleochroic haloes until his recent untimely death, but some of his colleagues are continuing his work. The great amount of information which Henderson was able to extract from these haloes is not as widely known as it should be.

At Laval University, L. Kerwin has built a mass spectrometer and hopes to use it for geological age work. GILL [1948a, 1949] at McGill University has made many contributions towards determining the structural and age relations existing between different areas of Precambrian rocks and has discussed these in the light of radioactive age determinations. A number of geologists from McGill University, Massachusetts Institute of Technology, Quebec Bureau of Mines, and Geological Survey of Canada have been and are still working upon the boundary between the Grenville and Keewatin provinces, about the exact nature of which there is still some dispute.

At the Geological Survey of Canada, ELLSWORTH [1932, 1950], who has made many notable contributions towards determining the ages of Canadian rocks by radioactive methods, is continuing his work with an enlarged group of colleagues including LANG [1950, 1951] and ROBINSON [1950]. They work closely with the Bureau of Mines [SPENCE, 1940].

Although COOKE [1947] and WILSON [1941a] are no longer with the Geological Survey, both of them continue to be actively interested in the structural approach to the age relations of Precambrian rocks. They were among the first to divide the Canadian Shield into geological provinces and to suggest age relations between them.

At McMaster University, THODE [in press] is pursuing a very active program of study of variations in the isotope ratios of some of the lighter elements. There are some aspects of this related to age.

At the University of Manitoba there is interest in radioactivity of rocks, but it has resulted in new methods of measuring quantities of radioactive elements rather than in age determinations [PRINGLE, ROULSON, and BROWNELL, 1950]. HARRISON [1951] and INNES [in press], among others, have considered the age relations between the different geological provinces in northern Manitoba and Saskatchewan.

Work largely by JOLLIFFE [1948] of Queen's University and J. F. HENDERSON [1948] and LORD [1942] of the Geological Survey has served to define the chief geological provinces in the northwestern part of the Canadian Shield. Jolliffe has stated the relations between them in a manner that has found general acceptance, and he has also collected data to give chemical age determinations of some of the rocks in that region.

This list is by no means complete, as almost every Precambrian geologist is concerned with age relationships, and more than 50 determinations of age of Canadian rocks made by chemical analyses for lead and uranium have been made by various men and published in the Reports of the Committee on the Measurement of Geologic Time and elsewhere. Many geologists have discovered stratigraphical and structural relationships which serve excellently for local purposes, especially in Proterozoic rocks, but these have not been sufficiently continuous nor have there been enough isotopic age determinations to enable any general agreement to be reached upon how best to subdivide the Precambrian as a whole [GROUT and Others, 1951].

The importance of obtaining a greater number of age determinations and ones that are more reliable has been evident for some time. It has been considered that the most reliable absolute method is by isotopic analysis (and preferably chemical analysis also) of lead from uranium-bearing minerals. Fortunately there are many localities in the Canadian Shield where at least small quantities of these minerals occur. Almost the only modern determinations of ratios of lead isotopes which have been published are those by NIER [1939, 1941] and by HOLMES and Others [1949, 1950ab].

This prompted E. F. Burton and N. B. Keevil to start to build a mass spectrograph for that purpose at Toronto, but the War interrupted their efforts. Work was started again by C. C. Gottlieb and E. C. Bullard, and the instrument was completed and put into operation in 1951 by C. B. Collins. It has a resolution of one mass unit in 300. To avoid the difficulties caused by introducing lead for analysis as lead iodide, Collins proposed and has been successful in introducing lead in the form of lead tetramethyl, which avoids those difficulties and speeds up the work, so that about 95 isotopic determinations of ages based upon the ratio of Pb^{206}/Pb^{207} were made by the spring of 1952 [COLLINS and Others, 1951; COLLINS and FREEMAN, in press].

A description of this new method of analysis is being prepared and details of the results will be published when some more determinations have been made. For those deposits at Eldorado Mine, Great Bear Lake, and Cardiff, Ontario, and Katanga, Belgian Congo, for which age determinations have been made both by Nier and Collins, the ages have been found to be in very close agreement.

The object of the work at the University of Toronto is to develop as many reliable age determinations as possible which can be used as standards. Particular attention is being paid to determining the exact geological relations of the specimens. At present fairly large specimens yielding a minimum of 50 mg of lead are being used. For Precambrian rocks this means either a small nodule about one-half inch in diameter of uraninite or a larger hand specimen of lower grade ore. It is realized that these requirements severely limit the number of localities from which material can be obtained, but it is considered that there are enough localities in the world and certainly in the Canadian Shield to make such a standard scale useful. It is known that other methods will have more widespread application, but the isotopic lead method used upon fairly large samples seems to offer the most reliable method for calibration. Contributions of suitable mineral specimens or of lead extracted from specimens (as $PbCl_2$) for age determination will be most welcome.

Proterozoic and Archean--The main divisions of Precambrian time have for a long time been regarded by some as Proterozoic (or Algonkian[a]) and Archean (or Archeozoic). As a matter of fact, over the Canadian Shield as a whole these divisions are not as useful as they might be because typical unmetamorphosed Proterozoic rocks occupy only relatively small and discontinuous areas of the Shield. In this respect the whole Canadian Shield differs markedly from that part of it in the United States around Lake Superior, which is mostly composed of Proterozoic rocks as is the African Shield also.

This twofold division is based upon the observation that in many localities, like the Grand Canyon of the Colorado, unmetamorphosed and gently dipping Proterozoic rocks lie beneath the Cambrian and upon a peneplained surface of older highly metamorphosed and deformed Archean rocks. LEITH and Others [1935] pointed out that there was no proof that the boundary between these rock types was everywhere of the same date, and they recommended that these names be used to distinguish types, not age, of Precambrian rocks.

It is known that degree of metamorphism is not necessarily related to age. Many rocks once regarded as Archean have been found to contain altered fossils or to pass unmistakably into those that do. These rocks have had to be transferred from early Precambrian to Paleozoic and even Cenozoic age. Highly deformed and altered rocks are everywhere found in the hearts of primary mountain ranges and probably Archean rocks are of this type, but old mountain roots have been peneplained and exposed over large areas in the shields.

Where eroded mountain roots are overlain by younger rocks there is commonly a great unconformity and the younger rocks are not altered. Where such rocks are non-fossiliferous they have been called Proterozoic, but this does not establish that they are everywhere late Precambrian. The recent discovery by Collins that some of the minerals from Proterozoic types of rocks in the Athabasca region are about 16×10^8 years old and are older than any ages for Archean type found in the Grenville province, will, if substantiated, either prove this view or require that all our ideas about the value of age determinations be revised.

If we follow the present view, then we can see that there are two great classes of rocks on continents, those that have been involved in the building of primary mountains and those that have not. Regardless of their age, we may call those rocks from the roots of primary mountains

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[a] Officially abandoned by the U. S. Geological Survey in 1938.--Ed.

"oronic" rocks. The Archean rocks are oronic rocks of Precambrian age or more precisely those which were involved in mountain building during Precambrian time. Oronic rocks everywhere form the basement or sialic crust of which the continents are composed. The rocks which have not been involved in any mountain building or only in the building of secondary mountains we may call "epeironic." They rest everywhere upon oronic rocks. Proterozoic rocks are the epeironic rocks of Precambrian age.

The distinction between primary and secondary mountains is that primary mountains have received accessions of hot fluids from the depths. They contain abundant gneisses and intrusive granites while secondary mountains do not. Secondary mountains always are associated with and are younger than the corresponding primary mountains and have formed upon the continental side of primary mountains. The Coast Ranges and Rocky Mountains of North America and the Western and Eastern Andes are examples of related pairs of primary and secondary mountains. It is believed that the Grenville and Huronian rocks and the rocks of the Labrador coast and of the Labrador trough are other older pairs of examples of related primary and secondary mountain roots. This distinction has been discussed in more detail elsewhere [J. T. WILSON, 1949a, 1951]. According to this view, continents have grown and the basements of them are made up of belts of mountain roots which have been added in succession around the margins of the original nuclei. The source of the necessary material and the rate of accumulation of continental material has also been discussed [J. T. WILSON, in press].

The alternative view to that advocated by Leith is to set an arbitrary date, say 10^9 years ago, and to define all more recent Precambrian rocks as Proterozoic and all older ones as Archean. This leads to great practical difficulties. There is good reason to believe that some undisturbed epeironic rocks are much older than this while many metamorphosed oronic rocks are younger. According to that classification, the appearance of the rock would no longer be grounds for telling whether it was Proterozoic or Archean. Instead, an age determination is needed, but these are scarce. Once an age has been obtained, it would be more useful to give the age than to merely classify the rocks as older or younger than some arbitrary date.

To sum up, the Proterozoic rocks form isolated and moderate-sized patches over the Canadian Shield as a whole. Each patch can be dealt with by normal stratigraphical methods, but they have not been correlated, and there is evidence that not all Proterozoic type of rocks are the same age or even that they are all younger than all Archean rocks. Practically every patch of Proterozoic rocks bears a faulted relation to Archean rocks along one margin. It is considered that the patches of Proterozoic rocks are related to provinces of Archean rocks and that the crux of the problem is how to subdivide the Archean.

Although ADAMS and BARLOW [1910], M. E. WILSON [1941b], THOMSON [1948, 1949], PETTIJOHN [1943], among others, have shown that with patience the local order of succession of Archean rocks can be worked out by large-scale mapping, the areas so far mapped in this manner are small. Illuminating and valuable as this work is, it has not yet led to any proposal for a general succession of Archean rocks applicable to the whole Shield. There is no evidence that it is likely so to do in the near future, for the correlations between these small, well-studied areas is uncertain.

Provinces of the Canadian Shield and their ages--At present a more fruitful line of attack stems from the recognition that the Shield is divided into geological provinces within each of which there is some local similarity of rock types, mineralization, and age, while between them there is little connection.

It has been known for a long time that the general lithology of some parts of the Archean in the Canadian Shield differs from other parts. The distinction was first recognized between the Grenville province and the Keewatin province. The boundary between them extends northeast from Lake Huron to pass south of Sudbury and into central Quebec south of Lake Mistassini. Other provinces have gradually been recognized by different geologists, and M. E. WILSON [1941a], COOKE [1947], GILL [1949], A. Holmes (unpublished map, 1948), and J. T. WILSON [1949ab] have made attempts to define the provinces for the whole Shield.

Naturally, there are points of difference and some of the boundaries are yet unknown. It is for that reason that the latest general maps of Canada [GEOLOGICAL SURVEY OF CANADA, 1945; DERRY, 1950] do not show these provinces, although both contain some hints of their existence and the tectonic map of Canada shows most of the known boundaries as probable fault zones.

The principal distinction so far noted is that there are two great areas or provinces in the

Canadian Shield characterized by abundant basic volcanic rocks with granites and granite gneisses, and that between and around these two areas are other regions without any great amount of basic volcanic rocks but consisting instead of gneisses and the altered equivalents of normal sedimentary rocks with small areas of granitic intrusives. The two provinces marked by volcanic rocks, now mostly greenstones, also contain some tuffs, greywackes, and small amounts of other sediments but no limestone nor pure quartzites. There is rich gold mineralization at many places in these two provinces.

One of these greenstone provinces, the Superior or Keewatin province, includes northwestern Quebec, that part of Ontario north of Lakes Huron and Superior, and eastern Manitoba. The other, the Yellowknife Province, lies north of Great Slave Lake and extends half way to Great Bear Lake as well as for an unknown distance to the northeast.

Evidence is accumulating that the boundary between provinces is generally a fault zone and in many cases that patches of Proterozoic rocks are downfaulted on one side of the boundary and hence overlie the Archean rocks of one province and bear indeterminate or uncertain age relations to the Archean rocks of the other province. An example of this uncertainty is the relation of the Huronian rocks to the Grenville along the Huron-Mistassini fault zone, although the fact that the Huronian rocks overlie the Keewatin is perfectly clear.

This idea of provinces in the Archean separated by fault zones has been strengthened by three suggestions. One is that each province also has a definite foliation pattern and that along the boundaries one pattern in some cases cuts off another [GILL, 1948b]. A second is that because these foliation patterns bear a striking resemblance to the foliation pattern already mapped in the heart of younger mountains, therefore these provinces may correspond to the roots of ancient mountains [J. T. WILSON, 1949a]. This idea had long been held in the case of the Grenville province along the St. Lawrence River [ADAMS and BARLOW, 1910]. Most important of all, when ELLSWORTH [1932] had made several dozen determinations of the age of radioactive minerals from pegmatites in the Grenville and Keewatin provinces he found that all the more reliable determinations in the Grenville province lay within a range (8 to 11 × 10^8 years) and that another age, a much greater one (18 × 10^8 years now regarded as probably 22 × 10^8 years), was obtained for specimens from the Superior Province.

Many determinations in the Appalachian region all gave younger ages (2 to 6 × 10^8 years), and Nier by isotopic analysis confirmed the correctness of these earlier determinations which had been based upon chemical methods alone. By analogy with the way in which pegmatites in the Appalachian region are of the same age as the period of mountain building of that province, it was natural to conclude that the ages of the Precambrian provinces corresponded to the periods when they were being built.

It seems likely that if there are a number of age determinations in a region, then some of the younger ages may not be diagnostic, but if a number of them, especially from pegmatites, cluster about a maximum age, then that may well represent the time when that area was being built into marginal mountains and was being transformed from a coastal shelf to a part of the continental block.

It is true that some hold that even the most correct determination of the age of a mineral only shows that the country rocks are indefinitely older and do not indicate that there is any relation between the age of the deposition of the mineral and of the mountain building because the mineral may have been deposited at any later date. This seems sometimes to be the case as, for example, with the uraninite of Paleozoic age found near Beaver Lodge Lake a few miles south of Great Bear Lake. The Ordovician rocks nearby are undisturbed and the mineral must have been introduced without mountain building or granite intrusion occurring in that area. On the other hand, all the evidence from the Appalachian determinations and much from isotopic determinations elsewhere suggests that most age determinations in any region are indicative of the time when the region was undergoing orogenesis, and this fits the usual theories of ore deposition.

<ins>The greenstone provinces as the most ancient nuclei</ins>--It has been suggested that the Proterozoic and Archean types of rocks do not represent a true division of Precambrian time. There is, however, another method of dividing Precambrian rocks, by their lithology, which may be valid. We have seen that at least two greenstone provinces can be distinguished in North America. There are clearly others in Australia and South Africa. In all cases the evidence suggests that they are the oldest rocks known. Perhaps we can distinguish between an early period of Precambrian time in which volcanic rocks were predominant and a later period in which normal sediments began to be deposited. It may be that this division is universal, although it is not yet proven.

The oldest rocks yet dated in North America are the pegmatites cutting Keewatin rocks in southeast Manitoba. These are about 22 × 10^8 years [NIER, 1939]. HURLEY [1949] has found similar if less certain ages in the eastern part of the Keewatin province. We may take it that if North America has grown by the accretion of new provinces, then one nucleus was the Keewatin province consisting of Keewatin lavas and tuffs cut by granite gneisses but lacking pure limestones and quartzites. In it there are a large number of areas of gold mineralization including Porcupine, Kirkland Lake, and Larder Lake mining camps.

Another nucleus of old rocks in North America is the Yellowknife province again marked by lavas and tuffs and abundant banded greywackes cut by granites and gneisses but also lacking pure limestone and quartzites. Again there are important gold mines. No radioactive minerals from this province have been dated but LORD [1942] and Wilson, who mapped the boundary with the Snare or Great Bear Province (age 14 × 10^8 years), agree that the Yellowknife Province is older and thus also perhaps about 20 × 10^8 years.

In Western Australia the oldest rocks are the Kalgoorlie Series of basic volcanics with associated iron formations, tuffs, and slates.

McKINSTRY [1945] has pointed out the striking similarities with the Keewatin rocks of Ontario and Quebec. In Western Australia as in Canada there are no limestones nor quartzites present but rich gold deposits. There are no age determinations from these rocks and few in Australia, but these rocks are usually classed among the oldest in Australia. Conversations with E. de C. Clarke and R. T. Prider at Perth and the frontispiece map in their book [CLARKE and Others, 1948] suggest that the Kalgoorlie rocks might be regarded as forming a nucleus or center, and that on every side there are younger rocks characterized by sedimentary gneisses or normal sediments. These include the Jimperding series and the young rocks west of the Darling Scarp on the west, and gneisses and whitestone series with an east-west strike mapped near Hopetown and Albany along the south coast by Clarke.

In South Africa the greenstone rocks which occur in contiguous parts of the Transvaal, Southern Rhodesia, and Swaziland are regarded as the oldest rocks and there is an age determination from them of 20 × 10^8 years [AHRENS, 1949]. These rocks consist predominantly of greenstones. They contain numerous gold deposits such as those at Barberton. There are no limestones but quartzites are not uncommon.

There is thus good evidence that the oldest rocks in each of these areas and perhaps in each continent are of the same peculiar petrological types and are marked by basic volcanics, gold mineralization, and lack of limestone. In each case the pattern of folding is also much the same for they form many narrow and steeply folded synclines striking in fairly sharply twisting bands which preserve a generally consistent direction over long distances like the east-west strike of the Keewatin rocks or the north-northwest strike of the Kalgoorlie series.

Although there are old rocks in Karelia in the Fenno-Scandian Shield, the writer is not familiar with them. They may not be of greenstone facies but neither do they appear to be so old as the other ancient rocks mentioned [HOLMES, 1948]. There may be a greenstone area in south India.

It is suggested that these greenstone volcanic areas which are all probably 20 × 10^8 or more years old represent the original nuclei of the continents. Normal sediments were only added around them once an initial and highly volcanic state in the Earth's growth was passed. If this is true, then it suggests that the Earth started with a uniform basic ocean floor with no continents, and that continents, like oceans and atmosphere [RUBEY, 1951; KULP, 1951] have grown during geological time as a result of the Earth being of such size and surface temperature that it could retain a hydrosphere and atmosphere with which to rework and shape the continents. According to this view, the chief source of additional continent material has been the andesitic volcanoes of mountain and island arcs, but the composition of the original volcanic rocks may have been altered both by erosion and sorting and by granitization.

Geological time scales--The only widely applicable, accurate, and reliable geological time scale yet discovered is that based upon the evolution of forms of life as shown by preserved fossils. This method, which we may call the biological time scale, has sufficed to divide the last 5 × 10^8 years or thereabouts into some 80 stages each a few million years long and each recognized all over the world.

It is true that this scale is relative, not absolute. It is not as precise as might be wished and sections of it are usually missing in any particular locality. Nevertheless, it is by far the best method of dividing geological time yet devised. The probable error in correlations is perhaps of the order of one per cent.

Radioactive age determinations within the Paleozoic and younger eras by which this scale could be put on an absolute basis are rather unsatisfactory. There are only three or four isotopic lead ages of minerals whose age is known relative to index fossils. Virtually all of these minerals are contaminated by much ordinary lead which reduces the accuracy of their determination. Some new minerals of Paleozoic age examined by C. B. Collins have also been found to be contaminated. Although the range in absolute time given to the biological time scale appears to be correct and although HOLMES [1947], BULLARD [1944], KNOPF [1949], and WICKMAN [1944] have examined the available data with the greatest care, they would be the first to claim that more determinations are very desirable.

There is one possibility which might still upset our concepts of absolute geological time. The isotopic lead time scale depends in part upon the half life of U^{235}. This was established by Nier, but recently there have been new determinations giving both of greater and lesser values. The latest conclusions do not change Nier's value, but should the value be altered by still more new work the whole time scale and especially the youngest part of it would be altered. Fortunately, this does not seem likely to happen.

The probable error of dating of the time of deposition of a very few of the most favorable radioactive mineral deposits such as Katanga may be as little as one per cent, but in most cases radioactive ages are not yet certain within better than ten per cent and are therefore much less accurate measures of absolute age than fossils are of relative age.

Accordingly, in the fossiliferous rocks, the relative ages are well established but the absolute time scale is not well fixed. In the Precambrian rocks the situation is to some extent reversed. The biological method depending upon fossils is useless whereas rocks of many ages have been found which can be roughly dated by radioactive methods. The attempt to extend the biological time scale backwards led to the distinction between the Proterozoic and Archeozoic (or Archean) eras which it has been suggested is not a valid division of time, however useful the distinction in rock types may be.

On the other hand, if the idea of provinces corresponding to mountain roots is kept in mind, we can consider the question of the subdivision of Precambrian time more clearly. Stille and others have advanced the view that mountain building has proceeded by a series of active episodes occurring at the same time all over the world with intervening periods of quiet. RUTTEN [1949] has opposed this, and GILLULY [1949] has pointed out that it is doubtful if the dating of episodes is accurate enough to establish it. No good reason for this contemporaneity of action has been advanced. It certainly does not appear to be true today, for along the belts active at present some parts like the Atlas Mountains or the British Columbia Cordillera are quiet while other parts like Indonesia are very active. If the contemporaneity of mountain building cannot be established in post-Cambrian time where there are worldwide fossil stages every few million years, then there is no chance of establishing it in Precambrian time. The probable error in dating most Precambrian rocks seems often to exceed 10^8 years or to be as great as the whole of Mesozoic time. At present there does not seem to be any method upon which to base any subdivision of Precambrian time that is within one or two orders of magnitude of being as accurate as the relative dating by fossils in Paleozoic time.

According to the writer's views the only natural way to subdivide Precambrian time is by means of the belts of mountains by which continents have grown. In the Canadian Shield these correspond to geological provinces. We believe, during the time from 2 to 6×10^8 years ago when the Appalachians were being built, that granites and pegmatites bearing radioactive minerals were emplaced. We know that mountain-building activity ceased in the Triassic period about 2×10^8 years ago, which is also the date of the youngest minerals. Would it not be reasonable to refer to that whole period of activity as the Appalachian "oron"? We know that the adjacent belt, the Grenville province, was active from 8 to 11×10^8 years ago. It can be referred to as the Grenville oron while the central and oldest is the Superior or Keewatin oron. The word "oron" is introduced to mean an eon or period during which primary mountain building occurred at one locality.

These are not universal divisions of time like the fossil stages. It is probable, for example, that the Athabasca oron overlapped parts of both the Grenville and Keewatin orons, and it is not suggested that in other continents there are units of time corresponding exactly to any of these. Nor are most orons precisely defined. For many we have only one age determination. For only one perhaps, the Appalachian, do we have a good closing date. For none have we yet a clear idea of the beginning. In spite of this indefiniteness it seems better to suggest that we no longer try to divide Precambrian time on the basis of a universal division between unmetamorphosed young Proterozoic and metamorphic old Archean, but to suggest we face the fact that we have as yet no good universal time scale for the Precambrian. The recognition of the existence and possible significance of belts or provinces in the continents each formed within a definite period of time should also be accepted. These views will be easier to accept and will seem less radical if we recall that Precambrian time is from three to six times as long as all subsequent time and if we realize that to grow continents it is only necessary to produce and rework into sediments less than one cubic mile of rock each year. Andesitic volcanic rocks are probably being produced at that rate today. Erosion and redeposition of sediments is certainly proceeding at that rate. Sorting and partial solution by the ocean waters and the effects of granitization may suffice to account for any difference which there may be in composition between andesitic volcanics and continental gneisses.

The idea has already been advanced that the atmosphere and oceans have grown during geological time. Perhaps the easiest explanation of the riddle of the continents is to suggest that they have grown in situ by the reworking of andesitic volcanic material produced from the depths by the same processes that we see in operation today and at about the same rates. According to this view, the Earth started with a basic crust without air, oceans, or continents and all of these have been added in ways which we can see in operation today. Volcanism at an average rate approximately equal to that of today would suffice to achieve it.

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(Manuscript received October 15, 1951; presented in part at the Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

HAIL-DAMAGE FREQUENCY DISTRIBUTIONS FOR IOWA, AND A METHOD OF EVALUATING THE PROBABILITY OF A SPECIFIED AMOUNT OF HAIL DAMAGE

Wayne L. Decker

Abstract--Hail may cause significant amounts of damage to crops in agricultural areas. Using data collected by the State Department of Agriculture in Iowa, a method is described for defining the frequency distribution of hail damage to agricultural crops. These frequency distributions can be used as a means of evaluating the nature of the hail hazard in various geographic areas. It is also possible to use the frequency distribution for estimating the probability of specified amounts of hail damage.

Introduction--Crop damage due to hail can often be of sufficient size to cause significant economic losses to agricultural areas. The occurrence of heavy hail during critical crop-developmental periods can almost entirely ruin crop prospects. Although the areas which receive the brunt of an individual storm are usually quite small, the total loss from an accumulation of such storms may be quite large. REED [1931] reported that in the year 1925 there were over seven million dollars of hail damage to crops in Iowa.

This paper presents a method for evaluating the frequency distribution of hail damage using data collected for Iowa. It also shows how these frequency distributions change in form from one area of the State to another.

In Iowa the greatest loss caused by hail is experienced by the large corn crop. According to ELDREDGE [1935], corn can recover from quite serious damage during its early growth stages, while hail coming in late July or early August, just prior to tasseling, may completely ruin all crop prospects. KALTON and ELDREDGE [1947] show that in the case of soybeans, Iowa's second leading cash crop, the greatest damage can be done to the crop when hail strikes during the period of pod development. This period is somewhat later than the period of greatest damage for corn, coming in mid-August.

Other crops such as the small grain crops, orchards, and truck gardens may also suffer seriously. But these crops are of lesser importance, as far as Iowa is concerned, so do not add significantly to the total hail damage.

Statistics on hail damage are almost entirely missing in a review of the literature. The attempts which have been made to present hail statistics are often faulty due to the lack of data. Many attempts have been made to use the number of days with hail reported as the basic statistic. Although these figures may be useful to people studying hail as a meteorological phenomenon, they are not too useful from an economic point of view. The most logical criticism of such statistics is that they do not give a quantitative method of separating hailstorms where little or no damage occurs from the hailstorms which produce large amounts of damage. Then the rather potent argument of the meagerness of the reporting networks can also be legitimately raised. None the less, DE RIEMER and ABBE [1898], HENRY [1917], and DAY [1928] present figures showing the average number of storms yielding hail. In a more recent report SHANDS [1947] gives a more complete description of this line of approach. Another method of obtaining hail statistics was reported by FLORA and BUSH [1917], and by WHITTIER [1923]. This method consists of soliciting figures of hail damage from hail insurance companies. Although this method provides the much-needed material, it fails to provide a complete set of data. ROTH [1949] describes the methods used by the Crop-Hail Insurance Actuarial Association in obtaining hail-damage statistics. This method combines the experiences of a group of Hail Insurance Companies. It would seem that this effort should yield a very complete set of hail-damage data.

Hail-damage statistics in Iowa--In 1923 the Iowa County Assessors began asking farmers how much damage to their crops had been caused by hail. The farmers gave their answers in dollar values. This practice was continued for 26 years, and REED [1928, 1931] gave preliminary reports of these data. At first guess, it might seem this would provide the best data on hail damage to Iowa farm crops. Unfortunately, there are several factors which decrease their value. In the first place, since the assessor visits the farmer after the cropping season is over, the farmer might easily forget small hail damages from which his crops nearly recovered. Then many farmers may not be able

to evaluate intelligently the amount of damage which hail caused to their crops. A third factor is associated with the fact that damages are reported in dollar values based on the agricultural prices of a particular year, making it impossible to compare years, that is, a hundred-dollar damage in an inflationary year like 1947 would not be comparable with a hundred-dollar damage of a deflationary year such as 1930. Also geographical distribution of dollar hail damage would depend on variation of value per unit acreage between different types of crops and on the season-to-season and geographical variation of the different-valued crops.

The third of these difficulties may be overcome quite easily by adjusting the figures to a common economic base. It was decided that the years 1910 through 1914 would be taken as the base years, and the Iowa index of grain prices received by farmers as published by the IOWA DEPARTMENT OF AGRICULTURE [1944] was used. By adjusting the data to this base, all figures were comparable.

Type of frequency distribution--In order to study the past records of adjusted-dollar hail damage, it would seem desirable to define the distribution of dollar hail damage. If a frequency distribution could be obtained, then it would be possible to (1) determine if it were skewed, and (2) form a basis of producing probability statements concerning expected hail damage. Thus, people dealing with hail-damage statistics, either from a meteorological or economic viewpoint, would have some idea of the type of data with which they were dealing.

It was suspected that the frequency distribution would change in form as the area under consideration changed in size. In order to study this feature it was necessary to make some adjustments for the size of area, since the entire State would of necessity have more damage than a unit within the State such as a county or a group of counties. It was arbitrarily decided to use the amount of adjusted-dollar hail damage per 1000 acres of land in crops as the index. The amount

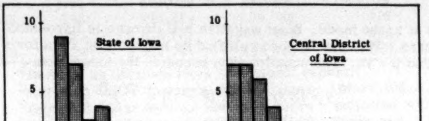

Fig. 1--Histograms of frequency of adjusted-dollar hail damage per 1000 acres for the State of Iowa, the Central District of Iowa, Polk County, and Washington Township in Polk County, 1923-1948

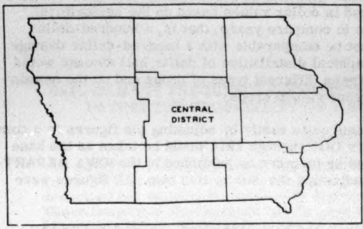

Fig. 2--Map of the nine crop-reporting districts of Iowa

of crop land as listed by the IOWA DEPARTMENT OF AGRICULTURE [1949] was used to adjust the data for size of area. Figure 1 shows histograms for the frequency of adjusted-dollar hail damage per 1000 acres of crop land for the State as a whole, for the Central crop-reporting district (an area comprising ten counties in central Iowa, see Fig. 2), for Polk County, and for Washington Township of Polk County. From these histograms, it can be readily seen that the distribution becomes markedly more skewed as the area becomes smaller. This is not surprising since the nature of the hailstorms is such that the probability of damaging hail striking a given point approaches zero, while for an area as large as the State of Iowa there is almost sure to be some damage in a given year. The important thing about these histograms is that the distribution, even for the State as a whole, is highly skewed. This makes the mean or average amount of damage a rather poor figure to use as a statistic, because the average value will not be exceeded half of the time.

From the plotted histograms in Figure 1, it can be seen that for small areas the distribution suggests a negative exponential. As the area under consideration becomes larger, there is a regular increase in the modal value. Since its flexibility permits both types of curves, it was decided to attempt to fit Pearson's type-III curve to the data. ELDERTON [1927] gives this function in the form

$$y = [(N)\, P^{P+1}/(a)\, e^{P}\ \Gamma(P + 1)]\, e^{-\gamma x}\ (1 + x/a)^{\gamma a}$$

where the origin is at a, the mode. Since negative hail damage is impossible, it is necessary to have the origin at zero. This can be accomplished by substituting x - a for x in the above equation and remembering that $p = \gamma a$. The equation then becomes the incomplete Γ-distribution

$$y = [N\, (\gamma)^{P+1} / \Gamma(P + 1)]\, e^{-x\gamma}\, x^{P}$$

where x is the adjusted-dollar hail damage per 1000 acres. By substituting (q - 1) for p the incomplete Γ-distribution becomes

$$y = [N\, (\gamma)^{q} / \Gamma(q)]\, e^{-x\gamma}\, x^{q-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

BARGER and THOM [1949] report that Thom has approximated the maximum likelihood estimates of q and γ. They reported Thom as approximating the estimate of q by the quadratic

$$12\, [\log_e \bar{x} - (1/N)\Sigma \log_e x]\, \hat{q}^2 - 6\hat{q} - 1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

and the estimate of γ as

$$\hat{\gamma} = \hat{q}/\bar{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

In the above equations $\bar{x}$ is the sample mean, and the circumflex designates the maximum likely estimates of q and γ.

Using these Maximum Likelihood estimates of the two parameters, it is possible to use the sample values to determine the frequency distribution for any given geographic area.

The geographic changes of the frequency distributions--The data available from the assessor's reports is in such form that the frequency distributions could be calculated for townships, counties, State districts, or for the State as a whole. Since there are 99 counties in the State and usually 12 townships in each county, computing frequencies for each of these would entail far too much work, although the same methods used in this study could be adapted to the smaller regions should it be desired. In order to obtain an idea of how the shapes and types of frequency distributions vary from one section of the State to another, one distribution for the entire State would be useless. It was decided for the purposes of this study to calculate the frequency distributions for the nine crop-reporting districts of the State shown in Figure 2.

Table 1--Computations of adjusted-dollar hail damage for 1000 acres for the Northwest District of Iowa

Year	Price index	Dollar damage[a]	Adjusted dollar damage[b]	Adjusted damage per 1000 acres[c]
1923	122	$105,557	$ 86,522	$ 27
1924	147	965,273	656,648	202
1925	161	697,182	441,330	136
1926	114	493,656	433,032	133
1927	136	758,510	557,728	172
1928	149	1,405,326	943,172	291
1929	138	1,584,822	1,148,422	354
1930	122	202,399	165,901	51
1931	77	97,890	127,130	39
1932	44	918,950	2,088,522	643
1933	54	1,213,200	2,246,666	692
1934	104	1,143,062	1,099,098	339
1935	129	282,885	219,291	68
1936	129	800,435	620,492	191
1937	161	237,981	147,814	46
1938	75	242,001	322,668	99
1939	73	380,352	521,030	161
1940	95	129,728	136,556	42
1941	106	145,930	137,670	42
1942	135	1,187,110	879,341	271
1943	172	4,450,324	2,639,723	813
1944	190	3,418,854	1,799,397	554
1945	185	486,099	262,756	81
1946	238	975,319	409,798	126
1947	330	4,816,901	1,459,667	450
1948	329	2,944,731	895,055	276

[a] Damage as obtained from assessors' report.

[b] Adjusted dollar damage = dollar damage / price index.

[c] Adjusted dollar damage per 1000 acres = (adjusted dollar damage / number of acres in district) × 1000. There are 3,246,000 farm acres in the Northwest District.

Table 2--Estimates of γ and q for the nine crop-reporting districts

District	$\hat{\gamma}$	$\hat{q}$
Northwest	0.0054	1.31
North Central	0.0112	1.38
Northeast	0.0099	1.14
West Central	0.0122	2.28
Central	0.0114	1.30
East Central	0.0087	1.02
Southwest	0.0119	1.51
South Central	0.0161	1.00
Southeast	0.0091	1.00

Using the yearly values of total hail damage in each of the nine crop reporting districts as a base, and adjusting these figures to the 1910-14 price index, an array of hail damage per 1000 acres of crop land can be easily constructed. Table 1 shows the process of adjusting these data for the Northwest District of Iowa.

From the data in Table 1, it is a simple matter to compute, by using (2) and (3), estimates of the parameters necessary to construct the frequency distribution. Statistics for these parameters are listed in Table 2 for all nine of the districts. Figure 3 shows histograms with the computed frequency distributions superimposed for each district within the State.

Note the changes in the form of the frequency distributions from one area of the State to another. As one moves across the State from east to west or from south to north, there is an increase in the modal values of the frequency distributions, and thus a decrease in the amount of skewness. The only exception to this general rule is in the west-central district. In the west-central district the modal value is larger than for any other district. According to the general characteristic of the curves, it might have been expected that it would be more skewed than the curve for the Northwest District. The question that arises is whether this is a discrepancy brought about by sampling error or faulty data, or if this curve is really characteristic of the area. SHANDS [1947] presents a map showing isolines of equal number of hailstorms per year, in which it may be noted that there is an area of maximum hailstorm occurrences in this general vicinity. A similar maximum area was also shown by DAY [1928]. It would seem that this sub-

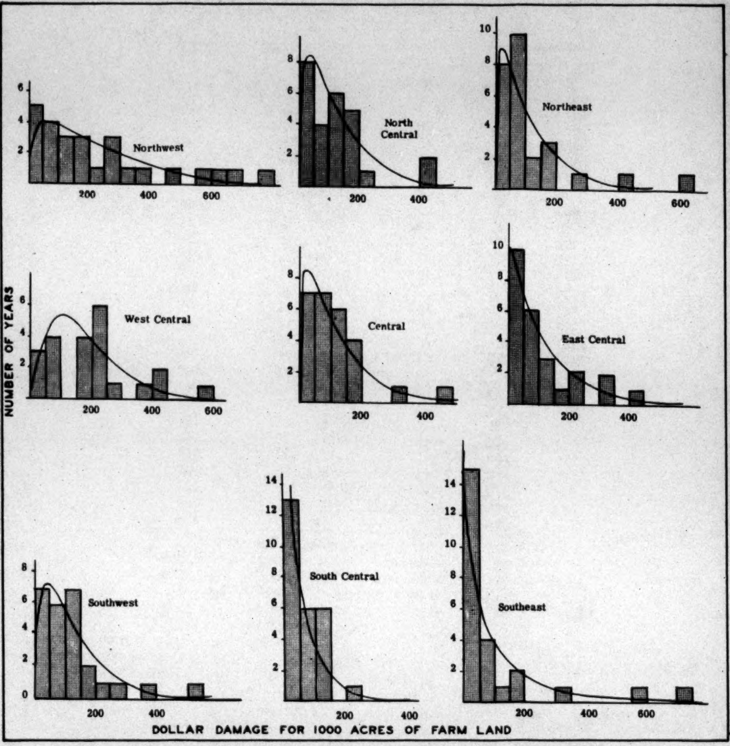

Fig. 3--Frequency distribution of adjusted-dollar hail damage per 1000 acres of farm land for the nine crop-reporting districts of Iowa, 1923-1948

stantiates the fact that this area has been exposed to more hailstorm activities, and thus suffered greater hail damage.

Also note that in the Southeastern Districts the curves become a negative exponential. This is significant, since it indicates that the probability of large amounts of hail damage becomes a minimum in that area.

It is desirable to know how well these curves fit the data. To this end the standard chi-square "goodness-of-fit" test was employed. Table 3 lists the chi-square values for the nine frequency distributions. Also listed in Table 3 are the approximate probability levels of the chi-square values. It can be seen that, in general, the curves fit the data far better than would be necessary to satisfy the goodness-of-fit criterion.

Hail-damage probabilities--The principal advantage of constructing frequency curves is that it facilitates the making of probability statements. In this case the areas under the curves in Figure 3 should be of great value to those people interested in hail damage from an economic point of view.

Table 3--Chi-square values for testing goodness of fit of the frequency distributions

District	Degrees of freedom	Chi square	Approximate probability level
Northwest	4	0.42	0.98
North Central	3	1.99	0.58
Northeast	3	5.89	0.90
West Central	4	0.59	0.96
Central	4	0.78	0.94
East Central	3	0.46	0.93
Southwest	4	3.12	0.55
South Central	3	0.65	0.89
Southeast	3	2.11	0.55

In order to facilitate the production of these probability statements it is advantageous to transform (1) into the arguments employed by PEARSON [1934], and use his tables. BARGER and THOM [1949] present a method for this transformation, which gives the probability of a value falling between o and x as

$$I = \int_0^{u\sqrt{P+1}} e^{-V} V^P \, dV \Big/ \int_0^{\infty} e^{-V} V^P \, dV \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where $P = \hat{q} - 1$, $V = \gamma x$, and $u = V/\sqrt{P+1}$.

Estimates of all the values in (4) can be made in terms of the parameters estimated in (2) and (3). Pearson's Tables of the incomplete Γ-function can then be used to estimate the probability of a value falling between o and x; in other words, the probability of receiving less hail damage than a specified amount. Actually the probability of receiving more damage than a specified amount would be more significant insofar as hail damage is concerned. This value would be simply found by subtracting the value obtained from (4) from unity.

Values of the probability of greater hail damage than specified are listed for the nine crop-reporting districts in Table 4.

Table 4--Probability of greater adjusted-dollar hail damage per 1000 acres than indicated for the nine Iowa crop-reporting districts

District	Adjusted-dollar hail damage per 1000 acres				
	25	50	100	250	500
Northwest	0.952	0.889	0.697	0.362	0.107
North Central	0.904	0.719	0.459	0.113	0.008
Northeast	0.841	0.643	0.453	0.108	0.010
West Central	0.986	0.939	0.772	0.276	0.027
Central	0.890	0.697	0.440	0.096	0.007
East Central	0.824	0.677	0.410	0.101	0.014
Southwest	0.923	0.750	0.486	0.119	0.008
South Central	0.670	0.446	0.200	0.019	.....[a]
Southeast	0.670	0.635	0.401	0.103	0.011

[a] Less than 0.0005.

From Table 4 it can be seen that the probability of sustaining damage as great as $500 per 1000 acres is very small. In the case of the eastern and southern districts this would not occur more than about one year out of a hundred. In the case of the South Central and Southeast Districts there is considerable chance (three years out of ten) of receiving less than $25 damage per 1000 acres. For the West-Central and Northwest Districts the probability of receiving appreciable amounts of damage is markedly greater than for the other districts of the State.

Summary--The use of the above-outlined method will result in the production of probability statements concerning hail damage to agricultural crops. If a particular area is of more interest than these generalized areas, the study could be adapted to those areas. On file in the Des Moines

Weather Bureau Office are hail-damage figures for each county and township within the State. This same general method should also apply to data obtained from other sources.

Acknowledgment--The author expresses his appreciation to C. E. Lamoureux, Official in Charge, Weather Bureau Office, Des Moines, Iowa; and to members of his staff for their cooperation and aid. Without their help this paper would not have been possible.

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(Communicated manuscript received January 4, 1951;
open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

GEOPHYSICAL STUDIES IN THE BEAUFORT SEA, 1951

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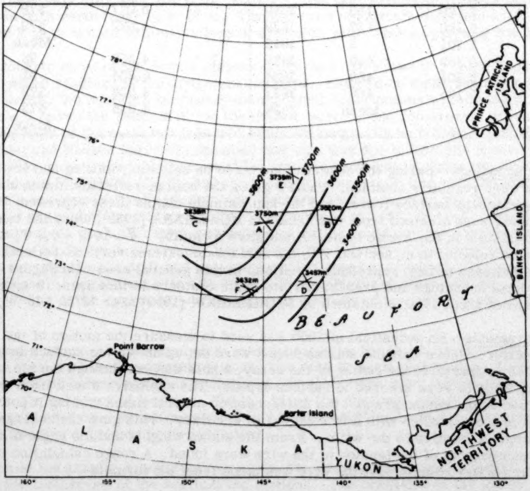
Abstract--Plane landings on the ice pack between latitudes 73° and 76°N, and between longitudes 135° and 150°W, allowed several geophysical studies to be made in an area unexplored since Storkerson's sled trip in 1918. Sonic soundings indicated ocean depths between 3400 and 3800 m. Measurements of the direction of ice movement correlated well with surface isobaric contours. Gravity-meter operations showed these studies to be feasible on the ice and indicated maximum vertical ice movements of about 0.05 cm in the period range five to 40 sec.

Introduction--During April 1951, six landings were made on the Beaufort Sea ice pack north of Barter Island, Alaska, by C-47 aircraft of the 10th Air Rescue Service, Detachment B, of Ladd Air Force Base, Alaska. The aircraft remained on the ice from four to six hours at each landing. Scientific studies made included seismic profiling, ice-movement studies, and gravity observations. The seismic studies will be reported on at a later date. Reported on here are the acoustic soundings, ice-pack movements, and gravity observations.

The locations of the landings were determined with a standard engineering transit, obtaining sun lines over as wide a time interval as possible at each landing. On two of the stations, clouds prevented good solar observations during the latter part of the stay, and the locations are somewhat doubtful. At these locations, use was made of the navigator's sextant observations, the sextant being corrected by comparison with the transit. Figure 1 shows the locations with the sun lines observed. In Table 1 are given the dates and locations of the stations. Calculated magnetic declinations, observed during the solar observations by orientation of the transit on the magnetic meridian, are also listed in Table 1.


78°
77°
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74°
73°
72°
71°
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3800m
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3600m
3500m
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3738m
E
3638m
C
3750m
A
3620m
B
3632m
F
3457m
D
PRINCE PATRICK ISLAND
BANKS ISLAND
BEAUFORT SEA
Point Barrow
Barter Island
ALASKA
YUKON
NORTHWEST TERRITORY
160°
155°
150°
145°
140°
135°
130°

Fig. 1--Ice landing locations and ocean depths

Table 1--Station locations and magnetic declinations

Station	Date	Lat. N	Long. W	Magnetic declination	Time A.S.T.
	1951	° ′	° ′	°E	h m
A	Apr. 8	74 46	144 55	41.6	11 02
B	Apr. 10	74 47	138 40	46.1	11 53
				46.3	16 00
C	Apr. 17	74 45	150 55	34.8	12 44
D	Apr. 20	73 22	141 20	43.1	09 47
				43.3	15 18
E	Apr. 22	75 53	142 10	45.9	13 19
				45.8	17 17
F	Apr. 23	73 11	146 05	37.3	10 17

Table 2--Reflection times and water depths

Station	Reflection time	Horizontal distance	Depth	Station	Reflection time	Horizontal distance	Depth
	sec	m	m		sec	m	m
A	5.081	186	3750	D	4.697	5	3462
B	4.779	131	3523		4.694	20	3459
	4.780	148	3524		4.693	150	3458
	4.777	142	3522		4.687	295	3452
	4.778	355	3519		4.686	345	3449
	4.775	256	3519		4.689	250	3454
	4.780	488	3516			Mean	3457
	4.781	355	3521	E	5.063	4	3738
	4.776	256	3519		5.064	1	3739
	4.783	488	3518		5.079	579	3738
		Mean	3520		5.085	579	3743
C	5.199	149	3840		5.085	579	3743
	5.199	149	3840		5.077	579	3737
	5.198	137	3839		5.077	579	3737
	5.201	8	3842			Mean	3739
	5.202	489	3835	F	4.930	98	3637
	5.201	489	3834		4.924	91	3633
	5.195	400	3832		4.934	573	3630
		Mean	3838		4.935	573	3630
						Mean	3632

Ocean soundings--During the observations made on seismic profiling and ice drift, bottom reflections were regularly obtained. Table 2 gives the bottom-reflection times obtained, corrected to surface-to-surface travel, and the approximate depths these represent. The average velocities used were obtained from the Tables of KUWAHARA [1939], using the temperatures and salinities available in the Arctic basin. A tentative formula, $\overline{V} = 1442 + 6.8\ T$, where $\overline{V}$ is the average sound velocity in m/sec and T is the surface-to-surface vertical-reflection time in seconds, is believed to best represent conditions in this general area. On Figure 1 are shown the results of these soundings and probable Arctic basin contours in this area. It appears quite likely that the depth of about 4700 m obtained by STORKERSON [1918] near 72°N, 146°W, is in error.

Ice movements--An acoustical method was used to measure the motion of the ice sheet relative to the water. Three vertical seismographs were set up on the ice surface in a triangular array about 250 m apart. In the center of the array, a hole was cut through the ice, and small TNT charges and weights were lowered to various depths. The explosive waves from the detonation were received at the seismographs, the differences in arrival times making it possible to locate the position of the explosion with a fair degree of accuracy. This gave the average direction of the ice movement relative to the water. From the known weights and the angle of the wire, the horizontal components of the tension in the wire were found. A rough calculation of the average velocity of the water column past the wire was made from the formula

$$V^2 = 2 H_f / (\rho A\ C_d)$$

Table 3--Ice-drift calculations and wind data

Site	Charge depth	Wire angle	H_f	Relative ice drift		Surface wind		Geostrophic wind	
				V	Direction[a]	V	Direction[a]	V	Direction[a]
	m	°	dynes	cm/sec	°	m/sec	°	m/sec	°
A	246	10.3	526	10.0	82	Not observed		10	112
B	244	2.3	116	3.9	64	Not observed		Small	. . .
C	24	18.2	106	11.3	125	12	100	21	145
	77	10.8	432	12.7	140	. .	. . .	. .	. . .
D	47	20.2	230	11.9	358	7	310	16	315
	50	5.3	181	10.1	354	. .	. . .	. .	. . .
	46	23.6	273	13.2	358	. .	. . .	. .	. . .
	44	27.9	318	14.6	361	. .	. . .	. .	. . .
E	25	2.3	64	8.0	301	7	250	11	320
	25	10.2	58	7.9	309	. .	. . .	. .	. . .
	49	10.3	113	7.9	322	. .	. . .	. .	. . .
F	11	9.8	27	7.8	258	8	260	11	315
	18	19.0	83	11.7	308	. .	. . .	. .	. . .

[a] Direction here refers to true direction from which the ice drifts or wind blows. Surface observations taken at height of 1.5 m above the ice.

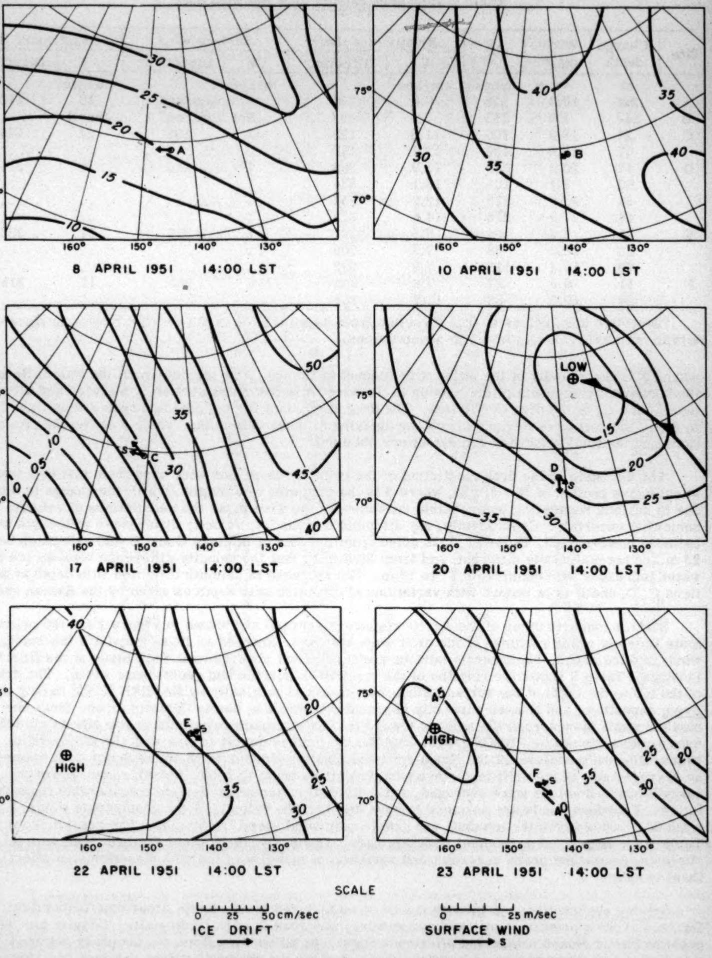
where V is the velocity of the water with respect to the ice, ρ is the density of the water, H_f is the horizontal component of the tension in the wire, A is the cross section of the wire and attachments, and C_d is the drag coefficient. The drag coefficient for cylindrical areas varies from 1.0 to 2.0 in the spread of Reynolds numbers applying to these velocities. Table 3 shows the observations and the final velocities and directions obtained.

The deviation of the drag coefficient of the twisted two-strand wire used from that of a theoretical cross section of $(1 + 2/\pi)d$, where d is the diameter of a single strand; the change in drag due to surface roughness; and possible curvature of the wire make the calculations of velocity somewhat uncertain. Consequently, any attempts to calculate velocity differences with depth are believed unwarranted. However, it is noted from the data of Stations C and E that velocities below 25 m are not materially different; and from Station F, that the velocity difference between ice and water increases with depth from 11 to 18 m. The increase in azimuth direction with depth at Stations C, D, and F is in accord with variations of direction with depth as given by the Ekman spiral.

Surface isobaric maps of the Arctic regions concerned are shown in Figure 2 for the approximate times at which readings of ice drift were obtained. Entered on these maps are the surface winds and ice drifts. No accurate surface meteorological observations were made at the first two landings. Table 3 shows the relation of the ice drift to surface and geostrophic winds. The drift of the ice to the right of the surface winds was observed originally by NANSEN [1902] during the Fram expedition, and Nansen correctly assumed that this was due to Coriolis force. Since the surface winds blow across the isobars toward the low-pressure areas, these two effects give the net result, as shown by ZUBOV [1945], that the ice drift is nearly in line with the geostrophic winds. Nansen's analysis of the Fram drift and Zubov's similar work on the Sedov drift showed an average ratio of ice drift to surface wind velocities equal to 1/50. In both cases, however, several hundred values were averaged, and individual ratios could deviate considerably from this figure. If surface winds are assumed to have one-half the velocity of the geostrophic winds, a ratio of 1/100 between the ice drift and geostrophic wind speed is obtained. The results shown in Table 3 are in general agreement with this ratio. Obviously, many other factors, such as tidal currents, permanent ocean currents, and variation of mobility of ice with direction, can affect these results.

Gravity observations--A gravity meter, manufactured by the North American Geophysical Co., and of the type used in oil exploration work, was read at each landing site. Gravity ties were made at Barter Island before and after each flight. At all but two sites, the accuracy obtained was well within the accuracy of location. Table 4 gives the observed values and free-air anomalies. These are based on a tentative value of 982.244 cm/sec^2 at Ladd Air Force Base, Alaska, as given by G. P. Woollard (private communication).

In addition to the gravity determinations made, the meter, a zero spring length, null-type instrument, gave indications of the vertical ice motions. The constants of the meter are approxi-



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75°
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35
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50
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05
0
S
C
LOW
D
E
HIGH
F
SCALE
0 25 50 cm/sec
ICE DRIFT
0 25 m/sec
SURFACE WIND
S

Fig. 2--Surface isobars and their relation to ice drift

mately as follows: natural period, 17 sec; damping, 0.7 critical; and sensitivity, 0.001 cm/optical scale division. Figure 3 shows the motion of the cross-hair on the optical scale as observed visually. Also shown are oscillations observed on the shore-fast ice about one-half mile offshore near Point Barrow, Alaska, where the ice thickness was 1.5 m and the water depth four meters. Maximum vertical ice motions in this range of periods appears to be about 0.05 cm, assuming

Table 4--Gravity measurements

Station	Lat. N	Long. W	Observed gravity	Free-air anomaly
	° ′	° ′	cm/sec^2	milligal
Barter Island	70 08	143 38	982.593	-26
B	74 47	138 40	982.857	- 6
C	74 45	150 55	982.845	-17
D	73 22	141 20	982.781	-15
E	75 53	142 10	982.901	-11

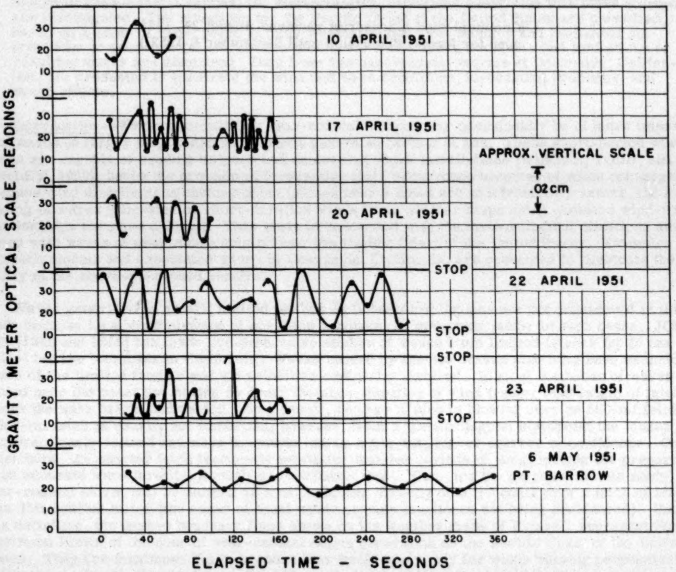

Fig. 3--Approximate vertical ice motion as obtained from gravity-meter readings

six-decibel drop per octave from natural period. ZUBOV [1945] refers to work by Bernstein who measured wind-driven oscillations up to 0.3 cm on river ice 80 cm thick with periods 20 to 180 sec.

Acknowledgments--The logistic support permitting these studies was provided by the 10th Air Rescue Service, Detachment B, from Ladd Air Force Base. The aircraft operations were under the expert handling of J. R. Whitmer, and it was owing mainly to him, Co-Pilot Otto, and others of the plane crew that successful landings and observations were made on all six attempts. Although the study of plane landings is out of the scope of this report, it is noteworthy that all landings were made within a range of ten miles of the previously determined locations.

Acknowledgments are also due to H. G. Dorsey, Jr., of the Arctic Weather Central at Elmendorf Air Force Base, for providing the weather maps used in the study of the ice drift.

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(Communicated manuscript received August 24, 1951;
open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

NEAR-COASTAL STORMS AND ASSOCIATED WAVES

D. K. Todd and R. L. Wiegel

Abstract--Because of the erosive action on beaches of high, short-period waves generated by near-coastal storms, the meteorological situations associated with these storms are investigated. The situations for the Pacific Coast of the United States are classified, based on a study of three years of daily weather maps. The monthly and latitudinal distributions of near-coastal storms are tabulated. Forecasting problems and limitations of resulting waves are discussed. Data from two near-coastal storms at Oceanside, California, are presented to illustrate the wind and wave conditions, forecasting problems, and beach effects.

Introduction--Waves generated by near-coastal storms may occasionally be of more importance than waves of larger amplitudes and periods generated farther at sea. These short-period wind-waves are capable of eroding beaches and destroying shore installations [WIEGEL, TODD, and KIMBERLEY, 1950]; hence the problem of forecasting their occurrence becomes of some consequence. Because wind distributions causing short fetches near a coast are of a transitory nature, the forecasting of waves generated by short-duration winds from weather maps using standard wind-wave relationships becomes difficult. This study is concerned with the meteorological situations associated with waves of near-shore origin along the Pacific Coast of the United States. Examples of two such storms and associated waves at Oceanside, California, are presented to illustrate the actuality of the above-mentioned results.

Waves generated by small, limited fetches as in bays and lagoons are not considered in this study because local configuration of coastline becomes the governing factor in such cases. JOHNSON [1948 and 1950] has made comprehensive studies of waves from limited fetches (up to ten miles) for two localities in California. Waves caused by the sea breeze also have been excluded because of the limited fetches and low velocities ordinarily involved. In most instances of waves generated near the coast and having no decay distance, duration of wind (rather than length of fetch) limits the wave height and period. As a result, whether a wind is blowing over an 800-mi fetch or a 300-mi fetch is usually not important; however, from a meteorological standpoint the longer fetch is more clearly defined and wave forecasts can be made with higher degrees of confidence. On the other hand, the shorter fetch frequently exists for smaller periods of time, making the preparation of an accurate wave forecast a much more difficult task. Then, for the purposes of this study, a near-coastal storm will be defined as a wind blowing directly onto the coast over a fetch of less than 350 nautical miles (the cases of local winds already mentioned not being considered). Using this definition, the dashed boundary lines shown on the weather maps of Figure 1 represent the maximum limits of the zone of near-coastal storm generation on the Pacific Coast of the United States. They are maximum limits because they would apply only for winds blowing perpendicular to the coast. In all other instances the 350-mi limit would lie entirely within this zone.

An analysis of past weather maps has been made to determine types and frequencies of meteorological situations associated with near-coastal storms. DONN [1949] made a study of weather conditions and short-period waves at Cape Cod as part of a more general investigation of ocean waves at that location.

Synoptic situations associated with near-coastal storms--In order to determine the meteorological situations associated with near-coastal storms, three years of Northern Hemisphere daily weather maps [JOINT METEOROLOGICAL COMMITTEE, 1943-1944] were examined. The years 1934, 1936, and 1938 were arbitrarily selected for this purpose and regarded as a random sample of the long-time distribution. All instances of near-coastal storms generated within the 350-mi zone off the Pacific Coast of the United States were noted, together with a brief description of each.

It was found that the situations could be classified into four general synoptic situations: pre-frontal, ridge off coast, low off coast, and post-frontal, as illustrated in Figure 1. The number of storms found and the breakdown into the four categories for each year are shown in Table 1. These data were subjected to a two-way classification test for independence using the statistic chi-square [DIXON and MASSEY, 1951, p. 187]. It was found that no significant differences existed in the dis-

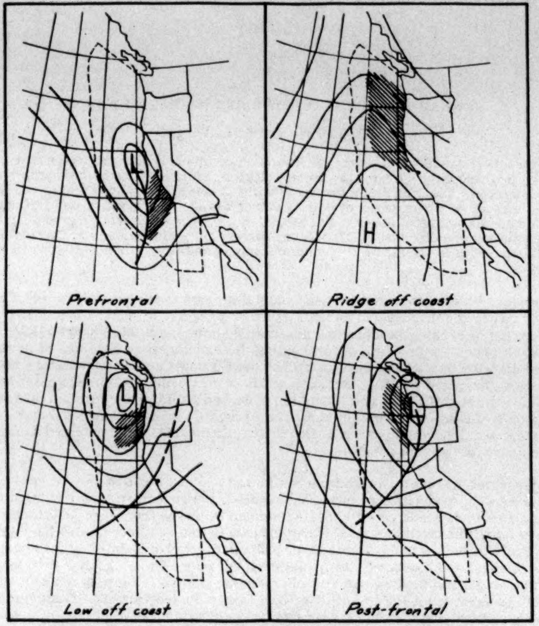


Fig. 1--Examples of four synoptic situations associated with near-coastal storms; the dashed lines delineate the maximum limits of the zone of near-coastal storm generation, and the cross-hatched areas represent particular generating areas for the situations shown

Table 1--Meteorological situations associated with near-coastal storms from daily weather maps

Situation	Number of storms			
	1934	1936	1938	Total
Prefrontal	18	13	14	45
Ridge off coast	6	4	7	17
Low off coast	2	3	6	11
Post-frontal	1	1	2	4
Totals	27	21	29	77

tribution of storms by situations for the three years, indicating the uniformity of the classification procedure from year to year.

The prefrontal situation, by far the most frequent of the four categories, develops ahead of a front oriented north-south or northeast-southwest off the coast and lying in a sharp trough. This situation produces a limited fetch immediately ahead of the front with generally southwesterly winds. As the front approaches the coast, the generating area decreases accordingly. This situation frequently is a transition from a longer fetch, extending beyond the 350-mi limit, to a shorter fetch, defined as a near-coastal storm, as the front approaches the coast. The second category, a ridge lying off the coast within the zone of generation, develops winds of limited fetch from the anticyclonic circulation. These winds, coming from the west or northwest, are generally weaker than those involving cyclonic circulations. The situation having a low located just off the coast constitutes the third category. Here the cyclonic circulation develops westerly and southwesterly winds. This situation is most effective with either a stationary, filling low having no pronounced frontal structure or a slow-moving low in which the frontal system has moved inland. This latter case is illustrated in Figure 1. It is important that a relatively sharp trough lie within the zone of generation in order that the shift of wind direction at the trough be large enough to limit the fetch to the eastward side of the trough only. The same limitation applies to the prefrontal case. This can be determined following standard procedures for delineating fetches [U. S. NAVY HYDROGRAPHIC OFFICE, 1951]. The final category, post-frontal, develops behind front and low centers that have moved inland together, leaving a portion of the cyclonic circulation still over the ocean. Westerly and northwesterly winds develop and continue until the low center has moved far enough inland to change the circulation pattern off the coast. This situation occurs most infrequently of the four categories, not because there are not many cases of post-frontal winds, but because the large majority of these winds have fetches which extend beyond the zone of near-coastal storm generation. The post-frontal case will only occur when the cyclonic curvature of the isobars is great enough to limit the fetch to less than 350 mi.

The categories defined here are, by no means, always clear-cut cases. For example, the primary distinction between the prefrontal and low-off-coast types is the presence or absence of a front in the trough. Any change in the continuity of the map analysis would cause a similar change in classification. It is entirely possible that one situation might be followed immediately by another, as from prefrontal to post-frontal. Also, two different situations may exist concurrently. If a large system were oriented northeast-southwest, a prefrontal near-coastal storm might occur on the California coast while, at the same time, a post-frontal or even a low-off-coast storm might be blowing onto the Washington-Oregon coast. The same degree of subjectivity that exists in determining fetches [U. S. NAVY HYDROGRAPHIC OFFICE, 1951] applies equally well in selecting near-coastal storm situations.

Distribution of near-coastal storms--In examining near-coastal storms along the Pacific Coast, the months of occurrence were noted. Because of the southward displacement of the Pacific polar front and the increased circulation during winter, it would be expected that more near-coastal storms occur at that time of year. This expectation is verified, as shown in Table 2. Here the total number of near-coastal storms for the three years studied is listed by months. It is worth noting that 70 pct of the storms occurred in the four-month winter period, December-March.

In an effort to determine the latitudinal distribution of these storms, the mean latitude of the intersection of each generating wind with the coast was found. Since this involved primarily a process of visual estimation, the number of storms counted for each degree of latitude in the sample period is not representative of the true distribution. However, by grouping the values into latitudinal zones, the data are indicative of the general distribution of near-coastal storms along the western coast of the United States. In Table 3 are tabulated the number of storms by two-degree zones over the range 33° to 48°N latitude. Although these data show a considerable increase in the number of storms in the northernmost zone, the small number of storms studied precluded attaching any statistical significance to this increase. Further studies are needed of storm occurrences in higher latitudes along the coast before any relationship can be determined. To find if any seasonal variation of near-coastal storms existed with latitude, the per cent of storms occurring during the maximum season, December-March, was computed for each zone (Table 3). No pronounced trend exists.

Table 2--Monthly distribution of near-coastal storms from daily weather maps

Month	Number of storms
January	15
February	12
March	10
April	4
May	2
June	2
July	3
August	1
September	2
October	5
November	4
December	17
Total	77

Table 3--Latitudinal distribution of near-coastal storms from daily weather maps

Latitude zone	Number of storms	Storms occurring in winter period
°N		pct
33-34	7	71
35-36	10	60
37-38	6	100
39-40	9	67
41-42	8	62
43-44	10	90
45-46	9	67
47-48	18	61
Total	77	

Other situations associated with near-coastal storms--The situations outlined heretofore were based on analyses of isobaric patterns. In addition to these, there are other phenomena related to near-coastal storms which are not subject to such ready detection from weather maps. For example, a squall line ahead of an approaching front is capable of generating important short-period waves if its primary direction is shoreward and if it has a fetch length of the order of 50 mi. Although such a situation might come under the prefrontal classification, it is doubtful whether the wind velocities forecast from the isobaric pattern alone would be large enough. The recognition of such phenomena is particularly difficult in oceanic areas, where surface reports are limited. Another problem is that of local packing of isobars near the coast. Here again the forecast would ordinarily be inadequate. This convergence phenomenon, common along coastal ranges such as in California, would tend to invalidate near-coastal wave forecasts unless detailed reports, as from observational aircraft, were available.

Table 4--Deep-water heights, periods, and breaker heights of waves generated by near-coastal storms

Minimum wind duration	Wind speed	Minimum fetch	Deep-water height	Period	Breaker height
hr	knots	naut mi	ft	sec	ft
6	20	35	5.7	5.6	6.0
6	30	50	10.7	7.2	11.2
6	40	55	16.0	8.7	16.8
12	20	90	7.5	6.8	8.1
12	30	120	15.0	9.1	16.1
12	40	140	23.5	11.0	24.9
18	20	170	8.4	7.6	9.3
18	30	210	17.4	10.3	19.0
18	40	250	28.0	12.6	30.0

Waves generated by near-coastal storms--To illustrate the magnitude of waves which can be generated by limited-fetch storms, some typical values have been summarized in Table 4. The deep-water wave heights and periods have been taken from standard wind-wave relationships developed by **SVERDRUP** and **MUNK** [1947], presented in the form of forecasting curves by **U. S. NAVY HYDROGRAPHIC OFFICE** [1945, 1951], revised by **ARTHUR** [1948], and further improved by **BRETSCHNEIDER** [1951]. The breaker heights were taken from **U. S. NAVY HYDROGRAPHIC OFFICE** [1944]. The values shown refer to unrefracted waves. The minimum wind durations shown are representative of a majority of the near-coastal storms investigated. In the situations involving a stationary low or ridge off the coast, these values might be exceeded considerably. Otherwise, an 18-hr duration may be regarded as above average. Thus, in the case of a front approaching the coast at 20 knots with prefrontal winds nearly perpendicular to the front and to the coast, the maximum duration would be about 17 hr within the zone of near-coastal storm generation. The minimum fetch distances have been listed in the third column of Table 4, and it can be noted that these distances are frequently exceeded in the situations discussed, indicating that minimum wind durations rather than fetch distances are normally the governing factor in short-period wave generation.

Forecasting waves from near-coastal storms--The forecaster assigned to forecasting near-coastal storm waves for the protection of shore installations must have available all data on weather conditions at sea. Forecasts will of necessity be of a short-range nature and must be based on last-minute data. When situations with a ridge or low off the coast occur, the fetch distances can be identified from map to map and detailed forecasts can be prepared. In cases of prefrontal and post-frontal winds, the time element becomes much shorter. Not only must frontal movements be forecast, but also wind directions and velocities both ahead of and behind fronts must be forecast. Existing surf conditions preceding near-coastal storms frequently tend to confuse the forecaster because these may be the result of long-period waves generated some distance at sea. Near-coastal storm waves are superimposed on these existing wave trains and usually become the significant waves because of their greater height. The location of frontal positions by 24-hr prognostic charts does not materially assist the wave forecaster. Pressure gradients and small variations in the isobaric pattern for a given area can only be determined by checking all available reports up to the last minute. Ship reports and aircraft weather observations are particularly important to the forecaster for storms arising from squall lines and offshore packing of isobars. These situations can seldom be accurately forecast unless such detailed information is available.

Table 5--Summary of surface winds observed at Oceanside, California, during two near-coastal storms

Storm of October 18-19, 1949			Storm of December 10, 1949		
Time PST	Direction	Velocity	Time PST	Direction	Velocity
h m		knots	h m		knots
13 10	SW	13	00 00	SW	10
13 30	SW	17	01 00	WSW	16
14 10	SW	14	02 00	WSW	18
14 30	WSW	13	03 00	WSW	20
15 10	W	14	04 00	WSW	24
15 30	W	16	05 00	W	32
16 05	WSW	17	06 00	NW	18
16 30	W	17	07 00	NW	22
17 07	WSW	17	08 00	NW	25
17 30	WSW	17	09 00	NNW	20
18 07	WSW	17	10 00	NW	24
18 30	W	20	11 00	NW	26
19 08	W	19	12 00	WNW	28
19 30	W	21	13 00	WNW	36
20 07	W	24	13 36	WNW	47
20 30	WSW	25	14 00	NNW	30
21 07	WSW	24	14 08	WNW	49
21 30	W	28	15 00	N	20
22 09	W	36	16 00	WNW	34
22 30	W	33	17 00	WNW	32
22 55	W	35	18 00	WNW	28
23 13	W	27	19 00	NW	14
23 28	WSW	23			
00 09	WSW	23			
00 30	WSW	25			
01 04	WSW	26			
01 30	WSW	25			
02 07	WSW	24			
02 29	WSW	23			
03 09	WSW	23			
03 28	W	22			
04 06	WNW	23			
04 29	W	21			
04 55	NW	9			

Examples of near-coastal storms and associated waves--Wind, wave, and beach records of two outstanding near-coastal storms which occurred at Oceanside, California, late in 1949 are available. These storms illustrate the wind and wave interrelationships, the forecasting problems, and the effects on beaches [WIEGEL, TODD, and KIMBERLEY, 1950].

The first storm occurred on October 18-19, 1949. Winds of over 20 knots were recorded at Oceanside for a nine-hour period from 19h 30m PST on October 18 to 04h 30m PST on October 19 (see Table 5). Wind directions were primarily WSW and peak gusts of 36 knots occurred near 22h 00m PST. This period of high winds was caused by the passage of a trough of low pressure extending southwestward from a warm-front type occlusion centered over southern Nevada. In Figure 2a the cold front, which is located in southern Arizona, passed Oceanside 36 hr previously. The trough lying north of Oceanside moved southeastward at ten knots and passed there at 04h 30m PST on October 19. Thus the high winds occurred in the nine-hour period immediately preceding the passage of the trough. This situation illustrates the cyclonic influence of an inland low center on nearby ocean areas and would be classified as post-frontal. The length of fetch was probably less than 100 mi. In Figure 2b is shown a portion of the prognostic weather map for 04h 00m PST, October 19 (5 1/2 hr after the surface map) prepared by forecasters of the Navy Weather Central, San Francisco, California. In comparing this map with the surface map, it can be seen that the overall synoptic situation is in agreement; however, the small variations of the isobaric pattern are not. This type of discrepancy is inherent in prognostic charts and limits the use of such charts for forecasting near-coastal storm waves. In this particular case the secondary cold front

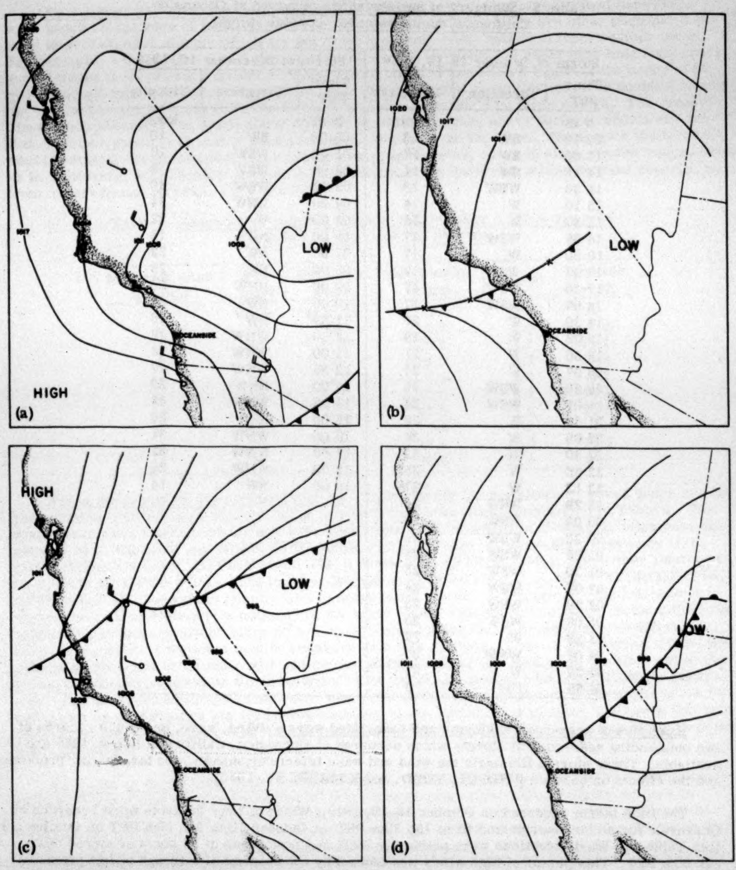


Fig. 2--Surface and prognostic weather maps associated with two near-coastal storms at Oceanside, California: (1) surface map at 22h 30m PST, October 18, 1949; (b) prognostic map at 04h 00m PST, October 19, 1949; (c) surface map at 22h 30m PST, December 9, 1949; (d) prognostic map, 04h 00m PST, December 10, 1949

expected by the forecaster is perhaps justifiable based on earlier maps, but its position for the time of the map is considerably north of the true position of the trough at that hour.

The second storm occurred at Oceanside on December 10, 1949. Winds of more than 20 knots occurred from 02 30m PST to 18h 30m PST, with a peak gust of 49 knots near 14h 00m PST (see Table 5). Wind directions were from WSW until 05h 00m PST, veering to NW until 11h 30m PST, and finally backing to WNW for the remainder of the period. The high winds were associated with a moderate cold front moving southeastward over southern California (see Fig. 2c). This front

moved southeastward at 30 to 35 knots and passed Oceanside at 05h 00m PST, as indicated by the marked wind shift. The major portion of the high wind period followed the frontal passage, placing this storm in the post-frontal category. The unusually high winds behind the front are believed to have resulted from localized convergence of the unstable air approaching the Coast Range, combined with the steep pressure gradient behind the front. The length of fetch in this storm was of the order of 300 mi. Figure 2d shows the corresponding prognostic chart (5 1/2 hr after the surface map). The chart pictures the actual weather situation well for that hour. The only discrepancy is the spacing of the isobars near the front in the vicinity of Oceanside, but this inaccuracy, as before, tends to introduce large errors in wave forecasts made from this chart.

Surf heights and periods for the two storms are shown on the running records presented in Figure 3. During both storms, periods dropped rapidly while breaker heights increased. This increase in heights is well illustrated by the actual thermopile wave-meter [ISAACS and WIEGEL,

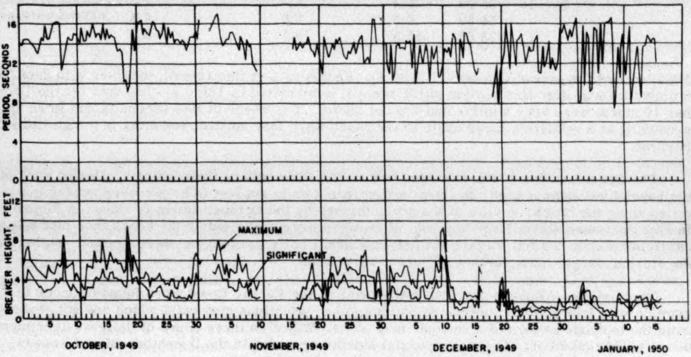
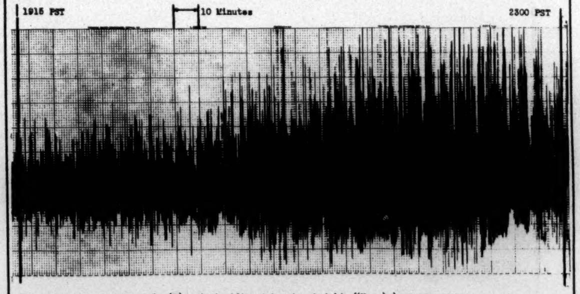


Fig. 3--Breaker records at Camp Del Mar, Oceanside, California, from October 1, 1949 through January 15, 1950; wave records were obtained by thermopile wave meter



Notes: Averages of periods (T) and significant breaker heights ($(H_{1/3})_B$) are for fifteen minute interval on each side of indicating lines.

1915 PST. T = 12.0 seconds, $(H_{1/3})_B$ = 4.0 feet.
2300 PST. T = 10.3 seconds, $(H_{1/3})_B$ = 8.0 feet.

Fig. 4--Mark V No. 1A record for 19h 15m to 23h 00m PST, October 18, 1949, Camp Pendleton, Oceanside, California

Table 6--Analysis of thermopile wave-meter record at Oceanside, California, for October 18-19, 1949

Date	Time PST	Average period	Significant breaker height	Maximum breaker height
1949	h m	sec	ft	ft
Oct. 18	00 10	13.4	3.7	5.4
	08 15	13.2	3.4	4.7
	04 59	10.2	3.2	5.2
	19 15	12.0	4.0	6.0
	23 00	10.3	8.0	9.2
Oct. 19	08 28	9.2	4.8	7.0
	14 50	9.8	3.6	4.8
	23 05	13.5	4.1	6.3

1950] record for part of October 18 (Fig. 4). An analysis of this record, together with data from both before and after this portion of the record, is tabulated in Table 6. The data for the December 10 storm were very similar and are not shown. The waves of this second storm were approaching at a relatively large angle to the beach when they spilled, resulting in a high littoral current.

The Oceanside beach was surveyed before and after both of these storms. During each storm the beach face was cut back. Scarps ranging from two to six feet in height were cut for many miles along the beach. A view of the scarp formed by the October storm is shown in Figure 5. During this same storm large amounts of seaweed were deposited on the beach face just below the scarp in many locations. Before both storms small bars, near shore, were present; while after the storms, larger bars, further offshore, were found.

Conclusions--Although near-coastal storms of the Pacific Coast were found to occur in conjunction with four general weather situations, other situations may exist which are not obvious from the isobaric pattern of a weather map alone. Based on three years of daily weather maps, it was found that 70 pct of all near-coastal storms occurred in the December-March season. No

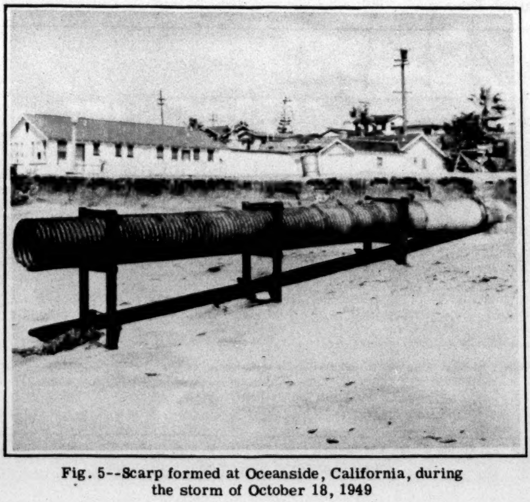


Fig. 5--Scarp formed at Oceanside, California, during the storm of October 18, 1949 significant differences were noted with regard to the number of storms occurring in different latitude zones. Forecasting these storms is difficult because of limited data ordinarily available and short durations involved. Prognostic charts alone do not provide sufficient information for accurate wave forecasting.

Short-period waves from near-coastal storms are capable of highly erosive action on beaches, causing scarps to form. During one such storm at Oceanside, other than the two mentioned above, the beach was eroded at one location to such an extent that a section of retaining wall and part of a road were undercut and seriously damaged during one night. These storms can be particularly critical on beaches because of the high percentage occurring during winter when beaches are in their normal narrow winter condition.

It should be emphasized that the results of this study are preliminary and based on only a relatively small sample of storms. Further studies are required to properly verify the findings presented herein.

Acknowledgments--The authors acknowledge the assistance of C. L. Bretschneider for checking the data in Table 4, and of H. L. Kimberley for furnishing the wind records of the two near-coastal storms. The material presented in this paper is part of extensive investigations of waves and related problems performed by the University of California at Berkeley for the U. S. Navy and the U. S. Marine Corps.

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(Communicated manuscript received July 23, 1951, and, as revised, January 7, 1952; open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

NUMERICAL ANALYSIS OF CONTINUOUS UNSTEADY FLOW IN OPEN CHANNELS

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Abstract--In this paper is presented a simplified method of solving Massau's equations of characteristics for unsteady flow having a continuous surface profile. This method eliminates much of the trial-and-error process formerly required.

Introduction--Although Massau's rigorous analysis of unsteady flow in open channels was published some 50 years ago, its practical application has not been very extensive; for, among other reasons, the application of Massau's method is handicapped by the tiring labor involved in the trial method of solving his equations of characteristics. By developing a scheme of using a constant time interval for each step of computation, it will be shown herein that the trial-and-error process can be avoided, except in the laying out of characteristics in the x-t plane. The latter operation, being guided either by the characteristics laid out in a previous step or by the initial condition, is generally easy to perform. Since the method has the distinct feature of using a constant value of Δt throughout a step of computation, it may be called the method of constant Δt.

A brief interpretation of the equations of characteristics will precede the discussion of the method itself.

Nomenclature--The following symbols are used in this paper.

A	Cross-sectional area
B	Surface width
c	$\sqrt{gy_m}$, celerity
15a, etc.	Point "a" on the t = 15-hr line in the x-t plane
K	$(g/2)\,(S_0 - S_f)\,\Delta t$
S_f	So-called friction slope
S_0	Bottom slope
t	Time
U	$(\sqrt{g}/2)\int (dy/y_m^{1/2})$ for a prismatic channel, $U = \sqrt{gy}$ for a rectangular channel
V	Mean velocity of flow
x	Distance along the channel
y	Depth of flow
y_m	A/B, mean depth of flow

The equations of characteristics--For simplicity, a rectangular prismatic channel will be considered. Corresponding to continuous unsteady flow in such a channel, two pairs of the equations of characteristics can be written, namely

$$dx/dt = V + \sqrt{gy} \qquad (1)$$

$$d\,(V + 2\sqrt{gy}) = g\,(S_0 - S_f)\,dt \qquad (2)$$

$$dx/dt = V - \sqrt{gy} \qquad (3)$$

$$d\,(V - 2\sqrt{gy}) = g\,(S_0 - S_f)\,dt \qquad (4)$$

If x and t are chosen as two axes defining an x-t plane (Fig. 1), there may be found on this plane a family of curves such as C of which the slope with respect to the t-axis at any point is given by (1). Only along this family of curves will (2) hold. Similarly, (4) will hold along curves D having a slope given by (3). Values of V and y corresponding to the intersection of C and D therefore can be found by solving (2) and (4) simultaneously.

Following the same line of reasoning, one can see that in the case of $V < \sqrt{gy}$ the conditions of flow at the upstream end of the channel can be found by solving (4) and

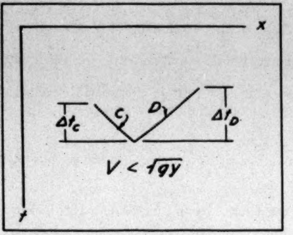


Fig. 1--The x-t plane

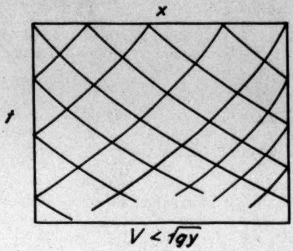


Fig. 2--"Subcritical" flow

$$\phi_1 (t) = 0 \ldots\ldots\ldots\ldots (5)$$

and those at the downstream end by (2) and

$$\phi_2 (t) = 0 \ldots\ldots\ldots\ldots (6)$$

ϕ_1 (t) and ϕ_2 (t) here being the boundary conditions at the ends of the channel (Fig. 2).

When $V > \sqrt{gy}$, all characteristics are directed downstream (Fig. 3), so that flow conditions at the downstream end can be determined by the equations of characteristics alone; no boundary conditions need be given. At the upstream end, however, flow conditions have to be given in order for the flow conditions downstream to be determinate. Theoretically the case of $V > \sqrt{gy}$ can be analyzed just as adequately as the other, although practically the possibility of disturbance due to the formation of standing waves would have to be considered too.

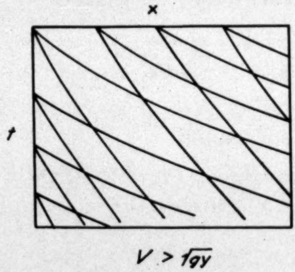


Fig. 3--"Supercritical" flow

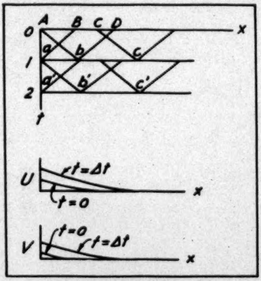


Fig. 4--Laying out the characteristics

The method of solution--As stated in the introduction, the proposed method is characterized by a constant value of Δt used in a step of computation. In order to accomplish this, a characteristic is generally not followed beyond one step and the values of V and U ($= \sqrt{gy}$) are plotted immediately after each step is concluded. Take, for instance, the first step of computation in which conditions of flow at the instant of t = Δt are to be computed from those at t = 0. The initial conditions being given, the distributions of V and U along the channel at the instant t = 0 are known. Basing on the values of V and U at A, the characteristics Ab can be tentatively laid out at a slope of $dx/dt = (V + \sqrt{gy})$ at A = (V + U) at A (Fig. 4). Similarly basing on the values of V and U at B and D, the characteristics Ba and Db can be drawn. By the method to be explained below, V and U at a, b, and all other points on the t = Δt line can be computed. These values are then used to recompute the slope of the characteristics by means of the formula

$$\text{Slope} = (V_0 + V_1)/2 \pm (\sqrt{gy_0} + \sqrt{gy_1})/2$$

Here $\Delta t = t_1 - t_0$, being assumed to be so small as to satisfy the conditions: (1) The characteristics in the x-t plane may be regarded as straight lines during each step of computation, and (2) $\int_t^{t+\Delta t} S_f\, dt \approx \bar{S}_f\, \Delta t$. If the characteristics laid out previously were found to be satisfactory, these newly found values of U and V are to be plotted against x to give the spatial variations at the instant $t = \Delta t$. With these variations in hand, the same procedure can be repeated to lay out the characteristics from $t = \Delta t$ to $t = 2\,\Delta t$.

The method to solve for y and V will now be explained in two parts: (1) for points within the channel, and (2) for points along the boundaries.

For points within the channel--Since $U = \sqrt{gy}$, the equations of characteristics (1) to (4) can be written (see Fig. 5)

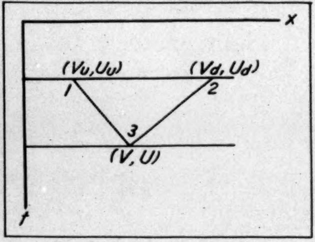


Fig. 5--A definition sketch

$$dx/dt = V + U \;\ldots\ldots\ldots\ldots (7)$$

$$d\,(V + 2U) = g\,(S_0 - S_f)\, dt \;\ldots\ldots\ldots (8)$$

and

$$dx/dt = V - U \;\ldots\ldots\ldots\ldots (9)$$

$$d\,(V - 2U) = g\,(S_0 - S_f)\, dt \;\ldots\ldots\ldots (10)$$

Integrating (8)

$$\begin{aligned} V + 2U &= V_u + 2U_u + g\,S_0\,\Delta t - g\int_t^{t+\Delta t} S_f\, dt \\ &\cong V_u + 2U_u + g\,S_0\,\Delta t - g\,[(S_{f1} + S_{f3})/2]\,\Delta t \\ &= V_u + 2U_u + (g/2)\,\Delta t\,(S_0 - S_{f1}) + (g/2)\,\Delta t\,(S_0 - S_{f3}) \\ &= V_U + 2U_U + K_U + K \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11) \end{aligned}$$

where $K_U = (g/2)\,\Delta t\,(S_0 - S_{f1})$ and $K = (g/2)\,\Delta t\,(S_0 - S_{f3})$. Similarly

$$V - 2U = V_d - 2U_d + (g/2)\,\Delta t\,(S_0 - S_{f2}) + (g/2)\,\Delta t\,(S_0 - S_{f3}) = V_d - 2U_d + K_d + K \;\ldots\; (12)$$

The advantage of keeping Δt constant is now evident, for K in (11) is thus made equal to that in (12). Subtraction of (12) from (11) leads to

$$U = [V_U + 2U_U + K_U - (V_d - 2U_d + K_d)]/4 \;\ldots\ldots\ldots\ldots\ldots (13)$$

which, being expressed in terms of known quantities, can be readily computed. Since K is a function of U and V, a plot of curves of constant K (Fig. 6) on the U-V plane can also be readily prepared. The plot will aid in the evaluation of K_u and K_d. With U found, the evaluation of V can be effected by a plot of V - K against V, using U as the parametric variable (Fig. 7).

Computations for the plot of Figures 6 and 7 are generally relatively simple, as can be seen in the solution of example.

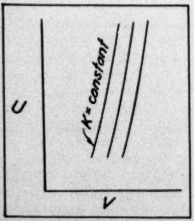


Fig. 6--Plot of K in terms of U and V

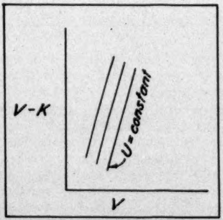


Fig. 7--Plot of U in terms of (V-K) and V

To sum up, U is to be computed by (13) whereas V is to be evaluated by a V-K against V plot.

For points along the boundaries--The method to be used depends on the form of boundary conditions given.

(1) If the boundary condition is of the form $y = f(t)$, the value of y and hence of U at any chosen instant t is known. V can therefore be evaluated as explained for points within the channel.

(2) If the boundary condition is of the form $V = f(t)$, V is known at any chosen instant. The only unknown involved in the quantities $(V + 2U - K)$ and $(V - 2U - K)$ is U. Either quantity may be plotted against U with V as the parametric variable. If (11) is to be used, a plot of $V + 2U - K$ is recommended; and if (12) is to be used a plot $V - 2U - K$ is recommended. U is to be evaluated by computing either of $(V + 2U - K)$ and $(V - 2U - K)$ by means of (11) or (12), and then entering the plot.

(3) For the boundary condition $V = 0$, U is given by

$$U = U_u + (V_u/2) + [(K_u + K)/2]\,\Delta t \ldots\ldots\ldots\ldots \tag{14}$$

or

$$U = U_d - (V_d/2) + [(K_d + K)/2]\,\Delta t \ldots\ldots\ldots\ldots \tag{15}$$

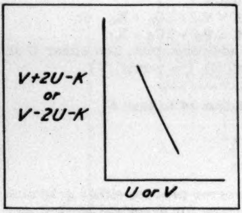


Fig. 8--Working curve for the boundary condition q = constant

(4) When the boundary condition is of the form q = constant, or UV = constant, $(V + 2U - K)$ or $(V + 2U + K)$ may be regarded as a function of either U or V. A plot as shown in Figure 8 may be prepared. From the equations, $V + 2U - K = V_U + 2U_U + K_U$ or $V - 2U - K = V_d - 2U_d + K_d$, the left-hand side can be computed. By using the plot, either of U and V may be evaluated. UV = constant furnishes another relationship to evaluate one of U and V from the other.

(5) If the boundary condition is given as $q = f(t)$, the same method of solution as (4) may be used, except that the corresponding plot will now consist of a family of curves instead of a single one.

It should be pointed out that $V + U$, $V - U$, $V - K$, $U + K$, and others may be used as the aforementioned plots. The choice is open to the individual computer.

Table 1 summarizes the methods to be used in the solution of V and U along a boundary.

Example--Let the flow in an infinitely long and very wide channel be initially uniform and amount to 50 cfs per ft. If a flood hydrograph of $y = 21.5 - 8.4 \cos(\pi t/48)$ ft be imposed at the upper end, route the flood through the first 200 mi for a period of 24 hr, given $n = 0.0297$, and $s = 1{:}5280$.

From the given data, compute

Normal depth and velocity: $y_0 = 13.1$ ft; $V_0 = 3.81$ fps = 26 mph
Initial celerity: $U_0 = \sqrt{gy_0} = 14$ mph
Boundary condition at the upper end:

$$y = 21.5 - 8.4 \cos(\pi t/48)$$

$$gy = U^2 = g\,[21.5 - 8.4 \cos(\pi t/48)]$$

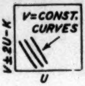
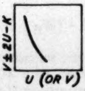
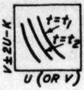
$$U = \sqrt{g\,(60/88)^2\,[21.5 - 8.4 \cos(\pi t/48)]}$$

$$= 3.87\sqrt{21.5 - 8.4 \cos(\pi t/48)} \text{ mph}$$

where t is in hr.

Adopting Δt = three hours

Table 1--Methods used in solution of V and U along a boundary

Case	Boundary condition	Additional plot	Remarks
1	$y = f(t)$	None	Direct solution Evaluation of V from the known value of U at the chosen instant
2	$V = f(t)$		Direct solution Evaluation of U from the known value of V at the chosen instant
3	$V = 0$	None	Direct solution $U = U_u + (V_u/2) + [(K_u + K)/2]\ \Delta t$ or $= U_d - (V_d/2) + [(K_d + K)/2]\ \Delta t$
		Do as in case 2	. . . .
4	q = const. or UV = const.		Direct solution $V + 2U - K = V_u + 2U_u + K_u$ or $V - 2U - K = V_d - 2U_d + K_d$ From the additional plot, find either U or V $V = (\text{const.}/U)$, $U = (\text{const.}/V)$
5	$q = f(t)$		Direct solution as in case 4
		None	Trial-and-error process either graphical or algebraic; for graphical method, see cases 2 and 3

V + U, V - U, V - K, K, U + K, and other values may also be used as the ordinate

---

$$K = (g/2)\ (S_0 - S_f)\ \Delta t$$

$$= 22.5 - 3750\ (V^2/U^{8/3})$$

Therefore

$$V = \sqrt{(22.5 - K)/3750}\ U^{4/3}$$

Basing on this equation, the computations shown in Table 2 can be made for the auxiliary plots, Fig. 9e and f.

Table 2--Sample computation

K	22.5-K	$\frac{22.5-K}{3750}$	U = 14		U = 16		U = 18		U = 20		U = 22	
			$U^{4/3}$ = 33.7		$U^{4/3}$ = 40.3		$U^{4/3}$ = 47.2		$U^{4/3}$ = 54.2		$U^{4/3}$ = 61.1	
			V	V-K	V	V-K	V	V-K	V	V-K	V	V-K
1.0	21.5	0.0758	2.55	1.55	3.05	2.05	3.58	2.58	4.11	3.11	4.63	3.63
0.0	22.5	0.0775	2.60	2.60	3.12	3.12	3.66	3.66	4.20	4.20	4.73	4.73
-0.5	23.0	0.0783	2.64	3.14	3.16	3.66	3.70	4.20	4.25	4.75	4.79	5.29
-1.0	23.5	0.0791	2.66	3.66	3.19	4.19	3.74	4.74	4.29	5.29	4.84	5.84
-1.5	24.0	0.0800	2.69	4.19	3.22	4.72	3.78	5.28	4.34	5.84	4.89	6.39
-2.0	24.5	0.0808	2.72	4.72	3.26	5.26	3.82	5.82	4.38	6.38	4.95	6.95
-2.5	25.0	0.0816	2.74	5.24	3.29	5.79	3.85	6.35	4.42	6.92	4.98	7.48
-3.0	25.5	0.0825	2.78	5.78	3.32	6.32	3.89	6.89	4.47	7.47	5.04	8.04

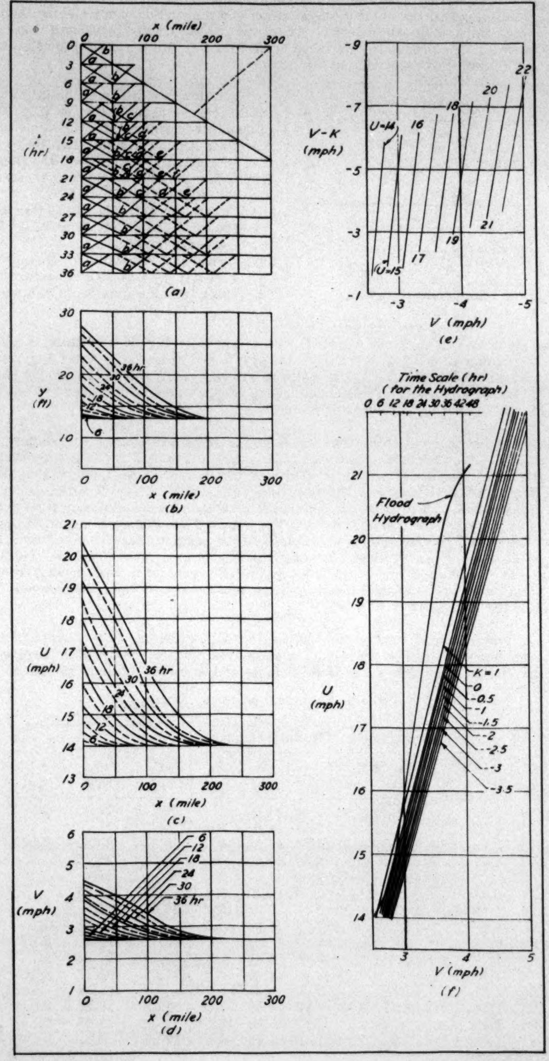


Fig. 9--Solution of the example

Computation procedure--Let the step of computation from t = 12 to t = 15 hr be taken for illustration, assuming that the velocity distribution and the surface profile along the channel have been previously computed at the instant t = 12 hr, so that the curves of V and U can be plotted as shown in Figure 9f. The procedure may then be divided into two parts: (1) for flow conditions at the inflow, and (2) for flow conditions in the channel.

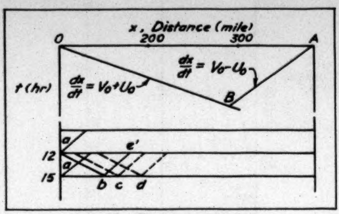


Fig. 10--Step computation from t = 12 to t = 15

For flow conditions at the inflow, sketch a characteristic to the point 15a (Fig. 10) in the general direction of AB. The slope of this characteristic should be somewhat flatter than that to 12a, because it is known from the flood hydrograph, Fig. 9f, that U will be increasing during the interval from t = 12 to t = 15 hr. The values of U and V at the downstream end of this characteristic are then found from Fig. 9c and Fig. 9d to be 2.77 and 14.45, and are entered in Table 3 under headings V_d and U_d. The value of K corresponding to $V_d = 2.77$ and $U_d = 14.45$ can be read from Fig. 9f. This value is entered as K_d in column 10.

Now the value of U at t = 15 hr can be directly read from Fig. 9f. By going through the computations for columns 12 and 15, the value of (V-K) is found to be 4.87. Enter Fig. 9e with V-K = 4.87 and U = 15.9 to obtain V = 3.21. The slope of the characteristic at 15a (where U = 15.9 and V = 3.21) can now be computed. The characteristic is subsequently to be readjusted, so that its slope is equal to the average of slopes at its ends.

For points in the channel, there are two characteristics intersecting each other. Let the solution of V and U at 15b be taken for instance. The values of V and U at 12a are known from a previous step of computation and, from these values, K can be found. All these values are then listed in columns 5 to 7 as V_u, U_u, and K_u. Similarly the values of V, U, and K corresponding to 12e are listed as V_d, U_d, and K_d. It is to be noticed that the subscript u denotes quantities pertaining to the upstream end of the characteristic $dx/dt = V + U$, and d denotes those quantities pertaining to the downstream end of the characteristic. Complete the computations from columns 11 to 14. Subtraction of column 14 from column 13 then gives U. Here for 15b, U = 8.18 - (-6.35) = 14.53. After U has been computed, V can be found as explained previously. With enough points determined, the values of V and U at the instant t = 15 hr can be plotted along the channel. One is thus ready to extend the analysis to t = 18 hr.

In this particular case of long channel having initially a uniform flow, the use of constant Δt affords a further possibility of saving labor. Let a horizontal line be drawn in Figure 9c from the top of the curve for t = 9 hr. This line intersects the curve for t = 12 hr at x = 40 mi. Also let a

Table 3--Computations in a channel

Point (1)	V (2)	U (3)	K (4)	Upstream point			Downstream point			V_u+2U_u $+K_u$ (11)	V_d-2U_d $+K_d$ (12)	$\frac{\text{Col 11}}{4}$ (13)	$\frac{\text{Col 12}}{4}$ (14)	Col 12 + 2U = V - K (15)
				V_u (5)	U_u (6)	K_u (7)	V_d (8)	U_d (9)	K_d (10)					
3a	2.66	14.17	-0.28	.....	....	....	2.60	14.0	0	0	-25.4	0	....	2.94
6a	2.75	14.41	-0.77	.....	....	....	2.61	14.0	0	....	-25.3	.....	....	3.52
6b	2.61	14.03	-0.05	2.66	14.17	-0.28	2.60	14.0	0	30.72	-25.4	7.69	-6.35	2.66
9a	2.85	14.76	-0.71	.....	....	....	2.67	14.19	-0.25	....	-25.96	.....	....	3.56
9b	2.66	14.05	-0.04	2.75	14.41	-0.77	2.60	14.0	0	30.80	-25.4	7.70	-6.35	2.70
12a	3.00	15.27	-0.87	.....	....	....	2.71	14.19	-1.0	....	-26.67	.....	....	3.87
12b	2.70	14.26	-0.42	2.85	14.76	-0.72	2.60	14.0	0	31.66	-25.4	7.91	-6.35	3.12
15a	3.21	15.9	-1.66	.....	....	....	2.77	14.45	-0.8	....	-26.93	.....	....	4.87
15b	2.79	14.52	-0.85	3.00	15.27	-0.87	2.60	14.0	0	32.67	-25.4	8.17	-6.35	3.64
18a	3.41	16.55	-1.76	.....	....	....	2.89	14.81	-1.2	....	-27.93	.....	....	5.17
18b	2.87	14.73	-1.10	3.21	15.9	-1.68	2.61	14.08	0	33.35	-25.55	8.34	-6.39	3.97
21a	3.62	17.25	-2.04	.....	....	....	3.00	15.12	-1.6	....	-25.64	.....	....	5.66
21b	3.01	15.25	-1.24	3.41	16.55	-1.76	2.69	14.22	-0.5	34.75	-26.25	8.69	-6.56	4.25
24a	3.83	17.95	-2.42	.....	....	....	3.17	15.86	-1.1	....	-29.65	.....	....	6.25
24b	3.11	15.57	-1.48	3.62	17.25	-2.04	2.75	14.45	-0.4	36.08	-26.55	9.02	-6.64	4.59

similar line be drawn in Figure 9d. If the latter line intersects the $t = 15$ hr curve also at $x = 40$ mi, a characteristic is to be drawn to 15c in Figure 9a starting from the $t = 12$ hr line at $x = 40$ mi. It can be seen that the values of V and U at 15c are equal to those known at 12b. Similarly V and U at 15d are equal to those known at b on Figure 10. Therefore, V and U at 15c and 15d can be determined without any computation. In fact, V and U at the intersections of dotted characteristics in Figure 9a are determined in this manner. As a result, only two sets of U and V need be computed in every step.

Table 3 gives all of the computations. Results are presented in Figure 9b and d.

Prismatic channel--In the case of prismatic channels, the equations of characteristics are

$$dx/dt = V \pm \sqrt{gy_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

and

$$dV \pm \sqrt{g/y_m}\, dy = g\,(S_0 - S_f)\, dt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

where $y_m = A/B$. By writing $2dU = \sqrt{g/y_m}\, dy$ the second pair of characteristics may be reduced to

$$d\,(V \pm 2U) = g\,(S_0 - S_f)\, dt$$

which is of the same form as (8) and (10). Therefore, the method of solution stated previously may be applied as well. The integral

$$U = (\sqrt{g}/2)\int (dy/y_m^{1/2})$$

can be evaluated functionally or numerically.

It should be pointed out, however, that now

$$dx/dt \neq V \pm U$$

Instead,

$$dx/dt = V \pm c$$

where $c = \sqrt{gy_m}$.

Remarks--This method has the following advantages: (1) It eliminates much of the trial-and-error process. (2) It affords better control of computations. More points may be computed in a strategic region. (3) Computed results are automatically presented as spatial variations at various instants, which is often desirable.

Since the method of characteristics is applicable as long as the governing equation is of the hyperbolic type, application of the numerical method presented herein is by no means limited to the case of open channel flow. An immediate example is the analysis of water-hammer phenomenon in a long conduit where resistance effect is important.

Although the method is semi-graphical, it may be converted to purely graphical, if so desired. Due to space limitations, details will not be given here.

Throughout this paper, U has been set equal to $\sqrt{gy}$. This is, however, quite optional. In fact, there are some advantages in writing $U = 2\sqrt{gy}$. For instance, the characteristics will then be represented on a U-V plane as straight lines inclined at ± 45° instead of 1:2 slope.

Acknowledgments--The writer desires to express his indebtedness to C. J. Posey for guidance in the study of unsteady flow, and for illuminating suggestions made in the preparation of this paper. Grateful acknowledgment is also due to A. Craya for many enlightening discussions.

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(Communicated manuscript received June 13, 1951; presented at the Fifth Annual Meeting of the Rocky Mountain Hydraulic Laboratory, Allenspark, Colorado, July 1950; open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

UNIT-HYDROGRAPH LAG AND PEAK FLOW RELATED TO BASIN CHARACTERISTICS

Arnold B. Taylor and Harry E. Schwarz

Abstract--Unit-hydrograph lag and peak-flow values have been empirically related to basin characteristics and to the duration of rainfall excess. Characteristic data for 20 basins ranging from 20 to 1600 sq mi of drainage area and located in the North and Middle Atlantic States were used in the study. Unit-hydrograph values were obtained from 65 rainfall excess periods.

The study indicates that the most significant basin characteristics were drainage area, length of longest watercourse, length to center of area, and equivalent main-stream slope. Graphs of the correlation studies are given and a method for determining equivalent main-stream slope is presented. A nomograph for the computation of synthetic unit hydrographs is included and is based on equations developed in the study. The data presented will aid in the determination of unit hydrographs for ungaged watersheds in similar areas.

General--The purpose of this paper is to present the results of studies of hydrograph lag and peak-flow values as related to basin characteristics and to the runoff duration. The object of the study was to reduce the uncertainties involved in developing synthetic unit hydrographs for ungaged basins and to present equations or graphs which will quickly give a synthetic unit hydrograph when the necessary basin-characteristic data are available.

The study was limited to 20 basins ranging from 20 to 1600 sq mi located in the North and Middle Atlantic States (Table 1). The basin-characteristic and unit-hydrograph data were obtained from preliminary copies of a recent publication of the CORPS OF ENGINEERS [1949].

The basins used in the study were selected from the same geographical region so as to reduce the variables involved and were limited to basins where the most basic data were available. Problems caused by uneven storm distribution were eliminated by selecting 65 hydrographs of runoff resulting from storms more or less uniformly distributed over an area.

The procedure for development of the relationships presented was to relate the elements of lag and peak flow of the unit hydrograph to rainfall-excess duration and in turn relate the coefficients and rate of change to applicable basin characteristics. The most significant basin-characteristic factors which affect the unit-hydrograph lag and peak-flow values were found to be drainage-area size, length of longest watercourse, length to center of area, and equivalent main-stream slope. These basin factors were the essential elements as used by SNYDER [1938] with the addition of a slope factor which is introduced and derived.

Terms, definitions, and derivations--The terms, definitions, and derivations used follow.

Terms and definitions

t_r	Unit rainfall duration in hours
t_R	Time in hours from beginning to end of computed significant rainfall excess
q_{pR}	Peak value of observed unit hydrograph in cfs per sq mi
t_{pR}	Lag in hours from the time the center of mass (centroid) of rainfall excess occurs to the time of the observed unit-hydrograph peak
L	Total length of longest watercourse, measured to basin boundary
L_{ca}	Distance along the main drainage channel from gage to computed center of gravity (centroid) of drainage area
D.A.	Drainage area in sq mi
S_{st}	Slope of a uniform channel having the same length as the longest watercourse and an equal time of travel
e	2.7183 = Base of natural logarithms

Table 1--Basic data on drainage basins

Stream	Gage location	Basin	Line	Basin characteristics				Storm characteristics			
				D.A.	L	L_{ca}	S_{st}	Storm date	t_{pR}	q_{pR}	t_R
				sq mi	mi	mi	ft/ft		hr	cfs/sq mi	hr
Charlotte Creek	Davenport Center, N. Y.	1	1	163	23	14	0.0037	May 22-23, 1942	13.4	27.6	8
			2					June 14, 1942	13.7	25.0	6
			3					Nov. 8, 1943	12.9	28.0	4
Contoocook River	Bennington, N. H.	2	4	186	20	7.7		Sep. 20-21, 1938	28.9	10.0	42
			5					June 24-30, 1944	23.0	9.8	15
Dog River	Northfield Falls, Vt.	3	6	76.1	14.7	5.0	0.0121	Nov. 8-11, 1943	6.2	57.8	6
			7					Sep. 18-19, 1945	6.0	74.4	3
Evitts Creek	Bedford Valley, Pa.	4	8	30.2	12	5.9	0.0069	Mar. 16-18, 1936	9.5	50.0	18
			9					Apr. 25-26, 1937	11.0	36.4	14
			10					Oct. 27-28, 1937	10.1	37.0	18
			11					July 29, 1939	4.3	97.0	2
			12					Apr. 19-20, 1940	9.0	34.9	10
			13					Apr. 9, 1942	11.0	30.1	10
			14					Apr. 18-19, 1943	10.5	34.2	24
Falling River	Brookneal, Va.	5	15	228	23	13		Jan. 18-19, 1936	14.9	33.3	18
			16					Aug. 18-19, 1939	14.6	29.2	12
Genegantslet Creek	Smithville Flats, N. Y.	6	17	83.1	16.6	8.3	0.0075	May 22, 1942	11.2	25.9	10
			18					Nov. 8-9, 1943	7.6	28.5	5
			19					May 7-8, 1944	7.9	28.5	9
			20					Apr. 26-27, 1945	6.9	30.2	8
Hoosic River	Williamstown, Mass.	7	21	132	23.8	9.0		May 9-10, 1941	5.3	37.5	6
			22					Nov. 8-12, 1943	5.0	36.7	6
Juniata River	Spruce Creek, Pa.	8	23	220	28.5	11.9	0.0046	Mar. 30, 1940	12.2	25.3	15
			24					June 4-5, 1941	11.6	27.7	9
			25					May 11-12, 1943	8.5	24.0	3
Lehigh River	Tannery, Pa.	9	26	322	34	16	0.0056	Aug. 23, 1933	15.4	21.7	9
			27					Nov. 8-9, 1943	11.0	23.0	8
North Branch Hoosic River	North Adams, Mass.	10	28	39	10.1	5.3	0.0277	Aug. 31-Sep. 3, 40	5.5	57.2	2
			29					May 9-12, 1941	7.5	54.1	3
			30					June 10-14, 1943	5.5	55.4	2
North Branch Potomac River	Cumberland, Md.	11	31	875	83	40.6	0.0036	Mar. 16-18, 1936	11.0	29.9	20
			32					Apr. 24-26, 1937	14.8	22.0	28
			33					Oct. 26-28, 1937	13.5	24.6	32

(Continued)

Derivation of slope equation

For each increment of stream length

$$t_1 = \ell_1/V_1 \quad \text{. . . . .} \quad (1)$$

where t_1 = travel time of flow through the increment of length, ℓ_1 = increment of length, and V_1 = average velocity of flow through the increment.

Table 1--Basic data on drainage basins (concluded)

Stream	Gage location	Basin	Line	Basin characteristics				Storm characteristics			
				D.A.	L	L_{ca}	S_{st}	Storm date	t_{pR}	q_{pR}	t_R
				sq mi	mi	mi	ft/ft		hr	cfs/sq mi	hr
North Branch Potomac River	Cumberland, Md. (concluded)	11	34	875	83	40.6	0.0036	May 15-16, 1942	15.2	26.2	8
			35					Oct. 13-16, 1942	18.5	16.4	40
Northeast Branch Anacostia River	Riverdale, Md.	12	36	72.8	15	7.3	0.0054	Aug. 8-9, 1942	9.5	31.4	8
			37					Oct. 15, 1942	11.2	30.4	2
			38					July 17-18, 1945	12.2	33.0	10
			39					July 26-27, 1945	7.8	38.2	2
Northwest Branch Anacostia River	Colesville, Md.	13	40	21.3	7	4	0.0042	Aug. 23-24, 1933	6.3	76.8	10
			41					Oct. 22-23, 1937	5.3	91.5	7
			42					Nov. 12-13, 1937	6.2	102.3	8
			43					July 26-27, 1945	2.2	127.7	1
Northwest Branch Anacostia River	Hyattsville, Md.	14	44	49.4	18	9.8	0.0038	Oct. 15, 1942	2.8	56.1	2
			45					July 17-18, 1945	5.5	44.1	8
			46					July 26-27, 1945	3.0	51.9	2
Otego Creek	Oneonta, N. Y.	15	47	108	24	13.6	0.0029	May 22-23, 1942	18.5	17.4	9
			48					May 26, 1943	12.0	26.7	2
			49					Sep. 18, 1945	18.0	22.6	6
			50					Oct. 1-2, 1945	16.5	20.7	8
Rapidan River	Culpeper, Va.	16	51	465	44	23	0.0018	Apr. 24-27, 1937	13.7	26.3	24
			52					Oct. 12-17, 1942	22.4	23.9	36
Rappahannock River	Fredericksburg, Va.	17	53	1599	78	46	0.0013	Apr. 24-27, 1937	24.5	17.8	30
			54					Oct. 12-17, 1942	28.7	14.4	36
Rappahannock River	Kellys Ford, Va.	18	55	641	42	28	0.0022	Apr. 24-29, 1937	15.5	24.1	30
			56					Oct. 12-17, 1942	23.1	21.1	36
Savage River	Bloomington, Md.	19	57	115	26	12	0.0119	Mar. 16-18, 1936	8.0	37.0	18
			58					Apr. 24-27, 1937	9.5	38.4	12
			59					Oct. 27-28, 1937	9.0	32.8	22
			60					June 3-5, 1941	7.8	40.7	10
Wills Creek	Cumberland, Md.	20	61	247	35	17.1	0.0063	Mar. 16-18, 1936	11.0	42.5	18
			62					Apr. 24-26, 1937	10.0	34.4	15
			63					Oct. 27-28, 1937	13.0	23.0	27
			64					Apr. 4-5, 1941	12.0	25.3	12
			65					Oct. 13-16, 1942	16.2	25.3	30

From Manning's formula, $V = 1.486\ r^{2/3}\ s^{1/2}/n$, and for each length increment $1.486\ r^{2/3}/n$ is determinate and equals K_1.

The equation for the velocity can therefore be written

$$V_1 = K_1\ S_1^{1/2} \quad \ldots\ldots (2)$$

The total travel time

$$T = t_1 + t_2 + \ldots + t_n \quad \ldots\ldots (3)$$

Substituting (1) and (2) in (3)

$$T = \ell_1/K_1\,S_1^{1/2} + \ell_2/K_2\,S_2^{1/2} + \ldots + \ell_n/K_n\,S_n^{1/2} \tag{4}$$

If equal increments of length are used

$$\ell_1 = \ell_2 = \ell_n = \ell \text{ and } \ell = L/n$$

Equation (4) therefore becomes

$$T = (L/n)\,[1/K_1\,S_1^{1/2} + 1/K_2\,S_2^{1/2} + \ldots + 1/K_n\,S_n^{1/2}] \tag{5}$$

Assuming a uniform channel having hydraulic elements equivalent to the average for the prototype channel, the travel time would be

$$T = L/V \tag{6}$$

and the velocity

$$V = KS_{st}^{1/2} \tag{7}$$

Substituting (7) for V in (6)

$$T = L/KS_{st}^{1/2} \tag{8}$$

Equating (5) and (8)

$$L/KS_{st}^{1/2} = (L/n)\,[1/K_1\,S_1^{1/2} + 1/K_2\,S_2^{1/2} + \ldots + 1/K_nS_n^{1/2}] \tag{9}$$

Solving for S_{st} this equation becomes

$$S_{st} = \left[\frac{n}{K/K_1\,S_1^{1/2} + K/K_2\,S_2^{1/2} + \ldots + K/K_n\,S_n^{1/2}}\right]^2 \tag{10}$$

The ratios K/K_1, K/K_2, . . . K/K_n will depart, generally, only slightly from unity and the average of the denominators will equal K by definition. These ratios have been assumed to be unity since the assumption is generally within the required accuracy of this computation. This makes the expression

$$S_{st} = [n/(1/S_1^{1/2} + 1/S_2^{1/2} + \ldots + 1/S_n^{1/2})]^2 \tag{11}$$

A more precise determination of S_{st} can be obtained if data are available to compute more accurate values of K.

Determination of the "lag" functions--Since the lag value of the unit hydrograph was observed to vary with the rainfall-excess duration, the observed unit-hydrograph lag value was plotted against the duration of rainfall excess as the first correlation step. Hydrographs of double duration were computed for each storm, and the lag and duration value as computed were used to increase the range of the plot. An equation of the type $Y = Ce^{mx}$ was found to give the best relationship of lag to duration and the curves of best fit were computed by the method of least squares for each basin and gaging station. The general equation for these curves is $t_{pR} = C'e^{m't_R}$, where m' is the rate of change of lag with storm duration and C' is the lag of the instantaneous unit hydrograph. Curves of lag-duration relationships which envelope the curves for about 75 pct of the basins studied are shown on Figure 1. The plot of m' against drainage area, length of watercourse, and length to center of area showed some degree of correlation, but only the relation between m' and the length of watercourse times the length to center of area (LL_{ca}) was considered usable, and is shown on Figure 2. A curve was fitted to the data eliminating from the computation seven values lying scattered outside the main grouping of points. The equation of this line is

$$m' = 0.212/(LL_{ca})^{0.36} \tag{12}$$

Note that the value $(LL_{ca})^{0.36}$ is similar to the $(LL_{ca})^{0.3}$ considered a characteristic value by SNYDER [1938].

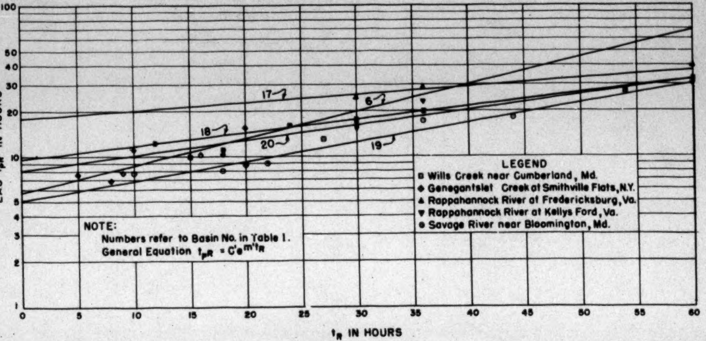


Fig. 1--Lag-duration relation

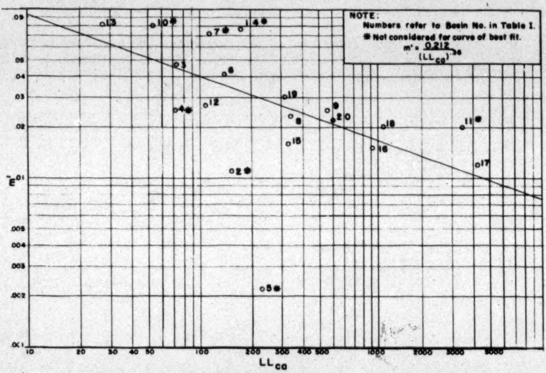


Fig. 2--Rate of change (t_{pR} vs t_R) related to LL_{ca}

The m′ values, which fall considerably above the line of best fit on Figure 2, are for basins with a large tributary joining the main stream near the gage. If the LL_{ca} value for each of these tributaries is used in plotting its position on the graph, each point would approach the line shown. It would seem to follow, therefore, that when dealing with a V-shaped drainage area, judgment is involved as to the manner in which the branches should be used for the determination of constants. Why the three values fall below the line of best fit is less apparent, but the drainage density or rather the lack of it may be the cause. However, enough data are not available to prove or disprove this.

The relation of C′ to basin characteristics was attempted by using the values of D.A., LL_{ca}, $\sqrt{S_{st}}$, and $LL_{ca}/\sqrt{S_{st}}$, and the relation between C′ and $\sqrt{S_{st}}$, as shown in Figure 3, was selected. This indicates that C′, the lag of the instantaneous unit hydrograph, is a function of the velocity of flow and the equation is

$$C' = 0.6/\sqrt{S_{st}} \qquad (13)$$

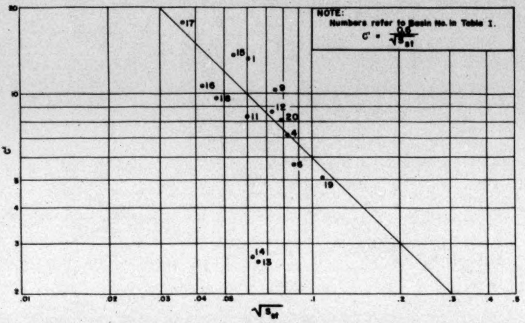


Fig. 3--Coefficient (lag vs t_R) related to slope factor

The controlling tributary in V-type drainage basins influences the C′ relation as noted in the m′ relation. For better results it was concluded that the values for LL_{ca} and S_{st} should be computed for that sub-area which gives the critical concentration for the basin.

Determination of peak discharge functions--The procedure used to determine the q_{pR} equations parallels the one used for the lag functions. An expression of the form $q_{pR} = C'' e^{m'' t_R}$ was found to represent the relationship between q_{pR} and t_R.

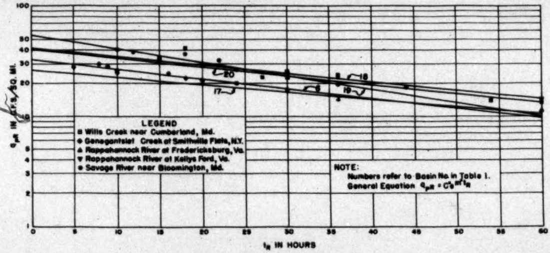


Fig. 4--Peak-duration relation

The unit-hydrograph peak value as related to the duration of rainfall excess is shown in Figure 4 for selected areas. The intercept with the ordinate C″ can be considered the q of an instantaneous unit hydrograph at the outlet, and m″ is the rate of change of q with the storm duration. Note that m″ has the opposite sign of m′, the lag rate of change with duration, but numerically the figures of m′ and m″ are similar for most of the basin studied. Therefore, m′ was plotted against -m″ and the deviation of m″ from the equation m′ = -m″ was recorded. The Δm″ deviation plus a constant, to make all values positive, was then plotted against S_{st} as shown on Figure 5 and the computed m″ equation is

$$m'' = 0.121\, S_{st}^{0.142} - 0.050 \ldots\ldots\ldots\ldots (14)$$

As m″ was determined to be

$$m'' = -m' + \Delta m'' \ldots\ldots\ldots\ldots (15)$$

and substituting (12) for m′

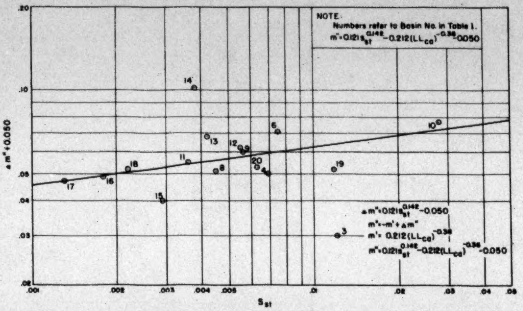


Fig. 5--Relation of deviation (+m′ = -m″) to slope factor

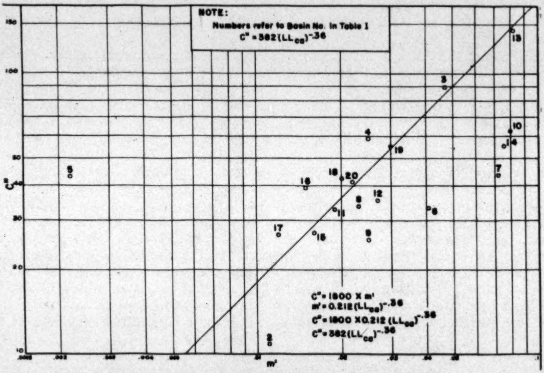


Fig. 6--Relation between coefficient (peak vs t_R) and rate of change (lag vs t_R)

$$m'' = 0.121\, S_{st}^{\,0.142} - 0.212\, (LL_{ca})^{-0.36} - 0.050 \qquad (16)$$

C″ was related to the different basin characteristics, and although a trend was usually apparent, only the relation between C″ and m′ gave a workable solution. This relation is shown graphically on Figure 6 and the equation

$$C'' = 1800\, m' \qquad (17)$$

was determined.

Substituting (12) for m′, C″ becomes

$$C'' = 382\, (LL_{ca})^{-0.36} \qquad (18)$$

Relating the C″ values directly to LL_{ca} values gave a much wider spread of the plot; therefore, the smoothed values of m′ were used.

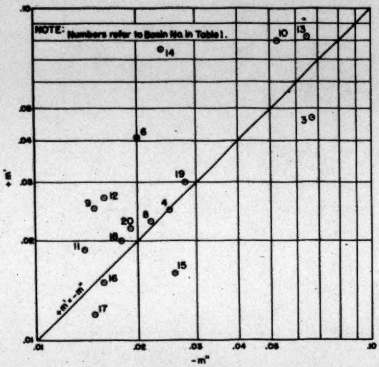


Fig. 7--Rate of change (lag vs t_R) related to rate of change (peak vs t_R)

Deviation of actual m'' values from $+m' = -m''$		
Basin	$\Delta m''$	$\Delta m''+0.050$
3	-0.020	0.030
4	0.000	0.050
6	+0.021	0.071
8	+0.001	0.051
9	+0.010	0.060
10	+0.027	0.077
11	+0.005	0.055
12	+0.011	0.061
13	+0.018	0.068
14	+0.052	0.102
15	-0.010	0.040
16	-0.001	0.049
17	-0.003	0.047
18	+0.002	0.052
19	+0.002	0.052
20	+0.003	0.053

Most of the points plotting off the line on Figure 2 also deviate from the trend as shown on Figure 7 and this indicates that the V-type drainage area also affects the peak-value relationships. However, the rate of change of lag with respect to duration is greater than the rate of change of the peak value with duration (No. 14, Fig. 7). An attempt to correlate values that deviate from the trend on Figure 2, other than V-type drainage areas, was made by introducing the variables of land slope and drainage density, but due to the lack of sufficient data no conclusions were reached.

The nomograph--The nomograph (Fig. 8) is a graphical solution of the equations developed and is included so that a synthetic unit hydrograph may be more readily obtained when the necessary basic data are available. Judgment and experience will be required in obtaining the basic data from the basin maps so that all hydrologic factors of the basin under study will be evaluated in obtaining a representative unit hydrograph. The scales marked (1) and (2) on Figure 8 have been adjusted at their extremities, due to the nature of the m'' equation, so that the values obtained from the nomograph in this range will be more accurate for conditions most likely attained, that is, small LL_{ca} values with corresponding steep slopes and large LL_{ca} values with flat slopes. However, the converse conditions (such as small LL_{ca} values with flat slopes) will give nomograph values that are not too accurate. For these converse conditions, it is suggested that the equations be solved mathematically.

Developing the hydrograph--A number of formulas for the determination of the base width of a unit hydrograph have been proposed. The one found most suitable in this area is

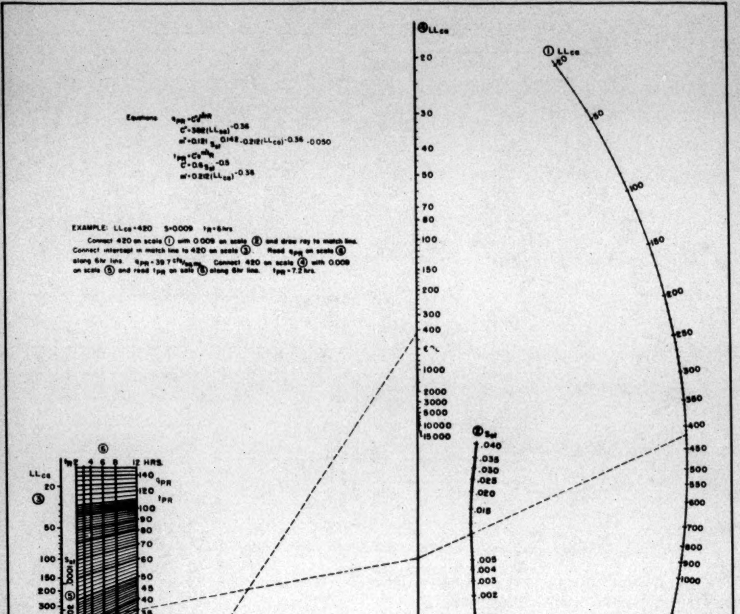
$$T = 5 (t_{pR} + t_r/2 \qquad (19)$$

or the nearest whole multiple of the time units. The complete hydrograph may be drawn using the base width, the peak height, and lag time computed as outlined in the preceding paragraphs, and the W_{50} and W_{75} relation as shown on Plate 9 of the CORPS OF ENGINEERS [1948]. W_{50} and W_{75} are the width of the hydrograph at 50 pct and 75 pct of its peak, respectively, as shown in Figure 10.

The following is an example of the procedure applied to Seneca River near Anderson, South Carolina (six-hour unit hydrograph).

(1) Available data show no outsized tributaries or other unusual features in the drainage area (Fig. 9). Therefore, the method described may be applied without modification.

(2) The values of drainage area, L and L_{ca}, were taken from the CORPS OF ENGINEERS [1949]. $L = 60$ mi, $L_{ca} = 29$ mi, $L \times L_{ca} = 1740$, and D.A. = 1026 sq mi.



Equations $q_{pR} = C' e^{m' t_R}$

$C' = 382 (LL_{ca})^{-0.36}$

$m' = 0.121\, S_{st}^{0.142} - 0.212 (LL_{ca})^{-0.36} - 0.050$

$t_{pR} = C e^{m t_R}$

$C = 0.6\, S_{st}^{-0.5}$

$m = 0.212 (LL_{ca})^{-0.36}$

EXAMPLE: $LL_{ca} = 420$ $S = 0.009$ $t_R = 6$ hrs

Connect 420 on scale ① with 0.009 on scale ② and draw ray to match line. Connect intercept in match line to 420 on scale ③. Read q_{pR} on scale ⑥ along 6hr line. $q_{pR} = 39.7$ cfs/sq.mi. Connect 420 on scale ④ with 0.009 on scale ⑤ and read t_{pR} on scale ⑥ along 6hr line. $t_{pR} = 7.2$ hrs.

Fig. 8--Nomograph for the determination of unit-hydrograph peaks from basin characteristics

(3) The equivalent main-stream slope was determined from a profile by breaking up the river into six ten-mile reaches as shown in Table 2.

Table 2--Determination of equivalent main-stream slope

Elevation	Δ elevation	Length	S_i	$\sqrt{S_i}$	$1/\sqrt{S_i}$
ft	ft	ft			
515					
590	75	52,800	0.001420	0.0377	26.51
610	20	52,800	0.000379	0.0195	51.28
630	20	52,800	0.000379	0.0195	51.28
670	40	52,800	0.000758	0.0275	36.36
740	70	52,800	0.001328	0.0364	27.47
910	170	52,800	0.003220	0.0567	17.64
					210.54

S_{st} = [Number of equal length reaches/ $\Sigma(1/\sqrt{S_i})]^2 = (6/211)^2 = 0.00081$

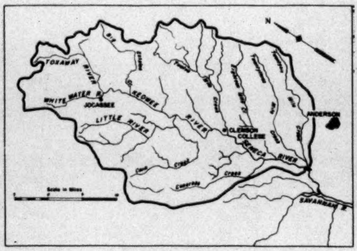


Fig. 9--Seneca River near Anderson, S. C.

(4) The next step in the determination of a synthetic six-hour unit hydrograph is the solution of the equations developed by using the nomograph (Fig. 8). Entering the nomograph with the values from subparagraphs (2) and (3) above, the values on the t_R = six-hour scale are t_{pR} = 22.0 hr, q_{pR} = 22.3 cfs/sq mi.

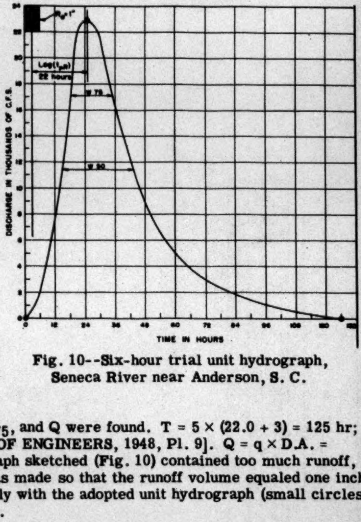


Fig. 10--Six-hour trial unit hydrograph, Seneca River near Anderson, S. C.

(5) To complete the hydrograph, T, W_{50}, W_{75}, and Q were found. T = 5 × (22.0 + 3) = 125 hr; Use 126. W_{50} = 28 hr and W_{75} = 16 hr [CORPS OF ENGINEERS, 1948, Pl. 9]. Q = q × D.A. = 22.3 × 1026 = 22,880 cfs. The first unit hydrograph sketched (Fig. 10) contained too much runoff, and, without changing the peak, an adjustment was made so that the runoff volume equaled one inch. This adjusted unit hydrograph compares favorably with the adopted unit hydrograph (small circles on Fig. 11) determined from 12 different storms.

A second example is given using the Savage River near Bloomington, Maryland (two-hour unit hydrograph).

(1) An examination of the drainage area shows no outsized tributaries (Fig. 12); therefore, the method described can be applied. From the drainage-area map and profile, values of L, L_{ca}, D.A., and S_{st} were determined and are L = 26 mi, L_{ca} = 12 mi, S_{st} = 0.0119 ft/ft, and D.A. = 115 sq mi.

(2) From the nomograph q_{pR} = 48 cfs/sq mi and t_{pR} = 6 hr was determined.

(3) From Plate 9, CORPS OF ENGINEERS [1948], W_{50} = 12 hr, and W_{75} = 7 hr; from (19), T = 5 (6 + 1) = 35 hr; use 36 hr. The hydrograph was sketched from these points and in this case no volume adjustment was necessary (Fig. 13). The derived hydrograph shows good agreement with the adopted unit hydrograph developed for this stream from the study of several storms.

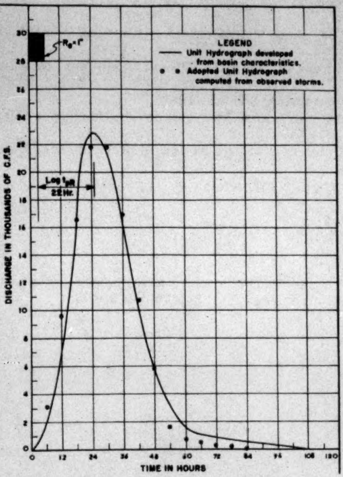


Fig. 11--Six-hour adjusted unit hydrograph, Seneca River near Anderson, S. C.

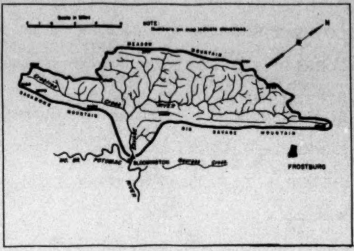


Fig. 12--Savage River near Bloomington, Md.

Discussion and conclusion--The relations developed were based on data from drainage areas of the Atlantic Slope, with most of the data obtained from basins in the Middle Atlantic area. However, relationships for basins in the same general locality deviate from the computed trend of the correlated data and indications are that the deviations are due to the arrangement of tributaries in the basin (basins 11 and 14, Fig. 2), and to steep land slopes with no tributary channels (basin 4, Fig. 2). The unit-hydrograph data were obtained from hydrographs of runoff resulting from moderate and major storms uniformly distributed over the basin, and synthetic unit hydrographs developed from the correlated data are applicable to storms of similar magnitude and distribution.

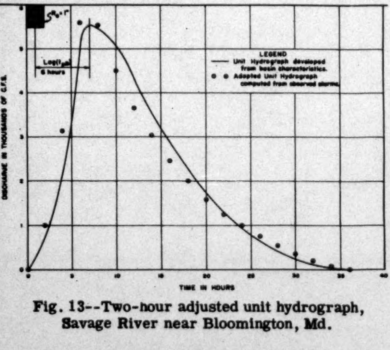


Fig. 13--Two-hour adjusted unit hydrograph, Savage River near Bloomington, Md.

The base width in time, Eq. (19), of hydrographs obtained from observed data may vary from basin to basin, but the equation will include within its limits all or most of the surface runoff for Atlantic slope basins.

The step correlation procedure indicates that other variables that have not been evaluated may be introduced whereby some of the uncertainties in synthetic unit-hydrograph determinations may be eliminated.

A usable synthetic hydrograph can be easily and quickly determined as outlined if V-type drainage areas are treated separately, and which, when applied to moderate or major storms evenly distributed over the basin, will give reasonably accurate hydrographs. However, the use of the nomograph or equations should be limited to the same geographical area used in the correlation unless the synthetic hydrograph is verified by application to actual storms.

Acknowledgments--The correlations presented in this paper were prepared as a part of a comprehensive compilation and study of unit hydrographs being conducted by the Washington District Corps of Engineers for the Office, Chief of Engineers. The opinions and conclusions presented in this paper are those of the authors and are not necessarily endorsed by the Corps of Engineers.

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(Manuscript received August 1, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

STRESS AND STRAIN IN THE OUTER SOLID SHELL OF THE EARTH

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Abstract--With comparatively simple means, expressions are developed for the distribution of the stresses and for both components of the displacements in radial and in tangential direction over extended portions in the thin solid shell of the Earth. The problem is solved for the distribution of stress and elastic strain in this thin, solid, spherical shell under the action of the tide-generating forces of the Moon and Sun by making use of the theory of thin, curved elastic shells. The stresses and the cyclically changing components of the elastic displacements in tangential direction permit drawing some conclusions concerning the permanent distortions to be expected in such a shell by considering the mechanical properties of the rocks under the high temperatures prevailing at the lower levels in the thin, solid shell of the Earth.

Introduction--Anybody who has enjoyed a view from the summit of a high mountain of the nearby and more distant ranges of mountains, as, for example, from the summit of the Gornergrat on the Monte Rosa Group and the commanding pyramid of the Matterhorn in the Alps of the Wallis in Switzerland, has been inspired to reflecting thoughts about the origin of the high mountain chains of the Earth and about the forces of nature that must have built them. The idea that the great mountain chains may have been lifted up from the plains through forces which must have acted deep under their hidden roots must have occurred to many an observer and lover of nature. The mechanically minded may have asked themselves the question, What kind of a force field could have pushed up the rock layers and brought them into their elevated positions during the past history of the Earth? Since, according to geological knowledge, mountain building has been going on in evolutions at intervals in the Earth's history concurrently with simultaneous sinking of the crust in broad belts in other localities, an inescapable conclusion in meditations of this kind is the probability that the upper layers of the Earth must continuously be stressed, pushed together, or stretched out and strained permanently, but not in a uniform manner, over large regions of the Earth's surface. It would seem to an unprejudiced observer that one of the fundamental questions of geology concerning the origin of these manifestations on a grand scale of deeper-rooted distortion phenomena in the outer solid shell of the Earth, is also of mechanical nature, requiring first, a search of the primary force field causing these distortion phenomena; and secondly, an analysis of the equilibrium states of stress and of strain generated by this force field in a comparatively thin shell of the solid rocks supported by the deeper substratum. A study of these states of stress may seem attractive for the purpose of a better understanding of the possible causes why certain broad regions of the Earth's crust were raised and frequently simultaneously pushed together in directions tangential to the surface of the Earth while other parts have been perhaps pulled out or stretched in some locally preferred direction causing the sinking of the ground and thus the formation of the vast troughs known as geosynclines. Space forbids a review in this paper of earlier and more recent attempts which have been made in similar directions which may perhaps be characterzied by saying that many of them were based on the assumption that the whole body of the Earth under the solid shell would behave either as a homogeneous incompressible fluid or as an ideally elastic, homogeneous, compressible solid sphere.[1] These investigations were also not concerned with the problem of determining the conditions in the outermost solid shell itself, which is the subject of this paper. A few recent investigators have adopted oversimplified assumptions to a considerable degree concerning the elastic, bending phenomena in the Earth's crust, with which they attempted to explain some phases of the folding in mountain chains and other phenomena, which are outside the scope of this paper.

Reference is, however, made to WEGENER's [1920; TAYLOR, 1910] continental drift theory, who has proposed a mechanism that could explain, among other facts, the origin of one of the most conspicuous features in the topography of the Earth, namely, the west-eastward and the south-northward trends of the two main systems of high mountain chains on the Earth. After the untimely death of Wegener on his last expedition to Greenland, his ideas were widely discussed, but, as far as this writer is aware, a general agreement about the continental drift theory has not been reached.[2]

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[1] Some also assume a number of discrete, concentric shells of such bodies. The reader is referred in this direction to the chapters in the books by JEFFREYS [1929] and GUTENBERG [1939] dealing with related questions. In a recent paper by TAKEUCHI [1950], the density and the elastic properties are considered to vary with the radial distance of a point in the interior of the Earth from its center.

Part 1

The thought having been expressed that mechanical facts must have caused the considerable shifting around of great parts of the outer shell of rocks of the Earth relative to each other and to its core, one might expect that some of these facts should throw some useful light on one of the fundamental questions of geology about the origins of the vast phenomena of the permanent changes of the levels of the continents discernible in horizontal dislocations and vertical movements and local concentration of the distortional permanent strains in the strata of rocks. The supposition must, however, be noted that these complex phenomena probably may not be attributable to a single mechanical cause, that is, to the action of a unique system of forces, and that probably more than one type of force field must have been at play in them. Investigations of this nature belong to a branch of geomechanics, aiming at a detailed analysis of the states of equilibrium of stress in the outer rock shell of the Earth which, it seems, has been developed comparatively little since the time of Kelvin.

This paper is an attempt in this direction endeavoring to obtain, with comparatively simple means, expressions for the distribution of the stresses and of the elastic and permanent strains over extended regions of the outer crust of the Earth and for the displacements of its points. The assumption is made that they are caused by a single, primary, bodily force field in the rock shell, namely, that of the tide-generating accelerations exerted by the attractions of the Moon and Sun. This field of forces seemed adequate for explaining several general overall trends observed in the distortion of the shell and for discussing the distribution of stress and strain under a unifying point of view if due consideration is given to the mobility of the outer shell as a whole, as well as in parts thereof, in consequence of the rapid decrease of the strength properties of the outer rocks with the increase of the temperature with depth. The following computations can represent only a first attempt in this direction, and it is conjectured that, for example, certain distortion phenomena of a more local nature, involved in such action as the formation of individual ranges of mountains, may have been caused by other types of force fields and influenced by processes which could not be covered in this paper.

The terms "the solid crust" or "the rock shell" hereafter frequently alternatively or synonymously used are meant to designate the outermost group of layers underground within a shell whose thickness h is considered small (50 to 100 km) compared to the mean global radius, $r = 6370$ km. The solid state is referred to mean that within the aforementioned thickness of layers the material has definable moduli of elasticity and rigidity of the same order of magnitude as, or smaller than, those of the rocks known on the surface of the Earth when the stresses act over prolonged periods of time. Within these depths the material is supposed to withstand either elastic or permanent deviatoral strains, or both, in addition to their hydrostatic mean values. We are thus concerned with the stresses and strains and with describing the distribution of the components of displacement in the normal and tangential directions relative to the substratum within a thin, hollow, solid, spherical shell having a thickness h which is quite small compared to its mean radius r_o under a force field, which, as we shall see, turns out to be rotationally symmetric. We may think of a small-scale model of the rock shell by considering a thin hollow metal sphere of one-meter radius having a thickness equal to 1/128 to 1/64 of its radius. Thin, elastic metal shells of spherical form or having a middle surface which is a surface of revolution are common elements in engineering construction; other forms are used in reinforced concrete buildings, in roofs, and other structures. Their problems have extensively been investigated by engineers. Among the pioneers of the theory of elastic curved shells LOVE [1906, chap. 23, 24] and Hans Reissner and Ernst Meissner [FLÜGGE, 1934] might be cited here. In the theory of thin, elastic shells one assumes that the normal and shearing stresses which act in directions parallel to the tangent planes to the middle surface of the shell vary linearly with the distance of a point from this surface. The stresses may be divided into a part uniformly distributed and in the remaining part which is then proportional to the distance from the middle surface. The former, the "membrane stresses," have a normal and a shearing force per unit of width of the section as their resultants, both of which lie in the plane tangent to the middle surface of the shell. The latter, the "bending stresses," balance the bending and twisting moments (per unit of width) which, in addition, are transmitted through the normal sections. These latter components of stress usually have significant values only near the edges of open shells or where one of the principal curvatures of the shell changes locally very rapidly. They must also be considered in stability problems.[3] In a thin shell like the rock shell of the Earth, which is a closed shell of uniform curvature, we may neglect entirely the bending stresses. Thus we shall be concerned in the following only with a determination of the membrane stresses in the spherical rock shell.

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[2] Wegener called attention to the "Polfluchtkraft" (force driving the continents towards the Earth's equator) which was proposed by W. Köppen [see WEGENER, 1920, p. 121] originating in the divergence of the equipotential surfaces in intermediate latitudes, around 45°, due to the ellipticity of the Earth's profile [for details, see GUTENBERG, 1939, p. 200]. This force would drive floating continents in intermediate latitudes on the northern hemisphere in southward direction, but could not explain a westward drift. Wegener mentions in one sentence on page 122 of his book that the westward drift could perhaps be attributed to tidal friction of bodily tides but has not elaborated more specifically on their effects.

It is well known in the theory of thin shells that, if the external forces acting on the shell are given, the distribution of the membrane stresses may be computed from the equilibrium conditions without having recourse to a law of elasticity or plasticity. It is important to note that this case just does not occur for the rock shell of the Earth. The weight of the rocks in the solid outer shell causes stresses in it which must increase with the depths in it. The stress in vertical direction increases proportionally with the depth if the density is assumed as constant. In the following considerations about the state of equilibrium of the rock shell we shall omit the constant field of the gravity forces as well as the stresses which it produces and which can independently be determined should it be required to superpose these stresses on the system of stresses with which we are concerned. If the solid shell, however, is deflected under the action of additional forces, it must displace the heavy magma under it. The slightest deviation from its spherical shape creates pressures in the substratum with which this latter resists against a lateral deflection of the solid shell. The outer shell may be compared with an ice sheet which floats on the surface of a lake. On account of its bending rigidity, an extended ice slab can carry additional concentrated loads such as the weights of skaters because of the buoyancy pressures produced in the water when the ice slab is bent downward. In this respect, the behavior of the rock shell of the Earth may be compared to that of elastic members resting on an elastic foundation behaving in an entirely analogous manner. It will therefore be necessary to consider the elastic distortion of the shell in the presence of these pressures.

Conclusions--Some interesting conclusions to which the following considerations led may be mentioned here.

(1) Although the distribution of stresses generated by the tidal system of bodily forces here referred to was derived from a purely elastic distribution of strains (based on the law of elasticity) which sweeps as a slow wave in continuous semidiurnal revolutions through the Earth's shell, it is obvious since these stresses possess certain mean values in a given locality which persist during the length of geologic epochs that they must generate also permanent strains when the long geologic periods of time are considered. These permanent strains will develop at the lower levels of the rock shell where the prevailing temperatures are the highest under comparatively small values of the differences between the principal stresses. The rocks at these high temperatures deform by creep in a viscous manner. But, as will be shown below, due to the easy deformability of the layers of rock in great depths a redistribution of the stresses must occur through the thickness of the outer shell resulting in much greater values of the stresses in the uppermost, coldest layers of the crust which will carry the major part of the resultant normal forces in the directions of the tangent planes to the middle sphere of the rock shell. If these permanent strains are negative the rock shell contracts, and if they are positive it is extended in circumferential directions. It is supposed that in a certain equatorial belt permanent contractions of moderate magnitude and, in the two polar caps, similar extensions persist over long times, the intensities of which vary with the geographic latitude β as the function $3\cos^2\beta - 2$ over the spherical rock shell in the regions such as are known as the "stable" continental masses or ocean floors. It must be remembered that on the stresses to which we refer always the hydrostatic distribution of pressures has to be superposed which balances the weights of the overlying rocks. Thus, the crust may suffer and can withstand positive permanent strains, that is, extensions in circumferential directions without fracturing (except in the uppermost layers where the ground has little cohesion any way and consists of loose

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[3] The equilibrium of thin members (bars, plates, shells) under compression forces may become unstable; slender bars or thin plates may buckle sidewise. The most common case is that of the straight bar under axial forces known as Euler's buckling case. This may occur also when the stresses under the applied load have reached the yield stress (plastic buckling). Cases of elastic or plastic buckling causing undulations in the packages of rock layers or of local buckling of the Earth's crust may perhaps exist and might perhaps show a certain analogy to what is known in engineering applications as buckling of bars on elastic foundation [HETÉNYI, 1946].

soil). If the stresses beneath the surface reach the yield point of the rocks corresponding to the low temperatures and to the comparatively low confining pressures prevailing in these coldest regions of the rock shell, the affected layers may have yielded plastically in addition.

Generally, thus, we see that under the steady (time independent) parts of the stresses produced by the system of the tide-generating forces, matter within the rock shell is pulled out from the two polar caps and flows extremely slowly toward the equator while a great passive viscous resistance exerted by the masses underlying the shell not accounted for in these computations retards these movements. In contrast to the form of an elongated ellipsoid pointing with its long axis toward the instantaneous position of the Moon in the case of the equilibrium tide on a fluid sphere, the bodily tide forces in a solid spherical shell tend to accumulate matter permanently around the entire length of the Earth's equator. This is due to the permanent character of one part of the distortions (of a viscous nature) through which a bulge is raised along an equatorial belt.

(2) The permanent parts of strain, it must be pointed out, may have not been able to develop as uniformly over long distances of the Earth's circumference and continuously over the long periods of time as was assumed in the preceding statements without causing new effects. The permanently stretched zones of the crust would become too long and the contracted regions too short to fit the practically unchanged dimensions of the substratum. One inevitable consequence of their continuous formation must be that new types of stresses are produced counteracting these strains through which the outer solid shell is fitted back to the given dimensions of the circumferences of the Earth's core. One could easily include some of these effects in the equations determining the stresses and make estimates about them, but this was not carried out in these investigations in consideration of other phenomena not susceptible of a mechanical treatment, such as differences in the nature of rocks influencing their strength properties over portions of the continents and ocean floors and the variation of the thermal gradients; glaciation, erosion, sedimentation, and other surface processes. (The migration of the poles, if accepted, must have had an especially profound influence in this respect.) In view of all these facts, one may conjecture that the permanent mean strains could not develop as purely as the simple visco-elastic equilibrium of the stresses in a spherical shell would permit prediction, which constitutes the subject of this paper. But one mechanical remark may at least be made, in spite of the admittedly great complexity of all these circumstances. There seem sufficient evidences available to assume, wherever for one reason or for several concurrent reasons (an increased value of the temperature gradient) the viscous, permanent contractions of the crust at the deep, lower levels would accumulate locally, that such excessive lateral contractions due to easier creep in the deep-seated, hot strata cannot form unless the upper, cold layers of rocks are displaced along deep thrust faults running more or less parallel to the surface accompanied by local, plastic distortion (folding). Since, because of the rigidity of the upper rock layers, these are unable to adjust their dimensions to the contracted lengths of the strata in great depths, the former must have been displaced as solid packages, pushed over the base on which they rested, or pushed horizontally together and folded to allow for the required space for the continuance of the permanent contraction of their roots at the lower levels of the crust where the general distortion originates through creep. Conversely, where "gaps" would tend to form, because one continental block would be displaced relatively to a neighboring block or parts of the ocean floor would become separated, zones in which the permanent strains are positive and extensions would form, sinking of the surface of the ground in comparatively narrow belts would tend to bridge the gap, the layers of rock would be fractured near the surface, joints, fault troughs, steep shear faults would result, to allow for the necessary space for the continuance of a deep-seated extension.

(3) It seems that the effects of the components of elastic displacement, u, of a point of the solid shell in the tangential directions have seldom, if ever, been considered in more detail. It will be shown below that on account of their cyclic nature the points of the shell describe spiralic curves and are moved back and forth relatively to the substratum by small amounts dependent on the geographic latitude and the position of the Moon. As the solid shell wobbles around in the planes of the horizon, it must drag the more mobile hot masses of magma around with it, and thus shearing strains of a viscous nature are caused in the magma layers under the solid crust. Since the only source available which can supply the energies lost through the viscous drag, apart from the comparatively very small amounts of elastic strain energy which are stored in the solid shell, is the kinetic energy of rotation of the Earth's interior mass, the friction through which the magma resists against the wiggling of the solid crust must retard the rotation of the Earth. This results in a moment around an axis which may not be situated too far perhaps from the axis of the Earth

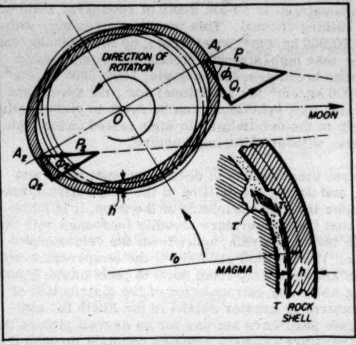

Fig. 1--Forces explaining the westward drift of the solid shell of the Earth

(coinciding with the latter exactly when the Moon stands over the equator of the Earth).[4] According to the law of action and reaction, a resultant couple of equal magnitude but of opposite direction must act on the solid shell itself, that is, in the same direction as the Earth rotates. This seems paradoxical or absurd; the outer, solid, thin rock shell should then act as a huge brake shoe sliding on the substratum and retarding its rotation while floating freely in space together with it, this reaction moment representing an inner moment for the rock shell. A brake shoe would have to be supported by some external point or forces. The classical theory of tidal friction by **DARWIN** [1911] explains the paradox. An external couple of forces exists, which balances the moment of the internal frictional forces originating in the drag of the magma core. This external moment is supplied by the difference between the greater value of the tide-generating attractions exerted by the Moon on the sharper curved portions nearest to the Moon (A_1, Fig. 1) of the rock shell and the smaller value of these forces in the bulge farthest from the Moon (A_2), in consequence of the fact that the solid shell is distorted by the tidal attractions, and has been elastically deformed. The bulged region having the maximum deflections (measured from the original spherical middle surface of the solid shell) which is nearest to the Moon (A_1) receives in its elastically bent condition larger attraction forces from the Moon than were postulated to act in its original, undistorted condition, and the region A_2, farthest from the Moon, receives smaller forces. Thus, the tangential component Q_1 in A_1 of the resultant attraction force P_1 is larger than the component Q_2 in A_2 for two reasons: (1) because $P_1 > P_2$ due to Newton's law of gravitation, and because the angle $\phi_1 < \phi_2$. Therefore, $Q_1 = P_1 \cos \phi_1 > P_2 \cos \phi_2 = Q_2$, and an external moment results tending to turn the solid shell relatively to the core in the opposite direction in which both the shell and the Earth's core revolve. It is noted that the two most sharply bent regions of the solid shell have taken new positions in A_1 and A_2 in Figure 1, in accordance with Darwin's tidal friction theory, that is, that they are no longer shown exactly under the Moon, where they would be found were no frictional forces at play, but they are as shown in Figure 1, in a retarded position, viewed by an observer of the tidal deflection of the shell on the Earth. The moment $(Q_1 - Q_2)\, r_o$ (r_o, mean radius of solid shell) tends to retard the velocity of rotation of the solid shell by some extremely small amount causing a westward drift of it relative to the core. Thus a system of shearing stresses τ, as indicated in the side sketch in Figure 1, may become evident in the layers nearest to the solid shell illustrating the drag created by the totality of all the small, cyclic, relative, tangential displacements u of the solid shell and which may supply the forces causing a westward drift of the continents which were sought by **WEGENER** [1920].

Some remarks concerning strength of solids--Before discussing the shell problem, it seems in order to summarize briefly a few facts concerning the mechanical properties of solids which influenced the following computations. The two important variables on which the strength depends, under the conditions encountered in depths between 0 and 100 km, are the mean pressure p and the temperature θ. They act in many details in opposite direction, the temperature having in the rock layers of the solid earth shell probably a greater influence on changes of some of these properties than has been heretofore assumed.

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[4] The directions of these shearing stresses in the lower bounding sphere of the solid shell must be tangential to the relative orbits of the points, but will deviate in the magma below. This raises an interesting but difficult problem of oceanography related to the action of wind on oceanic currents or on ice packs which is not likely to have an easy solution. The problem is the determination of the magnitude of the resultant moment and of its axis. In this connection, perhaps some striking features of the Earth's topography may be mentioned which may possibly have a relation to the effects of these underground frictional drags, namely, that both southern continents, Africa and South America, have a similar break in their western shore lines (the Gulfs of Guinea and of Peru, respectively). The peninsula of India again has such a break on the west side. Also, the turning of the shore line from south-north into the northeastern direction in North America may have possibly something to do with these underground currents.

Uniaxial compression stresses of the magnitude of 15,000 kg/cm^2 (equal to 214,000 lb/in^2 and equal to the mean (hydrostatic) pressures encountered in 50-km depth of rocks) for solid high-strength metallic materials of construction are nothing unusual. Thin tungsten wires may withstand stresses, even under tension, of 60,000 to 70,000 kg/cm^2; steel wires, up to 20,000 kg/cm^2. Although the natural rocks are very much weaker than high-strength metals (the compression strength under uniaxial compression at which rocks break are, for example, 1000-3000 kg/cm^2 for granite, 1000-6000 kg/cm^2 for basalt, 150-3000 kg/cm^2 for sandstone) and rock specimens under such conditions behave like brittle materials, rock specimens can be made to yield continuously by permanent, plastic deformation similarly to the ductile metals when tested under high confining lateral pressures at normal temperature, without suffering fracture.

Table 1--Variation of temperature and pressure with depth

Depth y	Temperature θ	Pressure p
km	°C	atm
0.1	3	30
1	30	300
10	300	3000
50	1500	15,000
100	3000	30,000

From observations in deep petroleum wells and mines, and during the drilling of the long railroad tunnels under the high mountains of the Alps, it is established that the temperature steadily increases with the depth in the rock strata, apart from the volcanic phenomena. The vertical gradient of the temperature with depth is known to vary from point to point on the Earth. Leaving aside any extrapolation of the distribution of the temperatures to greater depths in the Earth for simplicity, we shall here assume for an overall picture that the temperature gradient remains constant through the first 50 to 100 km depth of the Earth's crust. Assuming its value to be 3°C per 100-m depth, accepted by geophysicists, and an average specific weight of the rocks of 3000 kg/m^3, this would lead to the temperatures and mean pressures at the various depths shown in Table 1. Most of the known rocks of the Earth would, under atmospheric pressure, be molten around 1500°C.[5] Allowing for an increase of their melting temperature under high all-sided pressure, if they would not be molten at the depths of 100 km corresponding to 3000° C, certainly their parameters measuring their deformability under continuously (over long times) active small differences in the principal stresses must have changed to values characterizing a greater degree of mobility. This may refer also to the substratum just under the solid shell. These remarks are based on experimental evidences collected in the engineering laboratories during the past 20 to 30 years in tests done on metals. P. Ludwig has defined the term of the "homologous temperature" as the ratio T/T_m which facilitates a comparison of the strength properties of different metals or solids. T denotes the absolute test temperature, and T_m the absolute melting temperature. Metals seem to behave in analogous manner at the same homologous temperature. Alloyed steels lose much of their strength above $T/T_m = 1/2$; around 700-800°C they creep with quite appreciable rates of strain under a given stress held at a constant value which is a small fraction of the stress they would carry indefinitely without permanent deformation at room temperature and without breaking.

The behavior of metals as to the dependence of the rates of slow creep with which they deform on the stress at elevated temperatures may be illustrated in one or two examples.[6] From long-time creep tests made on a "stainless" steel (containing 18 pct chromium, 8 pct nickel, the rest iron) the creep rates, $w = d\epsilon/dt$ at 815°C under the given constant tensile stresses, were as shown in Table 2. These test values fitted well the hyperbolic sine speed law [McVETTY and NADAI, 1943, p. 735; PRANDTL, 1928, p. 85] relating the minimum creep rate to the stress

$$w = w_0 \sinh(\sigma/\sigma_0) \ldots\ldots\ldots\ldots (1)$$

where w_0 and σ_0 are material constants. The values of these latter were determined equal to

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[5] According to JOLY [1930, p. 41], basalts melt around 1170°C under atmospheric pressure. Concerning the increase of the melting temperatures of rocks with pressure and related questions having physico-geologic aspects, the reader must be referred to Joly and other special papers in an endeavor to limit the present considerations to mechanical questions. From these has to be excluded also the one in regard to a precise characterization of the state of aggregation of the substratum under the solid shell, whether it be considered as fluid or not. This seemed irrelevant in view of the greatly increased mobility with which the layers in these levels would yield and be able to deform under small differences of stress acting continuously due to the high temperatures prevailing at these depths.

[6] A 0.35 pct carbon steel increased its length during a testing time of 10,000 hours (= 1.14 years) while it was held loaded continuously under a tensile stress of 7500 lb/in^2 or 525 kg/cm^2 at a temperature of 454°C by approximately 5.7 per mil ($\epsilon = 0.0057$); copper at 200°C stretched approximately 0.1 pct during 1000 hours under a stress of 2500 lb/in^2 or 176 kg/cm^2. It would have elongated by one per cent in approximately 10,000 hours.

Table 2--Long-time minimum creep rates for stainless steel

Minimum creep rates reached $w = d\epsilon/dt$	Stress	
1/hour	lb/in^2	kg/cm^2
1.20×10^{-7}	1500	105.5
2.50×10^{-7}	2000	140.8
1.28×10^{-6}	3000	211
6.44×10^{-6}	4000	281

$$w_o = 2.0 \times 10^{-8} \text{ (1/hour)} \qquad \sigma_o = 620 \text{ lb/in}^2 = 43.6 \text{ kg/cm}^2$$

It is seen from (1) that while at the high stresses quoted in Table 2 the creep rate increases much faster than the stress (a characteristic behavior of the flow of ductile metals), the stress at sufficiently high values may be assumed to increase as the logarithm of the creep rate (since $\sinh(\sigma/\sigma_o) = e^{\sigma/\sigma_o}/2$ when σ/σ_o is large)

$$\sigma = \sigma_o \log_e (2w/w_o) \text{. . . . . (2)}$$

At comparatively small stresses the speed law converges toward a linear function

$$w = w_o \sigma/\sigma_o \text{ . . . . (3)}$$

that is, it may be stated that metals at elevated temperatures, and under sufficiently small stresses, flow as viscous materials. Viscous behavior for the stainless steel would be expected in a range of stress $- \sigma_o/10 < \sigma < \sigma_o/10$. This stainless steel would stretch at 815°C under the tensile stress $\sigma = \sigma_o/10 = 4.36$ kg/cm^2 which must be considered as a very small stress in engineering. The resulting strain would be: in one hour, $\epsilon = 2 \times 10^{-7}$ of one per cent; in one year, $\epsilon = 1.75 \times 10^{-3}$ of one per cent; and in 1000 years, $\epsilon = 1.75$ per cent. A time of 10,000 years in geologic phenomena would scarcely be considered as a long period, but an extension or compression of 17.5 pct which would accompany the creep of a stainless steel in such a long period around 800°C under four kg/cm^2 stress would certainly have to be considered as a very large one over large parts of the circumference of the Earth. Differential stresses and the strains of the order just mentioned seem yet much too great to be postulated at the lower levels of the solid shell, but the preceding example may perhaps illustrate the importance of the distortion of rocks due to creep at moderate depths, increasing even more at depths of 50 to 100 km where temperatures of 1500 to 3000°C exist. It should be remembered that rocks, generally speaking, are much weaker in their strength properties, break at lower stresses, and flow and creep easier under confining pressures than the stronger metals.

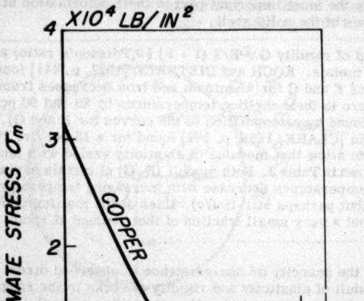

Fig. 2--Ultimate stress (stress at load maximum) of pure copper and aluminum in tensile tests made under constant strain rates (see text)

The effect of the temperature θ on the flow stress σ which is required to deform a specimen by straining it permanently with the same rate of strain, $d\epsilon/dt$, may be judged from Figure 2. In this figure a large number of tensile tests were represented by plotting the ultimate stress σ_m (the stress referred to the original cross-sectional area under which the load in tensile tests reaches its maximum value) as ordinates above the testing temperatures θ as abscissas for copper and aluminum. All the tensile tests represented in the upper curve for copper were made under the same strain rate $d\epsilon/dt = 7.7 \times 10^{-4}$ per sec; those in the lower curve, for aluminum, under $d\epsilon/dt = 8.5 \times 10^{-4}$ per sec. These curves bring out the great effect of the temperature on the force needed to deform a metal; the ordinates in both curves rapidly decrease to the value zero as the melting point is approached.

Within the linear ("viscous") range of the speed law relating the permanent rate of strain $w = d\epsilon/dt = \dot{\epsilon}$ to the stress σ, the ratio σ_0/w_0 in (3) expresses 3μ where μ is the coefficient of viscosity. Eq. (3) may also be written

$$\sigma = 3\mu\dot{\epsilon} \quad \text{.........................} \quad (4)$$

μ is known to increase with the pressure p and to decrease with the temperature, the latter effect perhaps being the more important at shallow depths in the solid shell. Unfortunately again very little is known from direct experiments about the values of μ for rocks at high temperatures. One can only conjecture the viscosity values at the lower levels of the solid shell.[7]

This lets us expect that rocks must deform continuously and creep wherever in the depths the shearing stresses and temperatures are sufficiently great. Assuming a homologous temperature equal to one-half (well below the one corresponding to the example for the stainless steel), and a melting temperature $T_m = 1600°$ for rocks, the absolute actual temperature $T = T_m/2 = 800°$ or $\theta = 527°C$ would be reached at a depth of 17.5 km where the pressure would be of the order of 5270 atm. It is scarcely probable that rocks would be deformed by plastic flow under confining pressures of this latter magnitude by a difference of pressures at these depths. This would require a pressure difference of a few thousand atmospheres which seems not likely in the Earth's crust. But the rocks would creep at the homologous temperature $T/T_m = 1/2$ due to their otherwise lower strength and cohesive properties compared to strong metals under quite small differences between the principal stresses when also the long times of geologic action are given due consideration. Thus, distortion by creep must play the most important part in their deformation at moderate depths and certainly at the lower levels of the solid shell.

The values of the moduli of elasticity E and of rigidity $G = E/2\ (1 + \nu)$ (ν, Poisson's ratio) are known to decrease with rising temperature for metals. KOCH and DIETERLE [1922, p. 441] found that the value of E and G for aluminum and iron decreases from the absolute zero to their melting temperatures by 86 and 90 pct, respectively (found by extrapolation of the curves for E and G). M. Fleischmann [CLARK, 1950, p. 171] found for a 16 pct Cr, 25 pct Ni, 6 pct Mo allow that modulus of elasticity varied with temperature as shown in Table 3. Both moduli (E, G) of metals having high melting temperatures decrease with increasing temperature toward small (but perhaps still finite) values at the melting point. These can be but a very small fraction of their values at room temperature.

Table 3--Variation of E with temperature (after M. Fleischmann)

Temperature θ	E
°C	10^6 lb/in^2
20	28.2
650	17.9
760	11.3
815	10.0

In view of the scarcity or non-existence of observed direct data on the moduli of elasticity and rigidity of rocks in the range of temperatures approaching their melting or softening points (other than the indirect conclusions reached as to their probable values in the Earth's crust derived from the velocity of propagation of the various types of earthquake waves), nothing remains than to assume tentatively that the values of the moduli of rocks decrease similarly to those for metals with increasing temperatures within the solid shell. In the following, an average value of E = 65,000 kg/cm^2 for the entire rock shell, has tentatively been assumed. This is equal to one-half of the value E = 130,000 kg/cm^2 which has been observed in elasticity tests on rocks at room temperature.[8]

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[7] Experiments on creep of metals have indicated that the creep rates are extremely sensitive to the slightest changes of the test temperature; they appear under a constant stress perhaps to increase as the terms in a geometric series if the temperatures increase according to an arithmetic series.

[8] Since the exact measurements of the elasticity of certain non-metallic solids by BACH and BAUMANN [1924] it has been well known that at room temperature certain rocks, cast iron, concrete, and other crystalline solids do not follow Hooke's law when tested in uniaxial tension or compression. Such materials show also a considerable elastic hysteresis; their inner structure is not as consolidated as that of the ductile metals. In many cases they have a porous structure. For many of them the modulus of elasticity decreases with increasing stress. The reference book of HÜTTE [1931, p. 699] gives: for granite, E = 129,000 kg/cm^2; for sandstone, 230,000 kg/cm^2;

As far as Poisson's ratio ν is concerned, we may be on safer ground by assuming it to be around 0.25 in the consolidated rocks near the surface, and 0.50 at the bottom of the rock shell where a near fluid state is reached. Therefore, in the middle surface of the rock shell the arithmetic mean of 0.375 seems a probable value [see also JEFFREYS, 1935].

Part 2

The following symbols are used in Part 2.

ϵ	Strain, unit elongation; ϵ_1, ϵ_2, ϵ_3 principal strains
σ	Normal stress; σ_1, σ_2, σ_3 principal stresses
τ	Shearing stress
p	Pressure
$\dot{\epsilon} = d\epsilon/dt$ (= w)	Rate of strain
u	Tangential displacement of shell (positive in direction in which angle α' increases, see Fig. 3)
v	Normal displacement, deflection of shell (positive in radially outward direction)
α, β	Geographic longitude and latitude
α'	Angle M_1OP of point P (α, β) on Earth, measured from point M_1 (see Fig. 3)
δ	Angle of inclination of ecliptic ($\delta = 23° 30'$)
r	Variable radius; r_1, r_0, r_2 outer, mean, inner radius of solid shell
h	Thickness of shell
y	Depth ($y = r_1 - r$)
E, G	Moduli of elasticity and rigidity
ν	Poisson's ratio
μ	Coefficient of viscosity
θ	Temperature, °C
ρ	Density
g	Acceleration of gravity
γ	Body force per unit of volume or a specific weight, kg/cm^3
ϕ	Angle of stress ρ_1 with a parallel of the Earth

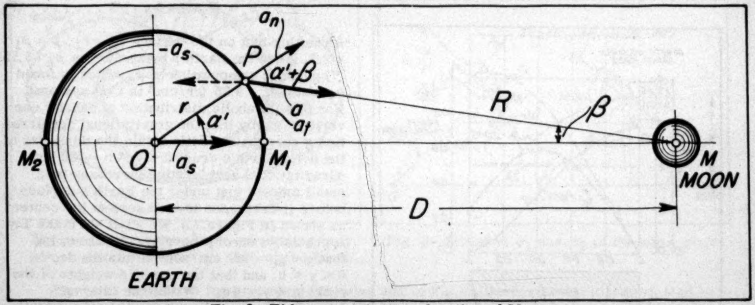

Fig. 3--Tide-generating acceleration of Moon

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for concrete, in compression E = 300,000 - 350,000 kg/cm^2. The values derived from seismic observations for rocks seem much higher; for example, GUTENBERG [1939, p. 347] gives for the granitic layer in continents the value for the modulus of rigidity $G = 3 \times 10^{11}$ dynes/cm^2 ~ 300,000 kg/cm^2. Assuming Poisson's ratio to $\nu = 0.25$, this would correspond to a modulus of elasticity for the upper rock layers equal to E = 750,000 kg/cm^2 which is 5.8 times greater than the value observed by Bach, quoted above. Such high values for E have only been observed for one artificial material, namely, fired porcelain, whose strength in compression is known to be as high as 7000 kg/cm^2 and modulus of elasticity 7 to 8×10^5 kg/cm^2 [see HÜTTE, p. 852]. Presumably the moduli of elasticity and rigidity of rocks computed from the velocity of propagation of earthquakes (shear or dilatational elastic waves) are considerably higher than the ones which would have been observed under static conditions, could these same rocks have been tested in a laboratory. In consideration of the many uncertainties involved in the computation of elasticity moduli from earthquake wave propagation and of the high temperatures prevailing at the depths of 50 to 100 km of the crust, our figure of E = 65,000 kg/cm^2 above in the text appears rather on the conservative side.

In order to determine the state of stress in the solid shell of the Earth, we need to know the body forces with which the Moon attracts the elements of mass of this shell. The solid crust is assumed as a hollow, spherical shell of material having an outer radius $r_1 = 6370$ km, an inner radius $r_2 = 6270$ km, and a mean radius $r_0 = 6320$ km. The thickness of it, $h = r_1 - r_2 = 100$ km, is assumed constant. An element of mass in a point P, having the polar coordinates r, α', in the interior of the Earth, if it were free to move, is accelerated relatively to the Earth by the attraction force of the Moon with an acceleration $-\boldsymbol{a}_s' + \boldsymbol{a}$, $\boldsymbol{a}_s'$ denoting the acceleration of the center of the Earth, and $\boldsymbol{a}$ that of the mass element at r, α' toward the Moon. The angles α' are measured from a polar axis coinciding with the line connecting the centers of the Earth O and Moon M (see Fig. 3). This axis intersects the surface of the Earth in two points M_1 and M_2, the Moon stands in the zenith above M_1. As in the theory of the equilibrium tides, we shall resolve the relative acceleration $-\boldsymbol{a}_s' + \boldsymbol{a}$ in point P into a normal component a_n in the direction OP and a tangential component a_t. According to this theory they are expressed by

$$\left.\begin{aligned} a_n &= (g_e/2)\,(\rho_m\, r_m^3/\rho_e\, D^3)\,(r/r_e)\,(1 + 3\cos 2\alpha') \\ a_t &= (3\,g_e/2)\,(\rho_m\, r_m^3/\rho_e\, D^3)\,(r/r_e)\sin 2\alpha' \end{aligned}\right\} \ldots\ldots\ldots\ldots \quad (5)$$

ρ_m and ρ_e denote the mean densities of the Moon and Earth, r_m and r_e the radia of the Moon and Earth, D the distance between their centers, and g_e the gravity acceleration on the surface (at $r = r_e = r_1 = 6370$ km) of the Earth ($g_e = 981$ cm/sec^2).

Denote the density of the rocks on the surface ($r = r_e = r_1$) of the Earth by ρ_1, at the bottom of the solid shell ($r = r_2$) by ρ_2, and the mean density of the rocks within the solid shell by ρ_0. Since the density increases with the depth $y = r_1 - r$, some assumption must be made about its distribution within the interior of the Earth. For the sake of convenience we have assumed that ρ increases toward the center of the Earth according to a parabolic distribution

$$\rho = \rho_c - (\rho_c - \rho_1)\,(r/r_e)^2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where ρ_c denotes the density in the center ($r = 0$) of the Earth. The mean density of the Earth ρ_e corresponding to (6) is found to be equal to

$$\rho_e = (2\rho_c + 3\rho_1)/5 \ldots\ldots \quad (7)$$

Assuming that on the surface $r_e = r_1$, $\rho = \rho_1 = 2.7$, since the Earth's mean density $\rho_e = 5.52$, (7) permits computation of ρ_c which is found equal to $\rho_c = 9.75$ (gr/cm^3 in CGS system). For this parabolic distribution of density one verifies easily that the gravitational acceleration g changes but little within the thickness of the solid Earth's crust $0 < y < h = 100$ km. (Gravitational acceleration increases by a small amount just under the Earth's surface before it decreases toward zero at the center as shown in Figure 4.) We shall not make any appreciable error, therefore, by assuming that $g = g_e = 981$ cm/sec^2 within the depths $0 < y < h$, and that the specific weights of the rocks are $\gamma = \rho g_e$. Within the interval $0 \leqq y \leqq h$ for ρ, a linear function of y may be substituted for the quadratic one in (6). This then determines the specific weights of rocks shown in Table 4. In the following computations

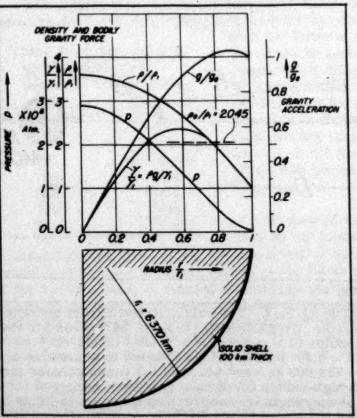

Fig. 4--Variation of density ρ, of gravity acceleration g, of gravity force per unit of volume $\gamma = \rho g$, and of pressure p in the interior of the Earth, assuming a parabolic distribution of density ρ

Table 4--Specific weights of rocks

Depth y	r	Specific weight	
		Value	Symbol
km		kg/cm^3 × 10^3	
0	r_1	2.70	γ_1
50	r_0	2.82	γ
100	r_2	2.94	γ_2

we prefer to adhere to the technical system of units in which the preceding specific weights were expressed in kg/cm^3 (kg meaning kilogram of force). Since the attraction force per unit of volume is the product of the acceleration with the density, the body forces through which the thin solid rock shell is loaded will be given by taking in (5) the radius r equal to the radius of the middle sphere $r_o = (r_1 + r_2)/2$ and after multiplying the components of acceleration a_n and a_t by the mean density ρ_o of the solid shell. Using the symbol γ to denote a body force or specific weight, let the constant term in (5) be denoted by the abbreviating symbol γ_o

$$\gamma_o = 2\,\rho_o\, g_e\, (r_o\, \rho_m\, r_m^3 / r_1\, \rho_e\, D^3) \qquad (8)$$

also $\gamma_n = a_n\,\rho_o$, $\gamma_t = a_t\,\rho_o$. Hence the body forces through which the hollow, spherical rock shell is loaded in the normal and tangential directions, are

$$\left.\begin{aligned} \gamma_n &= \gamma_o\ (1 + 3\cos 2\,\alpha')/4 \\ \gamma_t &= (3\gamma_o\ \sin 2\,\alpha')/4 \end{aligned}\right\} \qquad (9)$$

It will be convenient also to express the components of body force γ_h and γ_v in the directions parallel to the line connecting the centers of the Earth and Moon (OM in Fig. 3) and perpendicular to it (taking γ_h and $\gamma_v > 0$ when they have the directions indicated in Fig. 5). We have for

$$\gamma_h = \gamma_o \cos\alpha' \qquad \gamma_v = -\,(\gamma_o \sin\alpha')/2 \qquad (10)$$

Figure 5 reproduces their distribution in the solid rock shell.

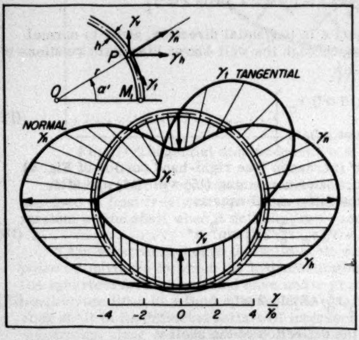

Fig. 5--Components of tide-generating body forces in the rock shell of the Earth

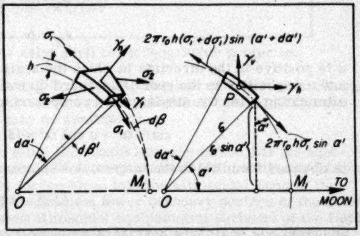

Fig. 6--Stresses σ_1 and σ_2 in the solid shell

As can be seen from the preceding formulas and in Fig. 5, the horizontal forces γ_h tend to elongate the hollow sphere in the direction of the Moon, and the vertical forces γ_v tend to flatten it along a circular belt by reinforcing the action of the former forces and tending to reduce the diameters of the shell in the directions perpendicular to the line connecting the Earth with the Moon. It is an elementary problem in the theory of shells to determine the membrane stresses in this spherical shell and the tidal elastic distortion it must suffer under this system of the force components γ_n, γ_t (or γ_h, γ_v). For this purpose consider an infinitesimal element in the hollow shell as reproduced in two views in Figure 6 and express the conditions of equilibrium of the forces including the stresses. In the shell, normal stresses σ_1 and σ_2 must act, the former ones along the great circles ("meridians") which converge in the two "poles" M_1 and M_2 in which the Moon stands in the zenith and nadir (see Fig. 3), and the latter ones along the "parallel circles" corresponding to this system of the circles α' = const. The values σ_1 and σ_2 are principal stresses and the two preceding families of circles represent the trajectories of principal stress in the solid shell. The two conditions of equilibrium may conveniently be expressed for the force components in normal and axial direction (α' = 0) and are

$$\left.\begin{aligned}\sigma_1 + \sigma_2 &= \gamma_n r_o \\ d(\sigma_1 \sin^2 \alpha')/d\alpha' &= \gamma_h r_o \sin \alpha'\end{aligned}\right\} \ldots\ldots\ldots\ldots \tag{11}$$

or after substituting here the values of γ_n and γ_h taken from (9) and (10)

$$\left.\begin{aligned}\sigma_1 + \sigma_2 &= \gamma_o r_o (1 + 3 \cos 2\alpha')/4 \\ d(\sigma_1 \sin^2 \alpha')/d\alpha' &= (\gamma_o r_o \sin 2\alpha')/2\end{aligned}\right\} \ldots\ldots\ldots\ldots \tag{12}$$

from which

$$\left.\begin{aligned}\sigma_1 &= \gamma_o r_o/2 = \text{const} \\ \sigma_2 &= \gamma_o r_o (3 \cos 2\alpha' - 1)/4\end{aligned}\right\} \ldots\ldots\ldots\ldots \tag{13}$$

indicating that the shell is stressed by uniform tensile stresses along the "meridians" and that two caps of the shell around the "poles" M_1, M_2, ($\alpha' = 0$, $\alpha' = \pi$) are stressed also along the parallel circles by tensile stresses, whereas the belt-shaped zone between these polar regions carries compression stress σ_2 along the parallel circles.[9]

The principal elastic strains ϵ_1 and ϵ_2 corresponding to these stresses are found equal to

$$\left.\begin{aligned}\epsilon_1 &= (\sigma_1 - \nu\sigma_2)/E = (\gamma_o r_o/4E)(2 + \nu - 3\nu \cos 2\alpha') \\ \epsilon_2 &= (\sigma_2 - \nu\sigma_1)/E = (\gamma_o r_o/4E)(-1 - 2\nu + 3 \cos 2\alpha')\end{aligned}\right\} \ldots\ldots\ldots\ldots \tag{14}$$

They are related to the components of displacement u in tangential direction, and v in normal direction, of a point P at the middle surface $r = r_o$ through the well-known kinematic relations of the theory of spherical shells

$$\left.\begin{aligned}\epsilon_1 &= (v + du/d\alpha')/r_o \\ \epsilon_2 &= (v + u \cot \alpha')/r_o\end{aligned}\right\} \ldots\ldots\ldots\ldots \tag{15}$$

u is positive in the direction in which the angle α' increases (see right-hand portion of Fig. 6) and v is positive in the radially outward direction. After combining (15) with (14) and after eliminating v for the displacement component u the differential equation

$$du/d\alpha' - u \cot \alpha' = [3(1 + \nu)\gamma_o r_o^2/2E] \sin^2 \alpha' \ldots\ldots\ldots\ldots \tag{16}$$

is obtained from which the tangential displacement u

$$u = -[3(1 + \nu)\gamma_o r_o^2/4E] \sin 2\alpha' \ldots\ldots\ldots\ldots \tag{17}$$

and from one of (15) the normal displacement or the deflection of the shell v

$$v = [(2 + \nu)\gamma_o r_o^2/4E](1 + 3 \cos 2\alpha') \ldots\ldots\ldots\ldots \tag{18}$$

is found.

Whereas u vanishes in the points M_1 and M_2 and on the great circle $\alpha' = \pi/2$, and has the greatest values along the circles $\alpha' = 45°$ and $\alpha' = 135° = \text{const}$, the deflection v has its greatest positive value in M_1 and M_2 and smallest (negative) value in the great circle $\alpha' = 90°/\text{const}$. The distribution of the elastic tangential displacement components u is shown in a stereographic projection for the northern hemisphere in Figure 7. According to (18), the sphere is distorted into an elongated ellipsoid of revolution.[10]

---

[9] The stress σ_2 for $\alpha' = \pi/2$ equals $\sigma_2 = -\gamma_o r_o$ which represents the absolutely maximum stress in the shell which is a compression stress. Its magnitude was estimated to $\sigma_2 = 0.1972$ kg/cm^2. At the apex or bulge of the shell, $\alpha' = 0$, $\sigma_1 = \sigma_2 = 0.0986$ kg/cm^2.

[10] The greatest positive elevation of the shell above the middle sphere $r = r_o$ in the bulged-out region ($\alpha' = 0$) was estimated equal to $v_{max} = (2 + \nu)\gamma_o r_o^2/E = 45.5$ m, the greatest negative deflection (for $\alpha' = \pi/2$) is one-half of this value $v_{min} = -22.75$ m, and the absolutely greatest tangential displacement (for $\alpha' = \pi/4$) equal to $u_{max} = -3(1 + \nu)\gamma_o r_o^2/4E = -19.8$ m. Whereas the

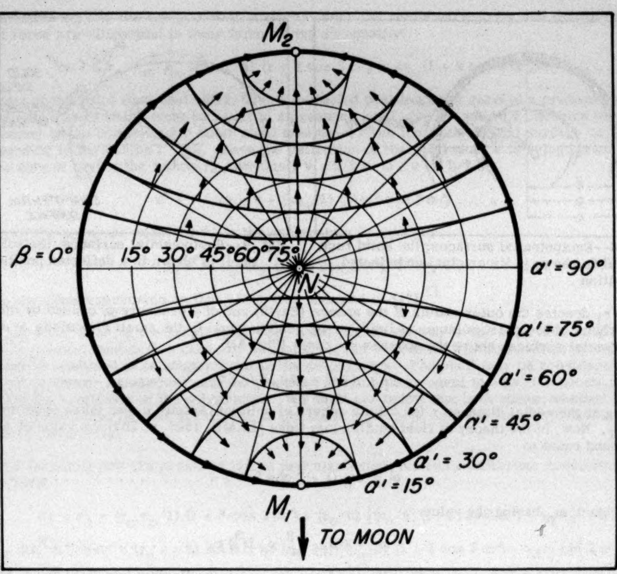

Fig. 7--Tangential displacements u in the solid shell in stereographic projection

In the preceding equations no external supporting forces were considered (the rock shell was assumed to float freely in space). In order to take account of the pressures exerted by the substratum on the shell when it deflects, two cases may be considered.

If the substratum below the solid shell were fluid, this would have the effect that the equipotential surfaces in which the hydrostatic pressure p has constant values will be distorted from the spherical form they would have under gravity forces alone into the ellipsoidal form of the equilibrium tides of a fluid sphere. The originally spherical lower boundary surface of the solid rock shell in its deflected form will intersect these ellipsoidal equipotential surfaces of the fluid substratum along certain curves. Along each of these curves the pressure p will have constant values which will increase toward the interior of the Earth. In a fluid sphere of uniform density ρ the total pressure in a spherical surface r = const will be the sum of the hydrostatic pressures p_g due to gravity

$$p_g = \rho g_e (r_1^2 - r^2)/2 r_1 \quad \text{. . . . .} \quad (19)$$

---

stresses σ_1 and σ_2 caused by the tidal forces have comparatively small values of the order mentioned in footnote 9, the maximum elastic displacements u and v are comparatively great on account of the huge dimensions of the rock shell having a mean radius of r_0 = 6320 km. Note that the deflection function v expressed by (18) has the remarkable property that the integral $\int vdA = 0$ (dA denotes an element of area of the middle sphere $r = r_0$), that is, that the total displaced volume under the deflected shell vanishes (just as it would under an equilibrium tide of a fluid sphere). It is perhaps noteworthy also that the elastic distortion of the solid shell seems not excessive, in spite of the low mean value of the postulated modulus of elasticity, E = 65,000 kg/cm^2. That such large deflections could exist is certainly out of any question, because they would produce tremendous floods through such large movements of the solid ground, that is, enormous tidal waves in the ocean.

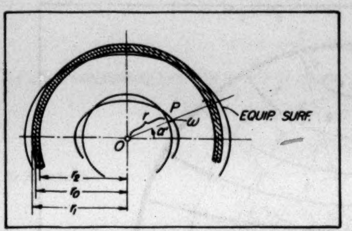

Fig. 8--Equipotential surfaces; the solid rock shell is shown in its original undeflected position

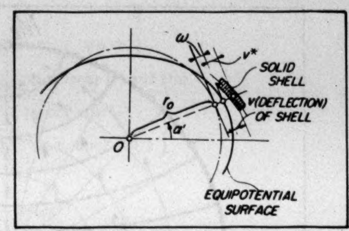

Fig. 9--Equipotential surfaces; the solid rock shell is shown in a deflected position

where r_1 denotes the outer radius of the sphere (Earth) and of a pressure p_f caused by the head (the weights of the short columns of fluid having heights equal to the small elevations ω of the equipotential surfaces above the spheres $r = \text{const}$ (Fig. 8))

$$p_f = \rho g \omega = \rho g_e r \omega / r_1 \quad \text{(20)}$$

where g at the radial distance r for a fluid sphere of uniform density ρ was taken equal to $g = g_e r/r_1$. Now, in the theory of fluid equilibrium tides [LAMB, 1945, p. 267] the value of ω has been found equal to

$$\omega = \omega_o (1 + 3 \cos 2\alpha')/4 \quad \text{(21)}$$

the constant ω_o having the value

$$\omega_o = (\rho_m r_m^3 / \rho_e D^3) r \quad \text{(22)}$$

Since, as previously stated, the effects of gravity under the Earth's own field are to be excluded in these computations of stresses in the rock shell, we need not include the gravity pressures p_g supporting the weights of the rocks above the sphere $r = r_2 = \text{const}$. In this latter lower boundary surface of the solid shell there will be a density ρ equal to ρ_2 and a pressure p equal to

$$p_f = \rho_2 g_e r_2 \omega / r_1 \quad \text{(23)}$$

which will act as a uniform distributed load not dependent on the deflections v of the shell in normal directions.

After substituting here ω and ω_o from (21) and (22), taking $r = r_2$, this pressure may be expressed by

$$p = p_o (1 + 3 \cos 2\alpha')/4 \qquad p_o = \rho_2 g_e r_2 \omega_o / r_1 \quad \text{(24)}$$

But $\rho_2 g_e r_2/r_1$ expresses the specific weight of the substratum material just below the solid shell ($r = r_2$) which has been denoted by the symbol γ_2. Thus we have the values of the constants p_o and ω_o in (24) in regard to the solid shell

$$p_o = \gamma_2 \omega_o \qquad \omega_o = (\rho_m r_m^3 / \rho_e D^3) r_2 \quad \text{(25)}$$

As soon as the solid shell deflects, it receives additional pressures equal to

$$-\gamma_2 v \quad \text{(26)}$$

which are proportional to the normal deflection v of the solid shell (measured from the middle sphere $r = r_o = \text{const}$, v being positive in the direction of increasing radii r). The total pressure on the concave side of the solid shell exerted by the substratum will therefore be the algebraic sum of the parts expressed by (24) and (26)

$$p - \gamma_2 v = (p_o/4)(1 + 3 \cos 2\alpha') - \gamma_2 v \quad \text{(27)}$$

Remembering that the equipotential surfaces under the combined gravity and tide-generating fields of force are ellipsoidal in their form having an equation

$$\omega = (\rho_m r_m^3/\rho_e D^3)(r/4)(1 + 3\cos 2\alpha') = \omega_o (1 + 3\cos 2\alpha')/4 \ldots\ldots\ (28)$$

an element of the solid rock shell (Fig. 9) in a deflected position must receive a pressure at $r = r_2$ from the substratum which must be equal to a pressure head $-\gamma_2 v^*$, where v^* denotes the normal displacement of the bottom of the outer shell measured from the equipotential surface ω, Eq. (28), corresponding to the radius $r = r_2$. Since the deflection of the solid shell v is being measured from the sphere having the radius r_o, obviously $v + h/2 = \omega + v^* + h/2$ or

$$v^* = v - \omega = v - (\omega_o/4)(1 + 3\cos 2\alpha') \ldots\ldots\ (29)$$

Therefore, the pressure received from the substratum is equal to

$$-\gamma_2 v^* = \gamma_2 \omega_o (1 + 3\cos 2\alpha')/4 - \gamma_2 v \ldots\ldots\ (30)$$

which is the same expression as that which was obtained in (27).[11]

On the other hand, if the substratum were solid, the first term on the right side of (27) or (30) will not be present, and in this case any deflection of the rock shell will be resisted by a pressure which may be assumed to be proportional to the deflection v. The shell may be considered "as resting on an elastic foundation" and the pressure may be taken equal to - kv, the constant k measuring the resiliance of the substratum. We shall see below that both cases, whether the substratum is considered fluid or solid, lead to the same equations, only the constants having to be interpreted differently.

After including now the pressure terms just mentioned, the two equilibrium conditions (12) take the form

$$\sigma_1 + \sigma_2 = (\gamma_o r_o/4)(1 + 3\cos 2\alpha') + (r_o/h)[(p_o/4)(1 + 3\cos 2\alpha') - \gamma_2 v] \ldots\ldots\ (31)$$

$$d(\sigma_1 \sin^2 \alpha')/d\alpha' = (\gamma_o r_o/2)\sin 2\alpha' + (r_o/2h)[(p_o/4)(1 + 3\cos 2\alpha') - \gamma_2 v]\sin 2\alpha' \ldots\ (32)$$

In order to reduce unnecessary writing, let us introduce three dimensionless constants defined as follows

$$k_o = \gamma_o r_o^2/Eh \qquad k_1 = p_o r_o/Eh \qquad k = \gamma_2 r_o^2/Eh \ldots\ldots\ (33)$$

The constant k_o measures the intensity of the tide-distorting body forces within the solid rock shell, k_1 the invariable part of the lateral pressures originating in a fluid substratum (also due to tide actions), and k measures the stiffness of the substratum or its resistance against the deflection v of the solid shell.

With the constants defined in (33), the two equilibrium conditions in the normal and axial direction of the solid shell take the form

$$\sigma_1 + \sigma_2 = (E/r_o)[(k_o h + k_1 r_o)(1 + 3\cos 2\alpha')/4 - kv] \ldots\ldots\ (34)$$

$$d(\sigma_1 \sin^2 \alpha')/d\alpha' = (E/2r_o)[k_o h \sin 2\alpha' + (k_1 r_o (1 + 3\cos 2\alpha')/4 - kv)\sin 2\alpha'] \ldots\ (35)$$

These have to be combined with the two relations

$$\sigma_1 - \sigma_2 = [E/(1 + \nu) r_o](du/d\alpha' - u \cot \alpha') \ldots\ldots\ (36)$$

$$v = (r_o/E)(\sigma_2 - \nu \sigma_1) - u \cot \alpha' \ldots\ldots\ (37)$$

---

[11] In the numerical values of the quantities given in footnote 12 the pressure p_o for the sake of convenience was computed from the expression $p_o = \rho_2 g_e r_2 \omega_o/r_1$ (Eq. 24) after identifying in it the product $\rho_2 g_e$ as γ_2 (conforming with the definition of the specific weight $\gamma_2 = \rho_2 g_e$ originally defined earlier) instead of letting, in accordance with the equilibrium theory of a tide of a sphere of a fluid of uniform density $\rho = \rho_2$, when $r = r_2$, correctly $\gamma_2 = \rho_2 g_e r_2/r_1$ as stated in the text above. This amounts to an error in the value of the constant p_o equal to $(1 - r_2/r_1) p_o$ or $1 - r_2/r_1 = 1 - 0.9841 = 0.0159$ or of -1.59 pct of the correct value $p_o = \gamma_2 \omega_o$.

expressing the elasticity law and the kinematic relation between strain and the displacement components u, v obtained from (14) and (15). These are four equations for the four unknowns σ_1, σ_2, u, and v. In dealing with them we may proceed in various ways. We may express the stresses σ_1, σ_2, and v in terms of the tangential displacement u

$$\sigma_1 = \left\{E/[2 + (1 - \nu)\,k]\,r_o\right\}\left\{(c/4)\,(1 + 3\cos 2\;\;')\right.$$

$$\left. + \,ku\cot\alpha' + [(1 + k)/(1 + \nu)]\,(u' - u\cot\alpha')\right\} \ldots\ldots\ldots\ldots\ldots \tag{38}$$

$$\sigma_2 = \left\{E/[2 + (1 - \nu)\,k]\,r_o\right\}\left\{(c/4)\,(1 + 3\cos 2\alpha')\right.$$

$$\left. + \,ku\cot\alpha' - [(1 - \nu k)/(1 + \nu)]\,(u' - u\cot\alpha')\right\} \ldots\ldots\ldots\ldots\ldots \tag{39}$$

$$v = \left\{[(1 - \nu)\,c/4]\,(1 + 3\cos 2\alpha') - u' - u\cot\alpha'\right\}/[2 + (1 - \nu)\,k] \ldots\ldots\ldots \tag{40}$$

where c stands as abbreviation for $c = k_o h + k_1 r_o$ and $u' = du/d\alpha'$. After substituting the preceding expressions for σ_1 and v in (35), the differential equation for u is obtained

$$L\,(u) = [(1 + \nu)/(1 + k)]\left\{ku + (3/4)\,[k_o\,h\,(4 + (1 - \nu)\,k) + 2\,k_1\,r_o]\sin 2\alpha'\right\} \ldots\ldots \tag{41}$$

where L (u) denotes a linear differential operator which is closely related to one more general such operator which was introduced by the late Ernst Meissner [FLÜGGE, 1934, p. 153] in the theory of thin shells of rotational symmetry

$$L\,(u) = du^2/d\alpha'^2 + (du/d\alpha')\cot\alpha' + (1 - \cot^2\alpha')\,u \ldots\ldots\ldots\ldots\ldots \tag{41a}$$

Thus a linear differential equation of second order for the tangential displacement u is obtained. It is a happy coincidence that in the present application to the solid Earth shell a particular integral satisfying the two additive terms appearing on the right-hand side of (41) represents the solution which is sought. One verifies that

$$u = -\,u_o\sin 2\alpha' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{42}$$

is a particular integral of (41). It satisfies one of the boundary conditions to be specified, namely, $\alpha' = 0$, $u = 0$. If one computes the distribution of the stresses σ_1 and σ_2 from (38) and (39) by making use of the function $u = -\,u_o\sin 2\alpha'$, one finds that a second condition of a rotationally symmetric distribution of stress in a closed shell, namely, that for $\alpha' = 0$, $\sigma_1 = \sigma_2$, is also automatically satisfied. It becomes, therefore, unnecessary to determine the complete forms of the integrals of the homogeneous differential equation L (u) = 0 which would, otherwise, require cumbersome developments in series. This considerable simplification of the task of finding the required solution is due to the fact that the solid rock shell is a closed shell for which certain otherwise prescribed boundary conditions drop out. Since

$$L\,(-\,u_o\sin 2\alpha') = 4\,u_o\sin 2\alpha' \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{43}$$

it is an easy matter to compute after substituting (42) in (41) the value of the constant u_o

$$u_o = 3\,(1 + \nu)\left\{[4 + (1 - \nu)\,k]\,k_o\,h + 2\,k_1\,r_o\right\}/4\,[4 + (5 + \nu)\,k] \ldots\ldots\ldots \tag{44}$$

and to evaluate (using (42) and (44)) the normal displacement or deflection v. Denoting its maximum amplitude by v_o it is found equal to

$$v_o = [4\,(2 + \nu)\,k_o\,h + (5 + \nu)\,k_1\,r_o]/[4 + (5 + \nu)\,k] \ldots\ldots\ldots\ldots\ldots \tag{45}$$

and the tangential and normal components of displacement of the solid shell are given thus by

$$\left.\begin{aligned} u &= -\,u_o\sin 2\alpha' \\ v &= (v_o/4)\,(1 + 3\cos 2\alpha') \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{46}$$

and the components of normal stress σ_1 and σ_2 by

$$\left.\begin{aligned}\sigma_1 &= (E/8\,r_0)\,[(c - k\,v_0)\,(1 + 3\cos 2\alpha') + 4\,u_0\,(1 - \cos 2\alpha')/(1+\nu)]\\ \sigma_2 &= (E/8\,r_0)\,[(c - k\,v_0)\,(1 + 3\cos 2\alpha') - 4\,u_0\,(1 - \cos 2\alpha')/(1+\nu)]\end{aligned}\right\}\ \ldots\ldots \quad (47)$$

The state of stress in the shell is composed of a uniform, biaxial tension or compression and of a state of pure shear expressed by the first and second terms on the right sides of (47) respectively. It may be resolved, after introducing the expressions for u_0 and v_0 from (44) and (45), into two additive parts which represent the contributions of the bodily tide forces (the part containing the constant k_0 as factor) and of the invariable pressures $p = p_0\,(1 + 3\cos 2\alpha')/4$ (the part having k_1 as factor). The stresses σ_1 and σ_2 are explicitly

$$\begin{aligned}\sigma_1, \sigma_2 = \Big\{E/8\,[4 + (5+\nu)\,k]\Big\}\Big\{(k_0\,h/r_0)\,[(4 - 3\,(1+\nu)\,k)\,(1 + 3\cos 2\alpha')\\ \pm 3\,(4 + (1-\nu)\,k)\,(1 - \cos 2\alpha')] + k_1\,[4\,(1 + 3\cos 2\alpha') \pm 6\,(1 - \cos 2\alpha')]\Big\}\ \ldots \quad (48)\end{aligned}$$

where the two upper + signs refer to the stress σ_1 and the two lower - signs to σ_2.

Due to the linearity of the expressions in the constants k_0 and k_1 for the maximum amplitudes of the elastic horizontal and vertical displacements of the rock shell u_0 and v_0 and for the stress components σ_1, σ_2, obtained in (44), (45), and (48), it is possible to evaluate their magnitudes caused independently through the effects: (1) of the presence of the bodily tide forces γ_0 acting in the rock shell, (2) of the invariable lateral pressures exerted by the tidal force field in the fluid magmatic substratum, (3) due to the further increase of the pressures when the shell deflects, and (4) to evaluate the effect of the stiffness of the substratum causing a great reduction of the elastic distortion of the solid shell. This may be done by evaluating the corresponding terms in the expressions for u_0, v_0, σ_1, σ_2 which depend on the three dimensionless, characteristic parameters k_0, k_1, and k.[12]

A glance at (44) and (45) discloses that the maximum amplitudes u_0 and v_0 of the horizontal and vertical movements of the shell are greatly reduced through the stiffness resistance of the substratum. It is noteworthy that the vertical component v_0, that is, the normal deflection of the spherical solid shell, is much more reduced in its magnitude than the horizontal component u_0 when the stiffness resistance of the substratum is duly considered. In regard to (45) or the expression for v_0

$$v_0 = [(2+\nu)\,k_0\,h + (5+\nu)\,k_1\,r_0/4]/[1 + (5+\nu)\,k/4]\ \ldots\ldots\ldots\ldots \quad (51)$$

obviously the first term in the numerator

$$v_0' = (2+\nu)\,k_0\,h\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (52)$$

represents the deflection caused by the bodily tide forces γ_0 of the shell (which has already been computed in (18) and in footnote 10), whereas the second term

$$v_0'' = (5+\nu)\,k_1\,r_0/4\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (53)$$

expresses the deflection by the invariable part of the pressure $p = p_0\,(1 + 3\cos 2\alpha')/4$. The former is $v_0' = 4.55 \times 10^3$ cm, the latter $v_0'' = 8.40 \times 10^4$ cm, and $v_0'' = 18.5\,v_0'$. However, neither v_0' nor v_0'' can develop on account of the counteraction of the rapidly increasing resistance to deformation - kv expressed through the presence of the stiffness constant k in the denominator of the complete expression (51) of v_0. The constant k having the great value k = 1809, the preceding partial deflections v_0', v_0'' and their sum must be reduced by the factor $1:[1 + (5+\nu)\,k/4]$ or 1:2430.

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[12] The values of the constants k_0, k_1, and k were computed by assuming $E = 65{,}000\ \mathrm{kg/cm^2}$ for the moduli of elasticity and $G = 23{,}630\ \mathrm{kg/cm^2}$ for the modulus of rigidity corresponding to a Poisson's ratio $\nu = 0.375$ (as explained in Part 1) for the mean radius of the rock shell r_0 = 6320 km, for the thickness h = 100 km, and for the specific weights γ_1 and γ_2 the values previously stated. The constant of the tidal body force per unit volume γ_0 defined in (8) was estimated at $\gamma_0 = 3.12 \times 10^{-10}\ \mathrm{kg/cm^3}$. The values of the constants p_0 and ω_0 (maximum elevation of equipotential surface or of a fluid tide in sphere of radius $r = r_2 = 6270$ km) given in (24) and (25) were found to be equal, $p_0 = 0.1017\ \mathrm{kg/cm^2}$ and $\omega_0 = 35.05$ cm. Using the preceding values for the various constants, k_0, k_1, and k were found from (33) equal to

$$k_0 = 1.916 \times 10^{-4} \qquad k_1 = 9.88 \times 10^{-5} \qquad k = 1809\ \ldots\ldots\ldots \quad (49)$$

and the maximum amplitudes of the tangential and normal displacement of the solid shell equal to

$$u_0 = 244\ \mathrm{cm} = 2.44\ \mathrm{m} \qquad v_0 = 36.4\ \mathrm{cm}\ \ldots\ldots\ldots\ldots \quad (50)$$

As mentioned in footnote 12, the shell obtains a maximum deflection v_o of magnitude

$$v_o = 36.4 \text{ cm} \quad \ldots\ldots\ldots\ldots \quad (54)$$

only. (The maximum elevation of an equilibrium tide in a fluid sphere having the radius $r_1 = 6370$ km would be found equal to $\omega_o^* = 35.2$ cm.)

In a discussion of the preceding equations we may mention two limiting cases which might be considered from a mathematical viewpoint, namely, when attributing to the stiffness constant k the two limiting values $k = 0$ and $k = \infty$, respectively. The first case, $k = 0$, has already been fully considered in (12) representing a shell floating freely in space. If, on the other hand, a solid substratum is postulated, as has been already stated, the constant k may be considered as the parameter, measuring the stiffness of such an elastic foundation with which the outer shell maintains connection. The larger the value of k, the more strongly the foundation resists a normal deflection. It is noteworthy, from a mathematical viewpoint, that the limiting case, $k = \infty$, has still a good mechanical significance even although it supposes that k increases indefinitely. The expressions for u_o, v_o, σ_1, and σ_2 converge when $k = \infty$ to definite values. Their numerical values, which were just computed after assuming certain tentatively estimated values of the various constants, represent a case which was approaching already the limiting case $k = \infty$ quite closely, as may be judged from the following limiting values of these quantities which differ but little from the previously found ones. In regard to (45), it is obvious, since k appears only in the denominator of the fraction expressing the value of v_o, that v_o must vanish when $k = \infty$, that is, the solid shell will not be deflected more in normal direction; the deflections v vanish

$$k \rightarrow \infty \qquad v = 0$$

From (44) we see, when $k \rightarrow \infty$, that the maximum displacement in tangential direction converges toward the value

$$u_o = 3\,(1 - \nu^2)\,k_o\,h/4\,(5 + \nu) = 3\,(1 - \nu^2)\,\gamma_o\,r_o^2/4\,(5 + \nu)\,E \quad \ldots\ldots\ldots\ldots \quad (55)$$

and the displacement u toward the function

$$u = -\,u_o \sin 2\alpha' = -\,[3\,(1 - \nu^2)\,\gamma_o\,r_o^2/4\,(5 + \nu)\,E] \sin 2\alpha' \quad \ldots\ldots\ldots\ldots \quad (56)$$

The shell is thus displaced only by horizontal movements in tangential directions to its middle surface. The values of the stresses σ_1 and σ_2 may be computed, when $k \rightarrow \infty$, from (36) and (37) after substituting in them for u the expression taken from (56). The stresses σ_1 and σ_2 converge toward the two functions

$$\left.\begin{aligned} \sigma_1 &= -\,[3\gamma_o\,r_o/4\,(5 + \nu)]\,[\nu + (2 + \nu)\cos 2\alpha'] \\ \sigma_2 &= -\,[3\,\gamma_o\,r_o/4\,(5 + \nu)]\,[1 + (1 + 2\,\nu)\cos 2\alpha'] \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (57)$$

When k increases and v decreases simultaneously indefinitely, the term kv in (34) and (35) becomes indeterminate. The term[13] $-\,(Eh/r_o^2)$ limes (kv) evidently represents that "hold-down pressure" p on the concave side of the shell which impedes its uplifting. After letting $p_o = 0$ (since a rigid substratum does not exert pressures of the kind considered in (24)), that is, assuming $k_1 = 0$, the hold-down pressure p may be computed either from (34) or (35) after substituting for σ_1 and σ_2 their values taken from (57). It is found equal to

$$p = -\,(Eh/r_o^2) \lim_{\substack{k = \infty \\ v = 0}} (kv) = -\,[(2 + \nu)/(5 + \nu)]\,\gamma_o\,h\,(1 + 3\cos 2\alpha') \quad \ldots\ldots\ldots \quad (58)$$

The principal stresses σ_1 and σ_2 given in (57) for the case $k = \infty$ may be expressed in the symmetric form

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[13] Eq. (34) expresses the condition of equilibrium of the force components acting on an element of the shell in radial direction which for a spherical shell of radius r_o and thickness h has the general form $\sigma_1 + \sigma_2 = q\,r_o/h$, where q denotes a pressure per unit of area of the middle sphere $r = r_o$. This "equivalent" pressure q may be considered as made up of the bodily forces γ_o represented by the first term on the right side of (34) (assuming $k_1 = 0$) and of a hold-down pressure $p = -\,Ekvh/r_o^2$ representing the last term in (34) which is the expression used in the text above.

$$\left.\begin{aligned} \sigma_1 &= \sigma + \sigma' \\ \sigma_2 &= \sigma - \sigma' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (59)$$

where σ denotes the mean stress $\sigma = (\sigma_1 + \sigma_2)/2$ and the terms $\pm\sigma'$ represent a state of pure shear. We have

$$\left.\begin{aligned} \sigma &= -[3\,(1+\nu)\,\gamma_o\, r_o/8\,(5+\nu)]\,(1 + 3\cos 2\alpha') \\ \sigma' &= [3\,(1-\nu)\,\gamma_o\, r_o/8\,(5+\nu)]\,(1 - \cos 2\alpha') \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (60)$$

Let

$$\sigma_o = 3\,(1+\nu)\,\gamma_o\, r_o/8\,(5+\nu) \quad \ldots\ldots\ldots\ldots \quad (60a)$$

Taking $\gamma_o = 3.120 \times 10^{-10}$ kg/cm^3, $r_o = 6.32 \times 10^8$ cm, $\nu = 0.375$, we have in kg/cm^2

$$\left.\begin{aligned} \sigma_o &= 0.0189 \\ \sigma &= -\sigma_o\,(1 + 3\cos 2\alpha') = -0.0189\,(1 + 3\cos 2\alpha') \\ \sigma' &= 0.0086\,(1 - \cos 2\alpha') \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (61)$$

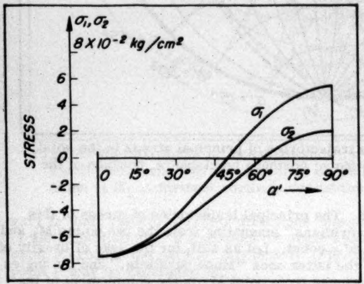

Fig. 10--Principal stresses σ_1 and σ_2 in the solid shell

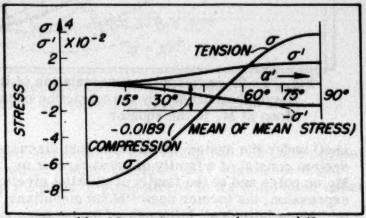

Fig. 11--Mean stress $\sigma = (\sigma_1 + \sigma_2)/2$ and stress σ' in the solid shell

These values have also been plotted in Figures 10 and 11, with the strains in Figure 12. Whereas, according to the stress formulas (13) (corresponding to the limiting case k = 0) the shell around the poles M_1, M_2, ($\alpha' = 0$ and $\alpha' = \pi$) was stressed by tensile stresses σ_1 and σ_2 having both positive values, according to (59) and (60) valid for the case k = ∞ and as can be seen in Figure 10, the shell is now, on the contrary, stressed in two caps of the sphere around the poles $\alpha' = 0$ and $\alpha' = \pi$ by compression stresses.[14] The compression stresses are predominant, having larger values than the tensile stresses through which the belt between the two caps is stressed. That compression stresses arise in the two caps is understandable since in the absence of any normal deflection v under the displacement field which was reproduced in Figure 7 for the tangential displacements u two broad, ring-shaped zones of the Earth shell are pulled by the Moon toward the points (poles) M_1 and M_2.

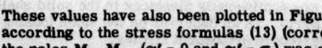

Fig. 12--Permanent strains $\epsilon_x = \epsilon_y$ in the solid shell dependent on geographic latitude β

This may perhaps be an appropriate place to add a few speculative remarks concerning the permanent strains which would be created in the solid

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[14] The meridian stress σ_1 vanishes when $\cos 2\alpha' = -\nu/(2+\nu) = -0.158$ for $\alpha' = 49°33'$, and the stress σ_2 vanishes when $\cos 2\alpha' = -1/(1+2\nu) = -0.572$ for $\alpha' = 62°25'$. The mean stress vanishes for $\alpha' = 54°44'$

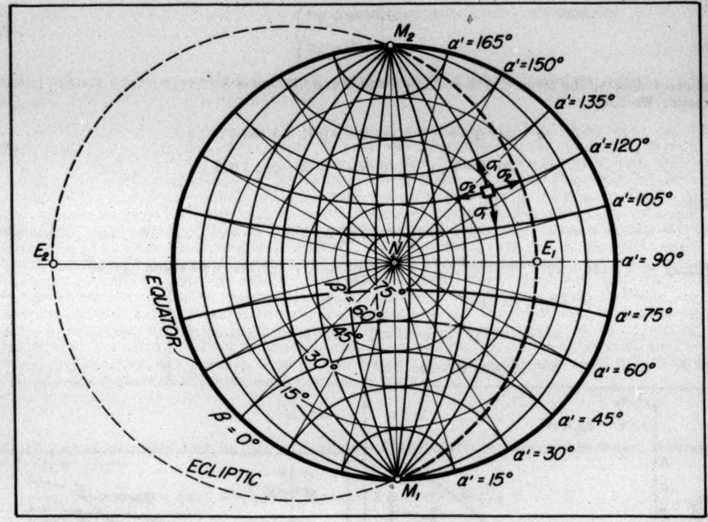

Fig. 13--Stereographic representation of the trajectories of principal stress in the solid Earth shell caused by the attraction of the Moon; northern hemisphere, position of the Moon at M_1 in the equator

shell under the system of stresses just discussed. The principal trajectories of stress of this system consist of a family of great circles or "meridians" emanating from the two points M_1 and M_2 as poles and of the family of parallel circles α' = const. Let us call, for the sake of brevity of expression, the former ones "Moon meridians," the latter ones "Moon parallels," and the angles α' "Moon latitudes" (although α' having been measured from point M_1 is the complement of the angle which should precisely be called a latitude). Thus the directions of principal stress σ_1 are the Moon meridians and those of σ_2 the Moon parallels (see Fig. 13 and 14). As the Earth rotates around its axis the Moon parallels and the meridians are continuously displaced in the solid shell also on account of the motion of the Moon around the Earth. To an observer on the Moon looking down on the Earth, they appear, however, as a fixed system of orthogonal circles.

Suppose, first, that the plane of the orbit of the Moon would coincide with the plane of the equator of the Earth. Then the two points M_1 and M_2 would lie in the Earth's equator and move on it by following the positions of the Moon. The observer on the Moon would see the surface of the Earth revolving past the Moon meridians and parallels, and for him the time of one full revolution of the Earth counted between two consecutive passages of some point chosen on the equator through the point M_1 would be T = 89428 sec.[15] Let α, β denote the geographic longitude and latitude of a point P of the Earth. During the time T the principal stresses σ_1 and σ_2 acting in point P (α, β)

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[15] If ω_e denotes the angular velocity of rotation of the Earth and ω_m the angular velocity with which the radius vector drawn from the Earth to the Moon rotates in stellar space, their difference $\omega = \omega_e - \omega_m$ expresses the angular velocity with which the Earth appears to revolve to the observer on the Moon. Point M_1 and the system of the principal stress circles turn with the same angular velocity ω in the opposite direction (westward) around the equator. Since $\omega_e = 7.29212 \times 10^{-5}\ \mathrm{sec}^{-1}$, $\omega_m = 2.6616 \times 10^{-6}\ \mathrm{sec}^{-1}$, $\omega = \omega_e - \omega_m = 7.02596 \times 10^{-5}\ \mathrm{sec}^{-1}$ the time of one Earth revolution $T = 2\pi/\omega$ = 89,428 sec. One solar day of 24 hours having T_s = 86,400 sec, $T = T_s$ + 3028 sec is 50 min 28 sec longer than one day.

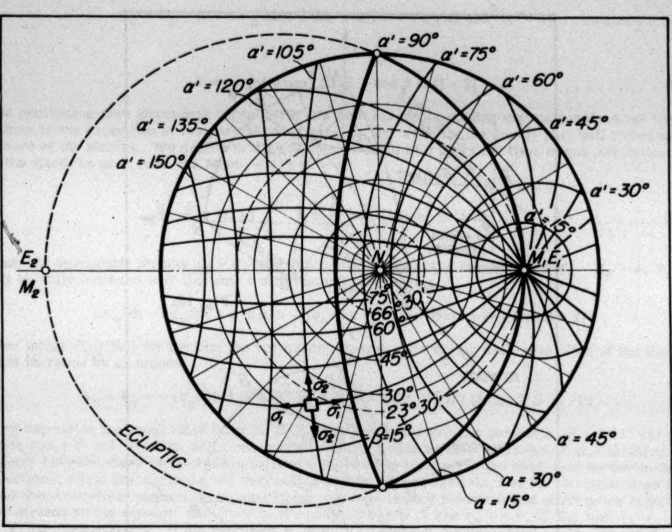

Fig. 14--Stereographic representation of the trajectories of principal stress in the solid Earth shell caused by the attraction of the Moon; northern hemisphere, position of the Moon at M_1 in farthest northern declination

vary in their values, and their directions change also periodically describing two complete cycles of their oscillations. If ϕ denotes the angle under which a Moon meridian M_1P passing through point P crosses the parallel circle A'B' (Fig. 15), the state of plane stress in the solid shell under point P may be expressed by

$$\left.\begin{aligned} \sigma_x &= (\sigma_1 + \sigma_2)/2 + [(\sigma_1 - \sigma_2)/2] \cos 2\phi \\ \sigma_y &= (\sigma_1 + \sigma_2)/2 - [(\sigma_1 - \sigma_2)/2] \cos 2\phi \\ \tau_{xy} &= [(\sigma_1 - \sigma_2)/2] \sin 2\phi \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (62)$$

The component α_x acts in the direction of the parallel circle (β = const), the component σ_y in the direction of the Earth's meridian (α = const) passing through point P, and the shearing stress τ_{xy} acts in these two directions. The question may be asked, What permanent components of strain may be caused during the time T of one revolution of the Earth during which σ_1, σ_2, ϕ describe just two oscillations in their values? Suppose that the permanent strains ϵ_x, ϵ_y, γ_{xy} were of a viscous nature. Then these strains increase with the following rates

$$\left.\begin{aligned} d\epsilon_x/dt &= (1/3\mu)(\sigma_x - \sigma_y/2) = (1/12\mu)[\sigma_1 + \sigma_2 + 3(\sigma_1 - \sigma_2)\cos 2\phi] \\ d\epsilon_y/dt &= (1/3\mu)(\sigma_y - \sigma_x/2) = (1/12\mu)[\sigma_1 + \sigma_2 - 3(\sigma_1 - \sigma_2)\cos 2\phi] \\ d\gamma_{xy}/dt &= \tau_{xy}/\mu = (1/2\mu)(\sigma_1 - \sigma_2)\sin 2\phi \end{aligned}\right\} \quad \ldots\ldots \quad (63)$$

where μ denotes the coefficient of viscosity. In an endeavor not to make the computation too involved, suppose that we could substitute, instead of the values of the principal stresses σ_1, σ_2

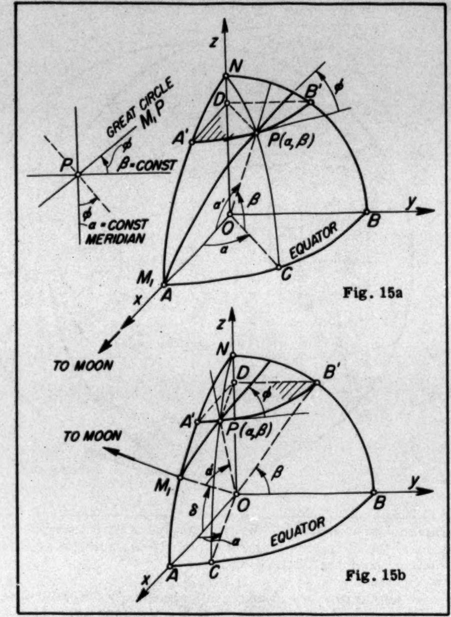

Fig. 15a

Fig. 15b

Fig. 15--Hour angle α, geographic latitude β, and angles α' and φ

as determined in (57) or (59), (60), and (61), their mean value $\sigma = (\sigma_1 + \sigma_2)/2$ by assuming that[16]

$$\sigma_1 = \sigma_2 = \sigma = - \sigma_o \ (1 + 3 \cos 2 \alpha') \ldots\ldots\ldots\ldots\ldots\ldots \quad (64)$$

Suppose that at a certain time a point P of the Earth situated on the parallel β has the position shown in Fig. 15a whereas simultaneously M_1 is situated on the Earth's equator. The angle of longitude α inscribed in Figure 15 may be considered as a relative hour angle of P with respect to the position of the Moon M_1. As the Earth rotates and point M_1 simultaneously trails the Moon along the equator, the hour angle α increases proportionally with the time t

$$\alpha = \omega t = (\omega_e - \omega_m) \, t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (65)$$

while P describes the parallel circle β = const. To any position (α, β) of P at time t there will correspond an angle α' ($\sphericalangle M_1OP$, Fig. 15) of Moon latitude for which

$$\cos \alpha' = \cos \beta \cos \alpha = \cos \beta \cos \omega t \ldots\ldots\ldots\ldots\ldots\ldots \quad (66)$$

The two equal values of the principal stresses of the biaxial state of stress in point P will be

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[16] This would amount to neglecting the terms $\pm \sigma'$ which are of comparatively small magnitude and to replacing the two curves for σ_1 and σ_2 of Figure 10 by a single curve having the ordinates equal to σ as plotted in Figure 11.

$$\sigma_1 = \sigma_2 = \sigma = -\sigma_0 (1 + 3\cos 2\alpha') = -2\sigma_0 (3\cos^2\alpha' - 1) \ldots\ldots\ldots\ldots \quad (67)$$

or

$$\sigma = -\sigma_0 [(3\cos^2\beta)(1 + \cos 2\omega t) - 2] \ldots\ldots\ldots\ldots \quad (67a)$$

The oscillating part (dependent on the term cos $2\omega t$) in the preceding expression does not contribute to the formation of the permanent strains ϵ_x and ϵ_y; only the steady part will cause an increase of the strains. We desire to know the mean value of σ over the time of one revolution T of the Earth as seen from the Moon, which is

$$\bar{\sigma} = (1/T)\int_0^T \sigma\, dt = -\sigma_0 [3\cos^2\beta - 2] \ldots\ldots\ldots\ldots \quad (67b)$$

Since the permanent strains $\epsilon_x = \epsilon_y$ according to (63) (since the stresses are $\sigma_x = \sigma_y = \sigma$, $\tau_{xy} = 0$) will steadily increase with the time t with the uniform rates

$$d\epsilon_x/dt = d\epsilon_y/dt = \sigma/6\mu = -(\sigma_0/6\mu)[(3\cos^2\beta)(1 + \cos 2\omega t) - 2] \ldots\ldots \quad (68)$$

after integrating (68) we see that the permanent strains $\epsilon_x = \epsilon_y$ for each revolution of the Earth must increase by an amount

$$\epsilon_x = \epsilon_y = -(\sigma_0/6\mu)(3\cos^2\beta - 2)T = -(\sigma_0/12\mu)(3\cos 2\beta - 1)T \ldots \quad (69)$$

They depend on the geographic latitude β of point P. There are two parallels $\beta = \pm 35° 16'$ (for which $\cos 2\beta = 1/3$) which will remain unstretched; a zone on both sides (south and north) of the equator between these two parallels suffers compression strains, in the east-west as south-north directions, equal in magnitude, but decreasing toward these parallels, and the two polar caps beyond these parallels sustain extensions which increase toward the north and south poles (Fig. 12). The strains on the equator $\beta = 0$ and at the poles $\beta = \pm\pi/2$ are $\epsilon_x = -\sigma_0 T/6\mu$ and $\epsilon_x = \sigma_0 T/3\mu$, respectively. If the viscosity μ (which is a complex function of the pressure p and the temperature θ underground and probably decreases with the depth y) were estimable, one could speculate about the amounts of strain which would accrue during longer epochs of geologic time by dividing them with the time T of one revolution of the Earth, using the daily rate of increase of permanent strain ϵ_x in (69). Since compression stresses predominate in the equatorial region of the Earth as can be seen in the σ curve in Figure 11, when the cosine curve $\sigma = -0.0189\,(1 + 3\cos 2\alpha')$ is swept along the Earth's equator $\beta = 0$ for a certain fraction of time also tensile stresses will act in the equatorial zone, the compression stresses must contribute more to the permanent strains $\epsilon_x = \epsilon_y$ than the tensile ones, with the result that the equatorial zone appears strained permanently by the value $\epsilon_x = -\sigma_0 T/6\mu$. The two polar caps are, however, strained continuously only by tensile stresses and thus their effect predominates in the resulting form of the ϵ_x curve near the polar region of Figure 12. The preceding conclusions, it must be remembered, were drawn by postulating that the plane of the orbit of the Moon would coincide with the plane of the equator, which is not the case. The plane in which the Moon moves around the Earth is inclined at a small angle with respect to the plane of the ecliptic.

It may be of interest to consider in a second approximation that the plane of the orbit of the Moon would coincide with the ecliptic. This is indicated in the two stereographic projections of the northern hemisphere reproduced in Figures 13 and 14 in which the ecliptic thought to represent the path of the Moon has been drawn as a circle $M_1E_1M_2E_2$ in a dashed, partially thick line in Figure 13 and by a partially heavy line in Figure 14 passing through E_1. The points M_1 (and M_2) representing the position of the Moon would move along the ecliptic circle inclined at an angle δ equal to 23° 30′ with respect to the equator ($\beta = 0$) in both figures. In Figure 13 the position of the Moon M_1 is shown just when it passes through the equator. The principal stress trajectories represented in Figure 13 have the position which was considered when arriving at (62) to (69). In Figure 14, however, the position of the Moon M_1 was assumed in its farthest northern declination coinciding with point E_1 of the ecliptic where the radius vector drawn from the Earth to the Moon is exactly inclined at an angle of $\delta = 23° 30'$ with respect to the plane of the equator. This has been shown also in Figure 15b in which the direction cosines of OM_1 are seen to be equal to $a_x = \cos\delta$, $a_y = 0$, and $a_z = \sin\delta$, and for OP equal to $b_x = \cos\beta\cos\alpha$, $b_y = \cos\beta\sin\alpha$, and $b_z = \sin\beta$. Hence, the cosine of the angle α' of the Moon latitude for point P is now

$$\cos\alpha' = a_x b_x + a_y b_y + a_z b_z \ldots\ldots\ldots\ldots \quad (70)$$

or

$$\cos\alpha' = \cos\delta\cos\beta\cos\alpha + \sin\delta\sin\beta \quad \ldots\ldots\ldots\ldots\ldots (71)$$

and

$$\cos^2\alpha' = \cos^2\delta\cos^2\beta\,[\cos\alpha + \tan\delta\tan\beta]^2 \quad \ldots\ldots\ldots\ldots\ldots (72)$$

After substituting this last expression in (67b) we find that the mean value of stress σ for one revolution of the Earth at the geographic latitude β is now

$$\bar{\sigma} = -\sigma_0 (3\cos^2\delta - 2)(3\cos^2\beta - 2) \quad \ldots\ldots\ldots\ldots\ldots (73)$$

and that the strains must increase during the time T by the amounts

$$\epsilon_x = \epsilon_y = -(\sigma_0/6\mu)(3\cos^2\delta - 2)(3\cos^2\beta - 2)\,T \quad \ldots\ldots\ldots\ldots\ldots (74)$$

In arriving at (74), it is understood that the assumption was used that the Moon would remain at its farthest northern declination in which M_1 coincided with E_1 in Figure 14 and would not change its angle of declination. Eq. (74) differs from (69) by the factor $(3\cos^2\delta - 2)$, which since $\delta = 23° 30'$ is equal to 0.523. Thus we see that all the strains ϵ_x if the Moon would linger in this most extreme position farthest from the plane of the equator would grow only approximately half as fast as while it stood exactly in the equator.

In summing up, then, we may state that, since the Moon stayed close to the equator during one-half of the time of its complete revolution around the Earth and during the other half close to its two farthest declinations (it does not make any difference in the effects it causes in the permanent straining of the shell, whether it was farthest north or south), the distribution of the permanent strains $\epsilon_x = \epsilon_y$ it creates over the entire Earth shell may be represented by (69) (or by the curve in Fig. 12) provided that their magnitudes (or ordinates) are reduced by a factor $(1/2)(1 + 0.523)$ or approximately to 75 pct of their values as given in (69). In other words, if the Earth was originally covered by a solid shell of uniform thickness and properties, which was probably not the case, material in it should gradually accumulate in an equatorial belt extending to ± 35° latitude at the expense of the thickness of the shell in the polar caps. A good portion of the highest extensive continental plateaus seem to be found in this belt; but many large mountain chains reach certainly beyond it.

A few remarks may now be added in regard to the elastic tangential movements of the shell of a periodic nature due to the tangential displacements u. Their magnitude was found to be proportional to the sine of twice the Moon latitude angle α' and their directions pointing always toward the two points M_1 and M_2. As the Earth rotates with respect to the Moon, a point P of the solid shell must, obviously, describe a certain curve relative to the Earth's interior. To find its shape it will also be necessary to determine the angle ϕ under which a Moon meridian is inclined with respect to the parallel β = const passing through point P (α, β). During a revolution of the Earth, this angle ϕ will continuously change. Two positions might again be considered, namely, when the Moon is standing in the equator and when it is at its farthest declination. In the first case, from trigonometry one finds

$$\tan\phi = \sin\beta/\tan\alpha \quad \ldots\ldots\ldots\ldots\ldots (75)$$

permitting then to evaluate the components of vector u in the directions of the parallel u_p and the meridian u_m of point P

$$u_p = u\cos\phi \qquad u_m = u\sin\phi \quad \ldots\ldots\ldots\ldots\ldots (76)$$

After making use also of (66) one finds that

$$\left.\begin{aligned} u_p &= -u_0\cos\beta\sin 2\alpha \\ u_m &= -(u_0\sin 2\beta)(1 + \cos 2\alpha)/2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (77)$$

indicating that the path which a point of the shell describes is a small ellipse

$$u_p^2/a^2 + (u_m + b)^2/b^2 = 1 \qquad \left\{\begin{aligned} a &= u_0\cos\beta \\ b &= (u_0/2)\sin 2\beta \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (78)$$

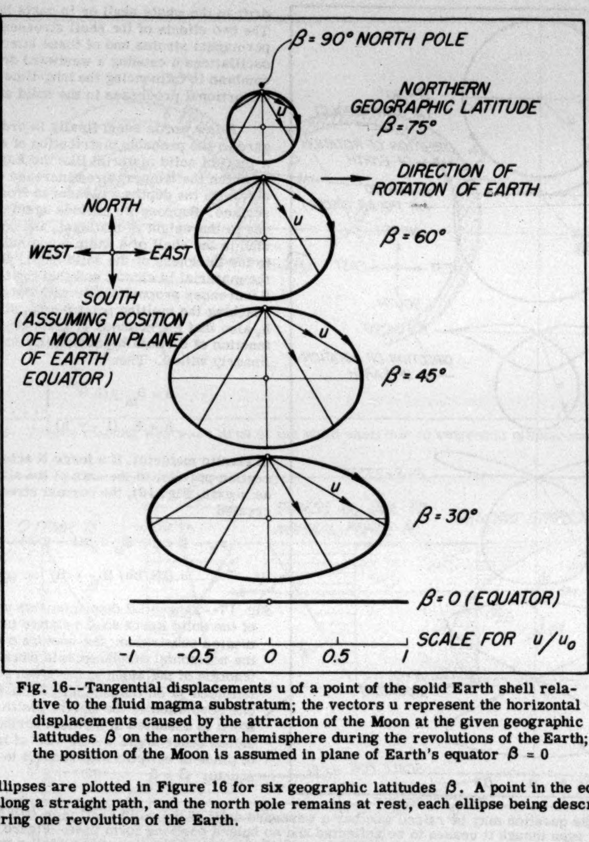
β = 90° NORTH POLE
NORTHERN GEOGRAPHIC LATITUDE
β = 75°
u
DIRECTION OF ROTATION OF EARTH
NORTH
WEST
EAST
SOUTH
(ASSUMING POSITION OF MOON IN PLANE OF EARTH EQUATOR)
u
β = 60°
u
β = 45°
u
β = 30°
β = 0 (EQUATOR)
SCALE FOR u/u₀
-1
-0.5
0
0.5
1

Fig. 16--Tangential displacements u of a point of the solid Earth shell relative to the fluid magma substratum; the vectors u represent the horizontal displacements caused by the attraction of the Moon at the given geographic latitudes β on the northern hemisphere during the revolutions of the Earth; the position of the Moon is assumed in plane of Earth's equator $\beta = 0$

These ellipses are plotted in Figure 16 for six geographic latitudes β. A point in the equator is moved along a straight path, and the north pole remains at rest, each ellipse being described twice during one revolution of the Earth.

The horizontal movements u become a little more involved in the second case, when the Moon is at its farthest northern declination, tan ϕ becoming equal to

$$\tan \phi = (\sin \beta \cos \alpha - \cos \beta \tan \delta)/\sin \alpha \quad (79)$$

Using (71) and (76), u_p and u_m may numerically be evaluated. The curves representing the movements of the solid shell in tangential direction are reproduced in Figure 17 for the geographic latitudes $\beta = 0$ (equator), $\beta = \delta = 23^\circ\ 30'$ (tropic of cancer), $\beta = 34^\circ\ 20'$ (producing two sharp cusps in curve), $\beta = 66^\circ\ 30'$ (polar circle), and $\beta = 90^\circ$ (north pole). In the plots of these curves in Figures 16 and 17, it was assumed that the Moon would remain in a fixed position. If its motion were also included, the curves would not be closed but spiral in shape. In Figure 17 it is seen that the north pole is moved around a circle of radius equal to 0.73 u_0. Although it is presumed that the solid crust is restrained in this elastic wiggling motion through viscous drags originating at the lower levels of the solid shell, the drags must be thought of as the origins of a westward

drift in the whole shell or in parts thereof.[17] The two effects of the shell stresses producing permanent strains and of these lateral periodic oscillations u causing a westward drift probably combine in influencing the long-time, complex, distortional processes in the solid shell.

A few words seem finally in order in regard to the probable distribution of stresses in a shell of solid material like the Earth's crust in which the temperatures increase considerably with the depths y measured from its outer surface. Suppose we exclude again the stresses due to the weight of material, and consider a slab in the shell of a width b and height h equal to the thickness of the solid shell. Suppose that the material is elastic and that the temperature θ increases proportionally with the depths y reaching the melting point $\theta = \theta_m$ at the depth h, also that the modulus of elasticity E being a function of θ would hypothetically decrease linearly with θ. Then

$$\left.\begin{aligned}\theta &= \theta_m\ y/h\\ E &= E_0\ (1 - y/h)\end{aligned}\right\}\ldots\ldots\ldots (80)$$

In elastic material, if a force N acts in the direction parallel to the axis of the slab (taken as x-axis, Fig. 19), the normal stress σ_x is created

$$\begin{aligned}\sigma_x &= E\,\epsilon_x' = E_0\,\epsilon_x'\,(1 - y/h)\\ &= (2N/bh)\,(1 - y/h)\ \ldots\ldots\ldots (81)\end{aligned}$$

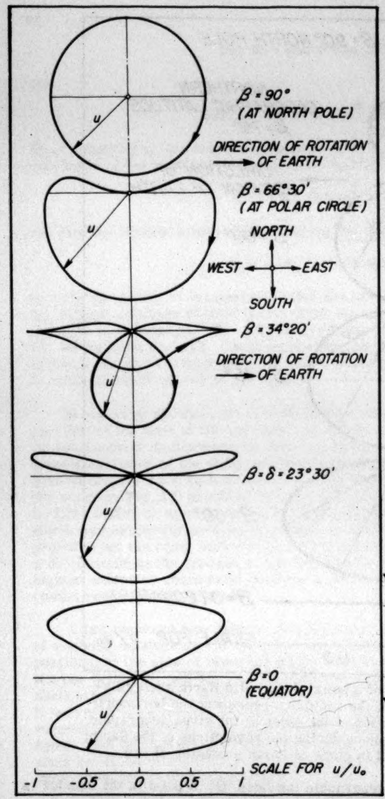

Fig. 17--Tangential displacements u of a point of the solid Earth shell relative to the fluid magma substratum; the vectors u represent the horizontal displacements caused by the attraction of the Moon at the given geographic latitudes β on the northern hemisphere for one revolution of the Earth; position of the Moon is assumed at farthest northern declination under angle δ = 23° 30′ of inclination of plane of ecliptic with respect to plane of equator β = 0

---

[17] The question may be raised whether a westward drift of the solid shell may still become possible even though it ceases to be deflected and no bulged portions form in it. Figure 18 is meant to illustrate the conditions in the case when the normal deflection of the shell v vanishes everywhere and the points of it are only displaced in tangential directions by the components of displacement u. Suppose that u reaches its greatest values u_0 in the points B, C, D, E in their positions indicated in the cross section of the shell in Figure 18. These four points are elastically displaced by the maximum amounts u_0 in the directions indicated by the small arrows u_0. Thus, material comprised within the two spherical caps corresponding to the quarter circumferences of the Earth BC and DE is very slightly condensed and within the zones CD and EB rarified (elastically). We see that the two caps have a greater and the remaining zones of the shell a smaller density ρ than was assumed in the preceding computation. Therefore, a new external moment $M = (Q_1 - Q_2)\,r_0$ is created through the attraction forces of the Moon, similarly as in the case which was previously described in Figure 1 making a westward drift possible. The second sketch on the right side of Figure 18 depicts in greatly exaggerated manner the very slight accumulation of mass around the points A_1 and A_2. The general, slow movement of the outer shell of the Earth relative to the substratum under the driving moment $M = (Q_1 - Q_2)\,r_0$ of the attraction forces of the Moon together with the periodic, elastic rarifications and condensations which sweep as a wave in semidiurnal revolutions through the shell is somewhat reminiscent of the manner a worm pushes its body in periodic convulsions forward on the ground.

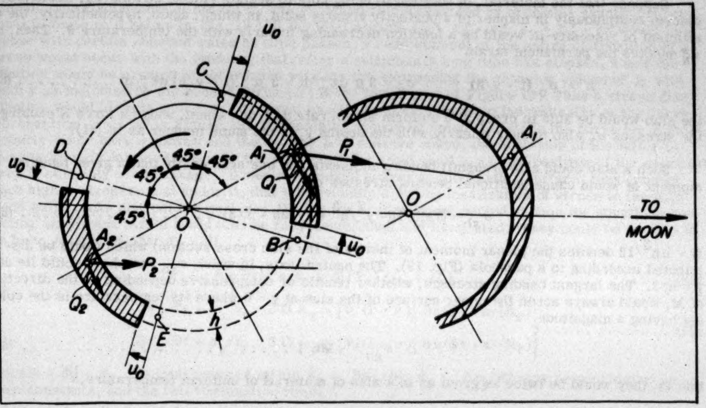
u₀
C
D
u₀
45°
45°
45°
45°
A₁
P₁
Q₁
O
B
u₀
A₂
P₂
Q₂
E
h
u₀
A₁
O
TO MOON

Fig. 18--Forces causing westward drift of the solid shell due to tangential displacements u

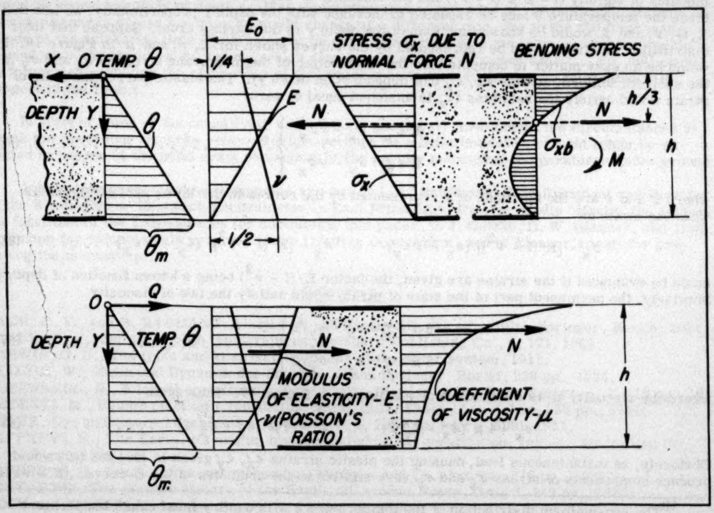
X
O TEMP. θ
DEPTH Y
θ
θm
E₀
1/4
E
ν
1/2
STRESS σx DUE TO NORMAL FORCE N
N
σx
BENDING STRESS
h/3
N
σxb
M
h
O
Q
DEPTH Y
TEMP. θ
N
MODULUS OF ELASTICITY-E
ν(POISSON'S RATIO)
N
COEFFICIENT OF VISCOSITY-μ
h
θm

Fig. 19--Stresses in a slab (in rock shell) in which the temperature varies

In pure tension (or compression), assuming a uniform strain $\epsilon_x' =$ const the stress σ_x in the slab would be linearly distributed along the thickness dimension and the resultant force N would act at the depth of one-third of the thickness h. The maximum stress is $\sigma_{x\,max} = E_0\ \epsilon_x' = 2\ N/bh$, that is, twice the mean stress.

Suppose that the material, in addition to being able to sustain elastic strains ϵ_x', would also deform continuously in manner of a perfectly viscous solid, in which, again, hypothetically, the coefficient of viscosity μ would be a function decreasing linearly with the temperature θ. Then, if ϵ_x'' denotes the permanent strain

$$\mu = \mu_o (1 - y/h) \qquad \sigma_x = 3\,\mu\, d\epsilon_x''/dt = 3\,\mu_o (1 - y/h)\, d\epsilon_x''/dt \quad \dots\dots\dots \quad (82)$$

the slab would be able to creep at a uniform strain rate $d\epsilon_x''/dt$ = const, under a force N causing the stresses σ_x also to vary linearly with the depths y, in the same manner as in (81).

Such a slab could also transmit bending moments and one sees easily that a given bending moment M would cause additional bending stresses σ_{xb}

$$\sigma_{xb} = (3\,Mh/I)(- y^2/h^2 + 4y/3h - 1/3) \quad \dots\dots\dots \quad (83)$$

($I = bh^3/12$ denotes the planar moment of inertia of the slab cross section) which would be distributed according to a parabola (Fig. 19). The neutral axis, in which σ_{xb} vanishes, would lie at $y = h/3$. The largest bending stresses, whether tensile or compressive depending on the direction of M, would always act in the upper surface of the slab at $y = 0$ where its temperature is the coldest having a magnitude

$$\sigma_{xb} = - Mh/I \quad \dots\dots\dots \quad (84)$$

that is, they would be twice as great as in a slab of material of uniform temperature.[18]

The preceding remarks might be generalized in regard to the actual distribution of stress in the solid shell. Suppose it were known how the modulus of elasticity E, Poisson's ratio ν, the modulus of rigidity $G = E/2\,(1 + \nu)$, and the viscosity μ depend on the temperature θ in rocks. Since the temperature θ may be assumed to increase with the depth y proportionally, we see that E, G, ν, and μ would be known functions of the depth y in the Earth's crust. Suppose that their distribution functions could be represented by the curves shown for E, ν, and μ in Figure 19. It would be an easy matter to compute also the distribution of the membrane stresses σ_x and σ_y in the shell for uniform strains ϵ_x, ϵ_y (not dependent on depth y). The elastic part of the state of strain would satisfy the relations (σ_z is assumed equal to zero)

$$\left.\begin{aligned} E\,\epsilon_x' &= \sigma_x - \nu\sigma_y \\ E\,\epsilon_y' &= \sigma_y - \nu\sigma_x \end{aligned}\right\} \quad \dots\dots\dots \quad (85)$$

where E and ν are the functions of y represented by the curves in the lower part of Figure 19. Thus the stresses

$$\sigma_x = [E/(1 - \nu^2)]\,(\epsilon_x' + \nu\epsilon_y') \qquad \sigma_y = [E/(1 - \nu^2)]\,(\epsilon_y' + \nu\epsilon_x') \quad \dots\dots\dots \quad (86)$$

could be evaluated if the strains are given, the factor $E/(1 - \nu^2)$ being a known function of depth y. Similarly, the permanent part of the state of strain would satisfy the law of viscosity

$$\left.\begin{aligned} 3\,\mu\,\dot{\epsilon}_x'' &= \sigma_x - \sigma_y/2 \\ 3\,\mu\,\dot{\epsilon}_y'' &= \sigma_y - \sigma_x/2 \end{aligned}\right\} \quad \dots\dots\dots \quad (87)$$

where the viscosity μ is the function of y indicated in Figure 19. Thus again

$$\sigma_x = 4\,\mu\,(\dot{\epsilon}_x'' + \dot{\epsilon}_y''/2) \qquad \sigma_y = 4\,\mu\,(\dot{\epsilon}_y'' + \dot{\epsilon}_x''/2) \quad \dots\dots\dots \quad (88)$$

Obviously, an instantaneous load, causing the elastic strains ϵ_x', ϵ_y' given in (85) and (86) would produce components of stress σ_x and σ_y very similar to the ordinates of the E-curve. (If ν were constant, the components of stress σ_x and σ_y would be represented by ordinates exactly proportional to those of E shown in Fig. 19.) If the total strains $\epsilon_x = \epsilon_x' + \epsilon_x''$, $\epsilon_y = \epsilon_y' + \epsilon_y''$ would increase with certain constant rates as time passes, a readjustment of the entire distribution of stress would occur with the tendency that, after a sufficiently long time has elapsed, a new distribution would be created in accordance with the law expressing the changing values of μ with depth y as indicated by the second curve (μ) in the lower part of Figure 19. Thus a stress distribution would form in which σ_x, σ_y would exactly be proportional to the ordinates of the curve representing $\mu = f(y)$ in Figure 19. Incidentally, it may be noted that since the curve for E most probably would have a convex and the one for μ a concave shape, the ordinates of the latter increasing to very large values near the surface of the solid shell $y = 0$, the position of the resultant force N certainly would change. It would be raised nearer to the Earth's surface, thus causing much higher stresses just under it, that is, obviously a strong localization of stress in the pack of the solid rocks nearest the surface. During the readjustment period of time the equations connecting stress and strain could also be fully established and integrated. They could be written in the form (assuming that $\sigma_z = 0$)

---

[18] The non-uniform distribution of the temperature θ with depth y must cause temperature stresses in such a slab which could perhaps be estimated if the dependence of the coefficient of thermal expansion α on the temperature were known for rocks. If the slab were free to move, it would be bent like a metal strip of a thermostat. But the weight of the layers would prevent this in the solid Earth's crust with the result that the temperature stresses might easily be estimated similarly as the stresses due to elasticity or viscosity by assuming that the strain ϵ_x = const (not dependent on y) if $\alpha = f\,(y)$ were known.

$$\partial s/\partial t + s/T_r = (6/T_r)\,(\dot{\epsilon}_x + \dot{\epsilon}_y) \quad \text{(89)}$$

$$\left.\begin{aligned} \partial\sigma_x/\partial t + \sigma_x/t_r &= 2\,G\,\dot{\epsilon}_x + [\nu/(1-\nu)]\,\partial s/\partial t + s/(3t_r) \\ \partial\sigma_y/\partial t + \sigma_y/t_r &= 2\,G\,\dot{\epsilon}_y + [\nu/(1-\nu)]\,\partial s/\partial t + s/(3t_r) \end{aligned}\right\} \quad \text{(90)}$$

where $s = \sigma_x + \sigma_y$, the total rates of strain $\dot{\epsilon}_x = \partial\epsilon_x/\partial t$, $\dot{\epsilon}_y = \partial\epsilon_y/\partial t$ may be considered as given constants, and the two "relaxation times"

$$t_r = \mu/G \qquad T_r = 3\,(1-\nu)\,t_r/(1+\nu) \quad \text{(91)}$$

would be known functions of depth y. It is interesting that in this kind of viscous readjustments of stress, the mean stress $s = \sigma_x + \sigma_y$ would change at a different rate, due to the different magnitude of the relaxation time T_r influencing an exponential time function term on which s would depend, than the stresses σ_x and σ_y, which also depend on the relaxation time t_r. Eq. (89) could be integrated supplying the right sides of (90) which then in turn could also be integrated with respect to the time t.

In spite of some of the simplifying assumptions previously mentioned , the circumstances to which the preceding remarks referred might perhaps be a little indicative of what might be expected to happen in the solid crust on account of the rapidly increasing temperatures under ground.

Acknowledgments--Acknowledgments are due to J. A. Hutcheson, Vice President and Director of the Westinghouse Research Laboratories in East Pittsburgh, Pa., for kindly offering the author the facilities of the Laboratories for completing this paper, to J. Getsko, D. W. Glasser, and Jean Hoffmann for valuable help received in the Drafting Department, and to Eleanor Lycett for preparing the manuscript.

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(Manuscript received April 10, 1951, and, as revised, September 12, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 2, 1951; open for formal discussion until September 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

DISCUSSION OF

"A GENERAL FORMULA FOR HYDROLOGIC FREQUENCY ANALYSIS"

by VEN TE CHOW

[Trans., v. 32, pp. 231-237, 1951]

E. J. Gumbel (441 Ocean Ave., Brooklyn 26, N. Y.)--Chow's comparison of different distributions used in hydrology raises a number of interesting problems. He reduces a variate, say, the flood, by subtracting the mean and dividing the difference by the standard deviation, a procedure which has long been customary in statistics. Conversely, the variate becomes a linear function of the reduced variate which he expresses by the return period. This procedure is perfectly general, and not limited to hydrology.

The author shows that the logarithmically normal distribution (Hazen's method) leads to results similar to those obtained by the distribution of extreme values if the skewness (the third moment about the mean divided by the third power of the standard deviation) is in the neighborhood of unity. It would be interesting to investigate the converse problem, namely: What are the

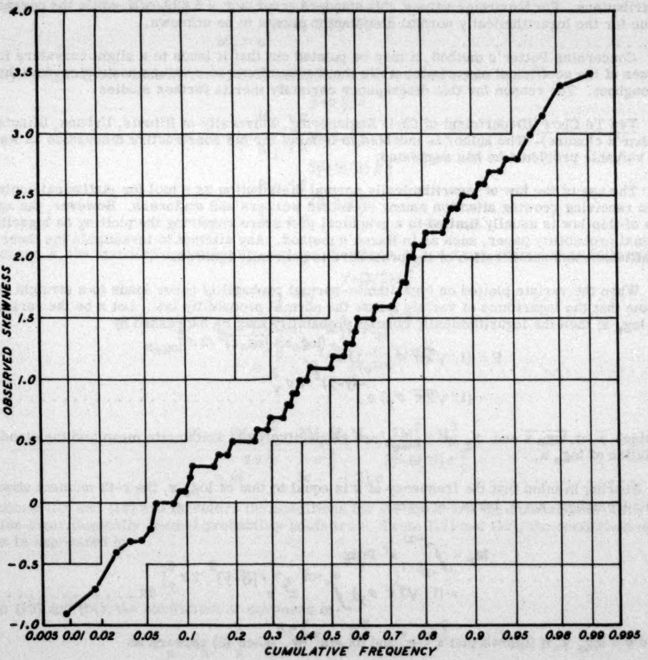

Fig. 10--The normal distribution of the skewness conditions for the mean and the standard deviation such that extremal distributions plotted on logarithmically normal paper also lead to straight lines?

The values of the skewness calculated by Hazen for 109 American streams have a mode in the neighborhood of unity. Chow disregards the enormous value 5.5 for the Arkansas River at Pueblo, Colorado, a procedure which is legitimate since this value departs from the 108 other observations by an amount which cannot be explained as due to chance.

The theory of extreme values postulates 1.139 for the skewness. This does not mean that every river with floods that can be analyzed by the theory of extreme values ought to have this skewness independent of the length of the record. The statistical meaning of the statement is instead: If we have a sufficiently large number of streams and a sufficiently large number of observed floods for each stream, the values of the skewness should be normally distributed about the mean 1.139. To test this statement, the 108 values, ordered in increasing magnitude, are plotted in the graph (see Fig. 10) on normal probability paper at the positions m/109. Evidently they are scattered about a straight line. The mean value is 1.237, and the standard deviation is 0.91. The departure of the mean from its theoretical value is due to the fact that the observations are based on small numbers, about 25 years. The difference in the means is not significant. Consequently, the 108 observed values give another proof for the validity of the theory of extreme values as applied to the floods, although the 1921 flood of the Arkansas River, which is responsible for the value 5.5, by far exceeded the expected value. It would be interesting to repeat the calculations of Hazen now, since the number of stations and the length of the records have increased.

Chow quotes Slade who calculated the standard error σ of the skewness for the Pearsonian distributions. For N extreme values, this standard error is $\sigma = 5.629/\sqrt{N}$, while the corresponding value for the logarithmically normal distribution seems to be unknown.

Concerning Potter's method, it may be pointed out that it leads to a slight curvature for large values of the coefficient of variation while the theory of extreme values postulates straight lines throughout. The reason for this discrepancy certainly merits further studies.

Ven Te Chow (Department of Civil Engineering, University of Illinois, Urbana, Illinois, Author's closure)--The author is indebted to Gumbel for his constructive discussion as well as the valuable problems he has suggested.

The use of the law of logarithmically normal distribution as a tool for statistical analysis has been receiving growing attention among scientific workers and engineers. However, the application of this law is usually limited to a graphical procedure requiring the plotting on logarithmic-normal probability paper, such as in Hazen's method. Any attempt to investigate the theoretical significance and qualification of the procedure is generally ignored.

When the variate plotted on logarithmic-normal probability paper leads to a straight line, it means that the logarithms of variate follow the normal probability law. Let x be the variate and $y = \log_e x$; then the logarithmically normal probability may be expressed by

$$P = (1/\sqrt{2\pi}\,\sigma_{\log_e x})\, e^{-(\log_e x - \overline{\log_e x})^2/2\sigma^2_{\log_e x}}$$

$$= (1/\sqrt{2\pi}\,\sigma_y)\, e^{-(y-\bar{y})^2/2\sigma_y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which $\bar{y}$ or $\overline{\log_e x}$ and σ_y or $\sigma_{\log_e x}$ are respectively the arithmetic mean and the standard deviation of $\log_e x$.

Bearing in mind that the frequency of x is equal to that of $\log_e x$, the r-th moment about an arbitrary origin, $x = 0$, takes the form

$$M_r = \int_{-\infty}^{+\infty} x^r\, P\, dx$$

$$= (1/\sqrt{2\pi}\,\sigma_y)\int_{-\infty}^{+\infty} x^r\, e^{-[(y-\bar{y})^2/2\sigma_y^2]}\, dx \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Since $y = \log_e x$, it follows that $x = e^y$ and $dx = e^y\, dy$. Then (2) appears as

$$M_r = (1/\sqrt{2\pi}\,\sigma_y)\int_{-\infty}^{+\infty} e^{(r+1)y-(y-\overline{y})^2/2\sigma_y^2}\,dy \;\ldots\ldots\ldots\ldots \quad (3)$$

Completing the square, the exponent of e becomes

$$\begin{aligned}(r+1)\,y - (y-\overline{y})^2/2\sigma_y^2 &= -(1/2\sigma_y^2)\,[y-(r+1)\,\sigma_y^2-\overline{y}]^2\\ &\quad + [(r+1)/2]\,[(r+1)\,\sigma_y^2+2\overline{y}]\\ &= -z^2/2 + [(r+1)/2]\,[(r+1)\,\sigma_y^2+2\overline{y}] \;\ldots\ldots\ldots \quad (4)\end{aligned}$$

where $z = (1/\sigma_y)\,[y-(r+1)\,\sigma_y^2-\overline{y}]$ from which $dy = \sigma_y dz$. Substituting this expression for dy and (4) in (3), the r-th moment is

$$M_r = (1\sqrt{2\pi})\,e^{[(r+1)/2][(r+1)\,\sigma_y^2+2\overline{y}]}\int_{-\infty}^{+\infty} e^{-z^2/2}\,dz \;\ldots\ldots\ldots\ldots \quad (5)$$

As the area beneath a normal probability curve and above its horizontal asymptote is equal to unity, or

$$(1/\sqrt{2\pi})\int_{-\infty}^{+\infty} e^{-z^2/2}\,dz = 1 \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Therefore, the r-th moment is

$$M_r = e^{[(r+1)/2][(r+1)\,\sigma_y^2+2\overline{y}]} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

By assigning the value of r = 0, 1, 2, and 3, the first four moments are

$$M_o = e^{\overline{y}+\sigma_y^2/2} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$M_1 = e^{2\overline{y}+2\sigma_y^2} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$M_2 = e^{3\overline{y}+(9/2)\,\sigma_y^2} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$M_3 = e^{4\overline{y}+8\,\sigma_y^2} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

The mean, $\overline{x}$, the standard deviation, σ_x, and the skewness, α_x, of x are computed as follows:

$$\overline{x} = M_1/M_o = e^{\overline{y}+(3/2)\sigma_y^2} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\begin{aligned}\sigma_x &= [(M_2M_o - M_1^2)/M_o]^{1/2}\\ &= (e^{\sigma_y^2}-1)^{1/2}\,e^{\overline{y}+(3/2)\,\sigma_y^2} \;\ldots\ldots\ldots\ldots \quad (13)\end{aligned}$$

and

$$\begin{aligned}\alpha_x &= (M_3M_o^2 - 3M_2M_1M_o + 2M_1^3)/M_o^3\\ &= (e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2)\,e^{3\overline{y}+(9/2)\,\sigma_y^2} \;\ldots\ldots\ldots\ldots \quad (14)\end{aligned}$$

Equations (12) and (13) are therefore the conditions for the mean and the standard deviation such that the logarithmically normal probability holds true. From (12) and (13), the coefficient of variation is expressed by

$$C_v = \sigma_x/\overline{x} = (e^{\sigma_y^2}-1)^{1/2} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

From (13) and (14), the coefficient of skewness is

$$C_s = \alpha_x/\sigma_x^3 = (e^{3\sigma_y^2} - 3e^{\sigma_y^2} + 2)/(e^{\sigma_y^2}-1)^{3/2} \;\ldots\ldots\ldots\ldots \quad (16)$$

Let us assume that the value of C_s is in the neighborhood of unity, or just take $C_s = 1$. Then, from (16), we get

$$e^{6\sigma_y^2} - 6e^{4\sigma_y^2} + 3e^{3\sigma_y^2} + 12e^{2\sigma_y^2} - 15e^{\sigma_y^2} + 5 = 0 \qquad (17)$$

Solving this polynomial for $e^{\sigma_y^2}$, $e^{\sigma_y^2} = 1, 1, 1, 1.105$ and two complex roots.

For $e^{\sigma_y^2} = 1$ or $\sigma_y = 0$, (12), (13), and (14) give $\bar{x} = e^{\bar{y}}$, $\sigma_x = 0$, and $\alpha_x = 0$ respectively. This is a point distribution, a degenerated case.

For $e^{\sigma_y^2} = 1.105$ or $\sigma_y = 0.316$, (12), (13), and (14) give $\bar{x} = 1.162e^{\bar{y}}$, $\sigma_x = 0.377e^{\bar{y}}$, and $\sigma_x = 0.0535e^{3\bar{y}}$ respectively. Accordingly, the coefficient of variation, $C_v = \sigma_x/\bar{x} = 0.325$. It is interesting to note that this value checks closely with Hazen's $C_v = 0.33$ for $C_s = 1$ [HAZEN, 1930, p. 188].

The computation of a theoretical K-T-curve for Hazen's method, when $C_s = 1$, may proceed as follows. Let

$$K_y = (y - \bar{y})/\sigma_y \qquad (18)$$

Substituting $y = \log_e x$ and $\bar{y} = \log_e(\bar{x}/1.162)$ in (18) and simplifying,

$$x/\bar{x} = 1 + (e^{K_y\sigma_y}/1.162 - 1) \qquad (19)$$

Comparing this equation with the general Eq. (7) in the original paper, we see that

$$C_vK = e^{K_y\sigma_y}/1.162 - 1$$

or the frequency factor is

$$K = (e^{K_y\sigma_y}/1.162 - 1)/C_v \qquad (20)$$

Substituting the numerical values of σ_y and C_v in (20),

$$K = (e^{0.316K_y}/1.162 - 1)/0.325 \qquad (21)$$

By (18), the integral of the probability function (1) becomes

$$\phi(P) = (1/\sqrt{2\pi})\int_{-\infty}^{K_y} e^{-K_y^2/2}\, dK_y \qquad (22)$$

This probability integral may be solved by means of any standard table for normal probability function. This $\phi(P)$ is related to the recurrence interval, T, by

$$\phi(P) = (T - 1)/T \qquad (23)$$

By means of (21), (22), and (23) the K-values for given values of T may be computed such as those given in Table 1. The K and T values for other values of C_s can be computed similarly.

In 1921, a table of Hazen's factor appeared [HAZEN, 1921, p. 21]. Regarding this table, Hazen wrote: "The factors (in the table) were reached by an approximate procedure that will not be described. They are used as a means of drawing smooth curves and do not replace, but are used as an aid to, the purely graphical procedure. If some member of the Society who enjoys mathematical problems would work out factors corresponding exactly with theoretical curves, it would be appreciated."

This table, however, was revised later [HAZEN, 1930, p. 188] by a procedure using an artificially prepared series of 25 terms each. The curves presented in Figure 3 of the original paper are based on Hazen's revised factors. For $C_s = 1$, the approximate Hazen's factors, the revised factors, and the corresponding theoretical values are listed in Table 1.

Table 1--Hazen's factors for $C_s = 1$

Frequency		Hazen factors		
		Approx.	Revised	Theoretical
pct	yr			
99	1.01	-1.50	-1.68	-1.80
95	1.05	-1.24	-1.34	-1.49
80	1.25	-0.82	-0.84	-1.04
50	2.0	-0.20	-0.15	-0.43
20	5.0	0.72	0.76	1.37
5	20	1.97	1.90	2.45
1	100	3.44	3.15	3.75
0.1	1000	5.70	4.95	5.35

Apparently, there are differences between Hazen's values and the theoretical values. Without further investigations on this subject, it is presumed that the differences are due to so-called "standard error." The computation of standard error by exact formulas is a very cumbersome job. Slade calculated this value for the Pearsonian distribution only by an approximate formula. The approximate formula for the standard error of the standard deviation of logarithmically normal distribution may take the following form

$$\epsilon(\sigma_y) = (1/\sigma_y)\,[C_v\,(8 + 106\,C_v + 332\,C_v^2 + 479 C_v^3)/12N]^{1/2} \quad \ldots (24)$$

For $C_s = 1$ in Hazen's procedure for computing revised factors the standard error of σ_y is found to be $\epsilon(\sigma_y) = 0.288$, by substituting $\sigma_y = 0.316$, $C_v = 0.325$, and $N = 25$ in (24). This error is 91.0 pct of the value of σ_y. The standard error of the coefficient of skewness may be considered as the differential error of C_s computed from (16). It is found that $\epsilon(C_s) = 3.53 \times 91.0$ pct or 321 pct. Therefore, it may be concluded for the time being, without further investigations, that the value of C_s is definitely uncertain because its standard error is approximately three times the value of C_s itself. As a result of this study a fair comparison between the logarithmically normal distribution having the value of C_s in the neighborhood of unity and the distribution of extreme values having $C_s = 1.139$ cannot be obtained. Supposing the theoretical K-T-curve and the relative position between the K-T-curve for $C_s = 1$ and the K-T-curve for $C_s = 2$ in Figure 3 of the original paper are correct; it is likely that the logarithmically normal distribution is close to the extremal distribution when the value of C_s lies somewhere between 1.00 and 2.00.

The author has noticed [UNIVERSITY OF ILLINOIS, 1949] the discrepancy between the results of Potter's study and the theory of extreme values for large values of the coefficient of variation, and he has found that this occurs when C_v is greater than about 1.00. This discrepancy may be explained in the following way. In hydrologic data it often happens that the largest value may have an actual recurrence interval which is several times greater (or smaller; however, this is unlikely in most cases as the results of Potter's study indicate) than the length of record. This can be readily observed from the plotting of a frequency curve where the point for the largest value deviates markedly from the general path of the curve of best-fit. Naturally, the use of such a value as the largest value in the computation would result in a higher value (or a smaller value) of C_v than that which the sample should indicate. In other words, this largest value would not be homogeneous with the rest of the sample. If this value is included in the computation, then the above-mentioned discrepancy would arise because the theory of extreme values cannot be satisfactorily applied to a non-homogeneous sample. It would be interesting to recompute the C_v-values without including the largest value in the computation in case such value in the data indicates a marked deviation from the frequency curve of best-fit. Anyhow, Potter's study is of great value because the number of cases for which the value of C_v is greater than 1.00 occurs only as a small percentage of the observed hydrologic events. This fact is evident from the data shown in Table 2. In this table, LANE and LEI's [1950] values of C_v are converted from their original values of "index of variability," equivalent to σ_y, by means of (15).

Table 2--Observed cases for C_v greater than 1.00

Source	Description	Range years of record	Range of last year in record	Observed cases		
				Total	No. for $C_v > 1$	pct for $C_v > 1$
HAZEN [1930]	Maximum daily floods	20-84	1911-1928	109	4	3.7
U.S.B.R. [1949]	Maximum peak discharge for western streams in U. S.	10-57	1922-1948	235	37	15.8
LANE and LEI [1950]	Mean daily flow in streams in U. S. for east of the 100-degree meridian	9-47	1910-1944	224	12	5.4

Some of Hazen's C_s-values have been revised by U. S. BUREAU OF RECLAMATION [1949]. In this revision, however, the use of data of broken series has been avoided where practical to do so. For example, the revised C_s for Arkansas River at Pueblo, Colorado, is 4.2 (compared with Hazen's 5.5), based on data of 36 years, 1909 to 1944 (compared with Hazen's 33 years, 1882-1914).

The author has computed the C_v and $\bar{y}$ values for one-hour rainfall intensities in various localities of the continental United States. Such computation was based on Yarnell's data [YARNELL, 1935]. Maps showing isohyetal lines of C_v and $\bar{y}$ are also prepared [CHOW, 1952]. The isohyetal map of C_v reveals the fact that the value of C_v for rainfall intensity of one-hour duration is generally less than 1.00. Only in Oregon and lower Washington the values of C_v are greater than 1.00, but less than 1.10.

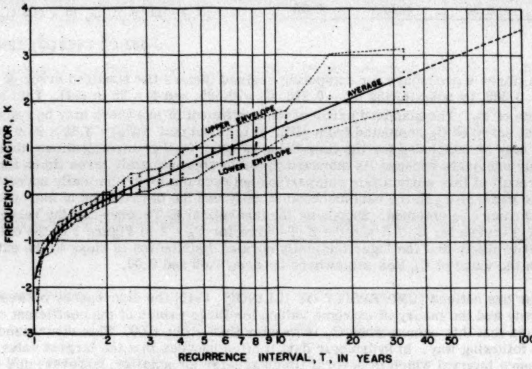

Fig. 11--T-K-curve for rainfall frequency study at Urbana, Illinois

Without employing any mathematical law to define the K-T-curve, an empirical K-T-curve may be developed from a group of hydrologic events which are found either of similar hydrologic frequency behavior or in the same region of hydrologic homogeneity. Figure 11 shows an empirical K-T-curve of this kind for the rainfall intensities at Urbana, Ill. This curve is derived from the 32-year record of rainfall intensities at Urbana of various durations. This is believed to be the K-T-curve most suitable for a general analysis of rainfall intensity frequency analysis at Urbana and its vicinity.

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REVIEWS, ABSTRACTS, AND BIBLIOGRAPHY

REVIEWS AND ABSTRACTS

NADAI, A. L., Theory of flow and fracture of solids, Volume I, Second Edition; Engineering Societies Monograph, McGraw-Hill Book Company, xxii + 572 pp., 1950.

This is the first volume of a proposed two-volume revision, bringing up to date Nadai's well-known book, Plasticity, published in 1931.

A great deal of progress has been made during the last 20 years in the theory and mathematical treatment of plastic deformation as well as in the experimental approach to its problems. Increasing temperature and pressure of operation in modern boilers and in jet engines have made problems of creep of metal parts of greater importance than ever before. The new edition aims at an improved treatment of the mechanics of permanent deformation of solids and so presents a more exhaustive and up-to-date discussion not only of problems of creep but also of such matters as methods of determining the more general mechanical laws of plastic deformation and fracture, data on tensile tests, finite homogeneous strain, limiting states of stress in solids, and strain and flow figures, including major results of recent experimental investigations made by the U. S. Government on the mode of yielding in steel, and related phenomena.

Much of the text has been rearranged and revised. Especially thorough is the revision of the treatments of the states of stress in permanently strained cylinders and discs and of the mathematical theory of the non-homogeneous states of plane plastic strain and of their surfaces of slip.

Several new chapters have been added on the theories of simple and composite substances based on types of strains and on the types of laws of deformation postulated. Other chapters have been rewritten and combined, however, so that the total number of chapters is almost the same as the number in the first edition covering the same subject matter.

An idea of the amount of material added to the revised edition is readily obtained by some comparative figures. The subject matter of volume I occupies 288 pages in the first edition; 559 pages in the second edition. This material is illustrated by 356 figures in the first edition, by 550 in the second, and many of the figures in the old edition have been replaced by new ones. That the mathematical treatments have been revised and extended is emphasized by the fact that in the first edition 35 symbols are listed; in the new edition, 65. The subject index has been expanded from 4 2/3 to 6 pages, which is still woefully inadequate for 559 pages of highly concentrated text.

In the first edition the final six chapters (Part II) were given over to a discussion of applications of the mechanics of the plastic state of matter to geology and geophysics. In the new edition these applications will be treated in volume II, as will also the flow of metals under concentrated pressure with applications to the forming processes of rolling and drawing, the theory of hardness, of residual stresses, of the forging of thin shells, of plastic buckling of thin plates, the principle of mechanical work, and examples of the flow of very viscous materials.

The scope of volume I can be indicated by the titles of the three parts into which the 37 chapters are divided: Part I, Deformation of solids, Analysis of stress and strain. Part II, The yielding of solids, particularly of the metals under simple states of stress. Part III, The elastic, the very viscous, the ideally plastic substance(s) and some of their generalizations.

It is rather surprising in a work of this scope and detail to find so little discussion of the mechanism of deformation in single crystal specimens and in individual grains of polycrystalline aggregates. This subject was treated in considerable detail, especially for metals, by Schmid and Boas in 1935, and Tertsch in 1949 prepared a more exhaustive summary with considerable emphasis on minerals as well as metals. Nadai makes no mention of preferred grain orientation developed by cold rolling, hot rolling, recrystallization, and other processes, although a considerable literature of this subject has grown up since the early 1930's. Most of this work is not merely descriptive; it explains the observed orientations in terms of direction of rolling (or drawing), temperature, the crystallographic mechanism of translation or slip, and the rotation accompanying slip in single crystals, etc. These studies contribute to our understanding of deformation; they provide information upon which to base, or with which to test, theories of the mechanism of plastic deformation and should, therefore, find some place in a treatise of this nature.

This second edition will be even more generally useful than its pioneering predecessor. It should be made widely available to metallurgists, physicists, physical chemists, engineers, geologists, geophysicists, and others who are interested in problems of the strength, plastic deformation and failure of metals, construction materials, and the geological building blocks of the Earth's crust.

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NOTES AND PERSONALIA

NOTES

Corrigendum--In the issue of the Transactions, October 1951, in the paper by Press and Ewing, page 678, the equation at the end of the paragraph opening the page was inadvertently incomplete. This should have read: $f_c = \beta_1/2H$.

SOUTHERN HOLIDAY expedition--An expedition tentatively called SOUTHERN HOLIDAY, to explore the eastern equatorial Pacific, is scheduled to leave San Diego aboard MV Horizon on May 15, 1952, returning about September 1. This expedition, part of a joint University of California - U. S. Navy Electronics Laboratory operation, will make an oceanographic study of the eastern equatorial region of the Pacific, making six hydrographic sections across the equator and carrying out a detailed study of the north equatorial current - counter current boundary between 95° and 115°W. In addition to hydrographic, meteorological, and chemical observations, biological observations and collections will be made along with a detailed bathymetric study of the region traversed. Ports of call will include Acapulco (Mexico), Puntarenas (Costa Rica), Talara and Callao (Peru), and the Galapagos Islands (Ecuador). Warren S. Wooster will be Scientific Leader of this expedition.

Report on projects in geochemistry--The Commission on Geochemical Localization of Elements of the International Union of Chemistry is gathering data on research and bibliographic projects in the field of geochemistry in order to serve as a central source of information on research in geochemistry. Those who are engaged in such work are urged to inform the Commission of the nature and scope of their work. Those in the Western Hemisphere should write to Michael Fleischer, U. S. Geological Survey, Washington 25, D. C., U. S. A., and those elsewhere to T. F. W. Barth, Geological Museum, Oslo 45, Norway.

Fifth Hydraulics Conference--Transportation of sediment has been chosen as the theme of the Fifth Hydraulics Conference to be held at the Iowa Institute of Hydraulic Research, Iowa City, Iowa, June 9-11, 1952. Further information can be secured by addressing the Institute.

Fellowships and assistantships in oceanography at Texas A. and M. College--Three fellowships and 20 research assistantships in oceanography are available to qualified graduates in physical and biological sciences and in engineering for 1952-53. The fellowships include the United Gas Fellowship in Engineering Oceanography $3,000, the Dow Fellowship in Chemical Oceanography $2,000, and the Texas A. and M. Research Foundation Fellowship in Physical Oceanography $1,500. Assistantships in these fields and in biological, geological, and meteorological oceanography provide $900 to $1,500 each. Fellows and assistants take standard curricula leading to the M.S. or Ph.D. degree with a major in some aspect of oceanography. Further information may be obtained by writing to the Head of the Department of Oceanography, College Station, Texas.

Canadian vessel for oceanographic studies--The Canadian Joint Committee on Oceanography, representing the Hydrographic Service, the National Research Council of Canada, the Fisheries Research Board, and the Royal Canadian Navy, has commissioned the former corvette Sackville as an oceanographic vessel to aid in studies in the Atlantic and Pacific Oceans.

Third National Surveying Teacher's Conference--The Third National Surveying Teacher's Conference, sponsored by the American Society for Engineering Education and the University of Illinois, will be held at Camp Rabideau, Minnesota, the University of Illinois Summer Surveying Camp, during the period August 3-8, 1952. Further information can be secured from Milton O. Schmidt, Surveying Building, University of Illinois, Urbana, Illinois.

Conference on aerial photography--A course on the theory and practice of aerial photography is to be given in Paris from June 23 to August 1, 1952 under the auspices of the National Geographic Institute. Interested persons may secure more information by addressing Le Directeur, l'Ecole Nationale des Sciences Géographiques, 35, Boulevard des Invalides, Paris 7^{e}, France.

Graduate work in meteorology at the University of Washington--The Department of Meteorology and Climatology at the University of Washington has now been authorized to grant the Ph.D. degree.

Increase in proved reserves of liquid petroleum and natural gas--The American Petroleum Institute has announced that an all-time peak in proved petroleum reserves was reached in 1951, this being 32.2 billion barrels of liquid petroleum, an increase of 2.7 billion barrels over 1950; and 193.8 trillion cu ft of natural gas, an increase of 8.2 trillion cu ft over 1950.

Voyage of oceanographic survey ships--The United States Navy oceanographic survey ships, USS San Pablo and Rehoboth, arrived in Dublin, Ireland, on February 13, 1952 engaged in survey investigations in the Atlantic. On board are Navy civilian scientists W. H. Littlewood, A. L. Brown, S. Saccoccia, R. L. Wilbrandt, J. Dail, A. Rarick, C. Hard, W. Magnitsky, H. Bernier, W. Thorp, C. Dudley, W. J. Jucksch, O. Dixon. The two vessels reached Bergen, Norway, on March 2, and Greenock, Scotland, on March 10.

Establishment of the Kirk Bryan Award--Through the efforts of students and friends of the late Kirk Bryan, geologist, geomorphologist, archeologist, and student of the Pleistocene, a Kirk Bryan Memorial Fund has been established in the Geological Society of America. The interest from this Fund will be used to finance an Award and stipend to be given the author of an outstanding published paper dealing with geomorphology or subjects related to the wide field of Bryan's interest. This Award will be made annually if a suitable paper is chosen, and will be presented at a meeting of the Geological Society of America. Contributions for this Fund are now being received by the Geological Society. Further details will be gladly furnished by the Secretary of the Geological Society of America, 419 W. 117th Street, New York 27, N. Y.

PERSONALIA

Archie Blake has accepted a position as head of the IBM Computing Section, Cornell Aeronautical Laboratory, Buffalo, N. Y.

Leo Otis Colbert, Director of the U. S. Coast and Geodetic Survey, retired, has been appointed Director of the Washington office of the Arctic Institute of North America, succeeding A. L. Washburn.

Morgan J. Davis, Vice President and Director in Charge of Exploration, Humble Oil and Refining Co., has been elected President of the American Association of Petroleum Geologists.

Earl G. Droessler, formerly with the Office of Naval Research, has taken up duties as the Executive Director of the Committee on Geophysics and Geography of the Research and Development Board of the Department of Defense.

Nicholas V. Feodoroff, Associate Professor of Civil Engineering at Manhattan College, has been named a Fellow of the New York Academy of Sciences.

John G. Gabelman has resigned his position as Geologist with the Colorado Fuel and Iron Corp. to accept one as Geologist with the American Smelting and Refining Co. in Salt Lake City.

George Gamow, Professor of Theoretical Physics at George Washington University, has gone to Japan on a lecture tour as one of four international figures invited by the Tokyo newspaper, Asahi, in connection with the Japanese Peace Treaty celebration.

William D. Johnston, Jr., Chief of the Foreign Geology Branch, U. S. Geological Survey, has been elected to foreign membership in the Academy of Sciences of Brazil.

J. Wallace Joyce, Deputy Science Adviser, Department of State, represented the United States as an observer at the British Commonwealth Scientific Official Conference held in Australia recently.

Raphael G. Kazmann has joined with Thomas J. Fricke to establish the firm of Fricke and Kazmann, Consulting Engineers, with headquarters in Stuttgart, Arkansas, dealing chiefly in water problems, both surface and underground.

J. P. Minton, Research Advisor, Field Research Laboratories of the Magnolia Petroleum Company, has been granted a special leave of absence, but will continue to serve the Laboratories on a part-time basis. He will make his residence with his son, M. C. Minton, 112 Welleslay Blvd., San Antonio, Texas.

Edmund M. Spieker, Chairman, Department of Geology, Ohio State University, delivered a series of lectures before various geological groups over the country under the auspices of the American Association of Petroleum Geologists. He lectured on the chronology of mountain building and the nature of the geologic time scale.

John C. Stevens, who has long headed the firm, Stevens and Koon, Consulting Engineers of Portland, Oregon, has organized a new firm, Stevens and Thompson, Engineers, upon the retirement of R. E. Koon.

Albert L. Washburn, until recently Director of the Washington office of the Arctic Institute of North America, has taken up duties as Scientific Director of the Snow, Ice, and Permafrost Research Establishment of the Corps of Engineers with headquarters in Wilmette, Illinois.

Paul Felix Neményi, Head of the Theoretical Mechanics Section of the Naval Research Laboratory, died March 1, 1952 at the age of 56. He was born in Fiume, then in Hungary, and studied civil engineering at the Institute of Technology of Budapest, receiving his degree in 1918. He pursued further studies at the Institute of Technology in Berlin, from which he took the Doctor of Science degree in mathematics in 1922. He worked as a structural and hydraulic engineer in central Europe and in England, and occupied teaching positions in several technical institutes. He came to the United States in 1938 and worked and taught at several engineering schools, taking up duties with the Naval Research Laboratory in 1949. He had been a member of AGU since 1940.

Charles I. Singer, New York physician and student of spas, died December 10, 1951 at the age of 55. He was born in Vienna, Austria, studied at the University of Budapest, and took his medical degree from the University of Prague in 1921. He had long been interested in the relation of climate to health, and the use of natural mineral waters and muds in biological and patho-biological problems and in the treatment of human ills. He had been a member of AGU since 1948.

Theodore Zuschlag, Vice President of the Magnetic Analysis Corporation, died in West Englewood, New Jersey, January 14, 1952 at the age of 56. He was born in Kassel, Germany, and took his baccalaureate degree in electrical engineering at the Technical Institute in Dresden in 1922. He worked at the Erda Geophysical Institute in Goettingen for three years, and came to the United States in 1925. From that date until 1928, he engaged in research work with the Tanmac Corporation in New York, then became Director of Research for the Swedish-American Prospecting Corporation. In 1931, he took up duties as Chief Engineer with the Magnetic Analysis Corporation. He had been a member of AGU since 1950.

Transactions, American Geophysical Union Volume 33, Number 2 April 1952

ADDITIONS TO THE MEMBERSHIP, SEPTEMBER 11, 1951 TO MARCH 10, 1952

Adams, Mr. Richard M., P. O. Box 1863, College Station, Tex. (ce)
Anderson, Mr. O. Kenneth, U. S. Weather Bureau, 657 Federal Bldg., Louisville 2, Ky. (c)
Anderson, Mr. Richard C., P. O. Box 74, Campus Sta., Socorro, N. Mex. (dh)
Autrey, Mr. C. B., Route 5, Many, La. (b)
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Bramson, Lt. Jay H., International Ice Patrol, Woods Hole, Mass. (ef)
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Ewart, Mr. George Y., Kekaha Sugar Co., Kekaha, Kauai, Hawaii (g)
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SPECIAL ANNOUNCEMENTS

Regional Meeting in Chicago, Illinois, September 11-12, 1952--As noted in previous Special Announcements [see Feb. 1952, p. 147], the Twenty-Eighth Regional Meeting will be held in Chicago, September 11-12, 1952. The Chairman of the Committee is W. S. Hamilton, Department of Civil Engineering, Northwestern University, who has taken over this assignment from Horace R. Byers who has been elected President of the American Meteorological Society and thus felt it desirable to give up this chairmanship. Interested persons should get in touch with Professor Hamilton or one of the other members of the Committee.

New Directory of Members--It is the plan to publish a new directory of members of AGU as of September 10, 1952, this to be part of the October 1952 issue of the *Transactions*. Members are urged to see that their correct addresses are a matter of record (as shown by the address label on the *Transactions*) in the office of AGU, especially as the September 10 date approaches.

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PROGRAM

THIRTY-THIRD ANNUAL MEETING

In the Building of the National Research Council - National Academy of Sciences,
and the Auditorium of the General Services Building,
Washington, D. C.

May 5-7, 1952

SCHEDULE OF SESSIONS

Day	Date	Time	Room 1[a]	Room 2[a]	Room 3[a]
	1952				
Monday	May 5	09h 00m	T. M. and E.	Volcanology	Oceanography
		14h 00m	Hydrology	Oceanography	T. M. and E.
		20h 00m	....	Evening Session	....
Tuesday	May 6	09h 00m	Meteorology[d]	Seismology	Hydrology
		14h 00m[b]	Hydrology	Geodesy	Meteorology
		20h 00m[c]	....	....	....
Wednesday	May 7	09h 00m	Geodesy	Tectonophysics	Hydrology
		14h 00m	General Assembly	....	....

[a] For key to room numbers, see page 312.

[b] In addition, there is to be a session in Oceanography, a Round Table on the Problems of the Gulf Stream System, in the Reading Room of the National Academy of Sciences at 14h 00m.

[c] The Smoker will be held in the Auditorium of the Cosmos Club, Madison Place near H St., N.W. from 20h to 22h.

[d] Symposium on the Upper Atmosphere, session held jointly by Sections of Meteorology and of Terrestrial Magnetism and Electricity and the American Meteorological Society.

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John A. Fleming, Honorary President
Walter H. Bucher, President
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J. P. Marble, Chairman
Woodrow C. Jacobs
Herbert S. Riesbol
Walter E. A. Ruska
Glenn E. Stout
Frank J. Veihmeyer

(Note: This program is paginated in the Transactions of 1952. Preserve for binding.)

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Terrestrial Magnetism and Electricity	J. W. Joyce	Victor Vacquier	L. R. Alldredge
Oceanography	A. C. Redfield	Roger R. Revelle	R. H. Fleming
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Hydrology	F. J. Veihmeyer	Walter B. Langbein	Ray K. Linsley
Tectonophysics	Robert Balk	Francis Birch	Wilbur S. Burbank

GENERAL MEETINGS OF THE UNION

EVENING SESSION

Monday, May 5, 1952, 20h 00m, Room 2 (Lecture Room, National Academy of Sciences)

Lecture: The Origin of the Elements
R. A. ALPHER and R. C. HERMAN
Fourteenth Award of the William Bowie Medal
(a) Citation by Walter D. Lambert, Chairman of the Committee on the Award
(b) Presentation by Walter H. Bucher, President
(c) Response
Social hour and refreshments

SMOKER

Tuesday, May 6, 1952, 20h 00m to 22h 00m, Auditorium of the Cosmos Club,
Madison Place near H Street, Northwest

This is our usual informal gathering where opportunity is afforded to renew old acquaintances and make new ones. No program is planned. Refreshments will be served. Tickets are 75 cents each, and will be sold in advance at the Main Registration Desk in the Building of the National Academy of Sciences - National Research Council, and by the Section Secretaries or others whom they may designate. Ladies are invited.

GENERAL ASSEMBLY

Wednesday, May 7, 1952, 14h 00m, Room 1 (Auditorium, National Academy of Sciences)

Lecture: Experimental and Theoretical Operations Research, Its Applications in Combat, Industry, and Science
ELLIS A. JOHNSON
Business Meeting (Effort will be made to keep this within our usual half hour. Please plan to attend.)
(1) Reports of officers and special and standing committees, including:
(a) General Secretary's report
(b) Report of the Committee on Resolutions (Mary C. Rabbitt, Chairman; Werner A. Baum; and Francis G. Wells)
(2) Unfinished business
(3) New business

MEETINGS OF THE SECTIONS

SECTION OF GEODESY

Tuesday, May 6, 1952, 14h 00m, Room 2 (Lecture Room, National Academy of Sciences)

Business Session

Reports of committees and other business

Scientific Sessions

(1) **J. E. LILLY and P. BRUNAVS:** Triangulation closure investigation (15 minutes)
(2) **SUMNER B. IRISH:** Mathematics of measuring the precision of a level (10 minutes)
(3) **LEWIS V. JUDSON:** Tape testing at the National Bureau of Standards (15 minutes)
(4) **A. D. SOLLINS:** Zenith camera observations in Liberia (10 minutes)

Wednesday, May 7, 1952, 09h 00m, Room 1 (Auditorium, National Academy of Sciences)

(5) **ARTHUR S. G. GRANT:** The application of ray tracing methods to the reduction of shoran to map distances (15 minutes)
(6) **CARL I. ASLAKSON:** Relative geoidal undulations from deflections of the vertical (15 minutes)
(7) **R. A. HIRVONEN:** Gravity anomalies and deflections of vertical above the sea level (15 minutes)
(8) **W. A. HEISKANEN:** The geophysical applications of the gravity anomalies (15 minutes)
(9) **C. H. G. OLDHAM:** Relation between gravity anomalies and Precambrian rock densities near Parry Sound, Ontario (10 minutes)
(10) **J. LAMAR WORZEL, G. LYNN SHURBET, and MAURICE EWING:** Gravity measurements at sea, 1951 (10 minutes)
(11) **B. F. HOWELL, JR. and KEEVA VOZOFF:** Gravity investigation in north-central Pennsylvania (10 minutes)
(12) **MANUEL MEDINA:** Geodetic work in Mexico from May 1951 to April 1952

SECTION OF SEISMOLOGY

Tuesday, May 6, 1952, 09h 00m, Room 2 (Lecture Room, National Academy of Sciences)

Business Session

Reports of committees and other business

Scientific Session

(1) **CHIA-SHUN YIH:** On the natural frequency of Rayleigh waves propagating in two dimensions (by title)
(2) **DEAN S. CARDER:** The continents and ocean floor as transmitting media of microseisms
(3) **L. H. ADAMS, J. W. GREIG, P. A. SCHERER, H. E. TATEL, and M. A. TUVE:** Studies of crustal structure by explosion seismic waves near the Mesabi Range and near Puget Sound
(4) **I. LEHMANN:** P and S at distances smaller than 25°
(5) **MAURICE EWING and FRANK PRESS:** Further studies of atmospheric pressure fluctuations recorded on microbarographs and long-period vertical seismographs
(6) **ROGER M. WILSON:** Dynamic testing of seismograph galvanometers
(7) **ELLIOTT B. ROBERTS:** Progress report on seismological activities of the U. S. Coast and Geodetic Survey (by title)

SECTION OF METEOROLOGY

Tuesday, May 6, 1952, 09h 00m, Room 1 (Auditorium, National Academy of Sciences)

Business Session

Reports of committees and other business

Scientific Sessions

Symposium on the Upper Atmosphere (Jointly with the Section of Terrestrial Magnetism and Electricity and with the American Meteorological Society)

(1) **D. H. MENZEL:** The origin and physical state of the Aurora Borealis (40 minutes)
(2) **R. J. HAVENS and OTTO BERG:** Density of the Earth's atmosphere at 219 kilometers (10 minutes)
(3) **FRANCIS S. JOHNSON:** High-altitude diurnal temperature changes due to ozone absorption (15 minutes)
(4) **MICHAEL FERENCE, JR.:** Temperatures in the Earth's upper atmosphere (15 minutes)

(5) **ALLEN G. WEISNER**: Determination of winds above thirty kilometers (15 minutes)
(6) **MYRON G. H. LIGDA**: Seasonal variations of high-level wind directions studied by means of long-enduring meteor trains (20 minutes)
(7) **R. J. HAVENS** and **H. SPITZ**: Atmospheric winds at 200 kilometers (10 minutes)
(8) **A. ARNOLD**: Turbulence in the stratosphere (10 minutes)

Tuesday, May 6, 1952, 14h 00m, Room 3 (Auditorium, General Services Building)

Cloud Physics (Jointly with the American Meteorological Society)
(9) **H. J. AUFM KAMPE** and **H. K. WEICKMANN**: Trabert's formula and the determination of the water content in clouds (15 minutes)
(10) **H. J. AUFM KAMPE** and **H. K. WEICKMANN**: Some properties of cumulus clouds (15 minutes)
(11) **RAYMOND WEXLER**: Precipitation growth in the Workman-Reynolds thunderstorm cell (20 minutes)
(12) **MYRON G. H. LIGDA**: Some preliminary results of a study of precipitation cell movement (20 minutes)
(13) **J. S. MARSHALL**: Precipitation trajectories and patterns (20 minutes)
(14) **ROSCOE R. BRAHAM, BEN K. SEELY**, and **W. D. CROZIER**: A technique for tagging and tracing air parcels (20 minutes)
(15) **FERGUSON HALL, T. J. HENDERSON**, and **STUART A. CUNDIFF**: Cloud seeding in the Sierra near Bishop, California (20 minutes)
(16) **ROLAND J. BOUCHER**: Results of cloud-droplet impinger measurements on Mount Washington, N. H. (15 minutes)

(Note: Sessions of the American Meteorological Society will continue Wednesday morning, May 7, 1952, through Friday afternoon, May 9, 1952, in the Auditorium of the Natural History Building of the National Museum at Tenth Street and Constitution Avenue, N.W., and in the Auditorium of the General Services Building. In arranging the above program and that of the American Meteorological Society, some papers transmitted to the American Geophysical Union for its program have been transferred to the program of the American Meteorological Society, in the interests of unity of program content. Likewise, some papers sent for the program of the American Meteorological Society have been included in the program above.)

SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY

Monday, May 5, 1952, 09h 00m, Room 1 (Auditorium, National Academy of Sciences)

Business Session
Reports of committees and other business
Scientific Sessions
(1) **E. O. SCHONSTEDT** and **H. R. IRONS**: NOL vector airborne magnetometer, type 2A (15 minutes)
(2) **L. HURWITZ**: Reduction of airborne magnetometer results (15 minutes)
(3) **GEORGE B. WALKER** and **PATRICK L. O'DEA**: Secular-change impulses (15 minutes)
(4) **E. J. CHERNOSKY**: The phased superposed epoch method of analysis and its application to geomagnetic activity (10 minutes)
(5) **E. J. CHERNOSKY, R. M. COON**, and **E. MAPLE**: A study of the short-period variation of the vertical component of the Earth's magnetic field at Tucson, Arizona (15 minutes)
(6) **A. W. SULLIVAN, H. M. VANVALKENBURG**, and **J. M. BARNEY**: Low-frequency atmospheric radio noise in Florida (15 minutes)
(7) **E. H. VESTINE** and **I. LANGE**: On variations of the geomagnetic field, fluid motions, and the rate of the Earth's rotation (15 minutes)
(8) **J. R. BALSLEY, A. F. BUDDINGTON**, and **J. J. FAHEY**: Titaniferous hematite and Ilmenohematite correlated with inverse polarization in rocks of the northwestern Adirondacks, New York (15 minutes)
(9) **G. R. WAIT**: Conductivity of the air resulting from radioactive matter in thunderclouds (15 minutes)
(10) **C. W. GARTLEIN** and **D. S. KIMBALL**: A graphic method of reporting aurora observations (15 minutes)
(11) **C. W. GARTLEIN**: Hydrogen and the aurora (15 minutes)
(12) **H. FREEBORN JOHNSTON**: Thesaurus of annual values at geomagnetic observatories (by title)
(13) **ELLIOTT B. ROBERTS**: Current magnetic work of the U. S. Coast and Geodetic Survey (by title)

Monday, May 5, 1952, 14h 00m, Room 3 (Auditorium, General Services Building)

(14) H. A. MILEY, E. H. CULLINGTON, and J. F. BEDINGER: Day sky brightness measured by rocketborne photoelectric photometers (15 minutes)
(15) V. F. HESS, W. F. BURNS, and W. D. PARKINSON: Gamma radiation from uranium X_2 (15 minutes)
(16) W. D. PARKINSON and V. F. HESS: Beta radiation from the ground (10 minutes)
(17) J. CARL SEDDON: Propagation measurements in the ionosphere by means of rockets (15 minutes)
(18) A. G. McNISH: Sweep-frequency oblique incidence experiments (15 minutes)
(19) A. H. SHAPLEY: North Pacific Radio Warning Service (15 minutes)
(20) R. W. KNECHT: Relationship between aurorae and sporadic E echoes (15 minutes)
(21) C. G. STERGIS and S. C. CORONITI: On the lifetime of atmospheric ions (15 minutes)

Tuesday, May 6, 1952, 09h 00m, Room 1 (Auditorium, National Academy of Sciences)

Symposium on the Upper Atmosphere (Jointly with the Section of Meteorology and the American Meteorological Society)
See Section of Meteorology for details of program.

SECTION OF OCEANOGRAPHY

Monday, May 5, 1952, 09h 00m, Room 3 (Auditorium, General Services Building)

Business Session
Reports of committees and other business
Scientific Sessions
(1) C. A. BARNES and JOHN H. LINCOLN: An hydraulic tidal model of the Puget Sound area (15 minutes)
(2) ROBERT G. PAQUETTE, JOHN H. LINCOLN, and C. A. BARNES: A precision water-level recorder for small-scale hydraulic models (10 minutes)
(3) JOSEPH E. O'HARE, QUICK H. CARLSON, and WILLIAM E. TAMBLYN: Some results of a tanker survey of the Gulf Stream (20 minutes)
(4) DALE F. LEIPPER and ROBERT O. REID: Preview of computed ocean currents in the Gulf of Mexico
(5) D. W. PRITCHARD: The effect of horizontal mixing on the absolute pressure field in a coastal plain estuary (20 minutes)
(6) LEWIS B. LAW: Sea-ice information (20 minutes)
(7) I. I. SCHELL: Longer period fluctuations in the ice of the North Atlantic-Arctic and in the precipitation (15 minutes)
(8) FRANCIS E. ELLIOTT, WILLIAM H. MYERS, and WILLIS L. TRESSLER: A comparison of the environmental characteristics of some shelf areas of eastern United States (15 minutes)
(9) GILBERT JAFFE and JAMES H. HUGHES: The radioactivity of bottom sediments in Chesapeake Bay (10 minutes)

Monday, May 5, 1952, 14h 00m, Room 2 (Lecture Room, National Academy of Sciences)

(10) JOHN LYMAN: The oceanographic activities of the Hydrographic Office, 1949-1952 (10 minutes)
(11) DALE F. LEIPPER: Oceanography at Texas A. and M. College, annual report 1951-52
(12) RICHARD H. FLEMING: Oceanographic activities at the University of Washington (15 minutes)
(13) S. A. LYNCH: History of geologic thought on Gulf of Mexico (15 minutes)
(14) R. S. ARTHUR, W. H. MUNK, and J. D. ISAACS: The direct construction of wave rays (15 minutes)
(15) WILLARD J. PIERSON, JR. and WILBUR MARKS: Power spectrum analysis of ocean wave records (20 minutes)
(16) WILLARD J. PIERSON, JR.: A unified mathematical theory for the analysis, propagation, and refraction of storm-generated ocean surface waves (part I) (20 minutes)
(17) E. ALLAN WILLIAMS and JOHN D. ISAACS: The refraction of groups and of the waves which they generate in shallow water
(18) ARTHUR SHAPIRO: The effect of a broken ice field on water waves

Tuesday, May 6, 1952, 14h 00m (Reading Room, National Academy of Sciences)

Round Table on the Problems of the Gulf Stream System

(19) HENRY STOMMEL: A brief review of Gulf Stream investigations from 1949 to 1951

(20) FREDERICK C. FUGLISTER: Recent temperature surveys of the Gulf Stream system

(21) WILLIAM L. FORD: Nature and origin of cold, low-salinity water adjacent to the Gulf Stream

(22) FLOYD M. SOULE: Some measurable variables which are related to the position of the northern boundary of the North Atlantic eddy

(23) WILLIAM S. von ARX: Oceanographic experiments in a rotating paraboloidal basin

(24) C. O'D. ISELIN: Future plans at the Woods Hole Oceanographic Institution for Gulf Stream investigations

SECTION OF VOLCANOLOGY

Monday, May 5, 1952, 09h 00m, Room 2 (Lecture Room, National Academy of Sciences)

Business Session

Reports of committees and other business

Scientific Session

(1) H. KUNO: Formation of calderas and magmatic evolution (15 minutes)

(2) R. E. WILCOX, O. GATES, and H. A. POWERS: Geology of the western islands of the Aleutian Arc (presentation by Mr. Powers) (12 minutes)

(3) T. P. THAYER: The tuff member of the Rattlesnake formation of eastern Oregon; an ignimbrite (12 minutes)

(4) E. WILLIAM HEINRICH: Paragenesis of veins of the sillimanite-group mineral (12 minutes)

(5) J. C. MAXWELL and P. VERRALL: Permanent expansion and increase in permeability of carbonate rocks on heating (presentation by Mr. Maxwell) (12 minutes)

(6) J. F. SCHAIRER and K. YAGI: The stability relations of iron cordierite and almandine garnet (12 minutes)

(7) R. J. UFFEN: A method of estimating the melting-point gradient in the Earth's mantle (12 minutes)

(8) L. H. AHRENS, W. H. PINSON, and M. M. KEARNS: Association of Rb and K in common igneous rocks and meteorites (presentation by Mr. Ahrens) (12 minutes)

(9) L. F. HERZOG, L. T. ALDRICH, W. HOLYK, and L. H. AHRENS: Variations in the Sr^{87} content of Sr extracted from biotite (presentation by Mr. Herzog) (12 minutes)

(10) L. A. SANDERMAN and C. L. UTTERBACK: The radium content of varved clays in the Seattle region (presentation by Mr. Utterback) (12 minutes)

(11) H. A. Slack: Field measurement of the radioactivity of rocks (12 minutes)

(12) ARNOLD C. MASON and HELEN L. FOSTER: Diversion of lava flows at O Shima, Japan (by title)

SECTION OF HYDROLOGY

Monday, May 5, 1952, 14h 00m, Room 1 (Auditorium, National Academy of Sciences)

Scientific Session

(R1) W. C. ACKERMANN: Report of Research Committee on Precipitation (10 minutes)

(1) E. E. SANDERSON and D. O. JOHNSTONE: Accuracy of determination of annual precipitation over a given area (presentation by Mr. Sanderson) (15 minutes)

(2) C. C. WARNICK: Experiments with windshields for precipitation gages (15 minutes) (Discussers: A. T. LENZ, W. T. WILSON, W. U. GARSTKA, and C. F. BROOKS)

(3) J. L. PAULHUS: Interpolation of missing precipitation records (15 minutes) (Discusser: JAMES SMALLSHAW)

(4) ALBERT S. FRY: Effect of reforestation upon hydrologic characteristics of Henderson County watershed (15 minutes) (Discussers: M. D. HOOVER and H. G. WILM)

(5) JAMES SMALLSHAW: Some precipitation - altitude studies of the Tennessee Valley Authority (15 minutes) (Discussers: T. W. KLEINSASSER and W. T. WILSON)

(6) SOL D. RESNICK: Analysis of results of rain-making projects in the western states (15 minutes)

Tuesday, May 6, 1952, 09 00m, Room 3 (Auditorium, General Services Building)

Business Session
- (a) Reports of officers
- (b) Report of Committee on Honors and Awards
- (c) Other business

Scientific Sessions

(R2) C. F. IZZARD: Report of Research Committee on Runoff (10 minutes)
(7) J. S. SWEET: Better definitions necessary in the development of unit hydrographs (15 minutes)
(8) W. D. POTTER: Rainfall and topographic factors that affect runoff (15 minutes)
(9) E. J. GUMBEL: Preliminary report on the statistical theory of droughts (15 minutes)
(10) LOUIS H. BEAN: Forecasting floods of the Columbia River by means of a short-cut method of time-series analysis (15 minutes)
(R3) FORREST L. RHODES: Report of Research Committee on Snow (10 minutes)
(11) M. W. NELSON, H. G. WILM, and R. A. WORK: Soil priming in relation to snow surveys and flood regulation (presentation by Mr. Wilm) (15 minutes)
(R4) H. V. PETERSON: Report of Research Committee on Land Erosion (10 minutes)
(12) G. M. BRUNE: Trap efficiency of reservoirs (presentation by C. E. Ghormley) (15 minutes)
(13) M. R. CARSTENS: Accelerated motion of a spherical particle (15 minutes)
(14) H. V. PETERSON and R. F. HADLEY: Physical features of an aggrading alluvial valley (presentation by Mr. Hadley) (15 minutes)

Tuesday, May 6, 1952, 14h 00m, Room 1 (Auditorium, National Academy of Sciences)

(R5) W. W. HASTINGS: Report of Research Committee on Chemistry of Natural Waters (10 minutes)
(15) H. A. SWENSON: Geochemical relationships of water in the Powder River Basin in Wyoming and Montana (15 minutes)
(16) W. H. DURUM: Relationships of the mineral constituents in solution to stream flow, Saline River near Russell, Kansas (15 minutes)
(R6) A. R. CROFT: Report of Research Committee on Evaporation and Transpiration (10 minutes)
(17) F. J. VEIHMEYER: Use of water by native vegetation versus grasses and forbs on range lands (15 minutes)
(R7) W. O. FIELD, JR.: Report of Research Committee on Glaciers (10 minutes)
(18) R. L. IVES: Anomalous glacial deposits in the Colorado Front Range area, Colorado (10 minutes)
(19) JOEL E. FISHER: The several sources of heat of a cold glacier (15 minutes)
(20) C. J. HEUSSER: Post-Wisconsin history of the region of the Juneau Ice Field as revealed by pollen analysis (15 minutes)
(R8) J. W. JOHNSON: Report of Research Committee on Dynamics of Streams (10 minutes)
(21) W. M. OWEN: Correlation between pipe flow and uniform flow in a triangular open channel (15 minutes)
(22) A. N. STRAHLER: Application of geomorphology to practical hydrology (15 minutes)
(23) L. B. LEOPOLD: Hydraulics in quantitative geomorphology - a progress report (15 minutes)

Wednesday, May 7, 1952, 09h 00m, Room 3 (Auditorium, General Services Building)

(R9) J. F. POLAND: Report of Research Committee on Ground Water (10 minutes)
(24) MAHDI HANTUSH: Three-dimensional flow in leaky aquifers (15 minutes)
(25) W. B. NELSON and H. E. THOMAS: Pumping from wells on the floor of the Sevier Desert, Utah (15 minutes)
(26) VAUGHN E. HANSEN: Complicated well problems solved by the membrane analogy (15 minutes)
(R10) L. L. HARROLD: Report of Research Committee on Infiltration (10 minutes)
(27) LEONARD SCHIFF: An exploratory study of the effect of head on infiltration rates under water-spreading conditions (15 minutes)
(R11) G. B. BODMAN: Report of Research Committee on Physics of Soil Moisture (10 minutes)
(28) S. IRMAY: On the capillary rise in a homogeneous medium composed of spheres, ellipsoids, or cylindrical fibers of different forms and sizes (by title)
(R12) J. L. HOUGH: Report of Research Committee on Lakes (10 minutes)

SECTION OF TECTONOPHYSICS

Wednesday, May 7, 1952, 09h 00m, Room 2 (Lecture Room, National Academy of Sciences)

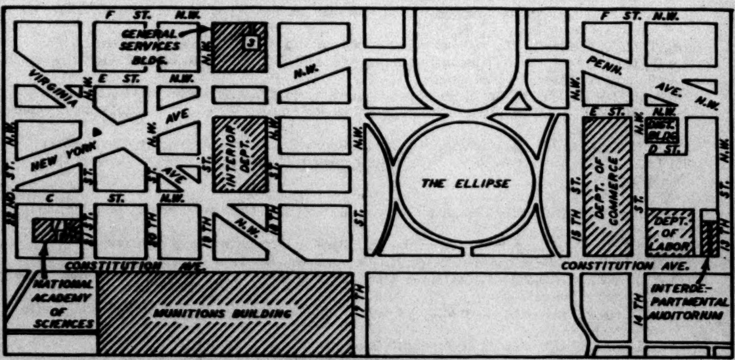
Business Session
Reports of committees and other business
Scientific Session
(1) KENNETH G. SMITH: Structure plan of clastic dikes (15 minutes)
(2) BYRON K. THOMAS: The use of the axial plane in the three-dimensional analysis of folds on Sugarloaf Mountain, Maryland (20 minutes)
(3) CHESTER R. LONGWELL: Is gravitational sliding important in orogeny? (30 minutes)
(4) J. TUZO WILSON: Structure, process, and stages of orogenesis (30 minutes)
(5) JAMES GILLULY: Geological Survey project for paleotectonic map of the United States (15 minutes)

NOTES

Room Reservations--Make hotel or room reservations directly with the hotel of your choice as soon as possible (see December 1951 issue, p. 954, for word concerning hotels at which block bookings have been made). Bear in mind that advance bookings on which deposits have not been made are cancelled two weeks in advance, and thus, during this interval, chances of making a reservation may be quite good. Even though no advance reservation has been made, it has been possible to arrange for quarters after arrival in Washington through the Housing Committee of the Washington Board of Trade (204 Evening Star Building, Washington 4, D. C., telephone REpublic 2600); but they may find it necessary to direct you to a tourist home.

Registration--All members and guests attending any session are requested to complete cards of registration at the Main Registration Desk at the entrance to the Building of the National Research Council - National Academy of Sciences, where Receptionists will be available to supply information regarding meetings, membership, telephone and mailing facilities, and other local matters. An auxiliary Registration Desk will also be maintained at the entrance to Room 3 (Auditorium, General Services Building). There is no charge for registration.

Map showing locations of meeting rooms


F ST. N.W.
GENERAL SERVICES BLDG.
3
E ST. N.W.
VIRGINIA
NEW YORK
AVE
N.W.
INTERIOR DEPT.
C ST. N.W.
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22ND ST. N.W.
21ST
20TH
19TH
18TH
17TH
CONSTITUTION AVE.
NATIONAL ACADEMY OF SCIENCES
MUNITIONS BUILDING
THE ELLIPSE
PENN. AVE. N.W.
E ST.
DEPT. OF COMMERCE
DIST. BLDG
D ST.
15TH ST.
14TH ST.
13TH ST. N.W.
DEPT. OF LABOR
INTERDEPARTMENTAL AUDITORIUM

Key to room numbers

1--Auditorium, National Academy of Sciences - National Research Council Building
2--Lecture Room, National Academy of Sciences - National Research Council Building
3--Auditorium, General Services (Old Interior) Building

Discussion--Within the available time, opportunity will be given for discussion from the floor following the oral presentation of a paper. Discussers are urged to prepare their comments for consideration for publication with the paper.

Resolutions--Chairman Mary C. Rabbitt (phone REpublic 1820, Ext. 4176) of the Union's Committee on Resolutions, desires to have this Committee fully advised by 15h 00m, Tuesday, May 6, 1952, of any resolutions to be submitted in order that this Committee may have opportunity to prepare them in final form for presentation at the Business Session of the Union on Wednesday afternoon, May 7, 1952. Each resolution presented to the Committee should include (a) names or original submitters, (b) whether presented to a Section, and action taken, and (c) whether action is desired in the Business Session or whether the resolution is to be regarded as being only for a particular Section or as one to be more effective as a general resolution of the whole Union. In addition to the Union's Committee on Resolutions, each Section has such a Committee. Section officers can supply information concerning these.

Conference Room--The Reading Room (west end of the first floor behind the Library) of the National Academy of Sciences is available by arrangement for conferences. See J. Putnam Marble, Chairman of the Committee on Meetings, for assignment.

Lounge--The Board Room (behind the Lecture Room), the Library (opposite the Lecture Room), and the Reading Room (behind the Library) (when not in use for conferences and committee meetings) of the National Academy of Sciences are available for the use of members to meet friends and to hold informal conferences. Those held in the Library should pay deference to the usual library regulations with regard to quietude.

Standard of time--The District of Columbia will be on Eastern Daylight Saving Time at the time of the meeting. Train schedules, however, are on Eastern Standard Time.

ABSTRACTS OF PROGRAM PAPERS

EVENING SESSION LECTURE

R. A. Alpher and R. C. Herman (Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Md.) The Origin of the Elements--A review will be given of the theoretical approaches to the problem of the formation and relative abundance distribution of the elements. The observational data on cosmic relative abundances as well as pertinent nuclear systematics will be discussed. The several theories to be outlined include one in which the abundance distribution is the result of a "frozen-in" thermodynamic equilibrium, another in which the elements result from non-equilibrium neutron-capture processes, a third in which elements are pictured as the fission products of large primeval nuclei, and a fourth involving a steady-state cosmology, where elements are continually being formed. The successes, failures, and cosmological implications of the several theories will be described.

GENERAL ASSEMBLY LECTURE

Ellis A. Johnson (Operations Research Office, The Johns Hopkins University, Chevy Chase, Md.) Experimental and Theoretical Operations Research, Its Applications in Combat, Industry, and Science--Operations research is a new name for a scientific discipline which has been growing for several centuries. It is the application of the scientific method to problems of action - of operations. It was successful in World War II in helping to solve combat problems and has been successful recently in Korea. It is currently being applied with success to industry and has real possibilities of application to agriculture and geophysics, in particular in ore prospecting. It is suggested that it could be applied with great value in the present crisis in science and education. Such a study needs a careful weighing of many ponderable and some imponderable variables, but might well lead to better solutions than those now current in the great universities.

SECTION OF GEODESY

(1) J. E. Lilly and P. Brunavs (Geodetic Survey of Canada, Ottawa, Canada) Triangulation Closure Investigation--Latitude and longitude closure equations are developed, taking account of the second powers of differences of latitude and longitude. In a test on a triangulation net, it is found that adjustment by means of equations based on first-order terms only leaves appreciable residual discrepancies, which are reduced to less than 0".001 by inclusion of second-order terms.

(2) Sumner B. Irish (Princeton University, Princeton, N. J.) Mathematics of Measuring the Precision of Leveling--The development of a formula expressing the precision of a level is followed from the fundamental theory and assumptions to the final equation. This equation is expressed finally in a nomograph for ease in use. The actual method of making the test is described. This includes the testing with or without the determination of the systematic error of the instrument and the testing for instruments with or without stadia cross hairs. The results achieved by this test for various types of levels will be mentioned. The comparison of test results with field experience will be made with a brief discussion of the reason for any variables. Conclusions and recommendations will be given regarding use and selection of levels for precise leveling.

(3) Lewis V. Judson (National Bureau of Standards, Washington, D. C.) Tape Testing at the National Bureau of Standards--The testing of geodetic base-line tapes and of steel tapes used by surveyors and engineers has been carried out by the National Bureau of Standards ever since it was founded. A special underground laboratory extending between the two oldest buildings was constructed, and a geodetic comparator and a steel tape-testing bench were installed. A large amount of tape standardization has been accomplished in this laboratory. Some improvements were found to be greatly needed but these were impossible to accomplish because of space limitations. The construction of a new building at the Bureau early in World War II made it possible to design and construct a new tape-testing laboratory. The paper will discuss the ideas underlining the design of this new laboratory and its equipment, and will outline what has been accomplished. Brief mention will be made of investigations now under way in regard to a possible new method of determining the coefficient of thermal expansion of base-line tapes, and will suggest other investigations that have been given some consideration.

(4) A. D. Sollins (U. S. Coast and Geodetic Survey, Washington, D. C.) Zenith Camera Observations in Liberia--The instrument, its operation, and the theory of computation are discussed briefly. The results of series of observations at eight stations in Liberia are presented.

(5) Arthur S. G. Grant (Meteorological Service of Canada and Shoran Group, Department of Mines and Technical Surveys, Geodetic Survey of Canada, Ottawa, Canada) The Application of Ray Tracing Methods to the Reduction of Shoran to Map Distances--Ray tracing methods are used to obtain formulas for the arc length of the shoran radio ray and for the corresponding geodetic map distance, in terms of the angle of inclination of the ray at the ground station, the height above the smooth Earth surface, and the vertical rate of change of the squared modified index of refraction. The method is applicable to cases where N^2 can be expressed as a multi-linear function of the elevation. Examples are worked out comparing the reduced distances with those obtained by the standard method.

(6) Carl I. Aslakson (U. S. Coast and Geodetic Survey, P. O. Box 243, Cocoa, Florida) Relative Geoidal Undulations from Deflections of the Vertical--Shoran trilateration in the Bahama Islands permits the determination of the deflection of the vertical with considerable accuracy. The deflections observed to date show a striking correlation with the ocean-bottom gradients and with the available free-air anomaly gradients. This suggests the computation of relative geoidal heights by using strategically placed deflection observations. This paper describes a method of computing the relative undulations which is promising.

(7) R. A. Hirvonen (Finland Institute of Technology, Helsinki, Finland, and Ohio State University Research Foundation, Ohio State University, Columbus, Ohio) Gravity Anomalies and Deflections of Vertical above the Sea Level--By the aid of Poisson's integral formula, the gravity anomalies, the potential of the disturbing masses, and the deflections of vertical above the sea level can be computed if the gravity anomalies at the sea level are known. This means a generalization of the well-known formulas of Stokes and Vening Meinesz. Some practical applications on the basis of presently available gravity observations are given.

(8) W. Heiskanen (Finnish Geodetic Institute, Helsinki, Finland, and Ohio State University, Columbus, Ohio) The Geophysical Applications of the Gravity Anomalies--The gravity anomalies are an important tool in the hands of the geodesists and geophysicists. For instance, the positive Bouguer anomalies of the oceans and the negative ones of the mountains show qualitatively that the isostatic equilibrium exists in broad lines. The isostatic anomalies are even able to localize the isostatically compensating layers and in such a way to determine the thickness of the "Earth's crust." Also local disturbing masses of different types are not too difficult to discover. With the rapid and accurate gravimeters beginning a new epoch in gravimetry, quite new views for detailed geophysical study of the behavior of the Earth's interior are to be presented. But the gravity anomalies must be used carefully. In some cases they too easily lead us to wrong conclusions.

(9) C. H. G. Oldham (Dept. of Physics, University of Toronto, Toronto, Canada) Relation between Gravity Anomalies and Precambrian Rock Densities near Parry Sound, Ontario--A gravity survey was conducted over an area of 2500 sq mi of Precambrian gneisses near Parry Sound. The Bouguer anomalies of the 98 gravity stations indicated the existence of an approximately circular-shaped positive anomaly of 44 mgal. Four hundred and fifty rock specimens were collected and their densities determined. A study of the gravity anomaly and rock densities indicated the existence of a basin-shaped structure with a nine-mile radius and a depth of 3 1/4 miles containing rocks with a representative density of 2.85 gm/cc. These may be correlated with an outcrop of basic gneisses. The latter are surrounded by a second basin-like structure containing rocks with a representative density of 2.74 gm/cc. These extend to a depth of seven miles. They were regarded as constituting a transition zone between the basic gneisses and the hybrid granitic gneisses which cover most of the area, and have a representative density of 2.69 gm/cc. The need for more detailed density sampling in such areas of Precambrian rocks is indicated.

(10) J. Lamar Worzel, G. Lynn Shurbet, and Maurice Ewing (Lamont Geological Observatory, Columbia University, Palisades, N. Y.) Gravity Measurements at Sea, 1951--The long-range gravity measurements at sea have been continued on board U.S. submarines. During 1951 gravity measurements were made as follows: on U.S.S. Quillback between New London, north of the Azores, to Gibraltar and Taranto, Italy; on U.S.S. Requin between Gibraltar, south of the Azores, and Norfolk, Va.; on U.S.S. Volador between San Diego, Pearl Harbor, and Japan. Free-air and Bouguer anomalies have been computed for the cruises of U.S S. Tusk and U.S.S. Conger I and II.

(11) B. F. Howell, Jr. and Keeva Vozoff (Pennsylvania State College, State College, Pa.) Gravity Investigation in North-Central Pennsylvania--A gravity survey of about 900 sq mi in Tioga County, Pennsylvania, is described. Within this area a positive anomaly of 22 mgal is observed. It is shown that the greatest part of this could be accounted for by a buried intrusion of diabase or peridotite.

SECTION OF SEISMOLOGY

(1) Chia-Shun Yih (Colorado Agricultural and Mechanical College, Fort Collins, Colorado, and University of Nancy, Nancy, France) On the Natural Frequency of Rayleigh Waves Propagating in Two Dimensions--When no external force is present at the surface of a semi-infinite solid, wave motion of a certain mode is possible only when the frequency has a certain value, which will be called the natural frequency of the given mode. In this paper two modes of motion are considered. In the one the motion is periodic in two perpendicular directions and with respect to time; in the other the time-periodic motion has cylindrical symmetry. For each of these modes a formula is given from which the corresponding natural frequency can be found. The two formulas obtained are formally identical to each other and to a corresponding one for Rayleigh waves propagating in one dimension.

(2) Dean S. Carder (U. S. Coast and Geodetic Survey, Washington, D. C.) The Continents and Ocean Floor as Transmitting Media of Microseisms--Microseisms which originate from the coast of southern Alaska and northwestern United States may be transmitted without an appreciable change in period across the continent as far as the Atlantic coast, but have not been recognized with certainty above the local background at Honolulu. Microseisms which originate from the coasts of Labrador and the Maritime provinces may be transmitted without an appreciable change in period to western North America including Alaska, but attenuation under the North Atlantic to Bermuda is high. Such microseisms are barely recognized at Bermuda only when the local background is very low. If this condition is general, microseisms which are well recorded on land would hardly be expected to originate from great distances at sea. The Rocky Mountain system, on the other hand, is not an effective barrier to microseisms.

(3) L. H. Adams, J. W. Greig, P. A. Scherer, H. E. Tatel, and M. A. Tuve (Carnegie Institution of Washington, Washington, D. C.) Studies of Crustal Structure by Explosion Seismic Waves near the Mesabi Range and near Puget Sound--Our deep seismic explorations in the Atlantic Coastal Region and the Appalachian Mountains show large and consistent critical reflections, indicating a distinct lower crustal boundary; these reflections could not be found in a large number of observations specifically seeking them in Southern California. No evidence of intermediate layers has been found in any of these regions despite intensive and critical searches. We have continued these explorations into Minnesota, centered about the Mesabi Range; and into Washington, centered about Puget Sound. Under Minnesota we have observed critical reflections indicative of a sharp lower crustal boundary. Its depth is greater than that found under the Atlantic coastal region. No evidence for intermediate layers was found. Seismic velocities are about the same as in the East. Although we searched, we found no significant evidence for critical reflections in Washington; the

crust in this area apparently is similar to Southern California. We found no evidence of intermediate layers. The lower crustal boundary, if it exists, appears to be too rough to give critical reflections.

(4) I. Lehmann (Geodaetisk Institut, Copenhagen, Denmark) P and S at Distances Smaller than 25°--A study was made of the P and S phases of three European earthquakes as recorded at distances up to about 25°. One of them was about 280 km deep. It was well recorded at distances around 20° where the P and S curves were found to bend gradually and have only one branch. Consequently, no abrupt change of velocity would take place at "the 20° discontinuity." An effect of a possible low velocity layer at some depth below the Mohorovicic discontinuity was not found in the records of this earthquake, whereas it was present in the other two. One of these was shallow; the other one seemingly occurred at the base of the crust. The effect was much more pronounced in S than in P. The layer, therefore, is believed to be "liquid."

(5) Maurice Ewing and Frank Press (Lamont Geological Observatory, Columbia University, Palisades, N. Y.) Further Study of Atmospheric Pressure Fluctuations Recorded on Microbarographs and Long-Period Vertical Seismographs--A buoyancy-compensated, long-period vertical seismograph has been developed that eliminates atmospheric pressure fluctuations as a major source of background noise and permits the study of weak long-period seismic disturbances. A microbarograph sensitive to pressure fluctuations in the period range 20 - 2000 sec, and compensated for seismic movements, has been constructed for study of atmospheric turbulence and sound wave propagation. Comparison of records from the buoyancy-compensated seismograph and the seismically compensated microbarograph suggests that very little ground motion is caused by direct coupling of atmospheric pressure fluctuations to the Earth.

(6) Roger M. Wilson (U. S. Coast and Geodetic Survey, Washington, D. C.) Dynamic Testing of Seismograph Galvanometers--A method of directly determining the amplitude and phase response of seismograph galvanometers is described and examples given. The tests are made under operating conditions and can be made with nearly any chosen operating constants and circuit arrangement. It is believed that the method should be applicable to all types of electromagnetic seismographs with the possible exception of the variable-reluctance type.

SECTION OF METEOROLOGY

Symposium on the Upper Atmosphere

(1) D. H. Menzel (Harvard University, Cambridge, Mass.) The Origin and Physical State of the Aurora Borealis--Recent observations by Gartlein and Meinel have shown that the Aurora Borealis displays broadened and violet shifted lines of the Balmer series of hydrogen. An astrophysical analysis of the emission of hydrogen gas in the neighborhood of the Earth indicates that the number of protons (and electrons) per cubic cm of the gas responsible for this emission is of the order of 10^6. An analysis of the effect that gas clouds would have on the Earth's magnetic field indicates that the latter can exert a focusing action. The corpuscular solar radiation responsible for the Aurora Borealis comes its way through the magnetic barrier of the Earth's field. The total energy carried by the ions stream may approach that of solar radiation itself. Hence over limited areas of the Earth's surface corpuscular radiation may play an important part in the heating and the dynamics of various portions of the Earth's atmosphere.

(2) R. J. Havens and Otto Berg (Naval Research Laboratory, Washington, D. C.) Density of the Earth's Atmosphere at 219 Kilometers--The density of the atmosphere was measured above White Sands, New Mexico, at 11h 04m, MST, on August 7, 1951. It was determined by measuring the pressure in an ion gage mounted on the side of a rolling rocket. This pressure oscillated with the frequency of the rocket roll, the magnitude of the oscillation being proportional to the atmospheric density, rocket velocity, and the sine of the angle between the rocket axis and velocity vector. A density of $1.0 \pm 0.2 \times 10^{-7}$ gm/m^3 was found at 219 km above sea level, while the density at 160 km was $1.1 \pm 0.5 \times 10^{-6}$ gm/m^3 compared to an average of $1.5 \pm 0.4 \times 10^{-6}$ gm/m^3 at 160 km on March 7, 1947 and January 22, 1948.

(3) Francis S. Johnson (Naval Research Laboratory, Washington, D. C.) High-Altitude Diurnal Temperature Changes Due to Ozone Absorption--The heating effect due to absorption of solar energy by atmospheric ozone was calculated from direct measurements of ozone concentrations and of the ultraviolet spectral intensities of sunlight made during a rocket flight at White Sands Proving Grounds, New Mexico, on June 14, 1949. Ozone concentrations were measured from 20 to 70 km by means of spectrographs. The solar intensities outside the Earth's atmosphere were measured from 3400 to 2100 Angstroms, and at longer wave lengths the Smithsonian data were used.

The heat absorbed by ozone in the Chappuis, Huggins, and Hartley bands was calculated over the altitude range 20 to 70 km. Assuming a constant rate of heat loss throughout the day and night, and no temperature change from day to day, the diurnal temperature change had a maximum value of 5.3°C at 48 km, and fell to 1°C at 31 km and 70 km, and to 0.1°C at 20 km. The temperature minimum and maximum at 70 km were found to occur at approximately sunrise and sunset, respectively. At 48 km the minimum and maximum occurred about an hour after sunrise and before sunset, respectively, and at 42 km the time difference was 1 1/2 hours.

(4) Michael Ference, Jr. (Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey) Temperatures in the Earth's Upper Atmosphere--Temperatures of the atmosphere up to 80 km have been obtained from five successful Aerobee rocket flights made at White Sands Proving Grounds, New Mexico, between July 1950 and June 1951. The mean temperature of a layer of atmosphere was determined by measurement of the average velocity of sound through the layer. The sources of sound were grenades programmed to explode at predetermined heights. The coordinates of the grenades were deduced from the positions of the grenade flashes recorded on a ballistic camera. The times of arrival of the sound waves at the ground were measured with a microphone array positioned to receive nearly vertically propagated rays. Temperatures at the maximum at 50 - 55 km are found to be lower than ground values. Some data are available on seasonal and diurnal temperature variations. Evidence is presented for the existence of marked dynamical effects at these high levels.

(5) Allan G. Weisner (Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey) Determination of Winds Above Thirty Kilometers--Twenty-seven values of wind velocity between 30 and 80 km have been determined from five Aerobee rocket flights made at night at White Sands Proving Grounds, New Mexico, between July 1950 and June 1951. The average wind velocity in a horizontal layer at a certain altitude is determined from the effect of the wind on a sound wave traveling downward through the layer. The sources of the sound waves were grenades, ejected and exploded along the upward path of the rocket. The positions of the grenades, the travel times of the sound waves to a point on the ground almost directly underneath the grenades, and the angles of arrival of the waves at the ground are the data required for the calculations. The wind directions are found to be easterly in summer and westerly in autumn and winter. The wind speeds appear to reach a maximum at 55 km, the largest measured speed being 102 m/sec.

(6) Myron G. H. Ligda (Massachusetts Institute of Technology, Cambridge, Mass.) Seasonal Variations of High-Level Wind Directions Studied by Means of Long-Enduring Meteor Trains--Records of long-enduring meteor trains collected by Olivier have been analyzed for evidence of seasonal wind-direction changes at high levels. Some weak variations of borderline significance have been found which apparently agree with the model of upper atmospheric circulation as proposed by Kellogg and Schilling. However, because of the nature of the phenomenon of long-enduring meteor trains and the conditions under which the observations are usually obtained, it is not believed that further investigations of this type will contribute significantly to our knowledge of upper atmospheric circulation.

(7) R. J. Havens and H. Spitz (Naval Research Laboratory, Washington, D. C.) Atmospheric Winds at 200 Kilometers--The wind at 200 km above White Sands, New Mexico, was measured at 11h 00m on August 7, 1951, by means of a pressure gage in a Viking rocket. This gage, mounted on the side of the rocket, would, in the absence of atmospheric winds, have produced a telemetered signal oscillating in phase with the rocket roll. The observed phase shift in this signal indicated the presence of a southeast wind of 80 ± 20 meters per second.

(8) A. Arnold (Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey) Turbulence in the Stratosphere--Indications of considerable turbulence at altitudes of 23 to 32 km have been observed by visually tracking radiosonde ballons with a telescope. On four separate occasions the radiosonde separated from the balloon. In one of these flights the sonde was seen to swing rather violently prior to separation. The possibility of deducing gust velocities from these observations will be discussed.

Session on Cloud Physics

(9) H. J. aufm Kampe and H. K. Weickmann (Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey) Trabert's Formula and the Determination of the Water Content in Clouds--Using cloud data collected in all types of cumulus clouds, it is shown that the constant in the Trabert equation is not actually a constant but varies with the character of the droplet spectrum. Trabert's formula consequently is not suitable in general for the determination of liquid

water content from measurements of visibility and droplet size. A method for determining liquid water content from these measurements will be discussed.

(10) H. J. aufm Kampe and H. K. Weickmann (Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey) Some Properties of Cumulus Clouds--Measurements have been made of extinction coefficient and of droplet size distribution during flights through cumulus humilis, cumulus congestus, and cumulonimbus clouds. The procedure described in the preceding paper has been used to compute the droplet concentration and hence the water content of these cumulus clouds. Data will be presented showing the variation with height above the base of the cloud of the droplet radius, droplet concentration and water content.

(11) Raymond Wexler (Mount Washington Observatory, Cambridge, Mass.) Precipitation Growth in the Workman-Reynolds Thunderstorm Cell--The theoretical growth of precipitation from the ice phase in a cumuliform cloud is analyzed. Results are compared with activities of thunderstorms observed from their inception by Workman and Reynolds [Bull. Amer. Met. Soc., v. 30, p. 142, 1949]. Agreement is found between the theoretical and observed movements of the radar echoes for reasonable assumed values of the updraft.

(12) Myron G. H. Ligda (Massachusetts Institute of Technology, Cambridge, Mass.) Some Preliminary Results of a Study of Precipitation Cell Movement--The motion of small, discrete precipitation cells (which are observed as showers at the surface) can be observed and recorded by radarscope time-lapse photography. By means of a special film viewer it is now possible to make accurate measurements of the duration, velocity, and direction of motion of large numbers of these cells with a reasonable expenditure of time and effort. Preliminary results comparing the motion of these cells with various other variables such as winds aloft and low-pressure center motion are investigated. Some interesting properties of the field of motion are shown and their probable significance discussed.

(13) J. S. Marshall (MacDonald Physics Laboratory, McGill University, Montreal, Que., Canada) Precipitation Trajectories and Patterns--Vertical sections of precipitation observed by radar frequently run to oblique streaks, as though generating elements continued to exist at some level. The consequent pattern relates the rate of descent of the particles to the winds aloft. On occasion the resulting pattern for snow is so nearly horizontal that it might readily be mistaken for truly stratus cloud. The velocity of the whole pattern is that of the generating element. Thus its measurement at any height determines the velocity of the generating element, even when that element is not visible. This velocity, compared with the upper-air winds, gives in turn the height of the generating element.

(14) Roscoe R. Braham, Ben K. Seely, and W. D. Crozier (New Mexico Institute of Mining and Technology, Socorro, New Mexico) A Technique for Tagging and Tracing Air Parcels--Successful tagging and tracing experiments have been carried out using an aerosol of fluorescent pigment as the tagging material. The aerosol "plumes" were explored with an airplane carrying impaction equipment which continuously collected pigment particles from the air traversed. The collected particles were identified and counted under the microscope, with ultraviolet illumination, and particle concentrations were computed as a function of the position of the airplane. If the flight plans were properly designed, information could be obtained on the size and structure of the aerosol plume. Significant particle concentrations were found at distances up to 107 miles from the dispersal point. The feasibility of the method was demonstrated, and analysis of the data obtained has yielded some information pertinent to the general problem of turbulent diffusion. Certain limitations were encountered and means are suggested for minimizing their effects.

(15) Ferguson Hall (U. S. Weather Bureau, Washington, D. C.), T. J. Henderson and Stuart A. Cundiff (California Electric Power Co., Bishop, Calif.) Cloud Seeding in the Sierra near Bishop, California--Cloud-seeding operations using dry ice dispensed from aircraft have been carried out by the California Electric Power Company in the High Sierra near Bishop, Calif., since February 1948. Limited visual observations suggest that snow showers can be produced from nonprecipitating orographic clouds by seeding, the snow reaching the ground at the higher elevations. From comparisons of the seasonal and annual runoff from the seeded watershed with that from adjacent areas it is estimated that the average annual runoff during the three years 1948, 1949, and 1950 was augmented by 9 pct with the 90 pct confidence limits being zero and 18 pct. Most of the increase appeared in 1949. Similar comparisons of snow pack did not indicate a significant increase in the Bishop area, but it is shown that such a result is not incompatible with the possibility that additional precipitation was produced which was not reflected in the snow surveys.

(16) Roland J. Boucher (Mount Washington Observatory, Cambridge, Mass.) Results of Cloud Droplet Impinger Measurements on Mount Washington, N. H.--The cloud-droplet impinger, developed and built at Mount Washington Observatory, is briefly described. Comparisons of the data obtained by this instrument with the multicylinder data are discussed and the advantages and limitations of the two instruments indicated. Results of the correlation of the liquid water content of clouds with the median drop diameter and with Σnd^6 are given.

SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY

(1) E. O. Schonstedt and H. R. Irons (Naval Ordnance Laboratory, Silver Spring, Md.) NOL Vector Airborne Magnetometer, Type 2A--The NOL vector airborne magnetometer, type 2A (VAM-2A), is an improved version of the NOL vector airborne magnetometer, type 1A (VAM-1A). A description of the VAM-1A and evaluations of its performance were presented at the annual meetings of the AGU of 1949, 1950, and 1951. The VAM-1A was restricted to areas in which the angle of dip of the Earth's magnetic field exceeded 45°. The VAM-2A is designed for world-wide use. The VAM-1A employs a pendulum as a vertical reference, whereas the field-measuring mechanism of the 2A is pendulously suspended. By pendulously suspending the 2A, the computations required to obtain the components of the Earth's magnetic field are considerably simplified over those required with the 1A. The field-measuring mechanism may be suspended in either of two positions. One suspension arrangement is for operation where the angle of dip is from 0° to about 70°. The other suspension is used when the angle of dip is between approximately 20° to 90°. Oscillations of the aircraft affect the vertical reference accuracy of a pendulum. Therefore, the angular data with which the direction of the magnetic field is determined is averaged over a 1 1/2- to 2-minute interval depending upon the flight characteristics of the plane. With the VAM-1A, the averaging process is carried out manually by scaling the recorder tapes. The VAM-2A has integrating-type circuits so that the angular data are averaged during flight. Direct-recording circuits are provided also so that periodic checks of the accuracy of the integrators may be made by the manually scaling process. Several electronic circuit changes have been made in going from the VAM-1A to the 2A in order to improve the stability of the instrument and to reduce its temperature coefficient. Synchro units are used in the 2A in place of the potentiometers of the 1A in the circuits with which the direction of the magnetic field is determined.

(2) L. Hurwitz (U. S. Coast and Geodetic Survey, Washington, D. C.) Reduction of Airborne Magnetometer Results--Reduction of observations obtained with the NOL vector airborne magnetometer, type 1A. Two-minute averages of vector components of the Earth's magnetic field are based on the measured attitude of the detector fluxgate, corrected for pitch and roll of the aircraft. Corrections for aircraft magnetism are determined from swings in localities of widely different vertical intensity. Punch-card computing methods are applicable to about 40 pct of the processing.

(3) George B. Walker and Patrick L. O'Dea (U. S. Coast and Geodetic Survey, Washington, D. C.) Secular-Change Impulses--Secular-change impulses in declination, dip, and horizontal intensity, derived from world-wide observatory data, tend to congregate around certain years (epochs). As indications of the character of secular change, they are well suited for use in the construction of isoporic charts. Using a selected group of observatories, precise impulses and epochs were determined. These promise to be of value in investigations of the nature of secular change.

(4) E. J. Chernosky (U. S. Naval Ordnance Laboratory, Silver Spring, Md.) The Phased Superposed Epoch Method of Analysis and its Application to Geomagnetic Activity--A description of the phased superposed epoch method of analysis is presented together with some results obtained from its application in an investigation of geomagnetic data. In this method, reference days are selected on the basis of the characteristics of pairs of days instead of individual days as in the Chree superposed method that is frequently employed in studies of time-variant phenomena. The characteristics of the first and of the second day relative to each other define a phase such as an increase, or a decrease, in activity as well as defining the levels within which the phase occurs. In this investigation the daily magnetic C-figures for the 15 years 1932-46 were classified according to their level of activity as disturbed (D), intermediate (I), or quiet (Q), and analyzed by both the phase and the Chree method on punch-card machines. The results obtained by the phase method for the entire 15-year period, considered in terms of the averaged C-values, were as follows: (1) In any sequence of at least two days that are in the same activity classification (D, I, or Q), the second day is quieter than the first. (2) The phases that recur strongest 26 to 28 days after their initial incidence are II, QQ, DD, and QD. (3) A long-period (about 20 days) exponential decrease of activity is indicated immediately following a QD sequence while a very small long-period increase in activity begins about three days after an II sequence of days.

(5) E. J. Chernosky, R. M. Coon, and E. Maple (Naval Ordnance Laboratory, Silver Spring, Md.) A Study of the Short-Period Variations of the Vertical Component of the Earth's Magnetic Field at Tucson, Arizona--Variations of the Earth's magnetic field in the frequency range of 0.01 to 1.0 cps with amplitudes of 0.01 gamma or larger have been measured at Tucson, Arizona, for a period of about one month in each of the summers of 1947 and 1948. The amplitudes of the variations were roughly inversely proportional to the one-half power of the frequency in this range. A preponderating activity was observed at 0.05 cps. A diurnal dependence was observed in that the lower frequencies (below 0.1 cps) occurred more often and with greater amplitudes during the day, while the higher frequencies (0.1 to 1.0 cps) occurred more often at night. A horizontal loop having an effective area of 2.7×10^8 cm^2 was used as a detector. The data presented are limited in the time covered, and the trends observed may not be representative of the entire year.

(6) A. W. Sullivan, H. M. VanValkenburg, and J. M. Barney (Engineering and Industrial Experiment Station, University of Florida, Gainsville, Fla.) Low-Frequency Atmospheric Radio Noise in Florida--On the basis of data obtained at frequencies of 15, 52, 189, and 530 kc at Gainesville, Florida, a study was made of diurnal and seasonal variations in average noise level, and of average frequency spectra. Both the diurnal and seasonal variation in noise intensity have been found to vary with frequency, and to be functions of propagation conditions. Empirical equations were fitted to the seasonal variations which described the time variation of noise of 15 and 52 kc fairly closely, while a less precise relation was obtained at the higher frequencies. The diurnal variation of the frequency spectrum of noise also indicated to some degree the effect of propagation conditions on noise level. It is pointed out that there is a possibility that the average noise spectrum in the frequency range 10 - 500 kc at the source of the noise is constant. It was found that noise level was almost directly proportional to wavelength during periods when the effect of propagation conditions was negligible.

(7) E. H. Vestine and I. Lange (Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.) On Variations of the Geomagnetic Field, Fluid Motions, and the Rate of the Earth's Rotation--Estimated time fluctuations in the rate of westward drift of the geomagnetic field, 1885-1945, are shown to correspond reasonably well with the previously unexplained systematic fluctuations in the Earth's rate of rotation derived by Brouwer. From this correspondence, though based on data in some respects deficient, it appears reasonable to conclude (1) that the source of the geomagnetic field lies within a large-scale fluid circulation within the central core, and (2) that this fluid circulation in the core may be considered to be real, since no other adequate large source needed to conserve angular momentum is available. As an independent check, a smaller time fluctuation in angular momentum derived from the northerly component of the drift of the geomagnetic field with the fluid can explain non-seasonal departures observed in the variation of latitude.

(8) J. R. Balsley, A. F. Buddington, and J. J. Fahey (U. S. Geological Survey, Washington, D. C.) Titaniferous Hematite and Ilmeno-Hematite Correlated with Inverse Polarization in Rocks of the Northwestern Adirondacks, New York--Aeromagnetic surveys have revealed the existence of 16 extensive areas (as much as a few thousands of feet wide and miles in length) of intense negative anomaly distributed throughout an area of more than 3000 sq mi in the northwestern Adirondacks. The rocks beneath the negative areas are microcline-rich gneisses formed by granitization of metasediments. Remanent magnetization was determined for samples of these rocks and found to be inverse in each specimen. Chemical and microscopic study of these rocks has shown that without exception the major and, in some, the only metallic mineral is ilmeno-hematite (titanium-bearing hematite with a little exsolved ilmenite intergrowth) or titaniferous hematite (TiO_2 up to about 10 pct). The metallic mineral forms 2.5 to 7 pct by weight of the rock and presumably is the source of the strong remanent magnetism. A review of the literature shows many reports of minerals (ilmeno-hematite, magnetite rich in magnesic-ferrite molecule, and titaniferous maghemite) other than magnetite in rocks producing negative anomalies or showing inverse remanent magnetization. A general conclusion is that one major factor in the origin of inverse remanent magnetization is the nature of the associated metallic mineral.

(9) G. R. Wait (5241 Broad Branch Road, N.W., Washington 15, D. C.) Conductivity of the Air Resulting from Radioactive Matter in Thunderclouds--The rate of ionization obtained by means of an unshielded ionization chamber near the ground will generally show an increase during a thunderstorm and appears to be due to radioactive matter brought down in connection with the rain. A more or less corresponding increase in the small-ion content and consequently in the electrical conductivity of the air would normally be expected. Such expectations are not realized and are possibly due to the abnormal increase in heavier ions at the time which would increase the destruction rate of the small ions. The intermediate and large-ion content of the air do show an in-

crease at such times. Possible reasons for such increases will be presented. These results have a bearing on the electrical conductivity of air in thunderclouds which must be considered in any suggestion concerning method of charge regeneration in thunderstorms. Ionization and intermediate-ion content measurements were carried out at and by the Department of Terrestrial Magnetism, Carnegie Institution of Washington.

(10) C. W. Gartlein (Cornell University, Ithaca, N. Y.) and D. S. Kimball (Yale University, New Haven, Conn.) A Graphic Method of Reporting Aurora Observations--The usual schemes of drawing pictures of auroral displays or writing a verbal description frequently convey too much or too little information. With the aid of members of the American Association of Variable Star Observers, a plan is being tested of reporting on a graphical scheme. Pictures are drawn using conventionalized symbols to represent the various auroral forms as they appear in the sky. These are presented on a coordinate system, with time as one variable. The location of the aurora over the Earth and its changes can be easily determined and correlation made between reports of many observers. It seems desirable to use this system for international reporting.

(11) C. W. Gartlein (Cornell University, Ithaca, N. Y.) Hydrogen and the Aurora--The patrol spectrograms taken at Ithaca during 1942-1951 show that hydrogen radiations have been present in more than 80 annual displays. Certain correlations are presented. The two-station study of hydrogen on September 30, 1950 and September 25, 1951 are discussed, as will the single record of November 3, 1951. Incoming proton velocities are shown to exceed 2500 km/sec.

(12) H. Freeborn Johnston (Central Radio Propagation Laboratory, National Bureau of Standards, Washington, D. C.) Thesaurus of Annual Values at Geomagnetic Observatories--At the Ninth General Assembly of the International Union of Geodesy and Geophysics held at Brussels, August 21 to September 1, 1951, the Association of Terrestrial Magnetism and Electricity established a new Committee on Thesaurus of Annual Values at Geomagnetic Observatories. The tabulated values are those that have been received during the 12 months ending April 30, 1952. In general, they are for current years though corrections of earlier published values are included where furnished by the cooperation of observers-in-charge. Several observatories are to be congratulated on supplying values for 1951.

(14) H. A. Miley (1629 Far Hills Ave., Dayton 9, Ohio), E. H. Cullington, and J. F. Bedinger (Air Force Cambridge Research Center, Cambridge, Mass.) Day Sky Brightness Measured by Rocketborne Photoelectric Photometers--Several experiments have been conducted since 1946 to measure skylight in the upper atmosphere. From the results it appears that sky-brightness measurements consist of (1) sunlight scattered by the atmosphere (particularly important in the lower atmosphere), and (2) day airglow (light originating in the upper air by other means than Rayleigh scattering). The scattered light values are large at the ground but decrease rapidly (almost proportional to atmospheric density) until at about 30 km the sky is relatively black, the measurements being about two to three per cent (depending on direction) of those recorded at the ground level. The scattered light becomes unimportant in comparison with airglow at about 35 to 40 km. Day airglow appears to be nearly constant in the range 40 to 130 km and is much larger than predicted by existing theories. Photometers employing interference filters gave day airglow values which are of the order of ten thousand times the corresponding night airglow values. Further evidence of large light values at high altitudes is given by photographs of clouds recorded by a rocketborne camera at an altitude of 70 km. The nature and exact location of these clouds are to be determined. From the photographs they appear near enough to have been in the cold layer at about 80 km.

(15) V. F. Hess, W. F. Burns, and W. D. Parkinson (Fordham University, New York, N. Y.) Gamma Radiation from Uranium X_2--The amount of ionization due to hard gamma rays from UX_2 (Pa 234) in equilibrium with a known amount of uranium was found by subtracting the calculated radiation due to RaC from the measured total hard gamma radiation. The work was done with a sample of pitchblende from Joachimstahl which had been supplied and analyzed by John Putnam Marble. According to his analysis, it is practically free from thorium. Corrections for self-absorption, geometry, and differential absorption in the lead filter are applied. Ionization is measured in terms of radium standards. The result obtained is, that of the hard gamma radiation from the products of uranium, 5.2 pct (± 0.4 pct) is from UX_2, the rest being from radium C. This figure is about twice as high as the previously accepted one. Nevertheless, it shows that the contribution of the gamma radiation of UX_2 to the total radiation from all radioactive products contained in ordinary rocks is insignificant and does not play any important role in the total ionization of the lower atmosphere by terrestrial radiations.

(16) W. D. Parkinson and V. F. Hess (Fordham University, New York, N. Y.) Beta Radiation from the Ground--Only rather rough estimates of the contribution of beta-ray sources in the ground to the ionization of the lower atmosphere are available. We have attempted to put these on a rather more experimental foundation by making measurements of the ionization due to beta rays at a height of eight cm above the surface of various types of rocks. Rough values of the beta-ray ionization due to a given quantity of material (that is, Eve's Constant for beta rays) have been determined in the laboratory. This enables a comparison to be made with the figure obtained from published values of uranium, thorium, and potassium contents of these rocks. The amount of soft radiation from the ground largely controls the distribution of the ion formation with height through the first few meters of the atmosphere, and is closely connected with the height variation of conductivity and space charge in this region.

(17) J. Carl Seddon (U. S. Naval Research Laboratory, Washington, D. C.) Propagation Measurements in the Ionosphere by Means of Rockets--Daytime measurements of electron density, ion density, and Earth's magnetic field were made during rocket flights at the White Sands Proving Ground, New Mexico. Data were obtained on March 7, 1947, January 22, 1948, and September 29, 1949. In each case a 4.274-mc signal and its 6th harmonic were radiated from the rocket to receiving stations on the ground. In the first two flights, the signals were received on dipole antennas; on the other flight, they were received on circularly polarized antennas with the magneto-ionic components separated by a cable-type Magic-T. The low-frequency signals were frequency-multiplied by six and heterodyned with the high frequency. The resultant beat frequency made possible the measurement of the indices of refraction. From these measurements the Earth's magnetic field intensity was checked at 105 km on one flight within one per cent of the theoretical value calculated from the ground value using the inverse cube law. Also, curves of effective electron density versus height were obtained for the first two flights up to about 100 km, with a few points at higher levels. At low altitudes, these curves are remarkably similar.

Best results were obtained from the third flight at 1000 hours MST September 29, 1949. A continuous curve was obtained of actual electron density from 55 to 145 km with a detection sensitivity of 10^3 el/cc. Most of the points obtained were checked by means of the ordinary ray reflected from the F-layer. The curve shows a "flat" D-region with a maximum measured density of 2500 el/cc. This layer extended from below 55 to 74 km. Above this altitude no charged particles were detectable until 81 km when ions were detected followed by an ion-electron mixture. The peak ion density (assuming N_2 + and O_2) was 2.8×10^8 ions per cc at 90 km. The ion bank rapidly dropped off to below the detection level of about 10^7 ions/cc at 96 km. An interesting feature is that an electronic layer existed just below the E-layer. It extended from 85 to 90 km, with a maximum of 7500 el/cc. The electron density was undetectable on either side of this layer. The rapid build-up of the E-layer began immediately above this layer, rising to a maximum value of 1.6×10^5 el/cc at 107 km. Above this level the density remained quite high, with no deep minimum such as was observed in the winter flight of January 22, 1948 at 1312 hours ($\sim 2 \times 10^4$ el/cc at 124 km with low values extending above 130 km). The data on the third flight indicated that mode coupling was occurring, but so far no numerical results have been obtained from this interesting phenomenon.

(18) A. G. McNish (National Bureau of Standards, Washington, D. C.) Sweep-Frequency Oblique Incidence Experiments--This paper presents a summary of preliminary results of an experiment of multi-frequency soundings at oblique incidence over a point-to-point path in an attempt to resolve discrepancies between calculated and observed maximum usable frequencies. A comparison of the observed maximum frequencies with those computed from ionospheric data obtained at the mid-point of the path indicates that the observed values are higher than the computed ones. This is particularly true at times of spread echoes. Indications are that maximum usable frequencies are best given by scaling the higher frequency part of the spread echoes. Sporadic-E propagation over the path has been obtained up to the maximum frequency used in the experiments - 25 mc. Comparison of observed frequencies propagated by sporadic E over the path with the maximum frequency return by sporadic-E reflections at the mid-point of the path suggests that in many cases the MUF factor for sporadic E is higher than that conventionally used.

(19) A. H. Shapley (National Bureau of Standards, Washington, D. C.) North Pacific Radio Warning Service--Many of the difficulties in maintaining radio communications in Alaska region arise, as is well known, from abnormal and rapidly changing height and electron-density distributions of the D, E, and F regions of the ionosphere, at altitudes of roughly 80, 110, and 200-400 km, respectively. The gross characteristics of the regions have since 1945 been predicted many months in advance with remarkable success. The shorter time variations are harder to unravel because of the large time and space variations encountered in the zone of intense auroral activity, and paucity of ionospheric sounding stations in the area. At the present level of understanding of the physical processes in the upper atmosphere, it is not possible to forecast in detail the ionospheric charac-

teristics throughout the Alaskan region and the resulting quality of radio transmissions. A more general forecast, one or more days in advance, of the average quality of conditions in a 12- or 24-hour period is being prepared systematically by the recently established North Pacific Radio Warning Service of the Central Radio Propagation Laboratory, National Bureau of Standards. Ionospheric geomagnetic, solar activity, and radio propagation observations are forwarded promptly to the forecasting center at Anchorage by cooperating agencies in Alaska and Canada, as well as by the CRPL in Washington. The statistical relationships between solar activity and magnetic disturbance and between magnetic disturbance and the various types of ionospheric and radio disturbance are the bases for the forecasts.

(20) R. W. Knecht (National Bureau of Standards, Washington, D. C.) Relationships between Aurorae and Sporadic-E Echoes--During March 1951 opportunity presented itself for a series of visual auroral observations to be made simultaneously with ionospheric soundings at Point Barrow, Alaska. Observations were made every 15 minutes during the dark hours of ten successive clear nights. Some 400 simultaneous observations were made. Aurorae were present during about 90 pct of these observations. Results of the analysis indicate that quiet aurora, 45° or higher above the horizon, is correlated with certain sporadic E-region echoes. The more intense the aurora, the higher the maximum frequency returned. The closer the auroral form is to the zenith, the shorter the range of the "Es" echoes. Active auroral displays are usually accompanied by a sharp increase in radio-wave absorption. Local geomagnetic variations coincident with auroral displays were also examined.

(21) C. G. Stergis and S. C. Coroniti (Geophysics Research Division, Air Force Cambridge Research Center, Cambridge, Mass.) On the Lifetime of Atmospheric Ions--By assuming that the agents of destruction of small atmospheric ions are (1) recombination with small ions of the opposite sign, (2) combination with large ions, and (3) combination with condensation nuclei, a theoretical expression has been obtained for the number of small ions as a function of the time after a given number of small ions has been produced. This expression shows that under certain atmospheric conditions an appreciable number of ions live as long as 1000 sec after production of the ions. Experiments have been conducted both in the laboratory and in the field to test this expression and good agreement has been obtained between theory and experiment.

SECTION OF OCEANOGRAPHY

(1) C. A. Barnes and John H. Lincoln (Dept. of Oceanography, University of Washington, Seattle, Wash.) An Hydraulic Tidal Model of the Puget Sound Area--An hydraulic tidal model of the Puget Sound area south of the Strait of Juan de Fuca has been constructed to a horizontal scale of 1:40,000. The desired accuracy of representation, and practical dimensions for tidal changes, water depths, and other factors necessitated a vertical distortion of about 35:1 or to a scale of 1/16 inch per fathom. The scale of 1.82 inches per nautical mile makes the overall dimensions of the model about 16 by 7 feet. The time scale of hydraulic models of this type is a function of the vertical scale. In the present model, one hour is 3.055 seconds. Because of the relatively deep water of the area and because of the exaggerated scale, many of the channels in the model are deeper than they are wide. The model provides a means for detailed study of the complex oceanography of the area including tidal currents, water exchange, and flushing. It also provides a means for readily studying the distribution and concentration of industrial wastes and domestic sewage from present or future sources. The model has been of value in biological problems, such as the drift of fish eggs, in which water movements are a factor. A six-constituent tide machine generates tides which may follow the prototype for any calendar period or may be set to generate repeating tides of a desired type. Since tidal currents and flushing rates are affected by the density gradients established by river runoff and salt-water inflow, provision has been made to duplicate these processes. A portable recording tide gage capable of resolving model tides equivalent to less than 0.5 ft has been developed. Instruments for recording density gradients and for measuring current velocities and direction have been designed and tested.

(2) Robert G. Paquette, John H. Lincoln, and C. A. Barnes (Dept. of Oceanography, University of Washington, Seattle, Wash.) A Precision Water-Level Recorder for Small-Scale Hydraulic Models--A water-level recorder with a precision of ± 0.002 inch and a range of 0.4 inch is described. The position of a fine wire probe is recorded once each second on a magnified scale at the instant the probe touches the water surface. Electrosensitive paper is used. The device is used as a tide recorder in a small-scale hydraulic model of Puget Sound.

(3) Joseph E. O'Hare, Quick H. Carlson, and William E. Tamblyn (U. S. Navy Hydrographic Office, Washington, D. C.) Some Results of a Tanker Survey of the Gulf Stream--An analysis is made of hourly Loran and meteorological observations of a group of oil tankers which traveled

the east coast between Key West and Cape Hatteras for a period of a year. The Gulf Stream is shown to have fluctuations of position and current speed which are apparently related to seasonal and tidal cycles. It is demonstrated that the meanders, known to exist to the north of Cape Hatteras, probably also exist as far south as the Florida Straits.

(4) Dale F. Leipper and Robert O. Reid (Texas Agricultural and Mechanical College, Dept. of Oceanography, College Station, Texas) Preview of Computed Ocean Currents in the Gulf of Mexico--During the period April 20 to August 23, 1951, the Department of Oceanography in cooperation with the U. S. Fish and Wildlife Service completed three oceanographic cruises systematically covering the Gulf of Mexico. These cruises of the MV Alaska were the first comprehensive ones furnishing data suitable for use in the computation of ocean currents. There were 125 hydrographic stations occupied. Data from these stations were analyzed, and computed currents are indicated by charts of dynamic topography referred to the 500-m and 1000-m surfaces. Several interesting features appear. One of primary importance is the outline of the eddy about 250 miles north of the west tip of Cuba. A subsequent cruise in January 1952 was planned and conducted to further observe the details of this eddy. The cooperative survey program is continuing. It is supported in part by the U. S. Navy Office of Naval Research and the Bureau of Ships.

(5) D. W. Pritchard (Chesapeake Bay Institute, The Johns Hopkins University, Annapolis, Md.) The Effect of Horizontal Mixing on the Absolute Pressure Field in a Coastal Plain Estuary--A review is made of recently reported studies by the author of the salt balance and dynamic structure in the James River. In this estuary the horizontal eddy frictional terms are negligible. Observations of velocity, temperature, and salinity are employed in the mean equation of motion to evaluate the absolute pressure field. In the James River the level pressure surface occurs at about mid-depth, with the pressure surfaces above the reference surface sloping downward toward the mouth of the estuary, and also downward from the right side of the estuary to the left. Below the reference level the pressure surfaces slope downward from the ocean toward the head of the estuary and also downward from left to right. In estuaries in which horizontal mixing is an important factor in controlling the salt balance, it is probable that the horizontal eddy friction terms also become important in the equations of motion. It is shown that in these estuaries the successive increase in relative value of the horizontal eddy friction term results in a successive increase in the depth of the level pressure surface, such that when the horizontal mixing becomes a dominant process, then the pressure surfaces at all depths slope downward toward the mouth of the estuary.

(6) Lewis B. Law (U. S. Navy Hydrographic Office, Washington, D. C.) Sea-Ice Information--A brief introduction is given discussing the production of the film, which features selected shots of ice forms in the polar regions, with a running commentary. Several spectacular views of events such as calving are shown.

(7) I. I. Schell (Museum of Comparative Zoology, Harvard University, Cambridge, Mass.) Longer-Period Fluctuations in the Ice of the North Atlantic - Arctic and in the Precipitation--Ten-year fluctuations in the ice limit of the North Atlantic - Arctic from Greenland (50° W) to beyond Novaya Zemlya (80° E) were compared by decade with the fluctuations in the annual precipitation of the corresponding decades in Iceland, southern Greenland, Spitzbergen, northern Norway, Sweden, and Russia. Higher precipitation was observed in these regions during periods when the ice retreated and less than normal precipitation when the uce extended farther south than usual. The fluctuations in the precipitation are presumed to be associated with a latitudinal shift in the North Atlantic storm track which in turn is associated with a northward or a southward displacement in the ice limits.

(8) Francis E. Elliott, William H. Myers, and Willis L. Tressler (U. S. Navy Hydrographic Office, Washington, D. C.) A Comparison of the Environmental Characteristics of Some Shelf Areas of Eastern United States--Selected areas of eastern United States are compared on the basis of oceanographic, meteorological, and biological phenomena. Significant similarities and differences in shelf areas off Charleston, South Carolina, Chesapeake Bay, Delaware Bay, Narragansett Bay, New York Harbor, and Boston, Massachusetts, are discussed. Bathymetry of these areas is illustrated by means of contoured charts. The oceanographic factors considered are sea-level variations, currents (tidal, non-tidal, and wind), seasonal variations of surface and vertical temperatures and salinities, swell conditions, transparency, and water color. Pertinent climatological factors such as wind speed and direction, air temperature, precipitation, and fog conditions are included. The nature and extent of fouling are the only biological phenomena discussed.

(9) Gilbert Jaffe and James H. Hughes (U. S. Navy Hydrographic Office, Washington, D. C.) The Radioactivity of Bottom Sediments in Chesapeake Bay--Very little is known of the natural

radioactivity exhibited by continental-shelf sediments off the eastern coast of the United States. A knowledge of the level of this activity, better known as the background, is most important if future surveys are to utilize radiometric techniques for the solution of oceanographic problems. Activities were determined for a group of samples from selected areas in the Chesapeake Bay area. The apparatus for making these measurements is described. Results are used to show the distribution of natural radioactivity in this bay.

(10) John Lyman (U. S. Navy Hydrographic Office, Washington, D. C.) The Oceanographic Activities of the Hydrographic Office, 1949-1952--During the past three years, work has continued in many phases of oceanography. A chapter on oceanography is now being included in the Sailing Directions, and a variety of oceanographic manuals, tables, and publications has been produced. The time-honored method of determining ocean currents through comparison of fixes and dead-reckoning has been refined through the employment of Loran and applied to study of the Gulf Stream. Operations in the Atlantic with the San Pablo and Rehoboth have continued, and new programs of observation have been undertaken. The two vessels participated in CABOT in 1950. A new program for surveying inshore areas of the United States has also been established, in collaboration with the Office of Naval Research, U. S. Coast and Geodetic Survey, and various universities and institutions.

(11) Dale F. Leipper (Texas Agricultural and Mechanical College, Dept. of Oceanography, College Station, Texas) Oceanography at Texas A. and M. College, Annual Report 1951-52--New research programs undertaken by the Department of Oceanography include studies of the chemistry of marine sediments and of the physical oceanography of shallow waters along the Gulf Coast conducted for the American Petroleum Institute by subcontract with the University of California, a project for the design of offshore structures and the development of oceanographic information needed for offshore oil operations - industrially sponsored, research on wave forces as they affect a hinged piling in shallow water for the Civil Engineering Research Laboratory of the U. S. Navy Bureau of Yards and Docks, a hydrographic and chemical survey of Corpus Christi Bay and connecting water bodies for the Reynolds Metals Company, and an analysis of air-mass modification over the Gulf of Mexico for the Air Weather Service. Programs previously reported are continuing except the marine pipe-line study which was completed. Three fellowships, $1500 to $3000, and 20 assistantships, $900 to $1500, are available for 1952-53.

(12) Richard H. Fleming (Dept. of Oceanography, University of Washington, Seattle, Wash.) Oceanographic Activities at the University of Washington--In the spring of 1951 the Department of Oceanography was established as a part of the College of Arts and Sciences. This marked another step in the development of Oceanography in the University of Washington. With authority to grant Bachelor of Science degrees, a comprehensive program of undergraduate courses has been formulated and these will be offered beginning in the fall of 1952. The graduate-course offerings have been broadened to meet the needs of graduate students who are now able to take Master of Science and Doctor of Philosophy degrees in oceanography. The author was appointed Executive Officer of the Department in the fall of 1951. The Department is housed in the Oceanographic Laboratories on the main campus. The Laboratory at Friday Harbor, for many years a part of the Oceanographic Laboratories, has been made a separate activity under the College of Arts and Sciences and will provide a broad program of instruction and research during the summer quarter.

(13) S. A. Lynch (Texas Agricultural and Mechanical College, Dept. of Geology, College Station, Texas) History of Geologic Thought on Gulf of Mexico--The history of geologic thought during the past century concerning the origin of the Gulf of Mexico as a whole, and its northern margin in particular, is presented. The accelerated work of the last two decades is discussed, and some suggestions for future study are made.

(14) R. S. Arthur, W. H. Munk, and J. D. Isaacs (Scripps Institution of Oceanography, University of California, La Jolla, Calif.) The Direct Construction of Wave Rays--In the construction of refraction diagrams for ocean waves, the rays (orthogonals) can be drawn more accurately if the approximate formula $\Delta\alpha = (\Delta L/L_{ave})\tan\alpha$, used by Johnson, O'Brien, and Isaacs in 1948 in the preparation of refraction protractors, is replaced by $\Delta\alpha = \sin^{-1}[(1 + \Delta L/L)\sin\alpha] - \alpha$. In these formulae α is the angle between wave crest and depth contour and L the wave length taken at the beginning of any contour interval; α and L are the corresponding changes over the contour interval. A modified procedure for the construction of rays is presented.

(15) Willard J. Pierson, Jr. and Wilbur Marks (College of Engineering, New York University, New York 53, N. Y.) Power Spectrum Analysis of Ocean Wave Records--Technique of power spectrum analysis originally developed by J. W. Tukey and R. Hamming for the analysis of noise

in electronic circuits are applied to ocean-wave records (according to methods presented by Pierson in the paper abstracted in paper (16), next below. The power spectra of sea-surface records are computed correctly from the power spectra of the pressure records. Statistical properties of the wave records are deduced and verified within the limitations of a linearized theory. The implications of the results with respect to the problem of wave forecasting and wave refraction are discussed. The relationship between the non-normalized autocorrelation function and the power spectrum is given and the results of Deacon and Seiwell are interpreted in connection with the results of the analyses.

(16) Willard J. Pierson, Jr. (College of Engineering, New York University, New York 53, N. Y.) A Unified Mathematical Theory for the Analysis, Propagation, and Refraction of Storm-Generated Ocean Surface Waves, Part I--The present concepts of ocean-wave forecasting, in particular the concept of significant height and period, are inconsistent with the spectrum analyses of Deacon and Klebba and with the theory of ocean-wave refraction. Waves are treated partly as classical waves and partly as non-conservative waves, and there are many points where the mathematics of classical waves are applied incorrectly to actual storm-generated waves. In this paper, adequate methods of analysis of a short-crested sea surface are given which depend upon a power spectrum in two variables, namely, wave frequency ($2\pi/T$) and wave direction (θ). A Lebesgue power integral for the Gaussian case is then shown to represent all of the known properties of a short-crested sea surface (except those due to non-linearity). The integral is given by (1).

$$\eta(x,y,t) = \int_0^{\infty} \int_{-\pi/2}^{\pi/2} \cos\left[(\mu^2/g)(x\cos\theta + y\sin\theta) - \mu t + \psi(\mu,\theta)\right] \sqrt{[A_2(\mu,\theta)]^2 \, d\theta \, d\mu} \quad \ldots . (1)$$

This representation of the sea surface is then applied to the problem of wave forecasting. Given $[A_2(\pi,\theta)]^2$ at the edge of the storm, and given the duration of the storm, the width of the storm, and the length of the fetch, it is then possible to forecast the power spectrum for any point in the area of decay at any future time. The transformation of sea into swell, the decrease of wave height with travel, and the period increases of ocean swell are all demonstrated by the theory.

Part I covers only the theory of wave analysis, and wave propagation (forecasting). Further work on wave refraction and on practical applications is in progress.

(17) E. Allan Williams (Santa Barbara College, University of California, Santa Barbara, Calif.) and John D. Isaacs (Scripps Institution of Oceanography, University of California, La Jolla, Calif.) The Refraction of Groups and of the Waves Which They Generate in Shallow Water--A process is described which may be of importance in beach and near-shore development and which may account for surging in the surf zone as well as other phenomena. It is shown how a group front may be refracted according to one law as it approaches a coast, then after reflection proceed seaward under quite a different law. This permits the outgoing wave to move in a direction that it could not ordinarily attain, and causes "trapping" under many conditions. Stoneley's theory of refraction of energy fronts is further developed to permit quantitative investigation of group refraction on various types of shores. On the basis of the assumed hypothesis, the outgoing surf beat may be totally reflected from deep water, then reflected from the beach, a process which can occur repeatedly and result in newly arriving groups with variable phase relationships. Theory and techniques are developed for the investigation of the surf beat path. The refraction of the reflected groups is shown to display regions of high convergence, divergence, and reinforcement.

(18) A. Shapiro and L. S. Simpson (U. S. Navy Hydrographic Office, Washington, D. C.) The Effect of a Broken Icefield on Water Waves--Methods of forecasting sea and swell in the open ocean have been presented by Sverdrup and Munk, and others. In this paper a method for forecasting sea and swell in a broken icefield is presented. It is shown that water waves entering an icefield are damped exponentially with increasing ice thicknesses. It is further shown that selective damping occurs in the icefield, the shorter period waves being more easily damped. Graphs are included which help to give a clearer picture of the results.

SECTION OF VOLCANOLOGY

(1) Hisashi Kuno (Geological Institute, Tokyo University, Tokyo, Japan; Visiting Fellow, Princeton University, Princeton, N. J.) Formation of Calderas and Magmatic Evolution--Japanese calderas may be classified into two types. The first type includes calderas formed by the collapse following the eruption of a tremendous amount of pumice. The second type includes those formed without connection with the pumice eruption. In both types, post-caldera eruption builds up cones or domes, usually inside the calderas but sometimes on their margin. Most of the pre-caldera lava consists of rock series originating through fractional crystallization of basaltic magma. In the first type the post-caldera lava comprises a rock series formed through contamination of the

same magma by granitic material, whereas in the second the post-caldera lava represents more advanced stage of crystallization of the pre-caldera lava. In the first type the explosive eruption of the pumice probably shattered the roof of the magma reservoir and facilitated the contamination, whereas in the second the subsidence took place along a simple ring fissure without fracturing of the subsided mass.

(2) Ray E. Wilcox, Olcott Gates, and Howard A. Powers (U. S. Geological Survey, Denver, Colo.) Geology of the Western Islands of the Aleutian Arc--The rocks of the western Aleutian Islands probably represent the island-arc basement on which are built the late Cenozoic volcanoes of the more easterly islands. They are extrusive and intrusive rocks of the circum-Pacific suite with subordinate pre-Quaternary marine sediments. The oldest rocks comprise several thousand feet of basaltic, spilitic, and keratophyric pillow lava and tuff, with interbedded chert and conglomerate. Soda granite intrudes part of the rocks; xenoliths of soda granite occur in some lavas. After deformation and uplift, approximately 1000 ft of conglomerate grading into 3000 ft of argillitic sediment containing Oligocene microfossils was deposited. Extensive intrusion of diabase was followed by major uplift; erosion produced sandstone containing detritus from both diabase and older rocks. The last igneous episode produced lava and east-west dikes of porphyritic andesite and quartz keratophyre. Faulting, uplift, and erosion were followed by extensive Pleistocene glaciation.

(3) T. P. Thayer (U. S. Geological Survey, Washington, D. C.) The Tuff Member of the Rattlesnake Formation of Eastern Oregon: an Ignimbrite--The Rattlesnake formation (Pliocene) in the John Day River basin of eastern Oregon and adjoining region to the south is marked by a prominent cliff-forming volcanic tuff member interbedded with gravels. The tuff occurs over a distance of 60 mi from east to west in the John Day structural trough; in Bear Valley and Murderers Creek south of the Strawberry-Aldrich Mountain range; and has been recognized in the flats around Paulina, south of the Ochoco Mountains. It appears to have covered the lowland parts of an area of about 2500 sq mi. In exposures in the John Day Valley the tuff is 50 to 75 ft thick, and shows features typical of an ignimbrite. It is ashy near the bottom, semi-vitreous to vitreous in the central and upper parts, and composed throughout of shards, clear glass, and pumice fragments. In most places the top has been eroded and covered by boulder conglomerates; the maximum thickness found is about 150 ft. The upper 25 ft of a 70-ft section in Murderers Creek has been devitrified to resemble a stony rhyolite. Xenocrysts of plagioclase and other minerals, including olivine, are common in the vitreous facies. Rude columnar jointing is characteristic. Angular fragments of basalt up to eight inches across, and other rocks, are numerous almost everywhere in the lower few feet of the tuff, and commonly occur in irregular layers at the very base, even where it lies on old soil zones far out in the old valley. Near old hill slopes the fragments occur from top to bottom, commonly in nests or irregular pipelike accumulations. These foreign fragments appear to be typical surface debris, and are not visibly altered. The contained rock fragments are conclusive evidence of extensive travel by the tuff. A source in the area south of the Ochoco Mountains is suggested by progressive increase in size and abundance of pumice fragments and increase in degree of vitreousness and devitrification toward the southwest. No evidence of a source in the John Day Valley has been found. It is postulated that the tuff flowed northward down the South Fork of the John Day River, probably at least 15 mi, and spread out in the John Day valley proper. The tuff is believed to have traveled 50 to 60 mi from its source to the east end of the John Day Valley. Nothing definite is known regarding extensions toward the south.

(4) E. Wm. Heinrich (University of Michigan, Ann Arbor, Mich.) Paragenesis of Veins of the Sillimanite-group Minerals--Quartz veins that contain kyanite, sillimanite, and andalusite, or combinations of these trimorphous minerals, occur in many parts of the United States as well as in other countries. Usually various accessory minerals such as muscovite, biotite, chlorite, staurolite, garnet, tourmaline, topaz, spinel, ilmenite, rutile, and corundum also are present. In a few deposits still rarer constituents have been recorded, including orthoclase, lazulite, triplite, pyrite and other sulfides, hematite, and gold. In some areas these quartz veins appear to be related closely to pegmatites that also contain accessory sillimanite-group minerals. Although in a number of deposits the wall rocks of the veins are kyanite- or sillimanite-bearing, the vein mineralogy, in general, is not a function of the wall rock composition. The host rocks vary from peridotite to granite and also include many types of metamorphics. In the country rock along the margins of the veins, sillimanite-group minerals have been concentrated and coarsened, and in some districts these minerals appear only near the veins and dwindle and disappear at various distances from their margins. The arrangement of the minerals in the veins commonly indicates that they were not formed under strong directed pressure. Pseudomorphic and other textural criteria show that in some deposits sillimanite formed later than kyanite or andalusite, presumably at lower temperatures. In most cases the evidence strongly suggests that these veins have been formed by the deposition of mineral material from aqueous solutions under relatively high

temperatures and pressures. To this hypothesis many investigators have subscribed; however, others have suggested that they have been formed by the processes of metamorphic differentiation. One of the chief problems involved is determining the source of the alumina concentrations, whose presence has been variously ascribed to reaction with adjacent rocks, to assimilation of aluminous minerals or rocks at depth or to crystallization of a body of granitic magma.

(5) J. C. Maxwell and Peter Verrall (Princeton University, Princeton, N. J.) Permanent Expansion and Increase in Permeability of Carbonate Rocks on Heating--Specimens of marbles, limestones, and travertine, heated at one atmosphere and at 5000 and 10,000 lb/in^2 confining pressures, all showed permanent expansion, and of the same order of magnitude even at the highest pressure. Permeability of samples to water solutions (red and blue inks) increased markedly with heating. The mechanism may explain apparent high permeability of carbonate rocks to ore-bearing solutions when these same rocks have very low permeabilities at room temperatures.

(6) J. F. Schairer (Geophysical Laboratory, Carnegie Institution of Washington, Washington, D. C.) and Kenzo Yagi (Tohoku University, Sendai, Japan) The Stability Relations of Iron Cordierite and Almandine Garnet--The compound $2FeO \cdot 2Al_2O_3 \cdot 5SiO_2$ decomposes incongruently to mullite, tridymite, and liquid at 1210 ± 10°C. This iron analog of cordierite ($2MgO \cdot 2Al_2O_3 \cdot 5SiO_2$) crystallizes with some difficulty from appropriate compositions in the system FeO--Al_2O_3--SiO_2 and has a small field on the liquidus surface. All mixtures of fayalite, iron cordierite, and silica begin to melt at a ternary eutectic at 1083 ± 5°C. The composition of almandine garnet $3FeO \cdot Al_2O_3 \cdot 3SiO_2$ lies in the field of hercynite ($FeO \cdot Al_2O_3$) and a liquid of almandine composition becomes completely solid at the eutectic at 1083°C. No almandine crystallized at any temperature from a glass of its own composition even when seeded with the natural mineral. Almandine from Botallack, England (91.3 per cent almandine) when heated began to decompose at an appreciable rate as low as 900°C and yielded a mixture of hercynite, iron cordierite, and fayalite.

(7) Robert J. Uffen (University of Western Ontario, London, Ont., Canada) A Method of Estimating the Melting-Point Gradient in the Earth's Mantle--Using the Einstein-Debye theory of solids and Lindemann's theory of fusion, the ratio of the melting point at various depths in the mantle to that at depth 100 km is computed from seismic data. It is found that the melting-point gradient decreases with depth. Using data for solid olivine, a value of 5300°K is found for the melting point at the core boundary, which gives an upper limit for the actual temperature there.

(8) L. H. Ahrens, William H. Pinson, and Margaret M. Kearns (Massachusetts Institute of Technology, Cambridge, Mass.) Association of Rb and K and Their Abundance in Common Igneous Rocks and Meteorites--Spectrochemical determinations of Rb and K in common igneous rock types (granite, diabase, basalt, gabbro, and ultramafics) and in chondrites are given. These determinations show that the ratio K/Rb remains virtually unchanged when proceding from rock type to rock type and in crustal igneous rocks pct K/pct Rb averages 90. In chondrites this ratio remains remarkably uniform (av. 100) and to within an assumed tolerance of ± 20 pct is equal to that in terrestrial igneous rocks. As neither K nor Rb enters metal and sulphide phases to a significant extent, one concludes that to within ± 20 pct the ratio K/Rb in the Earth as a whole is equal to that in meteoritic matter as a whole: It is believed that this is the first example of such a kind. The abundance of K in chondrites is relatively uniform and appears to be considerably less than that usually given in the literature. In ultramafic rocks (mainly dunite and serpentine) K abundance should be drastically reduced by an order of magnitude. As a result, heat production by K^{40} in a dunite mantle may be a small fraction, perhaps only 1/30 of that contributed by the U + Th series. The abundance of Rb is 0.035 pct in crustal igneous rocks and 0.0009 pct in chondrites.

(9) L. F. Herzog, L. T. Aldrich (Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.), W. Holyk, and L. H. Ahrens (Department of Geology, Massachusetts Institute of Technology, Cambridge, Mass.) Variations in the Sr^{87} Content of Sr Extracted from Biotite--Ca-Sr concentrates were obtained from about 20 biotites according to the procedure described by Ahrens in 1951. The concentrates were separated further, using resin ion exchange columns. Complete elimination of Rb from the concentrate and a partial separation of Ca and Sr was achieved in this step. The separated Sr fractions obtained from this treatment were analyzed with a Nier 60° mass spectrometer. The Sr^{87}/Sr^{86} ratio was measured by a double collection null method. We were able to determine this ratio consistently to 0.5 pct or better when 100 μg of Sr were available for analysis. The ratio Sr^{86}/Sr^{88} was also determined to see if variations in this ratio were present in these sources. No significant differences between the biotite Sr^{86}/Sr^{88} ratios and that in Sr obtained from Eimer and Amend (herein denoted as "normal" Sr) were found. The Sr^{87}/Sr^{86} ratio was found to vary from 0.712 to 2.19 corresponding to fractions of radiogenic Sr present of from 0.000 to 0.129 respectively. The amount of radiogenic Sr^{87} present must be combined with the Sr/Rb ratio of each biotite to make a significant age determination.

(10) L. A. Sanderman and C. L. Utterback (University of Washington, Seattle, Wash.) The Radium Content of Varved Clays in the Seattle Region--A pre-Wisconsin sequence of varved clays, exposed in Seattle, includes about 470 paired summer and winter layers. The radium content of each layer has been determined for a series of ten consecutive years and also for additional layers, some of which lie near the surface and some at greater depth. With the exception of one varve near the surface, any winter layer has a greater radium concentration than the adjacent summer layer. A weighted mean radium content expressed in terms of grams of radium per cc of dry material shows interesting variations.

(11) Howard A. Slack (Dept. of Physics, University of Toronto, Toronto, Canada) Field Measurement of the Radioactivity of Rocks--A gamma-ray counter suitable for measuring the radioactivity of ordinary rocks "in situ" is described. This instrument represents a considerable improvement both in sensitivity and portability on an earlier instrument reported in 1951. The instrument was calibrated against prepared standard radioactive concrete blocks and was used to investigate the radioactivity of the Kirkland Lake area in northern Ontario. The results on and around the Round Lake batholith in this area are compared with previous results over the same batholith. In addition, graphs are presented showing the distribution of radioactivity across the contact zones of alteration of this batholith. The characteristic distribution of radioactivity observed in these zones is assumed to be the result of highly radioactive pegmatitic or hydrothermal solutions which circulated in the contact zone at the time of crystallization or later and subsequent mineralizing solutions, which leached out a portion of the radioactive material.

(12) Arnold C. Mason and Helen L. Foster (U. S. Geological Survey, Washington, D. C.) Diversion of Lava Flows at O Shima, Japan--During the 1951 eruption of Mihara Yama on O Shima, an island near Tokyo, basalt flows were partially blocked by a concrete building. Lava filled the interior and poured out through doors and windows. Observations made at the time indicate that it is feasible to build walls that will protect inhabited areas from lava streams. Two types of pressure were exerted by the lava: hydrostatic pressure, and impact from the flow's momentum. The first is not great as it consists of the vertical height of the lava column up to a point of relief which usually is nearby. The second can be reduced by locating a wall where it does not face a steep slope, or below natural obstructions. Walls should not dam a flow because they would be overrun, but should be located in a topographic position that will cause the flow to be diverted into a safe channel.

SECTION OF HYDROLOGY

(1) Earl E. Sanderson and Don O. Johnstone (Dept. of Natural Resources, State of Ohio, Columbus, Ohio) Accuracy of Determination of Annual Precipitation over a Given Area--The limits within which the total precipitation as measured by a single gage represents the total precipitation over the surrounding area is a subject of great interest to hydrologists and meteorologists. The lack of adequate data from large areas has made detailed analytical studies impossible. The precipitation-gage network established during the period from 1935 to 1940 in the Muskingum River Watershed in Ohio by the U. S. Soil Conservation Service provides data for limited studies for a 8038-sq mi area. Records from 230 gages for a full year beginning October 1, 1939 are available. The data from these stations have been used by the authors to analyze the effect of gage spacing on the accuracy of determination of the amount of annual precipitation for areas ranging in size from a few square miles to 8000 sq mi in Ohio. Standard errors for gages range from 1.28 inches on an area of three square miles to 3.42 inches or 8.8 pct of the annual mean for 8038 sq mi. The study indicates that uniform spacing of gages will give more accurate results than random spacing, that is, five to seven gages equally spaced over a given area will give results comparable to nine gages spaced at random. Isohyetal maps drawn from random spaced gages give about the same degree of accuracy as Thiessen computations based on gages equally spaced.

(2) C. C. Warnick (University of Idaho, Moscow, Idaho) Experiments with Windshields for Precipitation Gages--Since 1948 the Engineering Experiment Station of the University of Idaho has been studying the principles, development, and use of high-altitude precipitation gages. This project has been carried on under cooperative encouragement from several of the Federal agencies. A slow-speed wind tunnel has been used to study the effect of wind on the catching characteristics of model storage precipitation gages. A simulated snowstorm was produced with sawdust and various gage designs without windshields and with different type windshields were studied. Special techniques with photography were used to evaluate the value and improvement in performance of various windshield designs. Field studies have also been conducted at two remote mountain locations on prototype gages and shields of similar design to those studied in the laboratory. Results indicate reductions in catch due to the action of winds and new shields proposed for use are showing an improvement in the consistency of catch both in the wind-tunnel studies and in the field studies.

(3) J. L. Paulhus (Cooperative Studies Section, U. S. Weather Bureau, Washington, D. C.) Interpolation of Missing Precipitation Records--In 1948, the U. S. Weather Bureau stopped the practice of interpolating missing precipitation records and adopted the policy of not including monthly or annual totals in any of its publications whenever any portion of the record was missing. In view of criticism by the users of precipitation data, a series of tests involving some 1200 storms was recently conducted to determine if the missing records could be satisfactorily estimated. As a result of these investigations, two methods of interpolation, namely, the normal-ratio and three-station average, are being put into use. Because of the short deadlines for the issuance of the monthly Climatological Data, estimated data, properly identified, will appear in the annual number only, beginning with the 1952 issue. In order to limit the magnitude of the errors of interpolation, annual totals including more than 25 pct estimated precipitation will not be published.

(4) Albert S. Fry (Tennessee Valley Authority, Knoxville, Tenn.) Effect of Reforestation upon Hydrologic Characteristics of Henderson County Watershed--The Henderson County project was established to determine the effect of certain erosion-control and reforestation measures on the water-yield, storm-runoff, and soil-erosion characteristics of a small watershed. The experimental watershed is an area of 88 acres in western Tennessee. Erosion had caused the loss of a large part of the original surface layers of soil, and gullies up to ten feet deep were numerous when the project was initiated. Continuous hydrologic measurements of precipitation, runoff, sediment load, and ground water have been maintained and are analyzed. During a five-year period from 1941 to 1946 the watershed was calibrated hydrologically with the existing vegetal conditions. A survey of the vegetal cover in 1941 showed that 50 pct of the watershed was idle and severely eroded, 23 pct was in forest, and the remainder was in eroding pasture or row-crop cultivation. In 1946 the watershed was subjected to extensive erosion-control and reforestation measures. These included fencing, flow-line staking, diversion ditching, brush paving, reaming of gully sides, contour furrowing, construction of numerous check dams, and the planting of more than 100,000 trees. Appraisal of the land cover in the area five years after the treatment showed that, except for a few limited patches with active erosion, an almost complete vegetal cover had been achieved which virtually controls soil erosion on the watershed. The paper summarizes the hydrologic effects of watershed treatment on the water yield, surface runoff, peak discharges, sediment load, and other factors.

(5) James Smallshaw (Tennessee Valley Authority, Knoxville, Tenn.) Some Precipitation-Altitude Studies of the Tennessee Valley Authority--The mountainous southeastern part of the Tennessee River valley is an area of sharp precipitation contrasts. Annual rainfall along the high boundary ridges ranges from 70 inches to as high as 125 inches. Precipitation in some shielded valleys a few miles to the northwest rarely exceeds 50 inches. Exceptions to the general rule of higher precipitation at the higher altitudes occur, however, and complicate the selection of sites for a representative rainfall station network. Three precipitation-altitude studies made cooperatively by the Tennessee Valley Authority and the U. S. Weather Bureau are discussed. In one study covering observations of a line of precipitation stations across the Great Smoky Mountains, a typical increase of rainfall with altitude is described. Results of another study, an investigation across sharp-crested Snake Mountain, North Carolina, show precipitation at the top of the ridge to be only 70 pct of that on the slopes a short distance below the crest. A similar situation is described on Clinch Mountain in Virginia. Seasonal effects are noted, with the most pronounced differences occurring in the October-March period. The low catch at the ridgetop stations is attributed to a combination of updraft and carry-over of moisture-laden air because of the steep slopes and narrow crest widths of the ridges.

(7) James S. Sweet (U. S. Weather Bureau, Washington, D. C.) Better Definitions Necessary in the Development of Unit Hydrographs--No uniform definitions or procedures for using unit hydrographs have been adopted. This causes inconvenience and sometimes confusion when same unit-hydrograph data are used by a number of hydrologists. Standard definitions are necessary for unit hydrograph, excess rainfall, and concentration time. Standard procedures should be adopted for base flow separation. Research should be made on proportionality of precipitation and unit hydrographs, and on the criteria of the size of areas to which unit hydrographs can be applied.

(8) W. D. Potter (U. S. Bureau of Public Roads, Washington, D. C.) Rainfall and Topographic Factors that Affect Runoff--Based on runoff records for the period 1938-1948, peak-rate probability curves are computed for 51 watersheds located in the Allegheny-Cumberland Plateau and ranging in size from 100 to 350,000 acres. A simple correlation is computed to determine the relationship between the peak-rate for a ten-year recurrence interval and area. The average standard error was 34.8 pct. Two rainfall factors representative of the period of runoff record are then introduced together with a topographic factor and a multiple correlation is computed. The average

standard error was reduced to 18.2 pct. Long-term rainfall factors are then computed and used in the multiple-correlation equation to determine the peaks that would have resulted from the peak-rate probability curves, had long-term runoff records been available. Relationships between ten-year peaks and those for 25- and 50-year frequencies are determined from the peak-rate probability curves and the Plateau is divided into two zones based on these relationships.

(9) E. J. Gumbel (441 Ocean Ave., Brooklyn 26, N. Y.) Statistical Theory of Droughts--The droughts are analyzed by the asymptotic theory of smallest values of a limited statistical variate. The extremal probability paper used up to now for the floods is used for the logarithms of the droughts. If the lower limit of the discharges is assumed to be zero, the probability function of the droughts becomes a straight line. If the lower limit exceeds zero, the three parameters in the probability function are estimated by the method of moments. A statistical criterion is given which decides whether the lower limit may be assumed to be zero or not. The droughts of 13 rivers analyzed by this procedure show a very good fit between theory and observations. Therefore, extrapolation is legitimate provided that the basic conditions will prevail.

(10) Louis H. Bean (Office of the Secretary of Agriculture, Washington 25, D. C.) Forecasting Floods of the Columbia River by Means of a Short-Cut Method of Time-Series Analysis--This paper has a twofold purpose. It first states and then illustrates that it is possible to construct long-term forecasts of natural phenomena from the internal evidence of the record itself. This is in spite of the fact that the usual methods of time-series analysis are not useful for long-term forecasting of natural phenomena such as year-to-year variations in weather, in crop yields per acre, the occurrence of floods, the annual fluctuations in maximum or minimum discharge of rivers, and similar events. My illustration shows, among other facts, that the flood of the Columbia River in 1948 that wiped out the defense city of Vanport, Oregon, could have been predicted a year in advance. Meteorologists cannot tell us today, from an analysis of the record, what the weather will be here or in Iowa a year from now. The Crop Reporting Board of the U. S. Department of Agriculture can tell us as of July 10 what the crop looked like a week or two earlier, on July 1. It cannot forecast what the actual harvest will be, since it does not and cannot predict the course of weather. Even Gumbel's forecasting formulations do not help us in year-to-year forecasting. They are intended to help predict the magnitude of the next extreme point on a record, not the particular year of its occurrence. I believe that, in spite of the seeming randomness in the monthly and yearly records of natural phenomena, there must be a great deal of orderliness in them. There is much evidence to support my view.

Year	Bean forecast[a]	Actual
	M cfs	M cfs
1949	680	620
1950	710	743
1951	600	597

[a] On the basis of the record ending with 1948.

The second objective of this paper is to illustrate how year-to-year fluctuations in a time series can be predicted by means of a short-cut method for revealing trends and patterns. The chosen illustration happens to be most timely for it is based on a record Gumbel called to my attention as one of his illustrations. A study of the annual maximum discharge of the Columbia River near The Dalles, Oregon, that occurs usually toward the end of May or the first part of June, indicates that the two greatest floods of the Columbia River of the past 75 years could have been predicted a year in advance, strictly on the internal evidence of the record. One flood occurred in 1894; the other was the 1948 flood. That study also led me to set down early in 1949 actual predictions for the years 1949 to 1953, inclusive. Three of these forecasts, included in the above table, already have proven out quite accurately. Over the past 60 years, the maximum discharge has fluctuated between 270,000 and 773,000 cfs, with the exception of the two extremes of 1,240,000 cfs in 1894 and 999,000 cfs in 1948.

(11) M. W. Nelson (Division of Irrigation, Soil Conservation Service, Boise, Idaho), H. G. Wilm (U. S. Forest Service, Washington, D. C.), and R. A. Work (Soil Conservation Service, Medford, Ore.) Soil Priming in Relation to Snow Surveys and Flood Regulation--In earlier analyses of techniques for water forecasting in the Columbia River Basin, both water yields and flood discharges were found to be highly correlated with the water content of snow as determined by snow surveys, and with the amount of precipitation which occurred during the spring. Studies described in this paper demonstrate the additional correlation obtained by including data on autumn precipitation in the forecast analyses. Recorded for the autumn preceding each spring flood, these data express the degree of "soil priming" in the Basin--the extent to which autumn deficits in soil moisture are replenished by precipitation before the snow-pack accumulates each winter. Data on this variable were included in a linear multiple regression equation, with snow water content and spring precipitation as the other independent variables. As compared to previous analyses, the inclusion of the soil-priming variable made a 40 pct improvement in the standard error of the forecasts, made on April 1 of each year. Smaller improvements in precision were observed for later forecasts, made on May 1 and May 15.

(12) Gunnar M. Brune (Soil Conservation Service, Milwaukee, Wis.) Trap Efficiency of Reservoirs--Forty-four records of reservoir trap efficiency and the factors affecting trap efficiency are analyzed. The capacity-inflow (C/I) ratio is found to offer a much closer correlation with trap efficiency than the capacity-watershed (C/W) ratio heretofore widely used. It appears likely from the cases studied that accurate timing of venting or sluicing operations to intercept gravity underflows can treble or quadruple the amount of sediment lost from a reservoir. Desilting basins, because of their shape and method of operation, may have trap efficiencies above 90 pct even with very low C/I ratios. Semi-dry reservoirs with high C/I ratios, like John Martin Reservoir, may have trap efficiencies as low as 60 pct. Truly "dry" reservoirs, such as those in the Miami Conservancy District, probably have trap efficiencies in the 10 to 40 pct range, depending upon C/I ratio.

(13) M. R. Carstens (School of Civil Engineering, Georgia Institute of Technology, Atlanta, Ga.) Accelerated Motion of a Spherical Particle--The force components on an oscillating sphere in a fluid medium were studied by experimental means. These force components on the oscillating sphere were separated into the inertial effective force and the dissipative (shear) force. In order to accomplish this experimental program, the writer assumed that the sphere motion was approximately simple harmonic motion, and he analyzed the data as if the motion were exactly simple harmonic motion. The experimental results were applied to an allied problem of a sphere in an oscillating fluid. The experimental results, as applied to this problem, were found to compare with the data of an experiment performed in Germany in about 1920. Extending the analysis further, the writer presents a simple analysis of the diffusion characteristics of suspended sediment in water.

(14) H. V. Peterson and R. F. Hadley (U. S. Geological Survey, Salt Lake City, Utah, and Washington, D. C.) Physical Features of an Aggrading Alluvial Valley--Valley alluvium, particularly that constituting the floor of ephemeral stream courses, is generally regarded as resulting from deposition during overbank flooding of shallow and probably uniformly small channels. It is suggested here that the deposition of some alluvial fills is characterized by simultaneous aggradation and degradation in valleys occupied by discontinuous gullies but with aggradation being the net result. A description is given of conditions found in Whitehead Creek, a minor tributary of Hat Creek, in the Cheyenne River Basin, Wyoming. Sediment movement in the past two or three decades has caused rapid aggradation in two reaches in the Whitehead Creek basin. The deposits occur as broad, low, fan-like forms which feather out downstream, and on which occur only shallow channels. These aggrading reaches are separated by actively eroding gullies six to ten feet in depth and varying from 30 to 120 ft in width. Aggradation in the active fans is in places as much as five feet and it has completely buried fence lines. From the age of the fences, it is known that this amount of deposition has occurred in the past 30 years. It appears that the fan farthest downstream is extending headward and, unless there is a change in the stream characteristics, will merge with aggradation extending downstream from the upper fan. Should this occur, the entire flood plain will have been raised by several feet in a period of a few years. There is evidence in many valleys of the Cheyenne River Basin that alluvial fills are being developed in a manner similar to that observed on Whitehead Creek.

(15) Herbert A. Swenson (U. S. Geological Survey, Washington, D. C.) Geochemical Relationships of Water in the Powder River Basin, Wyoming and Montana--Geologic factors largely determine the composition and the amounts of dissolved solids that are transported by the Powder River and its tributaries. The chemical character of the water in the basin differs widely from one stream to another and is variable in each stream from headwaters to mouth. Mountainous areas underlain by igneous rocks of Precambrian age contribute small quantities of soluble materials to the river system; the concentration of mineral solids may be less than 100 parts per million. Streams heading in upland country and draining a region that is largely underlain by Tertiary sandstone and shales transport salts in concentrations exceeding 2000 parts per million. The composition of mineral substance in the river water can be correlated with occurrence in the area of such rocks as granite, limestone, gypsum, and shale.

(16) Walton H. Durum (U. S. Geological Survey, Lincoln, Nebraska) Relationship of the Mineral Constituents in Solution to Stream Flow, Saline River near Russell, Kansas--Studies were made of the salinity relationships of ground and surface flow in the Saline River basin, Kansas, during the period 1945-1949 in connection with the program of the Department of the Interior for development of the Missouri River basin. Mineralization in the Saline River near Russell, Kansas, fluctuates rapidly from more than 4400 ppm in low water stages to approximately 200 ppm at flood stage. Hydrographs for the 1947 and 1948 water years reveal that surface runoff comprised approximately 65 pct of the total runoff for each year and carried an estimated 24 pct of the annual dissolved

solids discharge. The dissolved minerals in surface runoff were derived from surface contact, channel scour, and dilution of base flow. Ground water discharging to the Saline River and its tributaries carries strong concentrations of dissolved minerals that are leached from saliferous and gypsiferous shales and sands of the consolidated materials. Sodium, chloride, and sulfate are the most prominent constituents. Chloride concentrations of more than 1900 ppm were identified with low flow and the chloride ion is related hyperbolically to stream flow. Equations were developed by the least-squares principle to correlate between specific conductance of the water and several mineral constituents in solution.

(17) F. J. Veihmeyer (University of California, Davis, Calif.) Use of Water by Native Vegetation versus Grasses and Forbs on Range Lands--Soil-moisture records from plots on potential range lands in California on which native vegetation, mostly woody brush species, was left intact, and from adjacent plots which were denuded by burning, were obtained for several years. Samplings, by which the amounts of soil moisture could be measured quantitatively, were taken with soil tubes generally under extremely difficult conditions at intervals frequently enough to give a satisfactory picture of the water regimen of the area. Over 23 paired plots were used in the experiments. They were selected to give a wide variety of vegetative types, rainfall patterns, topography, and soils. At all of the sites the rain generally fell from October through March; most of it was in December, January, and February. The growth of the plants, therefore, was dependent to a large extent upon the amount of water that could be stored in the soil mantle. Following the fire, grasses and forbs usually succeeded the brush, but, with some species, several annual burnings were necessary before it was eliminated. The removal of the brush resulted in saving of moisture. The soil was at a higher moisture content in the burned plots than in the unburned at the end of the growing season; consequently, less water was required to restore the soil in the burned plots to its field capacity. In most cases the results led to the conclusion that appreciable savings of water could be effected by burning to substitute grasses or forbs which have forage value. The soil-moisture records show that the soil in the burned areas was wet to its field capacity as early and, in many cases, earlier than in the unburned ones, thus indicating that the permeability of the soil for water was not adversely affected by burning.

(18) Ronald L. Ives (Cornell Aeronautical Laboratory, Buffalo, N. Y.) Anomalous Glacial Deposits in the Colorado Front Range Area, Colorado--Reinvestigation of reported glacial deposits in the Colorado Front Range area, in locations above or exterior to those commonly classed as glaciated, has resulted in a doubling of the number of known deposits, and the finding of a rough gradation from glacial, near the Arapaho Massif; through reworked glacial, at distances of six or seven miles; to glacio-fluviatile and fluviatile at greater distances. From this evidence, one or more early Pleistocene glaciations are postulated, the parent ice body being of the ice-cap type, resting upon a gently rolling terrain of which only the so-called "peneplain" surfaces now remain. A similar large ice cap in the Laramie-Poudre area is suggested by scattered evidence between those rivers and the Laramie-Fort Collins highway.

(19) Joel E. Fisher (25 West 43rd St., New York 36, N. Y.) The Several Sources of Heat of a Cold Glacier--Mountain-climbing recollections of over 40 years, supplemented by comparisons of authenticated photographs of identical high peaks in the Alps, Himalayas, and Alaska, taken over an interval of 30 to 40 years, often reveal a marked increase, over this period, in névé accumulated at only high altitudes. This is in sharp contrast with the widespread retreat of low-altitude isothermal valley glaciers into which drain these same high altitude névés. A change of viscosity of ice, restricted to these cold névés alone, is offered as the explanation of this contradictory behavior. Such restricted change of viscosity could be attributed to minute changes of temperature of the undermost stratum of such high-altitude cold névé, inasmuch as a small change of the temperature of ice which is moderately below its pressure melting-point temperature will change its viscosity a hundred to a thousand-fold. The accumulation of heat, in any existing névé, from the overlying atmosphere and from internal friction of movement is concentrated chiefly near its upper surface; it results that any heat delivered to its under surface (from the internal heat of the Earth) will face a negative temperature gradient through such a névé and will therefore be forced to accumulate within a thin lamina of ice at the very bottom of such névé. This ever-present thin lamina of warmish ice, already of relatively low viscosity, is what provides the more fluid "bearing" by which the mass of névé may flow down-grade. Any minute change in the amount of internal heat, trapped in the basal lamina, will greatly change its viscosity, and will exert a powerful control on the velocity of flow of the entire overlying névé. In the event of a minute diminution of internal heat, the resulting retardation of the cold upper névé will increase its thickness, and yet, by starvation tactics, simultaneously cause the outrunning bottom glaciers to retreat.

(20) Calvin J. Heusser (Dept. of Exploration and Field Research, American Geographical Society, New York 32, N. Y.) Post-Wisconsin History of the Region of the Juneau Ice Field as Re-

vealed by Pollen Analysis--Pollen analyses of nine peat sections obtained from muskegs on the southwest flank of the Juneau Ice Field, Alaska, reveal the post-Wisconsin succession of vegetation and climate in the region. The Juneau Ice Field is today an area of approximately 700 sq mi, and its present size is attributed to the change of climate which occurred a little over 2000 years ago. Prior to this date, the glacier network of this ice field was presumably greatly decreased in size, since a warm and dry climate prevailed from 5000 to 2000 years in the past. Earliest pollen-bearing sediments are estimated at 8000 years and disclose considerable physiographic instability in the region between that date and 6000 years. Amelioration of climate and increased stability took place between 6000 and 5000 years ago. Uppermost trends of pollen profiles corroborate the present-day warming and general recession of glaciers. These studies further substantiate geological evidence for postglacial change of sea level, as well as late glacial or postglacial vulcanism in northern British Columbia.

(21) M. W. Owen (Dept. of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.) Correlation between Pipe Flow and Uniform Flow in Triangular Open Channel--A dimensionless open-channel resistance diagram based on an existing diagram given by Moody for circular pipes is presented. Data were taken on a 90° triangular flume in smooth and roughened condition from actual open-channel flow data. A comparison of the two diagrams presented is made by computing average velocities for several hypothetical channels. Correlation was found to be fair.

(22) Arthur N. Strahler (Dept. of Geology, Columbia University, New York 27, N. Y.) Application of Geomorphology to Practical Hydrology--Within a fluvial drainage basin, velocities of surface runoff, soil creep, and soil flowage are functions of ground-surface slope. Maps can be drawn to show (1) degree of slope present and (2) intensity of the down-slope component of gravitational shear stress. The first type of map uses lines of equal slope tangent (isotangents) and is therefore a form of first-derivative map. From this map it is possible to determine frequency distribution characteristics of slope within unit basins. The second type of map uses lines of equal sine of slope (isosines) and thus shows the distribution of the down-slope gravitational shear-stress component. From this map, frequency-distribution studies of shear-stress intensity can be made. By studying representative basins of given orders of magnitude under various types of geologic, climatic, and relief conditions, a general stress-intensity distribution function may be derived.

(23) Luna B. Leopold (U. S. Geological Survey, Washington, D. C.) Hydraulics in Quantitative Geomorphology; a Progress Report--The operation of fluvial processes produces or affects a large proportion of land forms. Fluvial processes may be more adequately understood by quantitative analyses using hydraulic principles than by the purely qualitative descriptions which characterize classical physiography. Hydraulic parameters are used to describe channel-shape characteristics of natural rivers and the change in channel shape downstream. It is demonstrated that measured suspended sediment load is closely related to river channel characteristics. A great similarity exists in these characteristics between rivers in very different physiographic settings. The same parameters have been measured in some actively cutting gullies or arroyos during periods of storm runoff. It is shown that changes in width, depth, velocity, and suspended sediment load with discharge are distinctly different in active gullies and in most rivers. These differences are shown to be of the type expected to exist between channel systems near equilibrium and those distinctly out of equilibrium or rapidly degrading.

(24) Mahdi Hantush (College of Engineering, Baghdad, Iraq) Three-Dimensional Flow in Leaky Aquifers--A solution is obtained for the potential distribution induced by a well, screened in and pumping at a constant rate from a circular artesian aquifer, which is overlain by another aquifer of a different permeability. The potential at the outer boundary, concentric with the well, remains constant. For all practical values of the parameters involved, the accuracy of C. E. Jacob's assumptions (1946) to reduce the three-dimensional flow to two-dimensional is verified by sample calculation for an assumed set of the system's parameters.

(25) W. B. Nelson and H. E. Thomas (U. S. Geological Survey, Salt Lake City, Utah) Pumping from Wells on the Floor of the Sevier Desert, Utah--Data collected at the Topaz War Relocation Center provide an excellent record of the development and subsequent disappearance of a cone of depression caused by pumping. This center occupied temporarily an area of natural ground-water discharge on the floor of the Sevier Desert in west-central Utah. Water for a population of 6500 was pumped from artesian wells which tapped aquifers of sand. Overlying these sand beds are lake-bed sediments too impermeable to yield water to wells, but apparently permeable enough to permit slow upward movement of water from the artesian aquifers. The water table is near the surface, and there is natural discharge by evapotranspiration, accumulation of "alkali" in the soil, and a need for drainage of irrigated lands. The wells were abandoned when the relocation center was closed, apparently because these handicaps were so great that the ground water could not be used profitably for irrigation.

(26) Vaughn E. Hansen (Utah State Agricultural College, Logan, Utah) Complicated Well Problems Solved by the Membrane Analogy--The two general types of well flow, confined and unconfined, are outlined for steady flow. The membrane theory is presented and the experimental equipment and results outlined. It is shown that the membrane analogy will yield solutions to complicated problems associated with both basic types of well flow. The principles of model design necessary to duplicate the existing boundary conditions are presented.

(27) Leonard Schiff (Division of Irrigation, Soil Conservation Service, Bakersfield, Calif.) An Exploratory Study of the Effect of Head on Infiltration Rates under Water-Spreading Conditions--Infiltration rates increased directly as head increased in infiltrometers and ponds under water-spreading conditions. There is an indication that the normal, relatively low infiltration rates of certain soils may be increased to higher possibly practical rates by providing sufficient head where feasible. Infiltration rate increases from 0.7 to 1.7, and 1.1 to 3.1 ft per day were achieved in ponds when heads were increased from 0.5 to 1.5, and 0.4 to 2.0 ft, respectively. These effects are discussed on the basis of the ratio of head to length of saturated soil column, H/L, in the Darcy equation. The length of saturated soil column was indicated by infiltrometers equipped with manometers located 1 1/2, 4 1/2, and 7 1/2 inches below the soil surface. Heads, equal to those frequently used in water spreading, were largely lost at a depth of 7 1/2 inches below the soil surface. These losses show the clogged conditions of the untreated soil at shallow depths and indicate a relatively short length of saturated soil column. Results obtained on the ponds may be reasonably representative of larger areas where the condition limiting infiltration is the surface soil. Results of varying the head on a pond, treated with cotton-gin trash, with high infiltration rates are also reported.

(28) Shragga Irmay (Hydraulics Laboratory, Institute of Technology, Haifa, Israel) On the Capillary Rise in a Homogeneous Medium Composed of Spheres, Ellipsoids, or Cylindrical Fibers of Different Forms and Sizes--The capillary rise h of a wetting liquid in a homogeneous porous medium composed of spheres, ellipsoids, or cylindrical fibers may be found from statistical considerations, the average upward vertical component of the capillary forces being equal to the weight of the liquid column h. In the case of grains of uniform size, h is found to be inversely proportional to their diameter and to the void ratio, as empirically verified (Atterberg, Terzaghi). If the grains are ellipsoids of revolution with vertical axis, h depends on their form: if planetary, it is less; if oblong, it is nearly the same as for spheres. In a medium of uniform structure composed of particles of different sizes, the equivalent diameter depends on the texture, but it is different from the usually assumed ten per cent less than value (Hazen). This explains the great variability of capillary rise as found by experimenters.

SECTION OF TECTONOPHYSICS

(1) Kenneth G. Smith (Dept. of Geology, University of Michigan, Ann Arbor, Mich.) Structure Plan of Clastic Dikes--Conclusions drawn from field observations are that vertical sandstone dikes of the White River Badlands, South Dakota, originated as open fractures that were filled with debris supplied by erosion of overlying formations. One existing explanation is that the dikes formed in deep dessication cracks which developed at the surface as the silts and clays of the Oligocene White River group became consolidated. Evidence from a recent study of a small area of clastic dikes is incompatible with the dessication-crack theory of origin. The pattern of intersecting dikes, when mapped, resembles a fracture system rather than having a hexagonal arrangement normally associated with dessication cracks. Shear fractures appear on the map, intersecting at angles of 54 to 71°, the acute bisectrix varying in strike from N45W to N54W. One dike occupies a major tension fracture, strikes N50W, and extends vertically through two beds of strikingly different lithology, a condition unlikely to occur with dessication cracks. South of the area, normal faults of small vertical displacement, trending northwest, suggest minor regional tension, which may well have caused the development of a fracture pattern as displayed by the dikes. If these fractures were open at the surface, subsequent filling from above would follow, resulting in numerous intersecting clastic dikes.

(2) Byron K. Thomas (Johns Hopkins University, Baltimore, Md.) Use of the Axial Plane in the Three-Dimensional Analysis of Folds on Sugarloaf Mountain, Maryland--Sugarloaf Mountain is a delicate topographic expression of tight overturned folds in ten lithologically similar quartzite beds. The steep plunges must be viewed as due to size variation of the anticlinal folds in relation to the horizontal plane. As the anticlines plunge, their fold axes leave the horizontal and steepen to produce crest line plunges up to angles approaching 90, but the folding can be related to the horizontal plane by determination of the axial plane. A method is suggested whereby stereographic net plots of cleavage poles, bedding poles, and drag-fold measurements can be used to establish the axial plane of the folds and the actual fold axes. The resulting net solution is a visual aid which can

be used in structural interpretation to much greater advantage than individual readings on outcrop drag-fold axes, cleavage-bedding intersections, or other lineations.

(3) Chester R. Longwell (Yale University, New Haven, Conn.) Is Gravitational Sliding Important in Orogeny?--There is a revival of interest, particularly among European geologists, in gravitational sliding as an orogenic process. Even some conservative students of tectonics favor this concept as an explanation of detached masses such as the Prealps. Van Bemmelen and others consider the process adequate for all thrusting and folding. Rich's "blister hypothesis" is one version of the concept. Some aspects of the problem are examined critically.

(4) J. Tuzo Wilson (Dept. of Physics, University of Toronto, Toronto, Canada) Structure, Process, and Stages of Orogenesis--Recent seismic exploration of the ocean floors and of continental shelves coupled with measured rates of erosion and probable rates of volcanism indicate that continents have grown greatly during geological time. The structure upon which mountains and continents have grown is the ocean floor beneath whose sediments and volcanic rocks lies what is presumably the original crust of the Earth. Recent reductions in estimates of radioactivity in the crust and increasing knowledge of its distribution make cooling and contraction of the Earth probable. Coulomb and Scheidegger have shown that this might be expected to lead to conical fractures in the crust and upper mantle. The shape and properties of these fractures can be readily equated to those observed for island and mountain arcs, thus providing a physical basis for the process of mountain building. Where primary mountains have been distinguished, they can be seen to follow a sequence of stages from single island arcs through mountain arcs to provinces of shields. Examples are given from the Cordilleran, Appalachian, and Grenville provinces together with their associated secondary mountains. Consideration of the elements of the Canadian Shield and of new age determinations there leads to the conclusion that Precambrian primary arcs have all been called Archean and Precambrian secondary arcs all called Proterozoic without regard to their absolute ages. A new time scale for the Precambrian based upon radioactive age determinations and orogenetic belts is required but its successful application requires more numerous and accurate age determinations.

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TRANSACTIONS
AMERICAN GEOPHYSICAL UNION

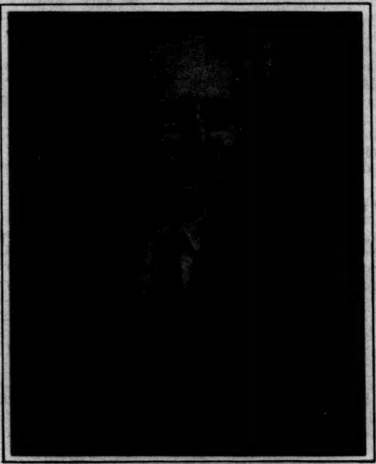
Volume 33 JUNE 1952 Number 3

FOURTEENTH AWARD OF THE WILLIAM BOWIE MEDAL

CITATION

Prepared and presented by Walter D. Lambert

Mr. President, Members and Friends of the American Geophysical Union: This Committee on the Award of the William Bowie Medal and, I suppose, its predecessor committees also, have tried to avoid too great concentration of the awards in any one special field of geophysics, one that might happen to be of special interest to the membership of the moment. In this instance, the question as to whether the recipient represented especially this or that section of our Union did not arise. The Committee chose a man whose work represents nearly all sections and goes even behind and beyond the special field of our Union. You will know why when I mention his name -- Harold Jeffreys, since 1946 Plumian Professor of Astronomy and Experimental Philosophy at the University of Cambridge.



Harold Jeffreys - Recipient, Fourteenth Award
William Bowie Medal - May 5, 1952

This Plumian Professorship was established early in the eighteenth century, and not all of the holders have been primarily astronomers in the sense of star-watchers, or experimental philosophers either. The first Plumian Professor was Roger Cotes, younger contemporary and friend of Isaac Newton. He is known primarily as a mathematician. I pass over the names of many holders, some well known, some less so, and mention the next-to-last holder, Sir George Horace Darwin, because he, like Jeffreys, was primarily a mathematical geophysicist. Next came Sir Arthur Eddington; he might be described as a mathematical cosmologist. Then came Harold Jeffreys himself, whose field is again applied mathematics.

It means no disparagement of his achievements to add that he had one great advantage that helped him to discuss successfully many difficult problems in many varied fields: He discussed them chiefly in the language of mathematics. The same mathematical method, for instance, the method of partial differential equations, may be applied to many seemingly diverse problems in diverse fields. Of course, for the different fields there are different boundary conditions, and here special knowledge is required. This knowledge Jeffreys has acquired and applied. To do this has meant cooperation with other workers, and you are well aware that cooperation of this sort is by tradition one of the criteria in the award of the William Bowie Medal. Jeffreys maintains that he has been on the receiving end of this sort of cooperation, but I maintain that he must have given to others quite as much as he has received. He is now in charge of the International Seismological Summary, which is both a cooperative enterprise of seismologists and their indispensable tool.

Another problem that must often arise in the deliberations of a committee like ours is this: Shall we be the first to recognize young genius by voting the award to a rising young scholar, or shall we play safe by bestowing the honor on an older man who has already received other honors? However, the question did not come before us in precisely this form. We simply said unanimously: "Here is our man." As a matter of fact, we are following in the wake of other bodies that have previously recognized Jeffreys' achievements. The following is a partial list, selected mainly to illustrate what has been said earlier, namely, that Jeffreys' scientific activities cover many fields.

In his university days he was one of the wranglers in the Mathematical Tripos and held the Isaac Newton Studentship. He has been a fellow of St. John's College since 1914. He has won prizes for his work not only in mathematics and astronomy but has received honors from meteorological, geological, and geographical societies. Our National Academy of Sciences and our own Geophysical Union have also honored him by making him respectively a Foreign Associate and a Corresponding Member.

Of his published books, I mention only three: (1) "The Earth," now going into its third edition. The book deals not only with the Earth as it now is but with the Earth as we suppose it to have been in the dim and remote past and with the mass of matter that in the dimmer and remoter past became our solar system.

(2) "The Theory of Probability," a treatise dealing not merely with the practical applications but with the fundamental philosophical questions that lie at the foundation of the theory.

(3) "Methods in Mathematical Physics," now going into its second edition. Here is another example of cooperation, of cooperation that began at home, for the work is the joint product of Mr. and Mrs. Jeffreys. Perhaps I should say of the Doctors Jeffreys, for Mrs. Jeffreys is a gifted scholar in her own right and lecturer at Girton College in Cambridge. The work covers a wide range of subjects from very practical advice on numerical calculations with interpolation formulas to the refinements of reasoning demanded by the modern school of mathematical rigor.

There are other books too and many papers in the *Proceedings* and *Transactions* of the Royal Society of London and in the *Monthly Notices* of the Royal Astronomical Society and its *Geophysical Supplement*. Just for variety, it might be mentioned that he has also written on plant ecology.

What sort of man is there behind all this formidable array of learning? Jeffreys was born in 1891 in the north of England. He attended a secondary school at Newcastle-on-Tyne, also Armstrong College there, now a part of the University of Durham, and then went to Cambridge. He lists his recreations as cycling, fishing, hill-climbing, and reading detective stories. He might have added music.

Perhaps we may gain some idea of the man from the quotations that he prefixes to the various chapters of his books, as Sir Walter Scott prefixed similar quotations, real or pretended, to the chapters of his novels. The chapters of Jeffreys' treatises don't read as easily as chapters from a novel. *Experto crede*, believe one who has tried it, or if not convinced, try yourselves. But Jeffreys' quotations are interesting in their own right and perhaps revealing.

Listed chronologically, they range from Homer and the Bible, the Old Testament and the New, down to Dante's "All hope abandon ye who enter here." This is prefixed to a chapter on stresses in the interior of the Earth. Then comes Shakespeare, Elizabethan and Restoration poets, and so on down to comparative moderns like Tennyson, Longfellow, and "Uncle Remus," and two really modern authors of "whodunits," for which Jeffreys has confessed a certain weakness.

In the books that I have examined the author who appears oftenest in the quotations that head the chapters is another mathematician, of a very different stripe from Jeffreys, and he is not quoted for his ideas about mathematics. That author is Charles Lutwidge Dodgson, otherwise Lewis Carroll, author of "Alice in Wonderland" and "Through the Looking Glass." Perhaps Jeffreys is slyly hinting that the more we know of our world, the "curiouser and curiouser" it becomes. Compare, for instance, the simple atom as physicists conceived it two generations ago with the modern atom, which seems to increase in complexity with every paper published on the subject.

But whether our studies make our world simpler or more complex, Harold Jeffreys has notably increased our knowledge of it and made it more interesting to think about. And so, Mr. President, I present him to you as the unanimous choice of our Committee to receive the William Bowie Medal for 1952.

PRESENTATION BY PRESIDENT WALTER H. BUCHER

Men are divided into two distinct groups by the basic motives that lie behind their life's work. This division is quite real, though the factors involved are intangible and can not be measured quantitatively. There is, on one side, the great majority whose actions are given direction and impetus by the needs and wishes of others; and, on the other side, the few who recognize no goals except those they set for themselves and who require no other drive but that which springs from within themselves. Here we find the creative men of science. Unlike many other leaders of men, they live absorbed in their own devices and withdrawn from the tastes of the market place, jealous of their time and energy, with one exception. Within his chosen field of research, the great man of science gives freely to his fellow workers thoughts as well as facts, sharing both time and energy as a matter of course. Without this whole-hearted cooperation and free intercourse of creative minds, modern science could not exist.

It is to honor this aspect of the work of great men in the field of geophysics that the William Bowie Medal was created. So far, it has been awarded to 13 men. The list begins with the man whose memory it keeps alive.

William Bowie (deceased)	1939
Arthur Louis Day	1940
John Adam Fleming	1941
Nicholas Hunter Heck	1942
Oscar Edward Meinzer (deceased)	1943
Henry Bryant Bigelow	1944
Jakob Aall Bonnevie Bjerknes	1945
Reginald Alsworth Daly	1946
Felix Andries Vening Meinesz	1947
James Bernard Macelwane	1948
Walter Davis Lambert	1949
Leason Heberling Adams	1950
Harald Ulrik Sverdrup	1951

To our deep regret, the fourteenth recipient, Professor Harold Jeffreys, can not be with us at this occasion. By good fortune, one of his former students is at present in Washington and has consented to receive the Medal for him, J. Roger Carter, First Secretary and Education Officer of the British Embassy. May I ask Dr. Carter to step to the platform?

This beautiful Medal bears the inscription:

"Unselfish Cooperation in Research"

Dr. Carter, it is with a deep sense of the privilege and honor involved that, in the name of the American Geophysical Union, I herewith bestow on Professor Harold Jeffreys this Medal and entrust it to you for transmittal to him with the greetings of all those here assembled.

RESPONSE BY J. ROGER CARTER

Mr. President: I have the honour to be present in this distinguished gathering on behalf of Professor Harold Jeffreys who, to his great regret, was prevented by his university duties and other pressing tasks from being present in person to receive the William Bowie Medal. As a one-time pupil of Professor Jeffreys and a member of his College in the University of Cambridge, I am glad to appear before you as his emissary on this notable occasion.

Your gracious act in bestowing this honour upon my countryman will give him great pleasure. He is one of those rare and remarkable men who have no thought for the lustre of their own achievements and who habitually rate others more highly than they rate themselves. To us, the award of this Medal is a fitting measure of his great stature as a scientist; to him, it will appear as a token of your friendship, an expression of warm feeling and regard, which, as though obedient to Newton's law, will quicken in him like feelings towards you. That this award should have been made by an American learned society will be an especial pleasure to him.

I recently read in the British scientific journal Nature a review by Harold Jeffreys of a recent work of geophysics by one of your eminent scientists containing reference to the work of many scholars. In these restless days of ours it is refreshing to look in upon a world in which national rivalry is utterly of no account and where the habit of co-operation on universally respected terms of frankness and objectivity has become as natural as it is necessary. Though the lessons of scientific co-operation are not easily applied to other fields of human activity, they are a rebuke upon our divisions of which we ought constantly to be aware. Your decision once again to award this Medal to a foreign scholar is a reminder of the universal polity of science and of the mutual confidence and respect, based upon common values, which prevail within the scientific family.

I would like now to read to you in his own words the response of Professor Jeffreys to the American Geophysical Union in their award of the William Bowie Medal.

ACCEPTANCE BY HAROLD JEFFREYS

The American Geophysical Union did me a great honour some years ago when it elected me a Corresponding Member. I find it somewhat overwhelming that I should now be selected for the Bowie Medal.

The name of the medal takes me back to 1923, when I first visited the United States and Canada. Dr. Bowie arranged my stay at the Cosmos Club, and besides his own considerable share in looking after me he introduced me to Day, Lambert, Bauer, Adams, Humphreys, and many other distinguished Washington geophysicists. The kindness I received on that trip at Washington and several other places is one of my lasting memories.

I have since visited North America in 1924, 1929, 1939, and 1950-51, and I should like to make this an opportunity of saying in public that the kindness of my various hosts has been boundless.

I have a further connexion with Washington in that I was born in the postal district of the original Washington, County Durham, from which the Washington family took their name. The form Wassingatun existed in 946 A.D. This is possibly the first time that a native of a place within a few miles of the original Washington has received a major award from an American scientific society.

I had some difficulty when I first heard of the award in understanding why I should be regarded as particularly cooperative, seeing that most of my work had been done in my own home. Possibly the construction of the seismological tables had something to do with it, since my partner, Bullen, and I based our work mainly on the work of about 500 observatories all over the world and on the elements of the earthquakes determined in the International Seismological Summary. The observations and the need for analysis never end, and I am glad to say that Mr. J. S. Hughes, who was with Turner from the start, is my right-hand man and the real leader now that I have myself become Director of the International Seismological Summary. But I cannot help feeling that as far as cooperation is concerned I have usually been at the receiving end. Dr. Lambert has spoken of my various fields of work, but versatility carries with it the consequence that I am not really an expert in any one of them, and I am greatly indebted to many people who have placed their special knowledge at my disposal when I have found myself in difficulties.

It is a particular pleasure to me that the citation should be by Dr. Lambert, an old friend and an endless source of information on matters connected with the Figure of the Earth and Earth Tides. I must thank him warmly for his kind words.

I am sorry that I cannot be present in person on this occasion, but I am glad to be represented by a former pupil, Mr. J. R. Carter, the brother of another former pupil, Prof. G. W. Carter, who is in turn the brother-in-law of a third, Prof. E. R. Lapwood, all distinguished in one field or another.

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

AN INVESTIGATION OF SWELL AND MICROSEISMS FROM THE HURRICANE OF SEPTEMBER 13-16, 1946

William L. Donn

(Contribution No. 56 of the Lamont Geological Observatory)

Abstract--Date from swell and microseism records associated with the hurricane of September 13-16, 1946 are compared. There appears to be no relationship between the period of the swell arriving at Cuttyhunk off Cape Cod and the period of microseisms recorded at nearby Weston Observatory. Time relationships tend to minimize the importance of swell or resulting surf as the microseism excitation factors. The origin of the microseisms is suggested as occurring directly beneath the storm.

Introduction--An attempt is made here to apply the methods of DEACON [1949] and DARBYSHIRE [1950] and most recently KAMMER and DINGER [1951] in comparing swell and microseism data from the same storm. A hurricane was used for this purpose rather than a larger cyclone since more precise locations could be made. This particular hurricane was selected owing to the pronounced variations in swell recorded from it which would therefore make obvious any correlation with microseism period that might be present. The swell records were made by an underwater pressure unit located at a depth of 78 ft off Cuttyhunk Island near the southwestern tip of

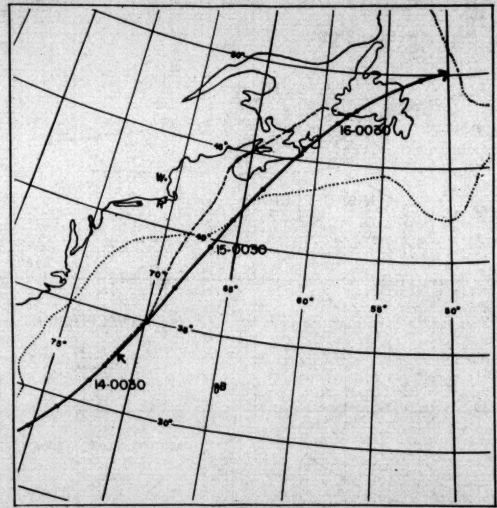


Fig. 1--Path of the hurricane from Sep. 13 to 16, 1946 (solid line) with center positions indicated every six hours (GMT); the broken line to the left of the track is a curve of microseism amplitude; the perpendicular distance between the path and the broken line is proportional to actual trace amplitude at that time; the dotted curve is the 1000-fm contour; B is Bermuda, R the wave recorder at Cuttyhunk, and W the seismograph station at Weston

Cape Cod (R on Fig. 1). Seismograms were obtained from Weston Observatory (W on Fig. 1), the nearest seismograph station.

Comparison of data--Figure 1 shows the path of the hurricane from September 13 to 16, with center positions marked for every six hours. It is apparent that the hurricane velocity increased as it traveled northeastward, and decreased temporarily near the end of its track. Figure 2 compares the measured period and amplitude of the microseisms recorded at Weston with the height and period of the swell recorded at Cuttyhunk. The technique of sampling and measuring the microseism and wave records is similar to that indicated earlier by DONN [1951a] and KLEBBA [1949], respectively.

After the usual increase of microseism period with increasing amplitude, the period became level at 3.5 sec, remaining so for the duration of the microseism storm. The earliest recorded storm swell at 24h 00m GMT, September 14, was 17 sec. Thereafter the swell period decreased to eight seconds, apparently a function of the azimuth of the hurricane [DONN, 1949]. A series of swell spectra made by machine analysis of the original wave records shows the narrow envelope of periods present (Fig. 3). The 11-sec swell (Fig. 2) at 18h 00m on September 14 was the last of the prevailing swell prior to the first arrival of hurricane swell which commenced abruptly with a much higher period. There is, further, no indication of the two-to-one ratio of swell period to microseism period reported empirically by DEACON [1949] and DARBYSHIRE [1950] and predicted theoretically by LONGUET-HIGGINS [1950] for microseisms supposedly originating from standing waves produced by interference of oppositely moving swell. This also contradicts the findings of KAMMER and DINGER [1951] who did not have actual recordings of swell for comparison with microseism dat

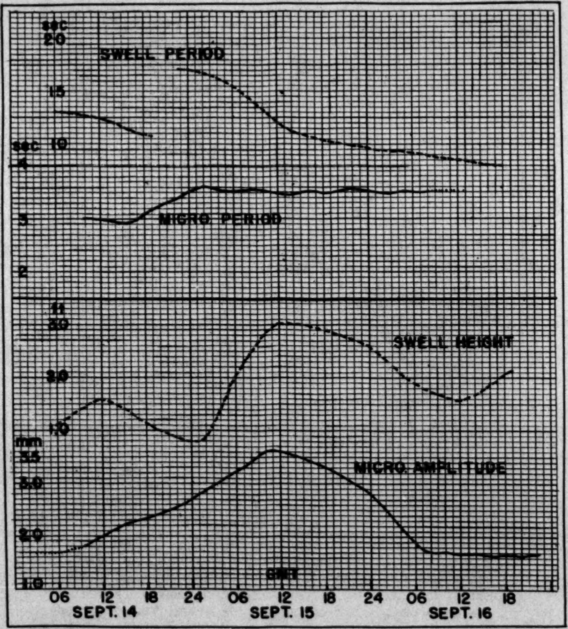


Fig. 2--Comparison of measured period and amplitude of microseisms at Weston (solid curves) with recorded swell height and period at Cuttyhunk (dashed curves)

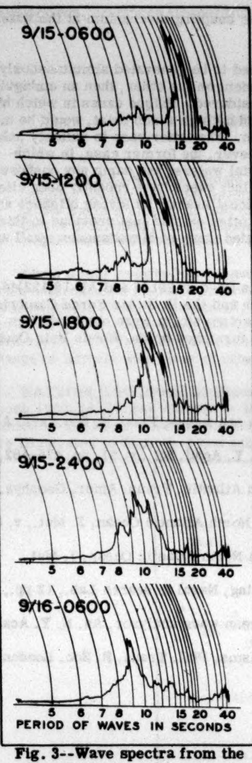


Fig. 3--Wave spectra from the Cuttyhunk records, November 26, 1941

The lack of any direct swell-microseism relationship is further strengthened by the time relation of the amplitude curves. The onset of the microseism storm was about 06h 00m, September 14, whereas the earliest storm swell reached the nearby coast between 18h 00m and 24h 00m, September 14. Further, the microseism storm declined to background level while storm swell was still being recorded at southeastern New England. This and later swell generated with the hurricane north of the station must have existed northward along the coast at points even closer to the seismic station until well after the microseism storm reached background level. This further conflicts with the findings of Kammer and Dinger that the duration of microseism storms parallels the duration of swell received from a passing or retreating hurricane.

All of the above tends to minimize also the effect of surf as the exciting agent. It might be argued that the early phase of the microseism storm was produced by swell or surf at some distance, possibly along the middle Atlantic Coast, most of which has gently sloping beach zones. But then it must be explained why the microseism storm did not continue while the swell was breaking on the rocky New England Coast instead of declining to background. Further, owing to the distribution of wind velocities in this hurricane, and its distinctly elliptical shape elongated in the direction of motion, the period of the swell that would have reached the middle coast zone is assumed to have been nine seconds, being a function of the storm azimuth [DONN, 1949]. From this information, plus the high forward velocity of the hurricane, it is calculated that the arrival of the swell along the East Coast would have barely preceded that at Cuttyhunk, and would have been well after the onset of microseisms. From a study of the weather maps it is estimated that the swell leaving the northwest quadrant of the hurricane when its center was at the 00h 30m position on September 14 would probably have reached the coast near Cape Hatteras first. Traveling 200 miles with a group velocity of 13.5 knots appropriate to a nine-second swell, such swell should have reached the above coast approximately at 15h 30m on September 14, or well after the commencement of microseisms.

Examination of the amplitude curve plotted on the storm path (Fig. 1) with respect to the hurricane position shows that the microseism storm increased in intensity fairly uniformly as the hurricane approached, and then decreased rather abruptly. The abrupt decline corresponds to the time when most of the hurricane passed over Newfoundland and surrounding shoals, and was also dissipating rapidly. Peak microseism amplitudes occurred with most of the hurricane over the continental shelf rather than a little earlier when it would have been closest to the station. This is similar to the behavior of other hurricane microseism storms suggested earlier [DONN, 1951b] as an evidence of a discontinuity affecting microseism propagation along the edge of the shelf in this area.

Further, by using appropriate group velocities, the position along the hurricane path corresponding to the time when recorded swell left the storm has been determined. The earliest swell to arrive, when projected back in time, thus left the storm approximately at the position shown by the arrow (Fig. 1). Hence the microseism storm had already commenced at Weston at approximately the time when the earliest recorded local hurricane swell was leaving the storm, and perhaps even earlier. This also tends to negate the possibility of progressive swell in shallow water off the coast being a generating factor.

Conclusions--These observations tend to minimize the importance of swell or surf as the exciting mechanism for this microseism storm. They suggest again that the microseism origin lies in the area directly beneath the storm, and results from some coupling mechanism at the water-air interface.

If swell (and resulting surf) and microseisms are assumed to be generated simultaneously by different mechanisms in a storm area, rather than one dependent on the other, then an ambiguity in origin will often exist, especially if time relations are considered. Unique cases in which high winds and no swell or surf exist, or in which swell or surf and no high winds exist, would be most helpful in discriminating between modes of microseism origin. The latter case is unlikely unless the swell or surf is the product of a very distant storm. However, the former case, in which strong offshore winds following a cold front spread over coastal waters, and damp any onshore swell that may exist, occurs very frequently. Many such distinct cases have recently been cited by DONN [1951ab] in which microseism storms commence almost exactly as strong offshore winds reach coastal waters, whereas only background microseisms often existed just previous to this, despite strong onshore swell-producing winds. A more extended study of simultaneous swell and microseism data is in progress.

Acknowledgments--This work was supported by Contracts N6-onr-29133 and AF 19(122)441 between Columbia University and the Office of Naval Research and the U. S. Air Force Cambridge Research Laboratories, respectively. Records for the microseism storm were obtained from Weston Observatory, and swell recordings and analyses were furnished by the Woods Hole Oceanographic Institution.

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(Communicated manuscript received October 8, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union **Volume 33, Number 3** **June 1952**

ON SECULAR CHANGE OF LATITUDE

Kennosuke Hosoyama

Abstract--Secular change of latitude is compared with secular change in other geophysical phenomena. An intimate correlation is found between the secular displacement of the Earth's mean pole and the change of height of mean sea level of the Atlantic Ocean. On the other hand, the Chandlerian amplitude and period show a parallel change with the amplitude of tidal variation of latitude. In both cases a geoid pulsation may be postulated.

About a half century has passed since the cooperative observation of latitude was commenced at the International Latitude Stations. During this period many important problems, such as determination of Chandlerian period, existence of mysterious Z-term, and so forth, were solved. But there remain many unsolved problems which have intimate connection with other geophysical phenomena. The following article is an attempt to compare and connect the observed secular change of latitude with those of other geophysical phenomena.

HATTORI [1947, 1949] of Mizusawa Latitude Observatory analyzed the observed data of the epoch 1900-38 at three stations - Mizusawa, Ukiah, and Carloforte - and discussed the secular motion of the Earth's mean pole and the periodic components of latitude variation. His results are illustrated in Figures 1 and 2ab. The Chandlerian periods in Figure 2a are the mean values at the three stations for the epochs 1900-06, 1907-12, 1913-19, 1920-25, and 1926-38, and the Chandlerian amplitudes in Figure 2b are the running means of six successive years of the mean values at the three stations. As shown in Figure 1, the Earth's mean pole was anomalously displaced in the period around 1920. On the other hand, the Chandlerian amplitude showed an exceedingly small value at about 1920 as shown in Figure 2, and also the period of Chandlerian motion became correspondingly shorter. From these facts it is assumed that some world-wide change of considerable amount might have occurred to the surface or to the interior of the Earth about 1920. It seemed worthwhile to seek some change of geophysical phenomena which would show a good correspondence to this peculiar change of latitude.

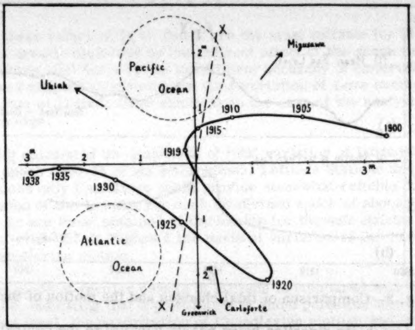

Fig. 1--Mean pole displacement

MARMER [1949] discussed the secular change of height of mean sea levels observed at New York, Seattle, and other tidal stations during a recent 50-year period. According to his paper there exists no large change in the Pacific Ocean, but a remarkable rise of mean sea level is observed in the Atlantic Ocean, and there appears also a sudden change before and after 1920 as if corresponding to the above-mentioned peculiar change of latitude.

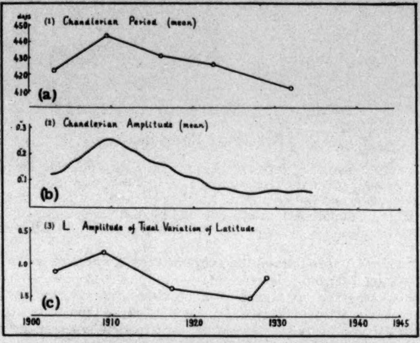

Fig. 2--Variations during the period

For convenience in comparing the results obtained by Hattori and Marmer, the following procedure was used. Concerning the mean sea level, the height differences between the values of six years' running mean at New York and Seattle were calculated, the mode of change of which is illustrated in Figure 3a. On the other hand, the motion of the Earth's mean pole was projected on the line (X-X) in Figure 1, its direction being nearly parallel to the central line of both oceans. The mode of motion of the mean pole on X-X is shown in Figure 3b. In Figure 3, it appears that the general mode of secular change of the height difference of the mean sea levels and of the position of the Earth's mean pole shows a good agreement. On this point a brief account has already been given [HOSOYAMA, 1951].

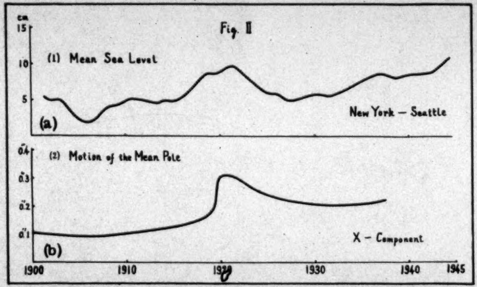

Fig. 3--Comparison of tidal changes and the motion of the pole

The following postulation may possibly explain this correspondence. Let us suppose that the observed change of mean sea level is not local but world-wide, caused by a gradual upheaval of geoid surface on the side of the Atlantic Ocean with a rapid change in the period about 1920. As a natural consequence, the Earth's mean pole moves towards the side of the Atlantic Ocean as shown in Figure 1. Moreover, if it is assumed that this pole shift is in the direction of diminution of the angle between the axis of figure and the axis of rotation of the Earth, then it should cause the diminution of Chandlerian amplitude and reduce the Chandlerian period. This is because the Chandlerian period T is expressed as

$$T = 2\pi\ [A/(C - A)]/[\omega\ (1 - \tau)]$$

where C and A are the moment of inertia of the Earth referred to the polar and equatorial axes respectively, ω the angular velocity of the Earth's rotation, and τ a function in product form of

the (C - A) and the product of inertia which originates from the discordance of the axis of figure with the rotation axis. As the axis of the figure approaches the rotation axis, the value of the product of inertia became smaller, and consequently also the value of τ when the value of (C - A) is assumed to be little disturbed in a first approximation. In the above formula the value of T may vary with ω and τ, taking [A/(C - A)] as a constant in a first approximation. At present, putting aside the problem of variation of the Earth's rotation speed, it may be possible that the diminution of τ caused by the approach of the axis of figure to the rotation axis results in the reduction of the Chandlerian period.

On the other hand, τ may be expressed in another form as

$$\tau = k\phi/(2\alpha - \phi)$$

where k is one of the Love numbers, α the Earth's ellipticity, and ϕ the ratio of centrifugal acceleration to gravity acceleration at the equator. The value of k is related to the density and elasticity distribution of the Earth's interior, and moreover to the geoid configuration. Any change which causes the geoid deformation and the pole shift and perturbs the Chandlerian motion may also change the values of k, α, and ϕ. The k-value can be calculated from the observed data on Earth tides in addition to being derived directly from the Chandlerian period. D, G, and L, the ratio of the observed amplitude to the theoretical in the observation of Earth tides with tiltmeter, gravimeter, and latitude variation respectively, are expressed as the following combinations of Love numbers

$$D \equiv 1 + k - h$$

$$G \equiv 1 - (3/2)\,k + h$$

$$L \equiv 1 + k - \ell$$

It cannot be said that these values of D, G, and L are the most suitable for the detection of change of k-value, as the D is greatly disturbed by the indirect effect of the ocean tides, the G by the scantiness of observations, and the L by the insufficient accuracy of observed data. But for the present the L-value was adopted to examine the time variation of Love numbers since it was derived from the same data of latitude observation as in the case of the analysis of the pole shift and the Chandlerian motion.

NISHIMURA [1950] calculated the amplitude of tidal variation of latitude from the observed materials during the epoch 1900-35 at six International Latitude Stations and discussed the value of L. Among six stations only Carloforte could provide somewhat reliable data for the examination of the time variation of the L-value for each subdivided epoch of about ten years. The values of L shown in Figure 2c are those obtained at Carloforte for the sub-epochs 1900-06, 1906-12, 1912-23, 1923-31, and 1923-35. In Figure 2 the mode of variation of the L-value appears to resemble that of the Chandlerian motion.

In conclusion, the mode of secular change of latitude is compared with that of other allied geophysical phenomena, indicating that a postulation of geoid pulsation may explain the pole shift, the change of mean sea level, the perturbation of Chandlerian motion, and moreover the change in the value of the Love number k, though it leaves much to be discussed. And, on the other hand, it would be desirable to examine other secular changes of geophysical phenomena, such as the annual seismic activity of the world, the displacement of the geomagnetic pole, and so forth, in connection with the secular change of latitude.

The writer expresses his cordial thanks to E. Nishimura for his kind advice throughout the preparation of this article.

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(Communicated manuscript received September 12, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

MAGNETIC ANOMALIES OVER OCEANIC STRUCTURES

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(Lamont Geological Observatory Contribution No. 57)

(As noted at the end of this paper, it was presented orally at the Twenty-Eighth Annual Meeting in 1947, and the paper was received for publication early in 1948. At that time, it seemed desirable to defer publication pending receipt of another paper then in progress reporting on actual observations over one of the areas here discussed. The latter paper has been delayed much longer than was anticipated but is now nearly complete, and tentative plans are to publish it in the Transactions of August, 1952. The importance of the airborne magnetometer as a new tool for the investigation of oceanic structure still deserves stress, and it is hoped that in any event the methods described in this paper may prove to be a valuable contribution despite the regrettable delay.--Ed.)

Abstract--The importance of the airborne magnetometer as a geophysical tool in the study of oceanic structures is stressed. Theoretical anomalies of the total magnetic vector are computed for typical oceanic structures to aid in the planning of aerial surveys. These structures are (1) a volcanic island, (2) a crustal fold associated with a negative gravity strip, and (3) a continental border.

Introduction--The introduction of the saturable-core airborne magnetometer as a geophysical tool should facilitate the study of large-scale oceanic structures. Oceanic magnetic surveys, which previously could be carried out only with great difficulty, can now be completed rapidly and with comparatively little effort. This paper deals with the theoretical anomaly in the total magnetic vector introduced by three typical oceanic structures. The structures considered are a volcanic island, a continental border, and a crustal fold associated with a negative gravity strip. A simplified cross section, based on earlier geologic or geophysical investigations, is assumed for each structure. The magnitude and extent of the theoretical anomalies should be especially useful in planning the flight lines for a magnetic survey of these structures.

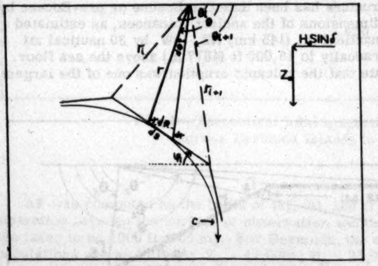

Fig. 1--Cross section of a two-dimensional structure, divided into line segments for the calculation of magnetic anomalies

The calculations were made on the basis of the well-known expression for the vertical and horizontal magnetic attractions ΔZ and ΔH of a distribution of magnetic charge induced by the Earth's field on the surface of a two-dimensional structure. From the equation for the attraction of an infinite line distribution of magnetic charge and the relations $ds \sin\theta = -r\,d\theta\cos\phi + dr\sin\phi$, $ds\cos\theta = dr\cos\phi + r\,d\theta\sin\phi$ (Fig. 1), one obtains

$$\Delta Z = 2\kappa \int_c (Z_0\cos\phi + H_0\sin\delta\sin\phi)(\sin\theta/r)\,ds = 2\kappa\int_c (Z_0\cos\phi + H_0\sin\delta\sin\phi)\,(\sin\phi\,(dr/r) - \cos\phi\,d\theta) \qquad (1)$$

$$\Delta H = -2\kappa \int_c (Z_0\cos\phi + H_0\sin\delta\sin\phi)(\cos\theta/r)\,ds$$

$$= -2\kappa\int_c (Z_0\cos\phi + H_0\sin\delta\sin\phi)\,[\cos\phi\,(dr/r) + \sin\phi\,d\theta] \qquad (2)$$

In these equations Z_0 and H_0 are the vertical and horizontal components of the Earth's field, ϕ is the angle of inclination of the line element ds, δ is the angle between the axis of the structure and magnetic north, r and θ are the magnitude and direction of the radius vector from the point of observation to the element ds, and κ is the magnetic susceptibility of the structure. The contour integral is to be carried out over the entire cross section of the structure. If we represent the cross section by a sequence of line segments, one of which is shown in Figure 1, (1) and (2) can be integrated over each segment, and the results can be summed over all segments to give

$$\Delta Z = 2\kappa \sum_i (Z_0 \cos \phi_i + H_0 \sin \delta \sin \phi_i)\left\{\cos \phi_i (\theta_i - \theta_{i+1}) + \sin \phi_i \log_e [(r_{i+1})/r_i]\right\} \ldots\ldots (3)$$

$$\Delta H = 2\kappa \sum_i (Z_0 \cos \phi_i + H_0 \sin \delta \sin \phi_i)\left\{\sin \phi_i (\theta_i - \theta_{i+1}) - \cos \phi_i \log_e [(r_{i+1})/r_i]\right\} \ldots\ldots (4)$$

where ϕ_i is the inclination of the i^{th} segment, and r_i, θ_i, r_{i+1}, θ_{i+1} define the radius vectors from the point of observation to the end points of the i^{th} segment. Since the total intensity F of the Earth's field is large compared to the anomaly ΔF, a good approximation of the total intensity anomaly will be its projection on F. Thus we may write

$$\Delta F = \Delta H \cos I \sin \delta + \Delta Z \sin I \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where I is the inclination of the Earth's field. Equations (3), (4), and (5) will be used as the basis for all subsequent derivations.

The magnetic anomaly of Bermuda--The information obtained in a magnetic investigation of an oceanic island such as Bermuda can be used to determine what portion of the island consists of igneous rocks. Proper interpretation of the magnetic data can reveal the thickness of the nonmagnetic cover. To aid in such an interpretation the theoretical total-intensity anomaly was computed for Bermuda, a volcanic island for which the geology is fairly well known.

Bermuda consists of a truncated volcanic peak upon which organic limestone sediments have been deposited. PIRSSON [1914] describes a geologic section based upon a bore hole, as follows: Chalky Bermuda limestone, surface to 245 ft (75 m) below sea level; oxidized volcanic matter, 245 to 455 ft (75 - 139 m); sand and gravel, 455 to 560 ft (139 - 171 m); dark gray lavas, below 560 ft (171 m).

More recently WOOLLARD and EWING [1939] placed the volcanics at depths of 243 and 273 ft (74 - 83 m) below sea level, on the basis of seismic-refraction profiles at two locations remote from bore hole.

The volcanic activity which formed the structure has been dated as Eocene or pre-Eocene by Pirsson on the basis of fossil evidence. The dimensions of the ancient volcanoes, as estimated from a study of bathymetric contours, are 90 nautical mi (145 km) NE - SW, by 30 nautical mi (48 km) NW - SE. The truncated cones rise gradually to 16,000 ft (4877 m) above the sea floor. According to Pirsson, these dimensions indicate that the volcanic eruption was one of the largest in the world.

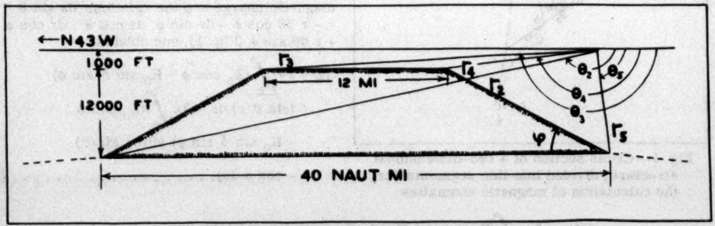

Fig. 2--Assumed cross section of Bermuda Islands

The relatively high magnetic susceptibility of the lavas as compared to the limestone overburden and the surrounding ocean results in a noticeable distortion of the Earth's field [see COLE, 1908]. A NW - SE cross section, derived on the basis of U. S. Hydrographic Office Chart 27, is shown in Figure 2. In order to simplify the calculations we have assumed that the magnetic anomaly is caused by the two-dimensional Trapezoid indicated by the heavy lines of Figure 2. It is believed that the error caused by considering the major slopes only and assuming the NE - SW extent of the structure to be infinite, is small.

The vertical magnetic attraction ΔZ and the horizontal magnetic attraction ΔH can be derived from equations (3) and (4), and are as follows

$$\Delta Z = 2\kappa\, Z_0 \left[\sin\phi \cos\phi \log_e (r_2 r_5/r_3 r_4) - \sin^2\phi\, (\theta_2 - \theta_3 + \theta_4 - \theta_5)\right]$$
$$+\, 2\kappa\, H_0 \sin\delta \left[\sin^2\phi \log_e (r_3 r_5/r_2 r_4) - \sin\phi \cos\phi\, (\theta_2 - \theta_3 - \theta_4 + \theta_5)\right] \quad \ldots\ldots (6)$$

$$\Delta H = 2\kappa\, Z_0 \left[\sin^2\phi \log_e (r_3 r_5/r_2 r_4) - \sin\phi \cos\phi\, (\theta_2 - \theta_3 - \theta_4 + \theta_5)\right]$$
$$+\, 2\kappa\, H_0 \sin\delta \left[\sin\phi \cos\phi \log_e (r_3 r_4/r_2 r_5) + \sin^2\phi\, (\theta_2 - \theta_3 + \theta_4 - \theta_5)\right] \quad \ldots\ldots (7)$$

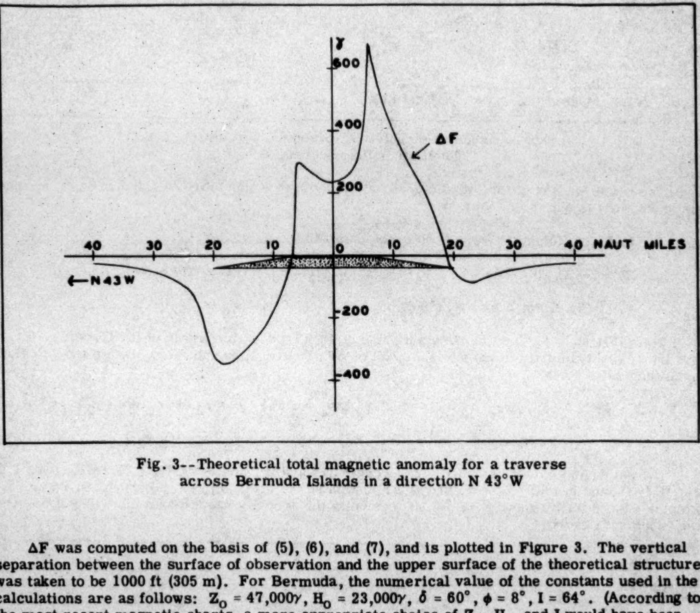

Fig. 3--Theoretical total magnetic anomaly for a traverse across Bermuda Islands in a direction N 43°W

ΔF was computed on the basis of (5), (6), and (7), and is plotted in Figure 3. The vertical separation between the surface of observation and the upper surface of the theoretical structure was taken to be 1000 ft (305 m). For Bermuda, the numerical value of the constants used in the calculations are as follows: $Z_0 = 47{,}000\gamma$, $H_0 = 23{,}000\gamma$, $\delta = 60°$, $\phi = 8°$, $I = 64°$. (According to the most recent magnetic charts, a more appropriate choice of Z_0, H_0, and I would have been $Z_0 = 49{,}000\gamma$, $H_0 = 21{,}300\gamma$, $I = 64\ 1/2°$.)

For the assumed susceptibility $\kappa = 0.01$ cgs units, the theoretical anomaly in the total magnetic field amounts to 230γ over the center of the structure. The anomaly rises as the flanks are approached and then drops very rapidly.

The anomaly will, of course, be affected by departures from the assumed configuration and by variations in the susceptibility of the volcanics. It is believed that the latter effect will be the largest source of error in these calculations.

The magnetic anomaly of the Continental Shelf--(The magnetic anomaly was recomputed in March 1952 for a structure based on more detailed seismic measurements than were available in 1947.) The structure of the Continental Shelf is the key to the solution of many significant geological problems. The edge of the continent marks the border between two provinces of contrasting geophysical properties and is therefore ideally suited for study.

Recent seismic profiles of EWING, WORZEL, STEENLAND, and PRESS [1950] across the Continental Shelf have traced the basement from just off the beach to the edge of the Continental Shelf. A sharp flexure in the basement was observed about 22 nautical mi (41 km) off the south shore of Long Island. The magnetic anomaly of this feature is computed in this section. The slopes indicated in the seismic profile form the basis of the simplified structure used in the derivation of magnetic-anomaly curves.

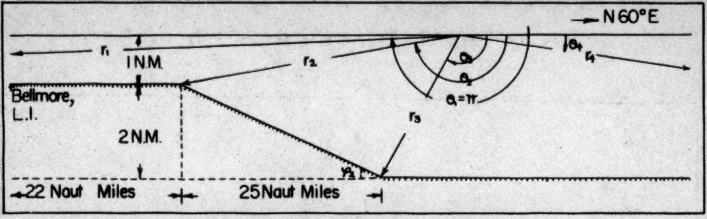

Fig. 4--Assumed structure of basement across Continental Shelf off Bellmore, Long Island

The cross section of the two-dimensional structure assumed for the calculation of the magnetic anomaly is shown in Figure 4.

The following formulas were used in the computation of ΔF

$$\Delta Z = 2\kappa Z_o\, [(\theta_1 - \theta_2 + \theta_3 - \theta_4) + \cos^2\phi_2\, (\theta_2 - \theta_3) + \cos\phi_2 \sin\phi_2 \log_e (r_3/r_2)]$$
$$+ 2\kappa H \sin\delta\, [\cos\phi_2 \sin\phi_2\, (\theta_2 - \theta_3) + \sin^2\phi_2 \log_e (r_3/r_2)] \quad \ldots\ldots\ldots\ldots\ldots (8)$$

(Note that $(\theta_1 - \theta_4)$ has the constant value of π for the entire length of the traverse. Since we are interested primarily in the variable part of ΔF, this term has not been used in the calculation of ΔZ and ΔF.)

$$\Delta H = 2\kappa Z_o\, [\log_e (r_3/r_2) - \cos^2\phi_2 \log_e (r_3/r_2) + \cos\phi_2 \sin\phi_2\, (\theta_2 - \theta_3)$$
$$+ 2\kappa H_o \sin\delta\, [\sin^2\phi_2\, (\theta_2 - \theta_3) - \cos\phi_2 \sin\phi_2 \log_e (r_3/r_2)] \quad \ldots\ldots\ldots\ldots (9)$$

The numerical values assumed for the constants are $Z_o = 54{,}000\gamma$, $H_o = 17{,}000\gamma$, $I = 72\ 1/2°$, $\phi_2 = 4\ 1/2°$, and $\delta = 72°$. The values of ΔF computed from (5), (8), and (9) appear in Figure 5. For the susceptibility contrast $\kappa = 0.002$ cgs units the anomaly has a maximum value of 60γ over the basement flexure.

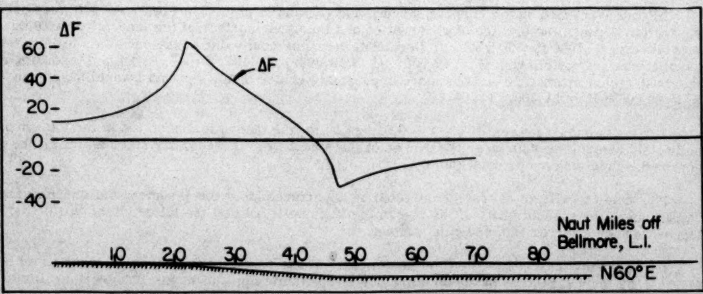

Fig. 5--Theoretical total magnetic anomaly for traverse across the Continental Shelf off Bellmore, Lond Island, in a direction N 60°E

The magnetic anomaly north of Puerto Rico--The celebrated discovery of negative gravity strips by VENING MEINESZ [1934] has been considered as one of the most important contributions to geology in the 20th century. Vening Meinesz first reported this phenomenon in the East Indies. Later work by Hoskinson, Ewing, and Hess [EWING, 1937] resulted in the delineation of another example which Vening Meinesz had discovered in the West Indies. The negative gravity strips are characterized by great length, relatively small width, and negative isostatic gravity anomalies of at least 175 mg.

Vening Meinesz explains these negative strips on the basis of his "buckling hypothesis." Under later compression the sial, or light upper layers of the crust, with a density of about 2.7, forms a root in the underlying sima, density 3.3, which is pressed down and aside. According to the hypothesis, the gravity minimum lies above the relatively light sialic roots.

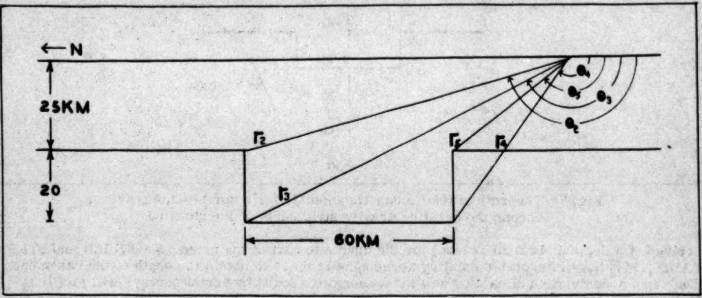

Fig. 6--Assumed cross section of the Earth's crust under the negative gravity strip north of Puerto Rico

Vening Meinesz calculated the dimensions of the root that would best satisfy the observed gravity anomaly. Assuming a density difference of 0.6 between sial and sima, he found that a rectangular cross section of 20 × 60 km, 25 km beneath the surface corresponds approximately to the curve of regional isostatic anomaly (Fig. 6).

The equations for the computation of the magnetic anomaly due to the two-dimensional root having a rectangular cross section are

$$\Delta Z = -2\kappa Z_0 (\theta_2 - \theta_3 + \theta_4 - \theta_5) + 2\kappa H_0 \sin\delta \log_e (r_3 r_5/r_2 r_4) \dots\dots\dots \quad (10)$$

$$\Delta H = 2\kappa Z_0 \log_e (r_3 r_5/r_2 r_4) + 2\kappa H_0 \sin\delta (\theta_2 - \theta_3 + \theta_4 - \theta_5) \dots\dots\dots \quad (11)$$

For the negative gravity strip north of Puerto Rico, the constants have the following numerical values: $Z_0 = 37{,}000\gamma$, $H_0 = 26{,}800\gamma$, $I = 54°$, $\delta = 90°$. The susceptibility contrast κ was taken to be 0.001 cgs units.

The magnetic anomaly ΔF computed on the basis of the gravity picture from (5), (10), and (11) is shown in Figure 7. If the observed anomaly fits the computed curve, we could conclude that the root and the surrounding material have a constant ratio of density and susceptibility, and that magnetization effects can exist at depths below 25 km.

Whether or not the proposed magnetic survey over Puerto Rico will reveal the existence of the root is problematical. If the high temperature which is known to exist 25 km within the crust makes the rocks at this depth non-magnetic, the root will not produce a magnetic anomaly. The folded rocks over the zone of buckling might have sufficient magnetic material to mask any effects originating at greater depths.

The thickness of the crust--VACQUIER and AFFLECK [1941] indicated how a computation of the average depth to the bottom of the Earth's magnetic crust could be made by a statistical study of local magnetic anomalies originating beneath the sedimentary layers. These investigators

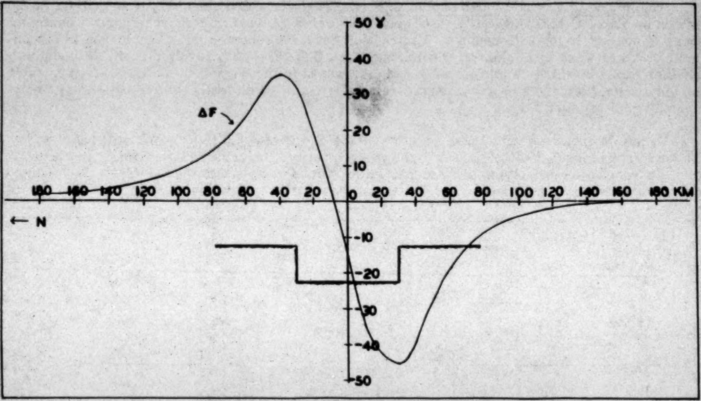

Fig. 7--Theoretical total magnetic anomaly for a north-south traverse across the negative gravity strip north of Puerto Rico

arrived at a depth of 12.5 mi (20 km) for the magnetic part of the crust. VACQUIER and STEENLAND [1947] have interpreted existing aeromagnetic maps to obtain the depth to the basement. The rapid accumulation of data on new local anomalies found by aerial surveys will facilitate further work along these lines.

Conclusions--The airborne magnetometer can be an effective tool in the location and study of oceanic and continental structures, especially in conjunction with other geophysical methods. Although the computation of theoretical anomalies are based upon over-simplified assumptions, such as constancy of susceptibility and simple geometric cross sections, the extent and magnitude of the theoretical anomalies are useful in plotting the flight lines of aerial surveys. Important information about the structure may be obtained from a study of the deviations between the observed and theoretical anomalies.

It is strongly urged that such aerial magnetic surveys be made of oceanic islands, continental borders, negative gravity strips, oceanic deeps, ridges, and trenches. These surveys would not require a great deal of preparation. They could be made, for example, on shake-down flights, or on flights whose primary purposes are transportation of instruments and personnel to and from areas where mineral and oil exploration is taking place.

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(Manuscript received January 7, 1948; presented at the Twenty-Eighth Annual Meeting, Washington, D. C., April 29, 1947; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

SIMILARITY SOLUTION OF A SPECIALIZED DIFFUSION EQUATION

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Abstract--The differential equation of diffusion when the wind velocity and the vertical and lateral diffusivities are power functions of height is

$$y^m \partial c/\partial x = D_1 \partial (y^n \partial c/\partial y)/\partial y + D_2 y^k \partial^2 c/\partial z^2$$

where x, y, and z are measured respectively in the down-wind, vertical, and cross-wind directions, and D_1 and D_2 are physical constants defined in the text. Exact solution of this equation for the case of a point source and for $m = k$ is presented in this paper. In the systematic search for this solution, dimensional analysis has been utilized to the optimum advantage. Although the solution is restricted to the special case $m = k$, it shows the important characteristics of atmospheric diffusion from a point source.

Introduction--Two-dimensional diffusion, when the wind velocity and the vertical diffusivities are power functions of height, has been extensively treated by O. G. SUTTON [1934], W. G. L. SUTTON [1943], FROST [1946], CALDER [1949], and YIH [1952]. Three-dimensional diffusion where lateral diffusivity must be considered has been treated by DAVIS [1947, 1950], and by O. G. SUTTON [1947] in the case of a point source, on the assumption that the variation of the wind velocity with height may be neglected. Thus in comparison with the two-dimensional phenomenon, the three-dimensional one has apparently received insufficient attention.

This paper is concerned with the atmospheric diffusion from a point source when the wind velocity and the vertical and lateral diffusivities are power functions of height, the exponents (m, n, k in the following) being at first left completely free. A mathematical solution is found possible for the special case

$$m = k$$

The differential system--With the origin at the point source, and the directions of x, y, and z defined as in the abstract, if the variation of wind velocity u with y (vertical) is expressed by

$$u/u_1 = (y/y_1)^m \qquad (1)$$

where u_1 is the wind velocity at y_1 and if the vertical and lateral diffusivities are respectively

$$A_v = A_1 (y/y_1)^n \qquad (2)$$

$$A_\ell = A_2 (y/y_1)^k \qquad (3)$$

where again A_1 and A_2 correspond to the height y_1, the equation of diffusion

$$u \partial c/\partial x = \partial(A_v \partial c/\partial y)/\partial y + \partial(A_\ell \partial c/\partial z)/\partial z \qquad (4)$$

can be written as

$$y^m \partial c/\partial x = D_1 \partial(y^n \partial c/\partial y)/\partial y + D_2 y^k \partial^2 c/\partial z^2 \qquad (5)$$

where c is the concentration of the quantity under diffusion and

$$D_1 = A_1 y_1^{m-n}/u_1 \qquad D_2 = A_2 y_1^{m-k}/u_1 \qquad (6)$$

The differential equation (5) is to be solved with the following boundary conditions

(a) $\partial c/\partial y = 0$ at $y = 0$
(b) $\partial c/\partial z = 0$ at $z = 0$
(c) $c \rightarrow c_0$ as $y \rightarrow \infty$
(d) $c \rightarrow c_0$ as $|z| \rightarrow \infty$
(e) $c \rightarrow c_0$ as $x \rightarrow 0$ for $y > 0$
(f) $c \rightarrow c_0$ as $x \rightarrow 0$ for $z > 0$

and the integral continuity equation

$$(g) \int_{-\infty}^{\infty}\int_{0}^{\infty} u\,(c - c_0)\,dy\,dz = Q = \text{constant}$$

where c_0 is the ambient concentration, and Q is the strength of the point source. Condition (a) stipulates that the ground is impervious to the quantity under diffusion, and (b) follows from symmetry and can be replaced by the more general condition that c should be an even function with respect to z.

The solution--To facilitate the systematic search for a similarity solution (Ähnlichkeitslösung), a dimensional analysis will be performed first, which, in conjunction with considerations of the powers of x, A_1, A_2, u_1, and y_1, will afford the most adequate transformation to be made in order that the solution will be the simplest. The pertinent variables are

$$c, c_0, Q, A_1, A_2, u_1, y_1, x, y, z$$

A dimensional analysis yields the relationship

$$(c - c_0)/c_0 = F\,(Q/A_1c_0x,\ u_1x/A_1,\ u_1x/A_2,\ y_1/x,\ y/x,\ z/x) \quad \text{(7)}$$

To obtain a similarity solution, one makes the following substitution:

$$\phi = (c - c_0)/c_0 = (Q/A_1c_0x)\,(u_1x/A_1)^{\alpha}\,(u_1x/A_2)^{\beta}\,(y_1/x)^{\gamma}\,f\,(\eta, \zeta) \quad \text{(8)}$$

where

$$\eta = (u_1x/A_1)^{p}\,(y_1/x)^{q}\,y/x \quad \text{(9)}$$

$$\zeta = (u_1x/A_2)^{r}\,(y_1/x)^{s}\,z/x \quad \text{(10)}$$

and where the exponents α, β, γ, p, q, r, and s are to be determined. Before proceeding further with the solution, it may be noticed here that the power of Q/A_1c_0x is unity in consideration of condition (g), and that a pair of fixed values for η and ζ defines a space curve which is the intersection of two parabolic cylinders

$$y = K_1\,x^{1+q-p} \quad \text{(11)}$$

$$z = K_2\,x^{1+s-r} \quad \text{(12)}$$

The set of all the curves defined by (11) and (12) for various values of K_1 and K_2 will be dense in the three-dimensional space under consideration. On any two such curves, the values of ϕ will always bear the same ratio for any value of x. This is the reason why the solution having the form of (8) is called a similarity-solution.

One now proceeds to determine the exponents in (8), (9), and (10). Substituting (8) in (5) and requiring equal powers in u_1, y_1, and x and equal joint powers in A_1 and A_2 one has

$$p = 1/(m - n + 2) \qquad q = (n - m)/(m - n + 2) \quad \text{(13)}$$

$$r = (k - n + 2)/2\,(m - n + 2) \qquad s = (k - m)/(m - n + 2) \quad \text{(14)}$$

so that

$$2p - q - 1 = 0 \quad \text{(15)}$$

$$2r - s - 1 = 0 \quad \text{(16)}$$

The exponents α, β, and γ are left undetermined by this procedure, and will be determined by condition (g) which gives, after (8) has been substituted into it

$$
\begin{aligned}
&\alpha + \beta + 1 - p\,(m + 1) - r = 0\\
&- m - q\,(m + 1) - s + \gamma = 0\\
&-1 - \alpha + p\,(m + 1) = 0\\
&- \beta + r = 0\\
&(m + 1)\,(- p + q + 1) + (- r + s + 1) + \alpha + \beta - \gamma - 1 = 0
\end{aligned}
$$

and

$$\int_{-\infty}^{\infty}\int_{0}^{\infty} \eta^{m} f\,(\eta, \zeta)\, d\eta\, d\zeta = 1 \qquad (17)$$

The five equations given by condition (g) involving the unknown exponents α, β, γ are not independent, and are satisfied by

$$\beta = r \qquad (18)$$

$$\alpha = p\,(m + 1) - 1 \qquad (19)$$

$$\gamma = m + s + q\,(m + 1) \qquad (20)$$

so that

$$\alpha + \beta - \gamma - 1 = - [p\,(m + 1) + r] \qquad (21)$$

With the exponents given in (13), (14), (18), (19), and (20) in terms of m, n, and k, substitution of (8) in (5) results in the following equation

$$
\begin{aligned}
&(\alpha + \beta - \gamma - 1)\, f + (p - q - 1)\, \eta f_{\eta} + (r - s - 1)\, \zeta f_{\zeta}\\
&\quad = \eta^{n-m} f_{\eta\eta} + n\eta^{n-m-1} f_{\eta} + \sigma^{-1-2r}\, \eta^{k-m} f_{\zeta\zeta} \qquad (22)
\end{aligned}
$$

where

$$\sigma = A_2/A_1$$

and where subscripts denote partial differentiations. In virtue of (15), (16), and (21), (22) can be written

$$- [p\,(m + 1) + r]\, f - p\,\eta f_{\eta} - r\, \zeta f_{\zeta} = \eta^{n-m} f_{\eta\eta} + n\eta^{n-m+1} f_{\eta} + \sigma^{-s}\, \eta^{k-m} f_{\zeta\zeta} \qquad (23)$$

This is the differential equation that has to be solved in general. For the case m = k, make the transformation

$$\xi = \sigma^{s/2} \zeta \qquad (24)$$

(23) can now be written.

$$- [p\,(m + 1) + r]\, f - p\,\eta f_{\eta} - r\, \xi f_{\xi} = \eta^{n-m} f_{\eta\eta} + n\eta^{n-m-1} f_{\eta} + f_{\xi\xi} \qquad (25)$$

(25) being in a form suitable for separation of variables, one assumes

$$f = Y\,(\eta)\, Z\,(\xi) \qquad (26)$$

Substitution into (25) gives

$$- [p\,(m + 1) + r] - p\,\eta\, Y'/Y - (\eta^{n-m}\, Y'' + n\;{}^{n-m-1}\, Y')/Y = Z''/Z + r\,\xi\, Z'/Z = \lambda$$

which can be written into the two separate equations

$$[p\,(m + 1) + r + \lambda]\, Y + p\,\eta\, Y' + (\eta^{n-m}\, Y'' + n\eta^{n-m-1}\, Y') = 0 \qquad (27)$$

$$Z'' + r\,\xi\, Z' - \lambda\, Z = 0 \qquad (28)$$

where the primes denote ordinary differentiation, with respect to η for Y and with respect to ξ for Z. The boundary conditions for (27) are

(h) $Y'(0) = 0$
(i) $Y(\infty) = 0$

and those for (28) are

(j) $Z'(0) = 0$
(k) $Z(\infty) = 0$

One first considers the system (28), (j), and (k). A first integration gives

$$Z' + r\xi Z - (\lambda + r)\int_0^{\xi} Z\, d\xi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28a)$$

the lower limit being chosen equal to zero since $Z'(0) = 0$. As $\xi \rightarrow \infty$, one has $Z \rightarrow 0$, $Z' \rightarrow 0$, and

$$\int_0^{\infty} Z\, d\xi \neq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

since otherwise the integral in (17) would vanish. On the other hand, $\int_0^{\infty} Z\, d\xi$ should be finite on account of (17). Consequently, if Z behaves as ξ^{-b} for large ξ, b must be larger than one so that $\xi Z \rightarrow 0$ as $\xi \rightarrow \infty$. If Z vanishes exponentially as $\xi \rightarrow \infty$, then also $\xi Z \rightarrow 0$ as $\xi \rightarrow \infty$. These are the only ways in which Z can vanish at infinity, and for each of these cases the first two terms of (28) vanish while the third one does not unless $\lambda = -r$ on account of (29). The satisfaction of (28) and (k) therefore requires

$$\lambda = -r \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

Thus there is only one single value for the parameter λ. With (30), integration of (28) gives

$$Z = K \exp(-r\xi^2/2) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

where K is an arbitrary constant to be determined by (17). Substituting (30) into (27) and multiplying throughout by η^m, one has

$$[(m + 1)/(m - n + 2)]\, \eta^m Y + [1/(m - n + 2)]\, \eta^{m+1} Y' + (\eta^n Y'' + n\eta^{n-1} Y') = 0$$

a first integration of which gives

$$[1/(m - n + 2)]\, \eta^{m+1} Y + \eta^n Y' = 0$$

the constant of integration being zero since $Y'(0) = 0$ and $Y(0)$ is finite. A second integration gives

$$Y = \exp[-\eta^{m-n+2}/(m - n + 2)^2] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

the constant factor being absorbed in K of (31).

The constant K can be determined by (17) which can be written as

$$K\sigma^{-s/2} \int_{-\infty}^{\infty} \int_0^{\infty} \eta^m \exp[-\eta^{m-n+2}/(m - n + 2)^2 - r\xi^2/2]\, d\eta\, d\xi = 1 \quad \ldots\ldots \quad (33)$$

Evaluation of (33) gives

$$\sigma^{-s/2} (2\pi/r)^{1/2} \int_0^{\infty} \eta^m \exp[-\eta/(m - n + 2)^2]\, d\eta = 1$$

or

$$(m - n + 2)^{1-2a} K \sigma^{-s/2} (2\pi/r)^{1/2}\, \Gamma(a) = 1$$

which gives

$$K = \sigma^{s/2} (r/2\pi)^{1/2} (m - n + 2)^{2a-1} / \Gamma(a) \quad \text{(34)}$$

where

$$a = (m + 1)/(m - n + 2) \quad \text{(35)}$$

and

$$\Gamma(a) = \int_0^\infty w^{a-1} e^{-w} \, dw$$

is the gamma function.

Equations (31), (32), (34), and (35) give the function f by means of (26), which in conjunction with (24) and (8), yields the solution. The exponents α, β, γ, p, q, r, and s being given in terms of m, n, and k by (13), (14), (18), (19), and (20), and σ having been defined to be A_2/A_1. As has been stated, the exponents n and m = k (and in fact also the parameter σ) are left free to be determined by measurements. The case of n = 1 - m has been treated by DAVIS [1950, pp. 71-73].

Acknowledgment--This work has been done in connection with an ONR project (Contract No. N90nr - 82401) of the Civil Engineering Department of the Colorado Agricultural and Mechanical College.

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(Communicated manuscript received September 10, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

SURFACE OZONE AT COLLEGE, ALASKA, FOR THE YEAR 1950

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Abstract--The daily average concentrations of ozone in the atmosphere near the surface at College, Alaska (latitude 65°N, longitude 148°W), were determined during 1950 by the absorption of ozone in a solution of potassium iodide and standard sodium arsenite. The results, which were generally higher and more erratic than those found at lower latitudes, ranged from one to 75 mg/100 m^3 of dry air at standard conditions to give a yearly average of 9.8 mg/100 m^3(or 4.6×10^{-8} molecules of ozone per molecule of air). The monthly averages showed a principal maximum in April, a secondary maximum in July, and a minimum in August.

Introduction--The presence of a greater concentration of ozone near the surface at higher latitudes was first definitely shown by DAUVILLIER [1934] at Scoresby Sound (latitude 70°N in eastern Greenland) during the Second Polar Year (1932-33). His results, which indicated large day-to-day fluctuations in addition to a seasonal variation, showed that a further study of the problem might possibly lead to valuable information on atmospheric circulation. In this paper the results of daily measurements of the concentration of ozone during 1950, together with the temperature and pressure data from the local weather bureau, are reported and comparisons of these data are made with results in other localities.

Method--Preliminary work on the chemical determination of ozone at College, Alaska, has been the subject of a previous publication in which the details of the method were given [WILSON, 1951]. Briefly, the procedure consists of passing air upward through a column that is loosely packed with chipped glass and kept moist with a buffered solution of sodium arsenite and potassium

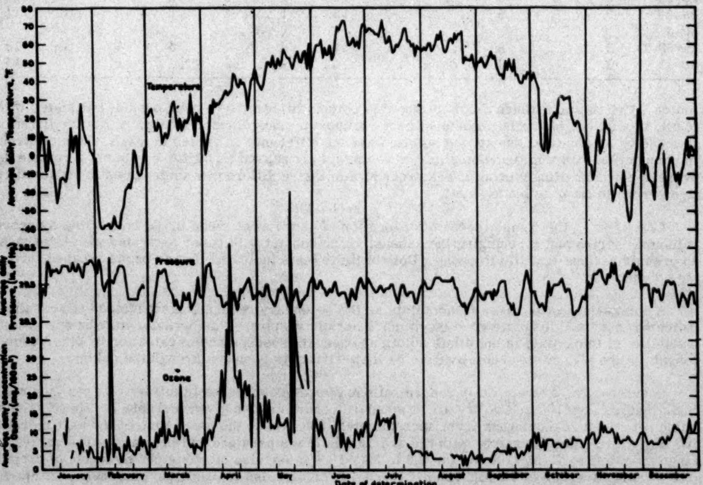

Fig. 1--Average daily determination of concentration of ozone, pressure, and temperature during 1950 at College, Alaska

Table 1--Ozone concentrations near the surface at College, Alaska, for 1950 expressed in milligrams per one hundred cubic meters (mg/100 m^3) of dry air at standard conditions, average of values taken from 08h 20m to 08h 00m

Day	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1- 2	..	3	7	7	18	14	9	5	4	10	9	10
2- 3	7	2	6	10	12	11	9	4	2	7	10	8
3- 4	..	6	6	11	11	9	11	4	3	10	10	6
4- 5	13	..	10	8	20	13	11	5	3	6	7	7
5- 6	..	8	7	11	13	9	14	3	3	6	8	8
6- 7	1	2	11	12	19	8	16	3	2	8	9	7
7- 8	12	5	9	15	11	6	12	3	4	5	13	7
8- 9	..	57	16	13	12	..	13	3	5	8	10	7
9-10	..	3	7	22	11	..	14	3	5	8	12	7
10-11	..	7	8	27	12	12	13	3	6	8	10	8
11-12	..	5	11	18	15	9	14	..	5	8	10	9
12-13	10	6	9	25	12	10	11	..	5	6	10	8
13-14	7	2	9	38	12	14	14	..	5	7	11	7
14-15	14	2	10	31	12	16	14	..	5	7	10	9
15-16	..	7	12	61	13	6	15	4	4	7	7	8
16-17	3	5	9	36	14	10	7	4	7	6	10	9
17-18	5	9	8	25	9	7	10	3	4	6	8	11
18-19	..	5	23	25	7	5	..	4	6	8	7	12
19-20	7	5	10	22	75	9	..	4	5	8	8	11
20-21	7	4	9	27	10	7	10	3	6	7	8	11
21-22	4	4	12	16	42	7	..	4	5	7	8	10
22-23	8	6	7	21	32	6	7	1	3	9	9	9
23-24	5	5	6	24	26	11	6	4	4	8	7	7
24-25	..	9	7	20	24	10	6	1	5	8	3	9
25-26	5	6	4	23	22	8	4	3	4	8	7	10
26-27	3	8	7	6	..	11	5	4	5	8	7	14
27-28	6	6	6	20	31	9	..	5	7	7	8	10
28-29	5	4	6	22	22	11	5	3	5	6	9	10
29-30	3	..	5	..	..	13	5	3	8	7	10	11
30-31 (or 30-1)	3	..	9	..	5	11	4	4	8	9	10	13
31- 1	3	..	8	..	13	..	4	2	..	8	..	13

iodide. The solution, which drips through the column twice in a 24-hour period, is titrated with iodine to a starch end-point to determine the amount of unoxidized arsenite. A similar titration is made on an identical, unexposed solution and the difference in values is taken. This difference, in conjunction with a measurement of the volume of air passed through the column, gives the daily average concentration of ozone. The procedure of taking differences is designed to reduce titration errors to a negligible amount.

Checks as to the completeness of absorption of ozone were made by (1) connecting a second column in series and (2) bringing the exhaust air in contact with paper saturated with tetrabase, an organic reagent specific to ozone. Both of these tests indicated that no ozone escaped through the column.

A comparison of the use of thiosulfate as the secondary reducing agent instead of arsenite indicated that the thiosulfate gave much more erratic results. This behavior may be due to the instability of thiosulfate in the dilute solutions used, its susceptance to oxidation by other components in the air, or its decomposition by light during its passage through the column.

Results-- The average daily concentrations of ozone expressed in milligrams per hundred cubic meters (mg/100 m^3) of dry air at standard conditions are given in Table 1. These values, which represent 24-hour runs from approximately 08h 00m to the same time of the next morning, are shown in Figure 1 together with the average daily temperature and pressure. The values range from one to 75 mg/100 m^3 (or 0.5 to 35×10^{-8} molecules of ozone per molecule of air) to give a yearly average of 9.8 mg/100 m^3 (4.6×10^{-8} molecules of ozone per molecule of air). A comparison with the results of a duplicate column over the last six months of the year showed that the values are reproducible to about one mg/100 m^3 for all but the higher concentrations of ozone.

While the air intake for the first seven months of the determinations was situated about three feet above the ground, the apparatus was shifted to the new Geophysical Institute building, and the intake was about 25 ft above the ground for the remainder of the year. The values obtained by different columns in the two locations were appreciably the same over the 12-day checking period.

A few consecutive 12-hour runs were made in the period July 8-14 with the result that the daytime (08h 00m - 20h 00m) value averaged almost 40 pct greater than that of the following night (20h 00m - 08h 00m). Although there was twilight all night long, the six-day tests showed only one evening value greater than that for the preceding day.

Discussion--The variations in concentrations of ozone from day to day are comparable to those found by DAUVILLIER [1934], but higher and more irregular than those found at lower latitudes [BOWEN and REGENER, 1951]. As the data represent the averages over 24-hour periods, it was not possible to show significant correlations with diurnal changes in local weather conditions. The only trend noted was the absence of high ozone values during periods of high pressure, which is in accordance with the theory [REGENER, 1938] that the amount of ozone transported to the surface should be dependent on atmospheric turbulence. Some instances have been observed, as on February 8-9, in which the high value of ozone seemed to accompany a subsidence of the tropopause rather than a large amount of mixing.

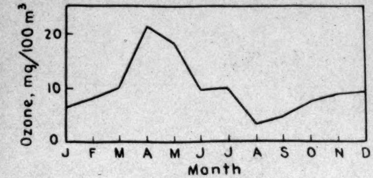

Fig. 2--Monthly averages of ozone concentrations

The graph of monthly averages of the concentrations, as shown in Figure 2, is similar to those for total ozone at comparable latitudes [MEETHAM and DOBSON, 1935; TONSBERG and OLSEN, 1944]. This similarity, which exists in the principal maximum in the spring, secondary maximum in the summer, and minimum in the autumn, may indicate the downward transport of stratospheric ozone in this region. However, as the surface ozone is probably reduced chemically by vegetation on the surface and foreign particles in the air, it is possible that the yearly variations would be affected by a ground cover of snow. This cover, which extended from late October through early April, would tend to move both the minimum and maximum of the surface variations ahead of the respective points in the total ozone curve as there was an absence of the reducing effect of the vegetation during a longer period.

The average concentration of ozone for 1950 (9.8 mg/100 m^3) represents a value that is comparable to the maximum concentrations found at the surface at lower latitudes and over twice as great as the average value found by BOWEN and REGENER [1951] in New Mexico. Compilations of CRAIG [1950] indicate that the amount of total ozone at latitude 65°N never exceeds that at latitude 35°N by more than 50 pct during the same month of the year. This relatively greater concentration of ozone near the surface in the Arctic probably arises from the colder temperatures, absence of atmospheric impurities, and the greater downward circulation of air.

Summary--The results of the daily determinations of the concentration of ozone at the surface have been presented with an outline of the method and comparison with values found elsewhere. This series of measurements is being continued, and plans are under way for determining the surface concentrations over shorter periods of time simultaneously with measurements of the total ozone. It is hoped that these future determinations will give definite information on the movement of ozone "clouds" in the Arctic atmosphere as well as help determine the seasonal circulation patterns. Furthermore, the possible relation between variations in the total amount of ozone and solar variability in the ultraviolet could be determined by a complete study of atmospheric ozone over a solar cycle.

Acknowledgments--The authors acknowledge the services of George Thirtle and Stuart Yaffe of this Laboratory who compiled a large amount of the data. The research reported in this paper has been made possible through support and sponsorship extended by the Geophysical Research Directorate of the Air Force Cambridge Research Laboratories under Contract W28-099 ac-445.

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(Manuscript received May 18, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1951; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

KINEMATIC VORTEX THEORY APPLIED TO THE MOTION OF ANOMALIES OF THE GENERAL CIRCULATION

Philip F. Clapp

Abstract--An attempt is made to apply the kinematic theory of vortex interaction, as exemplified by the von Karman "vortex street," to the problem of forecasting the motion of large-scale departure from normal centers as observed on northern hemisphere weather maps. The method has a small degree of success and points to some interesting characteristics of atmospheric vortices such as the failure of two systems to affect each other strongly when approaching too closely. The resemblance of this method to the two-dimensional numerical integration technique is pointed out, and suggests that the latter can be applied to problems of long-range forecasting.

Introduction--During "low-index" states of the general circulation, with cold cyclones at low latitudes and warm anticyclones at high latitudes, it becomes more difficult to forecast the behavior of upper-level flow patterns. In an attempt to overcome this difficulty in the preparation of long-range forecasts in the U. S. Weather Bureau, the kinematic theory of vortex interaction was applied on an experimental basis. A simple example of the application of this theory to a meteor-

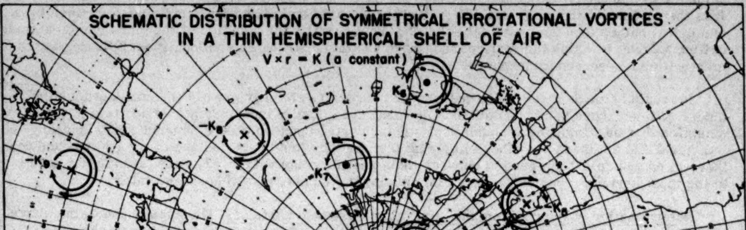

SCHEMATIC DISTRIBUTION OF SYMMETRICAL IRROTATIONAL VORTICES
IN A THIN HEMISPHERICAL SHELL OF AIR
$V \times r = K$ (a constant)
K_6
$-K_8$
$-K_9$
K_7
K_5
$-K_4$
Pole
K_{10}
K_2
$-K_{11}$
K_3
$V_1 = \frac{K_1}{r_1}$
K_1
$-K_0$
GREAT CIRCLE DISTANCE (r_1)
K_{12}
$-K_{13}$
U. S. DEPT. OF COMMERCE
WEATHER BUREAU
Date Prepared
For Period
Forecaster

Fig. 1--Schematic diagram showing anticyclonic (indicated by crosses) and cyclonic (dots) vortices embedded in a thin hemispherical shell of air; each vortex has its own strength (vr) indicated by K's with subscripts; anticyclonic strength considered negative; example of effect of one vortex on another indicated over the United States ological problem is afforded by the rotation of two hurricanes about a common center between them [HAURWITZ, 1946].

In applying the theory to the general circulation, a model is chosen in which each of the large-scale centers of action is a simple symmetrical irrotational vortex ("vr" vortex) whose wind field is felt in all parts of the atmosphere (considered as a thin spherical fluid shell), the wind speed decreasing directly with distance from the vortex center (see Fig. 1). The observed wind speed and direction at any point in the atmospheric shell is then the resultant of the effects of all the centers of action. Since this applies equally well at the center of any one of the vortices, the resultant wind at this point may be considered as a "steering current" which determines the instantaneous speed and direction of the given vortex.

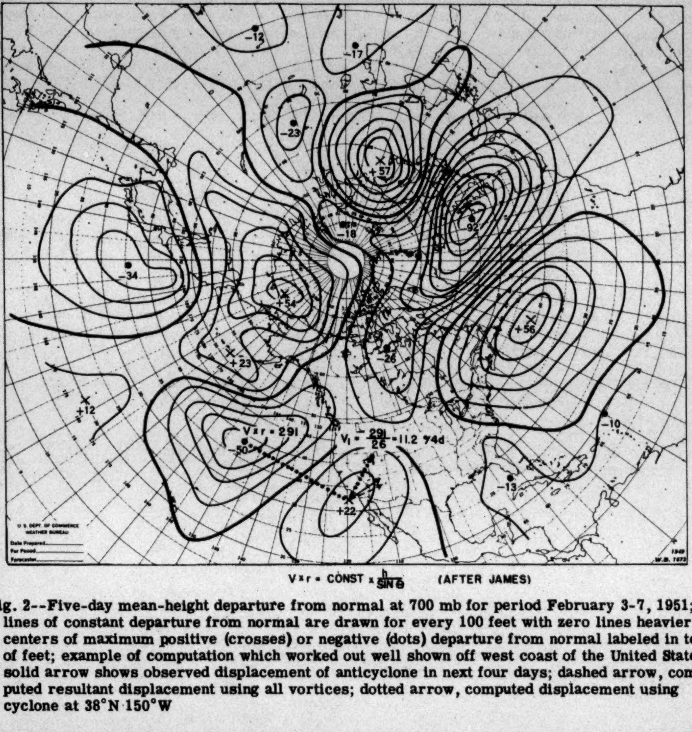
The resemblance between this model and the observed circulation at upper levels is improved by placing a strong cyclonic vortex at the pole. The flow field either as seen from a point in outer space or from the Earth's surface will then be characterized by a wave-like pattern. The relative behavior of the other vortices is assumed to be the same as though the polar vortex were absent, although in an absolute sense each will partake of the rotation about the polar axis.

An essentially similar model has been investigated theoretically, first by THOMSON [1883], and more recently by FLETCHER [1941] and STEWART [1943, 1945]. These authors have chosen a simpler model than that given above in that they have assumed an infinite plane fluid on a rotating or non-rotating disc. The vortices are all assigned the same strength (vr) and are arranged about the pole in concentric rings. These simplifications were made in order to reduce the mathematical problem so that analytical solutions giving the speed and stability of the vortices could be obtained. It is the purpose of the present study to derive the speed and direction of individual vortices by a numerical rather than an analytical solution. This permits the adoption of the more complex model outlined above, in which the vortices may have arbitrary strengths and locations, in better agreement with the observed atmosphere.

It should be made clear at the outset that in order to simplify greatly the computations the model outlined above has been deliberately made dynamically inconsistent. In addition to the assumption that the atmosphere consists of a polar vortex upon which are superimposed smaller individual eddies, there are others implying that the individual vortices are of the vr-type and that there is no dynamical interaction between them and the polar vortex. The last assumption leads to the most serious error encountered in applying the model, as will be shown later.

Determination of the position and strength of observed vortices--The application of the above circulation model with its kinematic simplifications to the determination of steering currents is quite straightforward, as explained in the opening paragraphs, and could be applied just as readily to the observed atmosphere provided the location and strength of the primary atmospheric vortices can be determined. At first glance this seems quite simple, since these could be identified with the large-scale centers of action as observed on hemisphere-wide sea-level charts. However, a major difficulty immediately arises, for on going to higher atmospheric levels the centers of action disappear and give way to open-wave systems. Even in extreme low-index situations, approximately half the major circulation features in the mid-troposphere consist of open waves. Thus it is clearly impossible to obtain in this way consistent results at different levels. This discrepancy is ascribable to the presence of the circumpolar vortex, which increases in intensity with height. In our chosen model it is necessary to remove this circumpolar current from the flow field in order to reveal the individual atmospheric vortices. (The method of doing this is described in the next paragraph.) When this is done, it is found that the residual flow pattern is approximately the same at different heights. This is especially true for the five-day or monthly mean charts, with which most of the current work has been performed. Also, the residual flow field is characterized by well-marked closed centers rather than by open waves.

When dealing with hemisphere-wide flow patterns at upper levels it is not possible to determine the true flow field, since wind observations are so sparse. Therefore, it is necessary to make use of geostrophic winds as obtained from constant pressure charts. The circumpolar vortex may then be approximated by a contour profile, giving the average height of the chosen pressure surface for each latitude circle from pole to equator. The residual (geostrophic) flow field may then be obtained by determining the field of height departure from this average profile. These will be called here "profile-anomaly" charts. This method was not used in this study because a similar procedure is more readily available. This consists of using hemispheric charts showing the field of height departure from its long-period normal (anomaly charts) which are prepared as a regular part of the forecast routine in the Extended Forecast Section. Comparison of five-day mean-anomaly and profile-anomaly charts for the same level and date shows that they are



$V \times r = \text{CONST} \times \frac{h}{\sin\theta}$ (AFTER JAMES)

Fig. 2--Five-day mean-height departure from normal at 700 mb for period February 3-7, 1951; lines of constant departure from normal are drawn for every 100 feet with zero lines heavier; centers of maximum positive (crosses) or negative (dots) departure from normal labeled in tens of feet; example of computation which worked out well shown off west coast of the United States; solid arrow shows observed displacement of anticyclone in next four days; dashed arrow, computed resultant displacement using all vortices; dotted arrow, computed displacement using cyclone at 38°N 150°W

quite similar with the single important exception that the profile-anomaly chart can never show a residual vortex centered at the pole. It should also be emphasized that the normal flow field is not strictly circumpolar in character, but has gentle undulations or waves superimposed on a general zonal current. Thus the normal geostrophic wind at any point may have a northerly or a southerly component. This must be kept in mind in discussing the results.

An example of a five-day-mean 700-mb anomaly chart for the period February 3-7, 1951, is shown in Figure 2. Having defined the location and sign (cyclonic or anticyclonic) of the primary vortices, it is next necessary to obtain an estimate of their individual wind fields before computations of displacement can be made. The wind field may be approximated by the geostrophic winds surrounding each vortex. However, this is only a crude estimate since the wind at any point is the resultant of all the individual wind fields. Even in the case of relatively small-scale hurricanes, it is not an easy task to separate the hurricane winds from those of surrounding larger-scale systems. This becomes even more of a problem when dealing with a group of systems all having the same general magnitude. Nevertheless, an attempt has been made to solve this problem by studying the contour profiles and geostrophic (or gradient) winds surrounding 22 large-scale vortices (11 cyclones and 11 anticyclones) as observed on five-day-mean 700-mb anomaly charts for 1949 and 1950. The vortices were chosen because they appeared to dominate the circulation over large areas. These observed contour profiles and wind fields were then compared with two hypothetical vortex profiles proposed by JAMES [1950] and with the Rankine vortex. It was assumed in all cases that the observed vortices are symmetrical, so that the contour profiles in both an east-west and a north-south direction through the vortex center were averaged together. In making the comparison it is most convenient to express all units non-dimensionally. Thus, contour departure from normal (h') is expressed as the ratio between its actual value (h) and that at the vortex center (h_o); wind speed (v'), by v/V_R, where V_R is the maximum speed; and radial distance from the center (r'), by r/R, where R is the radius of the maximum wind, or the "core radius."

One of the two hypothetical vortices of JAMES [1950] is the "sinusoidal" vortex, which he defined by the formulas

$$\begin{array}{ll} & p = p_\infty + (p_o - p_\infty)\,[1 + \cos\,(\pi r/2R)]/2 \qquad \text{when } r \leqq 2R \\ \text{and} & \\ & p = p_\infty \qquad \text{when } r \geqq 2R \end{array} \Bigg\} \ldots\ldots\ldots\ldots (1)$$

Here p is the pressure at any radial distance r, p_∞ is the pressure at infinity, and p_o is the pressure at the center. James' other vortex is the normal vortex, defined by

$$p = p_\infty + (p_o - p_\infty)\, e^{-r^2/2R^2} \ldots\ldots\ldots\ldots (2)$$

James points out that in practice it is necessary to define the pressure as a departure from normal. Thus in the above formulas $(p - p_\infty)$ and $(p_o - p_\infty)$ may be identified respectively with h and h_o, the contour departure from normal.

By using the geostrophic wind equation it can readily be shown that the product $V_R R$ is given by $\pi g\, h_o/4f$ for the sinusoidal vortex, and by $e^{-1/2} g\, h_o/f$ for the normal vortex, where f is the Coriolis force.

The anomaly contour profiles and geostrophic wind fields of these two vortices may be expressed non-dimensionally by the equations

$$\left.\begin{array}{l} \left.\begin{array}{l} h' = [1 + \cos\,(\pi r'/2)]/2 \\ v' = \sin\,(\pi r'/2) \end{array}\right\} \text{When } r' \leqq 2 \\ \qquad (h' \text{ and } v' = 0 \text{ when } r' \geqq 2) \\ \qquad (\text{sinusoidal vortex}) \end{array}\right\} \ldots\ldots\ldots\ldots (3)$$

$$\left.\begin{array}{l} h' = e^{-(r')^2/2} \\ v' = e^{1/2}\, r'\, e^{-(r')^2/2} \end{array}\right\} (\text{normal vortex}) \ldots\ldots\ldots\ldots (4)$$

The Rankine vortex is defined by its wind field as

$$\left.\begin{array}{l} v/r = \text{a constant, when } r \leqq R \\ vr = \text{a constant, when } r \geqq R \end{array}\right\} \ldots\ldots\ldots\ldots (5)$$

Here the constants become one when non-dimensional units are used.

If the winds are assumed to be geostrophic, the height-anomaly profile of the Rankine vortex is

$$\left.\begin{array}{ll} h - h_o = -\,(f\, V_R\, R/2g)\,(r/R)^2 & \text{when } r \leqq R \\ h - h_o = -\,(f\, V_R\, R/2g)\,[1 + 2\log_e\,(r/R)] & \text{when } r \geqq R \end{array}\right\} \ldots\ldots\ldots\ldots (6)$$

This cannot readily be expressed non-dimensionally since h does not become zero as r approaches infinity. However, it should be borne in mind that h may be measured from any arbitrary base since the Rankine vortex gives only the slope of the pressure profile. Defining h as zero when $1 + 2\log_e\,(r/R) = 2$, h_o becomes $(f/g)\, V_R R$, and the Rankine vortex can be expressed non-dimensionally as

$$\left.\begin{array}{ll} h' = 1 - (r')^2/2 & r' \leqq 1 \\ h' = 1 - (1 + 2\log_e r')/2 & r' \geqq 1 \end{array}\right\} \ldots\ldots\ldots\ldots (7)$$

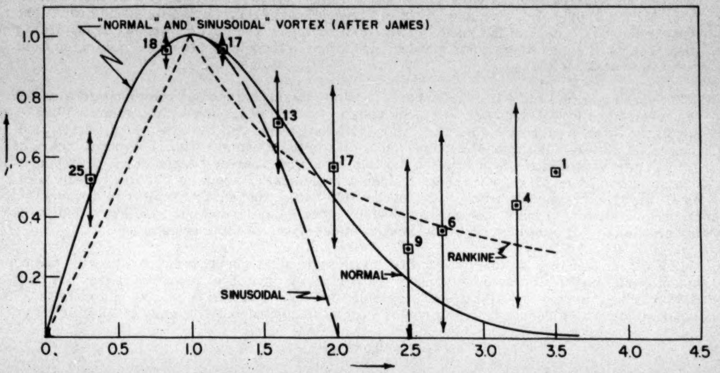


Fig. 3--The relationship of the average non-dimensional gradient wind to the radial distance for 22 observed 700-mb five-day mean-anomaly centers (squares with dots) compared to geostrophic winds for sinusoidal (dashed curve), normal (solid curve), and Rankine vortex (short-dashed curve); number of cases and standard deviations for each mean value are indicated by numbers and arrows near each square

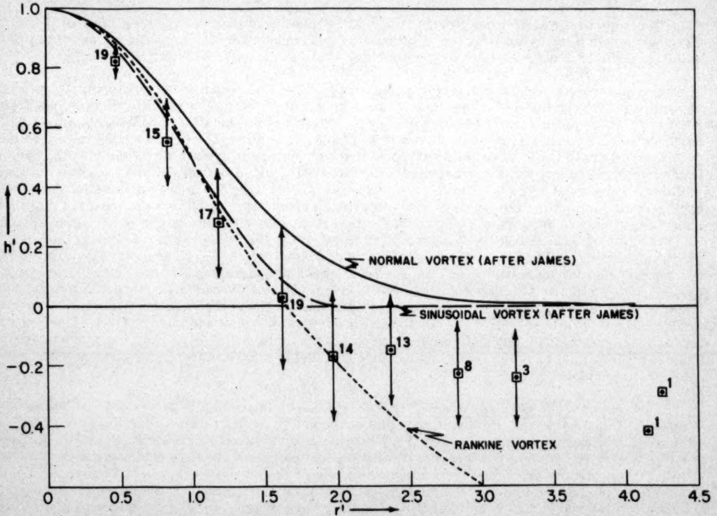


Fig. 4--The relationship of the average non-dimensional height departures from normal to the radial distance for same 22 anomaly centers shown in Figure 3; see title of Figure 3 for explanation

Graphs showing average values of v' and h' for the 22 observed vortices for intervals of r' from 0.2 - 0.6, 0.6 - 1.0, etc., are shown in Figures 3 and 4. Plotted on the same graphs are the non-dimensional values of the three hypothetical vortices. The vertical arrows drawn from the mean values of v' and h' are their standard deviations. The number of cases making up each average is also shown.

From Figures 3 and 4 it is seen that for radial distances up to and somewhat greater than the core radius (r' = 1) the observed mean values of h' and v' are approximated quite closely by any of the three hypothetical vortices. There is a suggestion that the observed gradient wind speeds lie closer to the normal and sinusoidal than to the Rankine vortex. Beyond a radius of 1.5 R, however, the agreement with the Rankine vortex is superior. A great deal of importance cannot be laid on this fact, since there is considerable scatter about the observed mean values, as shown by the vertical arrows. It is not possible to tell whether this scatter is due mainly to the non-uniformity of individual profiles (which means the failure of our hypothesis that the vortex profiles are all similar), or to the random effect of surrounding systems.

It will be assumed in what follows that large-scale atmospheric centers have wind and contour anomaly profiles resembling those of the normal or sinusoidal vortex up to the core radius. Beyond this point they are assumed to resemble the Rankine vortex so that vr is a constant. These assumptions enable the strength of a vortex $V_R R$ to be computed by the very simple formula

$$V_R R = \text{constant } h_o / \sin \theta_o \quad \ldots\ldots \quad (8)$$

Here, θ_o is the latitude of the anomaly center and h_o the height departure from normal at the center.

The constant term in (8) depends on the units used. In this paper V_R will be expressed in degrees latitude per four days, R in degrees latitude and h_o in tens of feet. The constant then becomes 4.50 for the sinusoidal and 3.48 for the normal vortex. In practice, an average value of 3.99 has been used.

To complete the statistical picture it should be mentioned that the average value of R for the 22 observed vortices is 9.5 degrees of latitude with a standard deviation of 2.6. The average value of V_R is 35.1 degrees latitude per four days with a standard deviation of 10.2.

Method for determining velocity of anomaly centers--The technique of preparing an estimate of the speed and direction of a selected anomaly center (for example, the anticyclone located off Southern California in Fig. 2) consists of several steps: First, locate and mark the central departures of all anomaly centers as shown in Figure 2. The strength of each vortex may then be determined very quickly using (8). Next, one of these centers is chosen (for example, the cyclone in the eastern Pacific) and the great-circle distance determined between it and the center whose motion is to be estimated. As shown in the example, the strength is simply divided by the great-circle distance to obtain the speed of the "steering current" due to this single vortex. (This is strictly true only if the vortex under consideration is outside the core radius of all other systems.) The direction of this current is at right angles to the great circle and in the proper cyclonic or anticyclonic sense. The dotted arrow shows this computed speed and direction. This procedure is then repeated for each vortex and the resultant speed and direction obtained (dashed arrow in Fig. 2). Theoretically all vortex centers in both hemispheres should be included. In practice this is not possible because analyses are not reliable below 20°N and a center south of this latitude would approach "infinite" strength due to the sin θ_o term in the denominator of (8). Thus, at low latitudes the geostrophic approximation is not valid. In addition, no center having a departure from normal of less than 100 ft is used because they seldom have much importance and are considered unreliable.

It is clear that the required great-circle distances and directions can be obtained by means of spherical trigonometry. However, the time consumed in making the computations would be prohibitive so that an approximate graphical method has been devised. This is based on two assumptions regarding the polar stereographic projections on which the anomaly fields are drawn. These are that great circles are straight lines and the scale of the map is everywhere the same. These assumptions permit the use of a simple transparent set of concentric circles on which the uniform distance circles and radial lines indicating the angles are drawn. By placing the center of the concentric circles on the chosen vortex with the zero-direction line pointing south, the direction and distance of each of the other vortices may be read very quickly. The direction of each steering current may also be quickly obtained by providing a scale on the concentric circles which automatically adds 90° to (for cyclones) or subtracts 90° from (for anticyclones) the actual direction.

Determination of the resultant velocity is facilitated by using an ordinary pilot-balloon plotting board. Computations of the resultant speed and direction for a single center by this short-cut method (called the "hodograph" method) require between 15 and 20 minutes, depending on the number of anomaly centers. (Average: 10 to 15 centers for five-day-mean charts.) Comparison with the exact spherical method shows that the approximations result in errors which are small in comparison with errors in measurements.

Selection of data and discussion of results--Figure 2 has been chosen because it shows an example that was quite well verified. The computed resultant speed, shown by the dashed arrow, agrees quite well with the observed displacement of the center in the following four days (solid arrow). In strict accordance with the chosen model, the normal wind velocity at the center of the anticyclone (representing the polar vortex) should be added to the resultant velocity. However, in this case this would result in a computed displacement too large by a factor of two and a direction too much from the west. A similar result was found for the other cases studied and will be discussed further below.

Not only does the resultant velocity, obtained by using all anomaly centers in the northern hemisphere, agree well with that observed in Figure 2, but it is considerably better than that computed using the single cyclonic center in the eastern Pacific (dotted arrow). This was found to be generally true for the other 25 cases investigated. In particular, computations made using only those centers in the immediate vicinity of the one under investigation were superior to computations using all centers in only six out of 25 cases, and in these six cases only a very slight superiority was indicated. This result may serve to emphasize once again the importance of remote circulation systems in their effect on local weather.

In selecting a case for study, the single requirement was made that its continuity be reasonably clear for a period of one week centered on the map on which the computation is made. Since two five-day-mean maps are prepared each week, this means that the center must be identified on three successive charts. Using this requirement, 25 centers (11 cyclones and 14 anticyclones) were chosen from five-day-mean 700-mb anomaly charts for the cold seasons (October to March) of the years 1949 to 1951. In measuring the observed velocities of these centers two other methods were used beside that of determining the displacement in the subsequent four days. One of these is a centered displacement, showing the average velocity for the week centered on the chosen map. This will be called the "full-week centered" displacement. The other is a "kinematic velocity" [NAMIAS, 1947, p. 41] which is a good approximation to the true instantaneous velocity of the mean vortex.

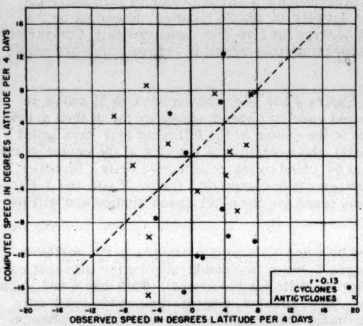
In testing the method, all computed and observed velocities were separated into west-to-east and south-to-north components. Computed and observed components were then related using scatter diagrams and correlation coefficients. The results are summarized in Table 1.

Table 1--Correlation coefficients showing relation between computed and observed motion for 25 selected five-day mean 700-mb anomaly centers

Type of observed motion	Correlation coefficients	
	W-E motion	S-N motion
Subsequent four-day displacement	-0.03	0.64
Full-week centered displacement	0.13	0.74
Kinematic velocity	0.12	0.50

The extremely poor relationship for the west-to-east motion will be discussed presently. Here, it is desirable to emphasize the generally better results obtained using the "full-week centered" displacement. Perhaps this is not too surprising in view of the fact that the large-scale features of the entire circulation are used in obtaining the computed speed. This might be expected to agree better with an observed displacement averaged over a finite time interval centered on the time of observation than with either the instantaneous velocity or the future displacement of the center.

Scatter diagrams for the full-week centered motion are shown in Figures 5 and 6. Note that the order of magnitude of the computed and observed motion is the same, despite the omission of the normal circumpolar current in each case. It can also be seen that the anticyclones appear to agree better with theory than do the cyclones, at least for the south-to-north motion. The greater



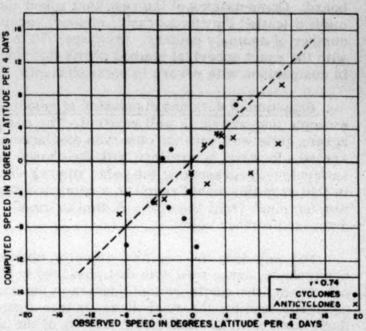


Fig. 5--West-to-east component of computed speed (called here the "hodograph method") in degrees latitude per four days plotted against corresponding observed speed obtained from the full-week displacement of each five-day mean-anomaly vortex centered on the time for which computation is made; data from cold seasons (Oct. - Mar.) of 1949-50 and 1950-51; dashed line is line of perfect agreement

Fig. 6--Graph for south-to-north component of displacement for same data as in Figure 5

success of the south-to-north as compared with the west-to-east computed speeds must be of considerable significance, and suggests at once that this has something to do with the omission of the normal circumpolar current, whose major component is from the west. However, the normal wind components at the centers of the 25 vortices correlate only 0.22 with the west-to-east and 0.03 with the south-to-north components of the full-week centered displacement.

The poor results of the east-west computations, as well as the lack of significance of the normal circumpolar westerlies, may be due to a serious failure of the assumptions. This may be due to the fact that the individual vortices embedded in the circumpolar vortex interact with the latter. They advect its vorticity field in such a way as to give each individual eddy a strong component towards the west. This can be seen most readily by considering a single concentric circumpolar ring of vortices. This ring consists of alternate cyclones and anticyclones all having the same strength. The "hodograph" method, when applied to this simple concentric ring will give zero east-west relative speed, since the effects of all the eddies on any individual one will cancel out. Thus, if there were now a circumpolar vortex superimposed on this field, each individual vortex would be computed to move eastward with the speed of the zonal current (U). Now suppose that the circumpolar vortex is in solid rotation with the angular velocity of the Earth. Its absolute vorticity at any point is then equal to the Coriolis parameter (f). Since this "Earth vorticity" is larger (more cyclonic) to the north, the relative wind field to the west of the cyclones (or east of the anticyclones) will cause a local increase of cyclonic vorticity. Conversely, the southerly flow to the east of the cyclones (west of the anticyclones) will cause a local decrease of vorticity. It can be seen that this will result in a "vorticity tendency field" which will move both cyclones and anticyclones to the west. In fact, the magnitude of this displacement is given approximately by Rossby's well-known dynamic component $-\beta L^2/4\pi^2$. Since it is known that this term is about the same order of magnitude as the advective term (U), it is easy to see why the east-west components work out poorly and why introducing the normal zonal wind merely gives displacements which are much too large.

Analysis of errors--One of the more interesting aspects of the present study is revealed by an analysis of errors. A histogram was constructed showing the number of cases as a function of the magnitude of the absolute vector error (the vector difference between computed and full-week centered observed velocity). This histogram is shown in Figure 7, and indicates a bi-modal distribution. This suggests that a given computation either verifies relatively well or extremely poorly. To find the reason for this, the frequency distribution of errors was divided in thirds (see bottom of Fig. 7). Eight cases having errors of less than six degrees latitude per four days were classified as good; eight cases having errors between six and 12, as fair; and nine cases having errors greater than 12, as poor. It was noted that the majority (55 pct) of the cyclones

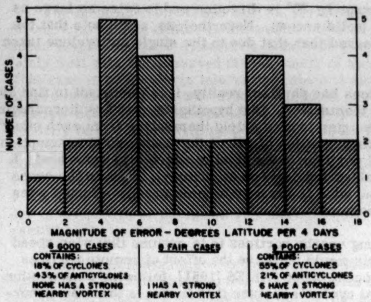


Fig. 7--Histogram showing number of cases plotted against absolute vector error in units of degrees latitude per four days; same 25 cases as in Figures 5 and 6

studied were in the poorest category of the three, while a sizable part of the anticyclones (43 pct) were classified as good. This suggests that the anticyclones are steered more readily than the cyclones, perhaps because they are less subject to development. This idea is strengthened when it is found that there is a slight tendency for systems undergoing the greatest deepening or filling during the week to have greater displacement errors than those undergoing small change in intensity.

The above conclusion may be too obvious, for closer examination reveals that of the nine poor cases, six are associated with at least one intense vortex located close to the one whose motion is computed. An "intense, close" vortex is defined as one having a central anomaly greater than 600 ft, and within 25° of latitude of the one under consideration. No situation at all resembling this is found among the eight good cases, and only one among the eight fair cases.

An example of a situation of this kind is shown in Figure 8. Here, an anticyclone with a central anomaly of 1120 ft is located 24° of latitude from a cyclone in the eastern Pacific. The re-

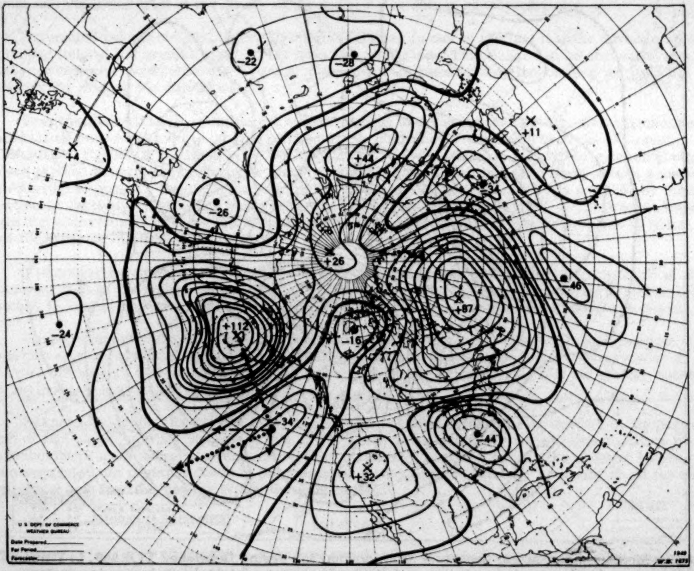


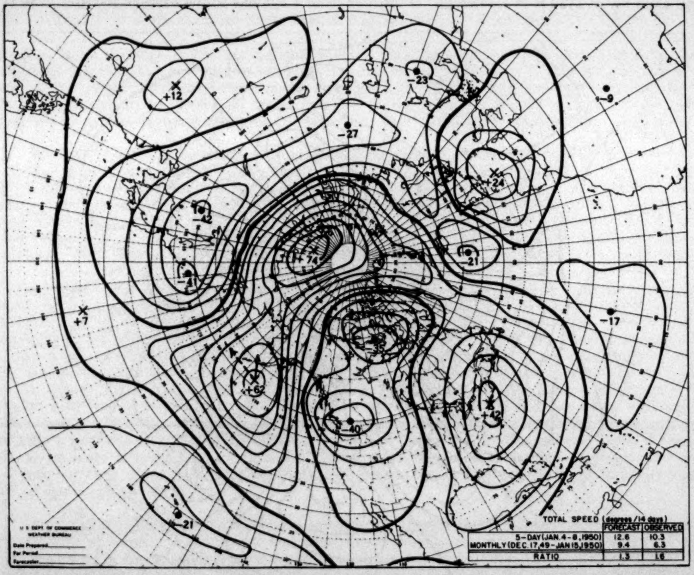
Fig. 8--Five-day mean-height departure from normal for period November 22-26, 1950; shows example of poor results due to influence of strong nearby vortex: for explanation of symbols, see Figure 2

sultant computed velocity (dashed arrow) is in error by 90° in direction and is twice as large as the observed displacement in the next four days (solid arrow). Nevertheless, again note that the resultant velocity is somewhat closer to the observed than that due to the single anticyclone taken alone (dotted arrow).

If this peculiar effect of strong nearby vortices has physical reality, it is important to find an explanation for it. Several possibilities suggest themselves. One hypothesis, perhaps discouraging if true, is that in some way, two vortices of unlike sign tend to shield themselves from each other's influence when they approach too closely. This is a negation of one of the fundamental assumptions made for our model--that a vortex will affect all portions of the fluid in which it is embedded. It will be recalled that the average core radius is about ten degrees of latitude. Thus, in the cases discussed here, it would appear that the individual wind fields may become greatly affected when two systems approach closer than twice the average core radius.

Another way to account for the effect of strong nearby vortices is to assume that wind speed falls off less rapidly than if vr is a constant. This would increase the effect of remote vortices and decrease that of nearby ones. Indeed, in a recent paper JAMES [1951], following a suggestion by MacDONALD [1942], points out that as tropical cyclones mature they increase the area of hurricane winds without increasing their central intensity greatly. Perhaps this finding applies generally to all large-scale vortex systems.

Kinematic method applied to different scales of eddies--An attempt has been made to apply the kinematic vortex theory to atmospheric eddies of different scales by using charts averaged over different time intervals. So far computations have been made on one monthly mean chart and one series of daily charts.



TOTAL SPEED (degrees/14 days)

	FORECAST	OBSERVED
5-DAY(JAN.4-8,1950)	12.6	10.3
MONTHLY(DEC.17,49-JAN15,1950)	9.4	6.3
RATIO	1.3	1.6

Fig. 9--Monthly mean-height departure from normal for period December 17, 1949, to January 15, 1950; example of computed and observed displacement for next 14 days shown near eastern Aleutians; comparison of order of magnitude of displacements for monthly and a corresponding (similar) five-day situation shown in box in lower right corner; see Figure 2 for explanation of symbols

The monthly computation (for the period December 17, 1949, to January 15, 1950) made on a strong anticyclonic center in the western Gulf of Alaska, is shown in Figure 9. The computed speed (expressed here in degrees of latitude per 14 days) is shown by the dashed arrow. It agrees fairly well with the observed displacement of the center in the following 14 days (solid arrow). This case is particularly interesting, since it shows how the method may allow for the different orders of magnitude in the velocity of eddies of different scales. In this case the five-day mean-anomaly chart for the middle of the monthly period (January 4-8, 1950) resembles the monthly anomaly field closely with respect to the location and sign of all the centers. Both the computed and observed displacements for the anticyclone in the Gulf of Alaska were also quite similar in direction. However, as might be expected, the observed speed of the five-day center was larger than that for the monthly. This is summarized in the box in the lower right-hand corner of Figure 9, together with similar data for the computed speed. The data shows that the reduction in the order of magnitude of the speed was forecast quite well. This gives further support to the physical reality of the method.

Similar computations were made on a series of daily charts in order to obtain the 24-hour speed and direction of a hurricane. This storm followed a rather erratic course in the Atlantic off the east coast of North America during the period September 4 to 12, 1949. While these computations were not very successful, they do indicate computed displacements of about the right order of magnitude. In view of the vast amount of work which has previously been expended on the problem of the steering of hurricanes, it is felt that this somewhat new approach to the problem might bear considerable further investigation.

Conclusions--In this paper evidence has been presented indicating that the elementary kinematic theory of vortex interaction can account for some of the behavior characteristics of large-scale atmospheric vortices (or eddies). Many of the properties of a simple kinematic model have been verified with two outstanding exceptions. These are the failure of the west-to-east computations, and the breakdown of the theory in the presence of strong nearby vortices.

As pointed out previously, the first of these failures is due to the fact that the advection of the vorticity field of the normal polar vortex has been neglected. This error can be corrected in a rather straightforward fashion, but only at the expense of considerably lengthening the time necessary for making the computations.

In connection with this problem the author was led to consider the two-dimensional numerical forecasting method which has been developed at the Institute for Advanced Study at Princeton [CHARNEY, FJÖRTOFT, and VON NEUMANN, 1950]. Indeed, this has a very close resemblance to the method described in this paper, despite the fact that the latter was arrived at by a seemingly different approach. Both methods attempt to forecast changes in circulation patterns by the advection of vorticity. Two of the basic differences between the methods are: In the present work only the major vortex centers (solid cores of primary vortices) are moved from place to place, and the steering current is obtained by an indirect procedure.

The numerical method has a great advantage in that it takes into account the advection of the vorticity field of the polar vortex. For this reason the author is currently applying the numerical method to large-scale patterns as found on five-day and monthly mean mid-troposphere charts.

Perhaps the greatest value of the present study is its suggestion that numerical forecasting methods may be applied to the larger-scale atmospheric systems having long time-periods. It is the characteristics of such systems which seem to form the principal basis of reasonably successful methods of long-range forecasting.

Acknowledgments--Among the many people who assisted in one way or another in the preparation of this paper, special thanks are due Jay S. Winston for critically reviewing and editing the manuscript; Eugene Aubert for suggesting the "hodograph" method of solution; Ragnar Fjörtoft, who suggested the physical cause of the principal error; Gordon Thiel for assisting in all the computations; and June Garner for typing the manuscript.

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(Communicated manuscript received October 17, 1951;
open for formal discussion until November 1, 1952.)

__Transactions__, American Geophysical Union Volume 33, Number 3 June 1952

WIND-INDUCED VERTICAL MOVEMENT OF THE WATER ON AN OPEN COAST

J. R. Longard and R. E. Banks

__Abstract__--The effect of winds on the vertical structure of the water column near the Atlantic coast of Nova Scotia is discussed. The analysis is based on twice-daily bathythermograms taken at __Sambro__ Light Vessel. This vessel is located in 50 fm ten miles offshore in the vicinity of Halifax, Nova Scotia. Strong easterly winds blowing for more than half a day are shown to cause such a shoreward movement of the light surface water that the bottom water may be entirely displaced for at least ten miles offshore. Evidence is presented that this is a wind-driven effect and is not necessarily due to the presence of a low-pressure area in the vicinity. Sea level as recorded on the Halifax tide gage confirms the piling up of surface water.

__Introduction__--The effect of the wind in producing upwelling or subsidence of the water close to a coast is discussed by SVERDRUP and Others [1942, pp. 501-502] and the application of this theory to the Scotian Shelf has been presented by HACHEY [1937]. Hachey's observations were made in Halifax Harbor, and it is probable that most observations of this phenomenon were made either in embayments or very close to the coast. It has recently been possible to observe this effect by continuous observations in more open water.

__Sambro__ Light Vessel is located ten miles off Halifax Harbor in water with a depth of 50 fm, as shown in Figure 1. This location offers a unique opportunity to study ocean-water conditions from a fixed station. Since August, 1949, Naval Research Establishment has had twice-daily bathythermographic observations made at this Light Vessel. At noon and midnight of each day a bathythermograph lowering to the bottom is made, and surface-water temperatures, winds, and other conditions are observed. In addition to this, the Light Vessel makes six weather reports daily for the Meteorological Service. As these data were available and as some points of correlation were observed, an investigation was made to find the relation between changes in the structure of the water column and the weather.

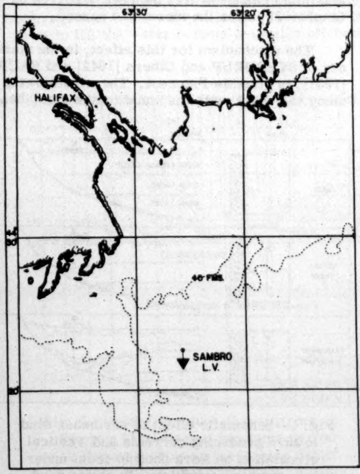

Fig. 1--Chart of Halifax approaches showing location of __Sambro__ Light Vessel where bathythermograph observations were made

__Procedure__--As an initial attempt, the maximum air-pressure gradients between Halifax and an arc 100 mi off the Nova Scotia coast, as shown on the daily weather maps, were plotted in parallel with the surface and bottom temperatures. No correlation was apparent, probably because the values taken from one map per day gave a far from complete picture of the changes in the pressure gradients.

A summary was then made of the high- and low-pressure areas that passed over eastern Canada during the period under investigation. It was immediately evident that pronounced changes in bottom temperature were associated with storms that passed either over __Sambro__ Light Vessel or within 200 mi of it such as to produce winds at the Light Vessel of Beaufort force 6 or more. Even a low as intense as a tropical hurricane appeared to have no effect on the coastal water conditions unless it passed near enough to cause such winds.

Since the winds at the position of the water-temperature measurement appeared to be the controlling factor, these were next investigated. To obtain an overall picture, the wind direction, wind speed, and surface and bottom temperatures were plotted in parallel. In all cases where winds had an easterly component and blew with force 6 or more for over half a day or with force 4 to 5 for several days, a marked increase in bottom water temperature occurred.

HACHEY [1937] found Halifax Harbor waters very susceptible to changes in the wind (or pressure) system. Winds of force 2 or 3 were sufficient to cause marked changes in the temperature of the waters within the harbor. However, there is no evidence that winds of this force produce any significant variation of temperature structure at the Light Vessel.

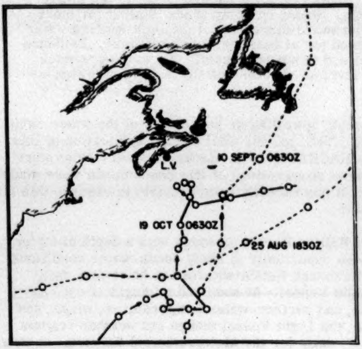

Fig. 2--Excerpt from weather map showing track of October storm (solid lines) and tracks of two hurricanes which did not affect the water structure at the Light Vessel (dashed lines); circles indicate positions of the storm centers as reported every six hours

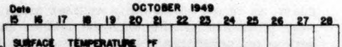

Fig. 3--Parallel plot of surface temperature, bottom temperature, wind direction and speed at Sambro Light Vessel during storm period in October, 1949, showing marked rise in bottom temperature and the fall of same after a change of wind direction

Wind effect from low-pressure area off coast--During the period October 19 to 25, 1949, the most marked change in water conditions occurred. This change was associated with an extratropical storm of hurricane origin, the path of which is shown in Figure 2, which also presents the paths of two hurricanes that did not affect the water structure at the Light Vessel. The center of this storm reached a point 150 mi SSE of Halifax at 08h 00m on October 19 and remained within 30 mi of this point until 15h 30m on October 20, at which time it commenced to move eastward. Figure 3 gives a synoptic picture of conditions at the Light Vessel during this period. For 2 1/2 days after noon of October 18, the wind blew from east to northeast, average force 5 to 8, with a maximum of 9. The bottom temperature increased from 35° to 57.5°F, the temperature of the surface water, and then returned to 37°. The surface temperature averaged about a three-degree drop during this period with some fluctuations. The maximum change of five degrees occurred on October 25, after the storm had passed.

The mechanism for this effect, in the manner of SVERDRUP and Others [1942] and HACHEY [1937], is given in Figure 4. The wind blowing along the coast, with the coast on the right-hand

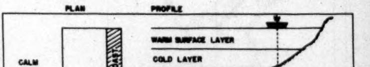

Fig. 4--Schematic effect of northeast wind toward producing currents and vertical circulation on Nova Scotian coast under late summer conditions; W shows wind direction and T direction of general transport of water side, causes a transport of water to the right of the wind. This effect leads to an accumulation of the warm surface water on the coast, setting up a helical circulation with a general transport of water along the coast. Because of the strong density gradients existing off the Nova Scotia coast in late summer, the circulation is largely confined to the surface layer. The underlying cold water which HACHEY [1935] defined as the "intermediate layer" is forced by the increased pressure of the surface water to recede deeper and hence farther offshore. Presumably the warm "bottom layer," which normally does not reach the Light Vessel, recedes farther offshore also. Figure 5, a bathythermograph profile from the Light Vessel data for the October, 1949, period mentioned above, shows how the thermocline between the surface and intermediate layer reaches progressively greater depths until the cold water is entirely displaced.

After the storm center has passed, westerly winds cause the surface water to move away from the coast. The cold water moves back again and, as HACHEY [1937] shows, this water then enters into the vertical circulation by upwelling close to the coast and lowering coastal water temperatures. This upwelling is indicated by the drop in surface temperature at the Light Vessel after the cold layer has been re-established there.

As evidence of the piling up of water on the coast, the effect of this storm on the tides, as recorded in Halifax Harbor, is also given in Figure 5. This shows a marked increase in tide levels over predicted values for the period of easterly winds blowing along the coast followed by a return to normal levels after the wind shift.

Wind effect from high-pressure area off coast--Easterly winds at the Light Vessel are usually caused by storm centers passing over the Scotian Shelf, so that it is difficult to distinguish between pressure effects and wind effects. However, in November, 1950, a storm remained over the Great Lakes for several days, held there by a high-pressure area off the Nova Scotian coast. This condition produced southeasterly winds of gale force on the Scotian Shelf. The effect on water structure at the Light Vessel is shown in Figure 6. Note that the water structure and tide heights changed under these wind conditions due to a "high" off the coast in the same manner as if the winds were due to a "low" off the coast, thus proving this to be a wind (or local pressure) effect, not necessarily associated with the passage of a storm center over the area. In view of this, it is suggested that the variation of sea level during storms which UNOKI [1950] shows to be caused by atmospheric pressure may really be wind-driven effects.

Conclusions--(1) Under late summer conditions, winds along the coast of force 6 or over are required to blow for at least a half day to cause sufficient piling up of surface water on the coast to fill the water column ten miles offshore to a depth of 50 fm.

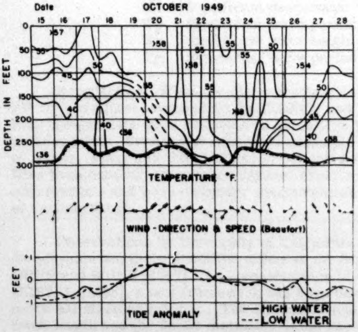

Fig. 5--Depth-time plot of isotherms at Sambro Light Vessel for same period as Figure 4 showing increasing depth of thermocline under influence of easterly winds and rise of thermocline after wind change

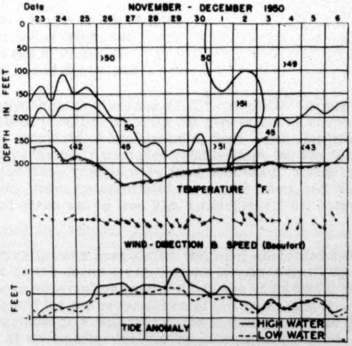

Fig. 6--Depth-time plot of isotherms at Sambro Light Vessel for November-December storm period showing increasing depth of thermocline under influence of southeast winds and return to normal after wind change

(2) This surface water displaces but does not mix with the underlying cold layer, forcing the cold layer to greater than normal depths.

(3) Tropical hurricanes passing by over 200 mi offshore, which do not produce such conditions, have no effect on the coastal water structure.

(4) The effect is wind-driven and is not necessarily caused by a storm center in the vicinity as evidenced by its occurrence with either high- or low-pressure areas off the coast.

Acknowledgments--The assistance is gratefully acknowledged of the Hydrographer, Department of Mines and Technical Surveys, Ottawa, in providing special tide data for Halifax Harbor and of the Meteorologist-in-Charge, Dominion Public Weather Office, Halifax, for meteorological data.

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(Manuscript received April 13, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1951; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

THE GENERATION AND DECAY OF WIND WAVES IN DEEP WATER

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Abstract--Data on the generation and decay of wind-generated gravity waves have been collected for several years by the University of California. These data, as well as the original data of Sverdrup and Munk, are analyzed and the results presented in dimensionless graphs suitable for use in wave forecasting. The dimensionless parameters presented by Sverdrup and Munk are suitable; however, new curves were constructed which include new data recently available. In order that the data on the decay of waves could be presented in a logical manner, a concept, based upon the following observations, was introduced: (1) Individual waves do not maintain their identity in deep water. (2) A spectrum of lengths and heights is present in both the fetch and decay areas. (3) At any particular decay distance the significant period decreases with time. (4) The significant period increases with decay distance in a manner different from that assumed by Sverdrup and Munk for their decay relationships. (5) Travel time depends upon the group velocity associated with the significant period at the end of the decay distance. It was found that the significant wave height and period at the end of the decay distance depend upon the length of the fetch and the height and period at the end of the fetch as well as upon the decay distance.

Symbols--The following symbols are used in this paper.

U	Average surface wind speed, ft/sec and knots
t	Duration of wind, sec
t_d	Duration of wind, hr
F	Fetch length, ft and nautical mi
D	Decay length, ft and nautical mi
g	Acceleration due to gravity, 32.16 ft/sec^2
H	Significant deep-water wave height, ft
T	Significant deep-water wave period, sec, $T = \sqrt{2\pi L/g}$
L	Wave length, ft, $L = gT^2/2\pi$
C_o	Deep-water wave velocity, ft/sec, $C_o = L/T = \sqrt{gL/2\pi}$
C_g	Group velocity, ft/sec, $C_g = C_o/2$
H_F	Significant deep-water wave height at end of fetch, ft
T_F	Significant wave period at end of fetch, sec
H_D	Significant wave height at end of decay, ft
T_D	Significant wave period at end of decay, sec

Introduction--When the dimensionless relationships C_oU versus gF/U^2 and gH/U^2 versus gF/U^2 were first proposed by SVERDRUP and MUNK [1947] to describe the generation of waves only meager data were available to determine the constants of integration of their semi-empirical relationships. Even fewer data were available to verify their proposed relationships for the decay of waves. Many new data have been collected since these results were first published. These data have been obtained from wave-channel experiments, from measurements on small lakes, and from observations and wave-recorder measurements of ocean waves (see Fig. 1 and Table 1 for sources of various data).

Observations by University of California investigators have shown that wind-generated waves do not maintain their identities either in the fetch or the decay areas, as was proposed by SVERDRUP and MUNK [1947]. As a consequence of these observations and the findings of BARBER and URSELL [1948], a new framework has been developed for the presentation of the decay data in a more satisfactory manner. The dimensionless variable D/F was introduced in order to relate the fetch length and the decay distances to the decay of waves after leaving the generating area, because the decay of waves appears to depend upon the relative decay and fetch distances.

No analysis was made of the effect of following or opposing winds. Because of the large amount of data available it is believed that, although some of the scatter of data was due to these winds, the resultant curves closely represent the case of waves traveling through a calm decay area.

Table 1--References used in Figure 1

Symbol	Reference
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×	**JOHNSON, J. W.**, The characteristics of wind waves on lakes and protected bays, Trans. Amer. Geophys. Union, v. 29, pp. 671-681, 1948.
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□ ▲ ▽ ●	**SVERDRUP, H. U., and W. H. MUNK**, Wind, sea, and swell; theory of relationships for forecasting, H. O. Pub. 601, 1947, App. 2, Table 2.
⁙	**JOHNSON, J. W.**, Relationships between wind and waves, Abbotts Lagoon, California, Trans. Amer. Geophys. Union, v. 31, pp. 386-392, 1950.
△	**COLUMBIA RIVER**, Data book of experiment M-1; waves at Columbia River Light Vessel, April 1934 to March 1935, Univ. of Calif., Berkeley (unpublished data book).
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○	**STUMP, R. S.**, Forecast and observed surf characteristics for three localities on the northern California coast for July, August, September 1944, Tech. Rep. H.E.-116-52, Univ. of Calif., Berkeley, February 2, 1945 (unpublished).
◐	**STANTON, T., D. MARSHAL, and R. HOUGHTON**, The growth of waves on water due to action of wind, Proc. R. Soc., A, v. 137, pp. 283-293, 1932.
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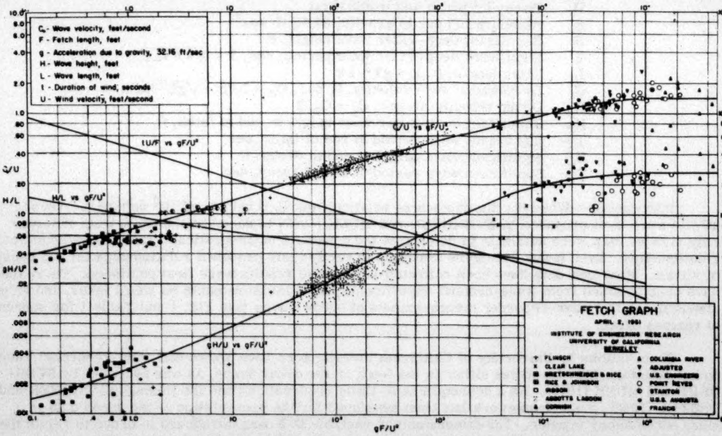

Fig. 1--Relationships between wind speed, wind duration, fetch, wave height, and wave velocity; see Table 1 for sources of data

Significant wave is intended whenever the wave height and wave period are mentioned in this paper, whether in the generation or decay area. The significant wave height is defined as the average of the highest one-third of the waves (where the number of waves which make up the highest one-third is obtained in the following manner: The average period of high groups of waves is determined, called hereafter the significant period; this significant period is then divided into the length of the observation time to obtain a wave number; one third of this number is defined as the number of waves to average in determining the significant wave height). Thus the term significant wave is associated with a statistical wave distribution rather than with the previous physical concept that the larger waves maintain their identity as they travel across the fetch or decay areas.

Generation of waves--New curves presenting the dimensionless relationships of C_0/U versus gF/U^2 and gF/U^2 versus gF/U^2 have been presented in Figure 1, using data collected by the University of California and the original data used by **SVERDRUP** and **MUNK** [1947]. These curves were determined by constructing the best-fit curves through the centroids of blocks of points, such that no irregularities existed in the curve of H/L (wave steepness) versus gF/U^2, constructed by using Figure 1, together with the formula $H/L = (gH/U^2)/[2\pi\,(C_0/U)^2]$ for particular values of gF/U^2.

The duration curve tU/F versus gF/U^2 also has been presented in Figure 1. This curve was determined assuming that for a given fetch length, waves generated by a particular wind velocity reach their limiting values in a particular time. This time increases as the fetch length increases, resulting in increased limiting values of wave height and period. For any particular fetch length and wind velocity, there is a duration of wind, after which no future increase in height and period occurs. This limiting time is the time required for the energy front associated with the significant waves to advance at the variable group velocity from the beginning of the fetch to the end of the fetch.

If one considers an increment of time Δt, and an increment of fetch length ΔF, a relationship can be established between group velocity Cg, time t, and distance F; this relationship can be represented by the equation

$$\int_0^t dt = \int_0^F dF/Cg \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

which can be solved for t by graphical integration using the relationship of C_0/U versus gF/U^2 (Fig. 1) and the formula $C_g = C_0/2$ (for deep water). The area under the curve of $1/C_g$ versus F between 0 and F represents t. However, it becomes difficult to obtain an accurate solution by this method.

A short section of the curve C_0/U versus gF/U^2 may be represented as a straight line on logarithmic graph paper, and the corresponding equation is

$$C_0/U = a\,(gF/U^2)^b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where a is the intercept of the line extended to $gF/U^2 = 1.0$ and b is the slope of the line segment. Solving (2) for $C_g = C_0/2$, one obtains

$$C_g = ag^b\,F^b\,U^{1-2b}/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and substituting for C_g in (1), one obtains

$$\int_0^t dt = (2/ag^b\,U^{1-2b}) \int_0^F dF/F^b \quad \ldots\ldots\ldots\ldots\ldots (4)$$

the solution of which is

$$t = (2\,F^{1-b})/a\,(1-b)\,g^b\,U^{1-2b} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Multiplying both sides of (5) by U/F gives the equation

$$tU/F = [2/a\,(1-b)]\,(gF/U^2)^{-b} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

For any particular straight segment of C_0/U versus gF/U^2, it is possible to solve for tU/F versus gF/U^2. In a similar way equations can be developed for other sections and the complete curve for the relation of tU/F versus gF/U^2 is determined. Equation (6) gives the limiting (time) conditions, or the minimum duration of the wind (Fig. 1). Figure 1 alone presents all the basic information on the generation of waves.

Decay of waves--Previous investigators [**BARBER** and **URSELL**, 1948; **SVERDRUP**, 1947]

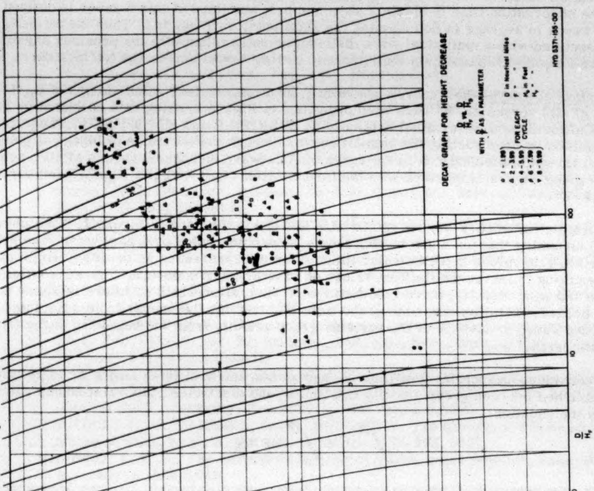

Fig. 2--Relationships between fetch, wave height at end of fetch, decay, and wave period at end of decay; see Table 2 for sources of data

have assumed that the wave spectrums recorded by wave recorders are caused by the superposition of numerous wave trains formed in the fetch owing to varying wind direction and speed. Recent laboratory experiments, performed at the University of California [JOHNSON and RICE, in press], have shown that height and period spectrums are formed by a constant wind blowing over a constant fetch under steady-state conditions. Thus, one must consider in what manner a finite group of waves of varying periods and heights propagates into undisturbed waters. A simple qualitative explanation might be made by considering the groups to consist of numerous waves of different amplitudes and periods. If each of these waves were to travel at some velocity which was dependent upon its "length," then one would observe that no individual "wave" would maintain its identity. The faster waves would move ahead of the main body of waves and the slower waves would drop behind. The train of waves would spread. The spread of the train would increase with increasing decay distance. Hence the "period," which is associated with the time it

Table 2--References pertaining to the source of wave data on the decay of waves used in Figures 2 and 3

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takes two successive "wave crests" to pass a fixed point, would increase with increasing decay distance while it would decrease with time at any fixed decay distance. This spreading is complicated by the fact that it depends not only upon the decay distance but also upon the original size of the disturbance (that is, the length of the fetch, the duration of the storm, the wind velocity, the movement of the storm, and other factors); hence it is necessary to study the decay of waves in regard to the decay distance relative to the fetch.

If a particular spectrum in the fetch may be represented by the period T_F, height H_F, and fetch length F, only one combination of wind velocity and minimum duration t_{min}, will produce this spectrum. Varying F will vary the spectrum whether H_F is changed or not.

The decay of waves is represented in this paper by a family of dimensionless relationships, one relationship for each different fetch length and significant wave at the end of the fetch, and the decay distance. Decrease of the significant wave height is expressed in terms of the dimensionless variables D/H_F versus D/H_D with D/F as the parameter. For the increase of period, the dimensionless relationship is D/gT_F^2 versus D/gT_D^2 with D/F as the parameter. Figures 2 and 3 show these relationships (see Table 2 for sources of data). In order to simplify the procedure, D has been given in nautical miles and the acceleration of gravity has been omitted. The variable used is dimensionally correct, except for the omission of g, the acceleration of gravity, because the graph applies to quantities expressed in nautical miles, knots, hours, and other terms. Figures 2 and 3 have been constructed from the compiled data. The abscissas of Figures 2 and 3 are D/H_F and D/T_F^2, respectively, with the ordinate of both figures being D/F. The reason the data were plotted in this manner was to make the parameter relationship more apparent than if the vertical scales had been D/H_D and D/T_D^2, respectively.

It was possible to fix the starting points of the curves of D/H_D (Fig. 2) and D/T_D^2 (Fig. 3), plotted on log-log scales, because

$$D/H_D \rightarrow D/H_F \text{ and } D/T_D^2 \rightarrow D/T_F^2 \text{ as } D \rightarrow 0$$

Since the curves representing D/H_D and D/T_D^2 (Fig. 2 and 3) were logarithmically spaced at the start, it seemed plausible that they should be so spaced throughout the range of values encountered in forecasting. In each case the family of curves constructed through the data was based upon this assumption.

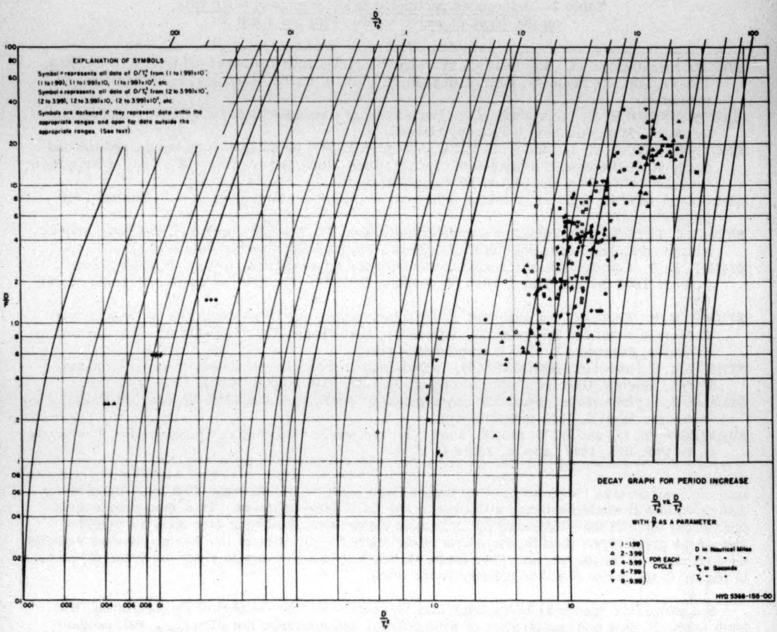


Fig. 3--Relationships between fetch, wave period at end of fetch, decay, and wave period at end of decay; sources of data the same as for Figure 2

The graphs in Figures 2 and 3 were constructed from the data in the following manner:

(1) All the data were plotted to a large scale with D/F and D/H_F, D/F and D/T_F^2, as the coordinates with the values of D/H_D and D/T_D^2 written within the plotting symbols.

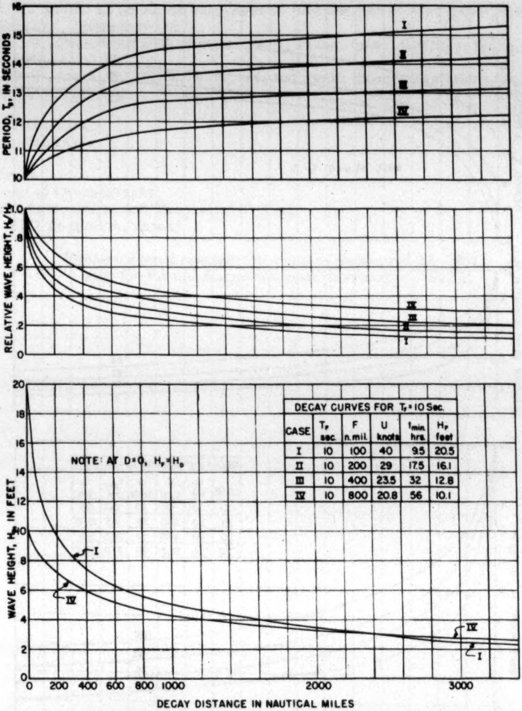
(2) For pictorial representation the data were assigned symbols which represented data within certain ranges of values of D/H_D and D/T, respectively (Fig. 2 and 3).

(3) If the data were within the appropriate ranges, the symbols were darkened. If the data were not within the proper ranges, the symbols were left open.

As an example in Figure 2, take the case of $D/H_F = 50$, $D/F = 1.60$, and $D/H_D = 1.95 \times 10^2$; as the point $D/H_F = 50$, $D/F = 1.60$ lies between $D/H_D = 1 \times 10^2$ and 1.99×10^2, and the value of D/H_D was between these two limits, the point was plotted as a darkened circle. Another case is $D/H_F = 34.9$, $D/F = 2.4$, and $D/H_D = 1.15 \times 10^2$; as the point $D/H_F = 34.9$, $D/F = 2.4$ lies between $D/H_D = 8 \times 10$ and 10^2, while the value of it was 1.15×10, the point was plotted as an open symbol.

The amount of scatter seems acceptable since the effect of helping and opposing winds would be to scatter the points. Most of the scatter is small; there are only a few cases of large discrepancies.

Decay curves were determined for $T_F = 10$ sec for four cases, I, II, III, and IV (fetch lengths of 100, 200, 400, and 800 mi respectively). These have been shown in Figure 4. Figure 5 shows decay curves for height for $H_F = 20$ ft for four cases, V, VI, VII, and VIII (fetch lengths of 100, 200, 400, and 800 mi, respectively). A study of Figure 4 shows that, for a given decay distance and a given fetch period, the shorter the fetch the greater the period increase. A study of Figure 4



DECAY CURVES FOR T_F = 10 Sec.

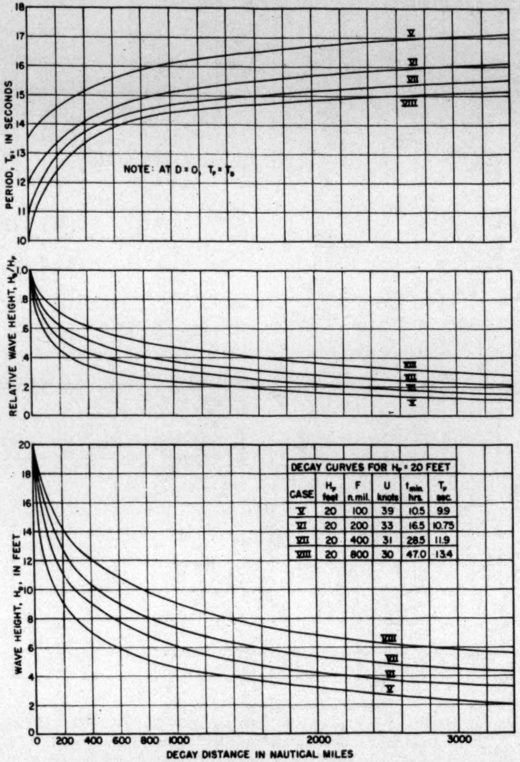
CASE	T_F sec.	F n.mil.	U knots	$t_{min.}$ hrs.	H_F feet
I	10	100	40	9.5	20.5
II	10	200	29	17.5	16.1
III	10	400	23.5	32	12.8
IV	10	800	20.8	56	10.1

Fig. 4--Relationship between T_F = 10 sec, F = 100, 200, 400, and 800 nautical miles, decay, and wave period at end of decay

also reveals that for the same fetch period the shorter the fetch the greater is the decrease in relative wave height, H_D/H_F. This is because greater wind velocities are necessary to generate a given period in shorter fetches; hence the waves are higher and steeper and so will decay more rapidly. It can be seen in Figure 4 that the curves representing the absolute values of the decay height for the two extreme cases I and IV cross, which is to be expected from the above reasoning.

Figure 5 shows the decay of wave height for a 20-ft significant wave for different fetch lengths. From this figure it can be seen that the greatest decrease in wave height is for the shortest fetch. Since the original value of H_F was 20 ft in all cases, it is obvious that these curves should not cross.

Conclusions--Figure 1 represents the generation of wind waves in a dimensionless manner. As far as the author knows, all the available data to date have been included. The decay graphs, Figures 2 and 3, were developed by introducing an additional dimensionless parameter, D/F. This was done because with the addition of the variable, F, the wave in the generation area is defined. The decay curves presented in this paper give a unique solution for each combination of period and height at the end of the fetch, whereas the decay relationship proposed by SVERDRUP and MUNK [1947] give a unique solution for the increase of wave period regardless of the wave height at the end of the fetch. Later, SVERDRUP [1947] proposed the existence of a special solution for



DECAY CURVES FOR H_F = 20 FEET

CASE	H_F feet	F n.mil.	U knots	$t_{min.}$ hrs.	T_F sec.
V	20	100	39	10.5	9.9
VI	20	200	33	16.5	10.75
VII	20	400	31	28.5	11.9
VIII	20	800	30	47.0	13.4

Fig. 5--Relationships between H_F = 20 ft, F = 100, 200, 400, and 800 nautical miles, decay, and wave height at end of decay

the decay of waves for each different fetch length.

More data would be desirable for further verification of the curves presented in this paper. This is particularly true for determining the effects of opposing and following winds, which were not included in this analysis.

Acknowledgments--The author is greatly indebted to M. P. O'Brien and R. A. Fuchs, whose ideas and comments made this paper possible. Credit is also given to R. L. Wiegel and D. K. Todd for their great assistance in the presentation of the material in the manuscript. This investigation was sponsored by the Office of Naval Research.

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(Communicated manuscript received September 14, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

BOTTOM PRESSURE FLUCTUATIONS DUE TO STANDING WAVES IN A DEEP, TWO-LAYER OCEAN

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(Contribution No. 582 from the Woods Hole Oceanographic Institution)

Abstract--The pressure fluctuation in a deep ocean, due to short-period surface waves, is calculated by means of a simple physical model. It is shown, as was pointed out earlier by Longuet-Higgins, that the pressure under a standing wave varies at twice the frequency of the wave and with an amplitude proportional to the square of the wave amplitude, and inversely proportional to the wave length, but that the pressure under a progressive wave is constant.

A similar calculation of the pressure fluctuation under a standing internal wave gives similar results, but the amplitude is diminished by the factor $(\Delta\rho/\rho)$.

Numerical calculations, using typical wave amplitudes, show that pressure fluctuations of about 30 cm may occasionally be expected under six-second surface waves, but that internal waves, because of their large wave lengths, will have little effect on the pressure in the deep ocean.

The purpose of this paper is to investigate the fluctuations of pressure at the ocean bottom due to surface and internal waves. LONGUET-HIGGINS [1950], by a rigorous analysis of the case of short-period waves, has shown that progressive waves cause no pressure fluctuations in deep water, but that in the presence of standing waves the bottom pressure varies with twice the frequency of the surface waves. We will not repeat his calculations, but will show how his results can be derived in a more elementary manner, and apply the same method to calculate the bottom pressure fluctuations due to short-period internal waves.

Under long-period waves the pressure can be calculated with sufficient accuracy from the height of water, on the basis of the usual hydrostatic assumptions, so this case requires no special consideration.

Free surface waves--We consider water of density ρ which is motionless except in the neighborhood of the surface. Let (x, y, z) be Cartesian coordinates with the origin on the mean surface and the z-axis directed vertically downward, and let P and V be the pressure and velocity vector. We will further assume that ρ is a constant at great depths. Then the equation of motion is

$$dV/dt = -\operatorname{grad}(P/\rho - gz) \qquad \ldots\ldots (1)$$

Therefore, there can be no pressure gradients, except for the hydrostatic term, in the deep water where V is zero. But P may vary with time providing it is the same at all points in deep water. We utilize this fact to calculate P by considering the motion of the column of water between the surface and the plane $z = z'$, and of such great lateral extent that the motion of water through its boundaries is negligible compared to the total mass. We denote by ζ the elevation of the free surface above the mean surface.

As has been pointed out by Longuet-Higgins, the center of gravity of this water column will rise and fall as a result of wave motion on the surface, except in the case of pure progressive waves. Vertical motions of the center of gravity can result only from the vertical components of external forces acting on the water mass. These forces are due to gravity, the external atmospheric pressure, and the pressure P at $z = z'$. Let z_0 be the depth of the center of gravity of the water column, M its mass, A its horizontal cross-sectional area, P' the dynamic pressure at $z = z'$, and P_0 the surface (atmospheric) pressure.

Then, by definition

$$P' = P - Mg/A - P_0 \qquad \ldots\ldots (2)$$

$$M = \int_A \int_{-\zeta}^{z'} \rho \, dz \, dS \qquad \ldots\ldots (3)$$

$$z_0 = (1/M) \int_A \int_{-\zeta}^{z'} \rho z dz dS \tag{4}$$

By Newton's second law

$$M\, d^2 z_0/dt^2 = \int_A (P_0 - P)\, dS + Mg \tag{5}$$

$$M\, d^2 z_0/dt^2 = A\,(P_0 - P) + Mg = -AP' \tag{6}$$

if we assume the surface pressure to be constant and take z' great enough so that P is essentially independent of x and y. We will further assume for the moment that ρ is constant. Then

$$P' = -(M/A)\, d^2 z_0/dt^2 \tag{7}$$

$$P' = -(\rho/A)\,(d^2/dt^2)\left(\int_A \int_{-\zeta}^{z'} z dz dS\right) \tag{8}$$

$$P' = (\rho/2A)\,(d^2/dt^2)\left(\int_A [\zeta^2 - z'^2]\, dS\right) \tag{9}$$

$$P' = (\rho/2)\,(d^2/dt^2)\,\overline{\zeta^2} \tag{10}$$

where $\overline{\zeta^2} = (1/A)\int_A \zeta^2\, dS$.

This is the result obtained by Longuet-Higgins in a more rigorous analysis of the wave motion.

Internal waves--We will now apply the same technique to a two-layer ocean. Taking the origin of coordinates at the free surface as before, let the density and average thickness of the upper layer be ρ' and h', and those of the lower layer be ρ and h. The height of the interface above the plane $z = +h$ will be denoted by ζ, and the elevation of the free surface above $z = 0$ by δ'. Equation (4) then becomes

$$z_0 = (1/2M) \int_A \left[\rho\left\{z'^2 - (h - \zeta)^2\right\} + \rho'\left\{(h - \zeta)^2 - \zeta'^2\right\}\right] dS \tag{11}$$

$$P' = -(1/2A)\,(d^2/dt^2)\left(\int_A \left[\rho\left\{z'^2 - \zeta^2 + 2h\zeta - h^2\right\} + \rho'\left\{\zeta^2 - \zeta'^2 - 2h\zeta + h^2\right\}\right] dS \right. \tag{12}$$

$$P' = (1/2)\,(d^2/dt^2)\left(\rho\,\overline{\zeta^2} + \rho'\,[\overline{\zeta'^2} - \overline{\zeta^2}]\right) \tag{13}$$

$$P' = (1/2)\,(d^2/dt^2)\left(\rho'\,\overline{\zeta'^2} + \overline{\zeta^2}\,[\rho - \rho']\right) \tag{14}$$

since $\overline{\zeta} = \overline{\zeta'} = 0$ by definition and $d^2h/dt^2 = d^2z'/dt^2 = 0$.

Pressure variations under harmonic waves--Equations (10) and (14) give the relation between the bottom pressure and the shape of the surface. However, this relation is more clearly seen if the surface motion is represented by harmonic waves. Because of Fourier's theorem, this results in no loss of generality.

We assume, then, that the surface displacement can be represented by the Fourier series

$$\zeta = \sum_{n=1}^{\infty} [A_n \sin (k_n x + \ell_n y + \omega_n t + \delta_n)] \tag{15}$$

where $\omega_n^2 = g\,(k_n^2 + \ell_n^2)^{1/2}$. Then

$$\zeta^2 = \sum_{n=1}^{\infty} A_n^2 \sin^2 (k_n x + \ell_n y + \omega_n t + \delta_n)$$

$$+ 2 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_n A_m \sin (k_n x + \ell_n y + \omega_n t + \delta_n) \times \sin (k_m x + \ell_m y + \omega_m t + \delta_m) \ldots (16)$$

$$\zeta^2 = (1/2) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [A_n \sin (k_n x + \ell_n y + \omega_n t + \delta_n) + A_m \sin (k_m x + \ell_m y + \omega_m t + \delta_m)]^2 . (17)$$

We see that ζ^2 is a sum of terms, each containing a pair of harmonic components. This means physically that the harmonic components interact in pairs, and characterizes the phenomenon as one of the second order. It will be necessary, therefore, to calculate the bottom pressure fluctuations due to the interaction of two harmonic components. The effect of any arbitrary surface motion can then be calculated by summing over all pairs of functions which occur in the Fourier expansion of ζ. Furthermore, the arbitrary phases δ_n may, without loss of generality, be made to vanish by a suitable choice of the origins of time and space. We therefore take

$$\zeta = A \sin (kx + \ell y + \omega t) + B \sin (k'x + \ell'y + \omega't) \ldots \ldots \ldots \ldots (18)$$

where

$$\omega^2 = (k^2 + \ell^2)^{1/2} \ldots \ldots \ldots \ldots (19)$$

$$\omega'^2 = (k'^2 + \ell'^2)^{1/2} \ldots \ldots \ldots \ldots (20)$$

k, ℓ, k′, ℓ', are arbitrary wave numbers, and A and B are arbitrary amplitudes. Then

$$\zeta^2 = A^2 \sin^2 (kx + \ell y + \omega t) + B^2 \sin^2 (k'x + \ell'y + \omega't)$$

$$+ 2\,AB \sin (kx + \ell y + \omega t) \sin (k'x + \ell'y + \omega't) \ldots \ldots \ldots \ldots (21)$$

But

$$\sin^2 (kx + \ell y + \omega t) = [1 - \cos 2 (kx + \ell y + \omega t)]/2 \ldots \ldots \ldots \ldots (22)$$

The first term on the right is independent of t, while the second has an average value of zero.

Therefore, the third term in (21) is the only one which will contribute to $(d^2/dt^2)\,\overline{\zeta^2}$. We now use the trigonometric identity

$$2\,AB \sin (kx + \ell y + \omega t) \sin (k'x + \ell'y + \omega't)$$

$$= AB \left[\cos \left\{(k - k')x + (\ell - \ell')y + (\omega - \omega')t\right\} - \cos \left\{(k + k')x + (\ell + \ell')y + (\omega + \omega')t\right\}\right] \quad (23)$$

But the right side of this equation has a mean of zero unless $k = k'$, $\ell = \ell'$, or $k = -k'$, $\ell = -\ell'$, either of which implies $\omega = \omega'$ because of (19) and (20). If $k = k'$, $\ell = \ell'$, (23) is replaced by

$$2\,AB \sin^2 (kx + \ell y + \omega t) = AB [1 - \cos 2 (kx + \ell y + \omega t)] \ldots \ldots \ldots \ldots (24)$$

with the constant mean AB. This is to be expected, since the center of gravity of a progressive wave moves horizontally, but not vertically.

We find, therefore, that a single progressive wave, or a linear combination of progressive waves with different wave lengths or directions of propagation, will not result in pressure fluctuations in the deep ocean.

On the other hand, if $k = -k'$, $\ell = -\ell'$, (23) becomes

$$2\,AB \sin (kx + \ell y + \omega t) \sin (-kx - \ell y + \omega t) = AB [\cos 2 (kx + \ell y) - \cos 2\omega t] \ldots (25)$$

Therefore

$$(d^2/dt^2)\,\overline{\zeta^2} = -AB\,(d^2/dt^2) \cos 2\omega t = 4\,AB\,\omega^2 \cos 2\omega t \ldots \ldots \ldots \ldots (26)$$

and from (10)

$$P' = 2\,\rho AB\,\omega^2 \cos 2\omega t = 2\,\rho g\, AB\,(k^2 + \ell^2)^{1/2} \cos 2\omega t \qquad (27)$$

We see that there are no pressure variations unless there exists in the surface waves a pair of harmonic components of equal frequency but opposite directions of propagation, which is the condition for standing waves. In this case the pressure will vary with twice the frequency of the surface waves. This is, of course, to be expected for a second-order effect and is supported by physical reasoning, since the center of gravity of a standing wave rises every half cycle and reaches its minimum elevation each time the surface passes through the mean surface.

If there are two layers of different density, the situation is more complicated, since there are two modes of vibration, in each of which both surfaces are in motion, but with different amplitudes. We consider first the high-frequency mode, analogous to free-surface waves, in which the two layers move together. For this mode the frequency is given by the usual relation

$$\omega'^2 = gk \qquad (28)$$

where k is the wave number in the direction of propagation. In view of our previous result that pressure variations arise only from the interaction of a pair of waves traveling in opposite directions, which holds for internal waves as well, we can, without loss of generality, assume the waves to be traveling along the x-axis. If we let

$$\zeta' = A' \sin(kx + \omega' t) + B' \sin(kx - \omega' t) \qquad (29)$$

then we have at the interface [LAMB, 1932]

$$\zeta = A' e^{-kh'} \sin(kx + \omega' t) + B' e^{-kh'} \sin(kx - \omega' t) \qquad (30)$$

Proceeding as before, but using (14) instead of (10), we readily find

$$P' = 2\,\omega'^2 A' B' [\rho' + \Delta\rho\, e^{-kh'}] \cos 2\omega' t \qquad (31)$$

$$P' = 2\, gk\, A' B' [\rho' + \Delta\rho\, e^{-kh'}] \cos 2\omega' t \qquad (32)$$

where

$$\Delta\rho = \rho - \rho' \qquad (33)$$

When the surface layer is deep compared to the wave length ($kh' >> 1$), we have

$$P' \cong 2\, gk\, A' B' \rho' \cos 2\omega' t \qquad (32a)$$

the same result as that obtained for a one-layer ocean, which is to be expected since in this case the lower layer is unaffected by the wave motion.

When kh' is small

$$P' \cong 2\, gk\, A' B' [\rho - \Delta\rho\, kh'] \cos 2\omega' t \qquad (32b)$$

which shows the damping effect of a shallow surface layer.

For the other mode of vibration [LAMB, 1932, p. 372]

$$\omega^2 = gk\,\Delta\rho / (\rho \coth kh' + \rho') \qquad (34)$$

and

$$\zeta'/\zeta = -\Delta\rho\, e^{-kh'}/\rho' \qquad (35)$$

Let

$$\zeta = A \sin(kx + \omega t) + B \sin(kx - \omega t) \qquad (36)$$

and

$$\zeta' = -A\,(\Delta\rho/\rho)\,e^{-kh'}\sin(kx+\omega t) - B\,(\Delta\rho/\rho)\,e^{-kh'}\sin(kx-\omega t) \tag{37}$$

Using (14) again, we find

$$P' = 2\,\Delta\rho\,AB\,\omega^2\,(1-e^{-kh'})\cos 2\omega t \tag{38}$$

$$P' = 2\,gk\,AB\,(\Delta\rho)^2\,(1-e^{-kh'})(\sinh kh'\cos 2\omega t)/(\rho\cosh kh' + \rho'\sinh kh') \tag{39}$$

$$P' = 2\,gk\,AB\,(\Delta\rho)^2\,(1-e^{-kh'}-e^{-2kh'}+e^{-3kh'})(\cos 2\omega t)/(\rho+\rho'+\Delta\rho\,e^{-2kh'}) \tag{40}$$

When the surface layer is thin, the effect due to the free surface nearly cancels that due to the interface and (38) becomes

$$P' = 2\,g\,AB\,(\Delta\rho)^2\,k^3\,h'^2(\cos 2\omega t)/\rho \tag{41}$$

On the other hand, when kh' is large, the motion of the surface is negligible and the pressure has its maximum value

$$P' = 2\,gk\,AB\,(\Delta\rho)^2(\cos 2\omega t)/(\rho+\rho') \tag{42}$$

Comparing (40) with (32), we might expect the effect of internal waves to be small compared to that of surface waves when $\Delta\rho$ is small. But the amplitudes of internal waves are generally larger than those of surface waves by a factor of the order of magnitude of $\Delta\rho/\rho$. The ratio of the two effects will therefore depend on k, and since the wave lengths usually observed in internal waves are much greater than those observed in free-surface waves, the effect of free-surface waves will predominate in the bottom pressure fluctuations, as can be seen by reference to the numerical calculations below.

One further possibility must be discussed, that is, the interference between a free-surface mode and an interfacial mode of equal wave lengths but different frequencies ω and ω' given by (19) and (20) (with $\ell = 0$). We take

$$\zeta' = A'\sin(kx+\omega' t) - B\,(\Delta\rho/\rho')e^{-kh'}\sin(kx+\omega t) \tag{43}$$

$$\zeta = A'\,e^{-kh'}\sin(kx+\omega' t) + B\sin(kx+\omega t) \tag{44}$$

Neglecting terms that are independent of time, we have

$$\overline{\zeta'^2} = -(k\,AB\,\Delta\rho\,e^{-kh'}/2\pi\rho')\int_0^{2\pi/k}\left[\cos(\omega'-\omega)t - \cos\left\{2kx+(\omega+\omega')t\right\}\right]dx \tag{45}$$

$$\overline{\zeta'^2} = -A'B\,(\Delta\rho/\rho)\,e^{-kh'}\cos(\omega'-\omega)t \tag{46}$$

$$\overline{\zeta^2} = A'B\,e^{-kh'}\cos(\omega'-\omega)t \tag{47}$$

$$\rho'\,\overline{\zeta'^2} + \Delta\rho\,\overline{\zeta^2} = 0 \tag{48}$$

$$\therefore P' = 0 \tag{49}$$

Therefore, interaction of the two modes of vibration cannot produce pressure changes in deep water.

Application to bottom pressure gages--In the design of bottom pressure gages and the interpretation of data obtained from them, it will be useful to have some crude estimates of the periods and amplitudes to be expected. The amplitudes (G) may conveniently be expressed in cm of water. Then

$$G = P'/(\rho g\cos 2\omega t) \tag{50}$$

For the spectrum of surface waves we take a "typical" distribution of wave heights given by MUNK [1947]. This distribution has a maximum height (crest to trough) of four feet, at a period of eight seconds. Since we wish to estimate the effects of the largest waves which are commonly encountered, we have arbitrarily doubled the wave heights; that is, we have plotted Munk's wave heights as amplitudes. This spectrum is shown in curve 1 of Figure 1. We have used only that part of Munk's spectrum below a period of one minute, since longer waves do not satisfy the

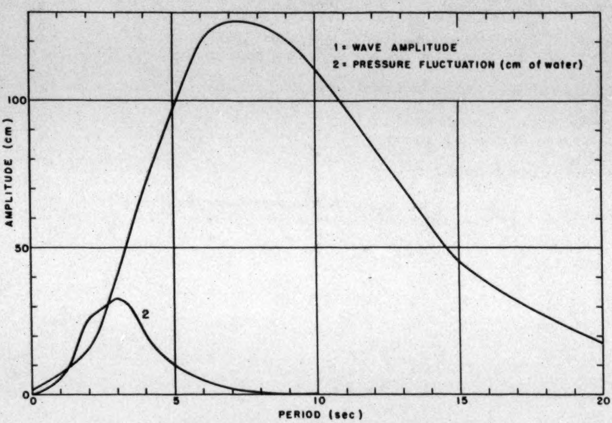

Fig. 1--Typical wave amplitudes and pressure fluctuations

condition for surface waves (L < 2h). From (27) (with $\ell = 0$) and (50) we find

$$G = 2\,AB\,k \qquad (51)$$

$$G = 2\,A^2 rk \qquad (52)$$

where $r = B/A$ is the standing-wave ratio. It is difficult to guess a typical value for r in the deep ocean. It is probably very small except near a moving storm or an island with a very steep shoreline. In any event, r can never exceed unity. We will assume in this calculation that $r = 1$, with the understanding that our results will represent upper limits to the effects that might be observed.

When the data of curve 1 are substituted for A in (52) with $r = 1$, we obtain the values of G plotted in curve 2. The greatest pressure fluctuation obtained is 32 cm of water at a period of three seconds. For this curve the periods are those of the bottom pressure, that is, half the wave periods.

Typical data on internal waves are more difficult to obtain, but it appears that oscillations of the seasonal thermocline at $h' = 200$ m ($\Delta\rho/\rho \sim 3 \cdot 10^{-3}$) occur with periods of about 1000 sec and amplitudes in the neighborhood of 30 m, while the deep thermocline at about 1000 m ($\Delta\rho/\rho \sim 5 \times 10^{-4}$) oscillates at tidal periods with amplitudes not exceeding 100 m. The deep ocean pressure fluctuation due to waves on the seasonal thermocline is about 10^{-3} cm. The periods observed in the deep thermocline correspond to wave lengths in excess of 8×10^4 m, much greater than the depth of the ocean, so our method of calculation cannot be used in this case. A simple hydrostatic calculation shows that in this case the first-order pressure variation has an amplitude of about 1.2 cm in phase with the internal wave.

We can conclude, therefore, that a bottom pressure recorder with a sensitivity of, say, five cm of water, will record no effects due to internal waves, and that free-surface waves may possibly cause pressure fluctuations with periods of less than 30 sec, with a maximum amplitude not exceeding 32 cm of water at a period of three seconds.

Acknowledgments--I should like to express my gratitude to Henry Stommel and Arnold B. Arons of the Woods Hole Oceanographic Institution for their valuable advice in the preparation of this paper. This work has been supported by public funds administered by the Office of Naval Research under Contract No. N6onr-27701.

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(Communicated manuscript received September 17, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union · Volume 33, Number 3 · June 1952

ON THE DETERMINATION OF TRANSMISSIBILITY AND STORAGE COEFFICIENTS FROM PUMPING TEST DATA

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Abstract--This paper presents a graphical procedure for determining the formation constants of an artesian aquifer from pumping test data. The procedure is based on the principle that the coefficient of transmissibility is determined by the ratio of the drawdown to its rate of change with respect to the logarithm of the time since pumping started, or $s/(\delta s/\delta \log_{10} t)$, use being made of the non-equilibrium theory. The computation of formation constants may be performed in two ways, and their results should be checked against each other. A numerical example is given to illustrate the application of the procedure.

Introduction--In quantitative studies of ground-water hydrology for the development of water resources in an artesian aquifer it is essential to determine the field values of transmissibility and storage coefficients since these two so-called "formation constants" are generally considered as the physical indexes of the aquifer characteristics, and are used as the bases for the theoretical prediction of the future yield of ground water in storage. This fact has been strongly recognized in applying the science of ground-water hydrology to engineering problems, particularly in the engineering office where, in order to determine the field values of these constants, the engineers are seeking reliable and simple procedures that are adaptable to the solution of practical problems.

The coefficient of transmissibility indicates the rate at which an aquifer will transmit water under a unit hydraulic gradient. It is defined by THEIS [1938, p. 894] as "the number of gallons of water which will move in one day through a vertical strip of the aquifer one foot wide and having the height of the aquifer when the hydraulic gradient is unity."

The coefficient of storage characterizes the ability of the aquifer to release water from storage as the head declines. It is defined by JACOB [1940, p. 576] as "the volume of water of a certain density released from storage within the column of aquifer underlying a unit-surface area during a decline of head of unity."

The field values of these two coefficients can be determined by the method of non-equilibrium theory developed by THEIS [1935], in which theory the drawdown produced by a well pumping at a constant rate from an areally extensive artesian aquifer of uniform thickness and physical properties is given by

$$s = (114.6\, Q/T)\, W(u) \quad \ldots\ldots (1)$$

where s = drawdown in feet, Q = constant discharge of the pumped well in gallons per minute, T = coefficient of transmissibility in gallons per day per foot, and W (u) is a so-called well function. The well function is an exponential integral and can be expanded in the form of an infinite series as

$$W(u) = \int_u^{\infty} (e^{-u}/u)\, du = -0.577216 - \log_e u + u - u^2/2 \cdot 2! + u^3/3 \cdot 3! - \ldots\ldots (2)$$

in which e = base of natural logarithm and u is an argument expressed as

$$u = 1.87\, r^2\, S/Tt \quad \ldots\ldots (3)$$

where r = distance of the observation well from the pumped well in feet, S = coefficient of storage as a ratio or fraction, and t = time since pumping began in days.

When the values of formation constants are given, the computations of drawdown may be made by (1), (2), and (3), and are even simpler using a nomograph [CHOW, 1951], or a special chart or slide rule. However, the reverse procedure would present real difficulties. Theoretically speaking, if the drawdowns at two or ~~more observation~~ points at given times, or the rate of change of drawdown at a given observation point, as available from the pumping test data, are known, then the values of formation constants, S and T, are determined by (1), (2), and (3). Yet, owing to the fact that one of the two unknowns, T, occurs twice in the equation, once as the divider of the exponential integral and once in the denominator of the argument, a direct mathematical solution is extremely difficult. A cut-and-try solution is theoretically possible, but due to the wide and uncertain range of estimation and the laboriousness of computation such solution is by no means practical. Within the author's knowledge, two graphical methods for the solution of (1), (2), and (3) for T and S have been proposed respectively by Theis and Jacob.

Theis has suggested to WENZEL [1942, p. 88] in 1937 and to JACOB [1940, p. 582] in 1938 a graphical method of superposition for the solution of S and T from (1), (2), and (3). By this method, values of drawdown, s, are plotted against values of r^2/t, or of $1/t$ if there is only one observation well involved, on logarithmic paper to the same scale as the "type curve," resulting in a curve of the observed data. The "type curve" is a curve showing the relation between the argument u and the function W (u). It can be proved that the curve of the observed data should be a certain portion of the type curve, bearing the same slope or the same rate of change of curvature. The curve of the observed data is then superimposed on the type curve in such a way that the coordinate axes of the two curves are kept in parallel. A position is found by trial whereby most of the plotted points of the observed data fall on a segment of the type curve. With the curves in the superimposed position, an arbitrary point is chosen on the segment of fitness. The corresponding coordinates of the match point on the two curves, W (u) and u on the type curve and s and r^2/t, or $1/t$, on the observed curve, are determined. Substituting these coordinate values in (1) and (3), the values of T and S can be computed.

JACOB [1946] has found that when the argument u in (1) is small, that is, r is small and t is large as shown by (3), a plot of the drawdown s against the logarithm of the elapsed time t since pumping began should approximate a straight line. From the slope of this straight line, or the drawdown difference per log cycle of time, Δs, and the time intercept on zero-drawdown axis, t_o, the formation constants, T and S, may be easily computed by the following two simple formulas

$$T = 264\, Q/\Delta s \qquad (4)$$

$$S = 0.3\, T\, t_o/r^2 \qquad (5)$$

Theis' method of superposition offers a very ingenious solution to the mathematical difficulties described previously. On the other hand, at the risky expense of accuracy, Jacob's method of approximation possesses a great advantage of simplicity. However, each method has its drawback when used alone. For the method of superposition, as the curvature of the type curve is very flat in the portion which is often most used in practical applications, the fitting of curves by trial is far from exact, and the resulting values obtained by this method would involve errors of personal judgment to a great extent. For the method of approximation, an erroneous answer is often reached when the plotted curve appears to be a straight line to the eye, while, in reality, it is far from the straight-line approximation. This latter case is commonly encountered when the time of pumping test is insufficient and consequently the test fails to produce data that could cover the region of approximation.

As a result of the above-mentioned drawbacks, the present practice is to use the method of approximation when the error involved in the straight-line approximation is tolerable, or to use the method of superposition when the error is significant. To follow this practice, the engineer must use his own judgment. Otherwise, he should be given a knowledge of the error that would be introduced by the application of the method of approximation to any given problem. This knowledge is to be presented in this paper.

Mathematically it can be shown that a semi-logarithmic plot of the relation of W (u) to u will display the same curvature of a semi-log plot of s versus t. The former plot will reveal that for values of u less than about 0.01 the plot practically approximates a straight line. Hence, this property is also true for the latter plot which is required by the method of approximation. By (24) and (25), to be derived later, for u less than 0.01 the method of approximation would introduce errors of less than one per cent in T and errors of less than four per cent in S. For the procedure described in this paper, a tolerable error of five per cent is assumed, and the corresponding value of u should be less than 0.05.

Moreover, experience has shown that the formation constants are gradually changing during the time of pumping. This phenomenon becomes evident particularly in the case of severe pumping or at the early stage of the pumping test, under which conditions the sudden release in pore water pressure would cause compression in formation that affects the realizable value of constants.

When this happens, it is difficult to fit properly the observed curve to any portion of a type curve. The procedure described in this paper, however, will help the computer to find the apparent value of formation constants at any instant as the pumping goes on.

Theory--From Theis' non-equilibrium theory (1), (2), and (3) are established. From Eq. (3)

$$\log_e u = \log_e (1.87\, r^2 S/T) - \log_e t \qquad (6)$$

Differentiating u, $\log_e$ u, W (u), and s of (3), (6), (2), and (1) with respect to $\log_e$ t, the resulting expressions are

$$\delta u/\delta \log_e t = -u \qquad (7)$$

$$\delta \log_e u/\delta \log_e t = -1 \qquad (8)$$

$$\delta W(u)/\delta \log_e t = -\delta \log_e u/\delta \log_e t + (\delta u/\delta \log_e t)(1 - u/2! + u^2/3! - \ldots)$$

or, from (7) and (8),

$$\left.\begin{aligned} \delta W(u)/\delta \log_e t &= 1 - u(1 - u/2! + u^2/3! - \ldots) \\ &= 1 - u + u^2/2! - u^3/3! + \ldots \\ &= e^{-u} \end{aligned}\right\} \qquad (9)$$

$$\delta s/\delta \log_e t = (114.6\, Q/T)\, \delta W(u)/\delta \log_e t$$

or, from (9),

$$\delta s/\delta \log_e t = (114.6\, Q/T)\, e^{-u} \qquad (10)$$

Converting the natural logarithm to the common logarithm, (10) becomes

$$\delta s/\delta \log_{10} t = (264\, Q/T)\, e^{-u} \qquad (11)$$

Dividing (1) by (11),

$$s/(\delta s/\delta \log_{10} t) = W(u)\, e^u/2.3 \qquad (12)$$

Let

$$F(u) = s/(\delta s/\delta \log_{10} t) \qquad (13)$$

Then

$$F(u) = W(u)\, e^u/2.3 \qquad (14)$$

Therefore, for a given value of u, the values of W (u) and F (u) can be computed by (2) and (14) respectively. Consequently, a relation between F (u), W (u), and u can be plotted as shown by Figures 1 and 2. Conversely, when F (u) is known, the values of W (u) and u can be found from these figures. Note on Figure 1 that W (u) is practically equal to 2.3 F (u) when W (u) is greater than 4.0, or when F (u) is greater than 1.74. From Figure 2, this corresponds to the condition that u is less than 0.01.

From (1) and (3), with values of W (u) and F (u) being given, the coefficients of transmissibility and storage can be computed by

$$T = 114.6\, Q\, W(u)/s \qquad (15)$$

$$S = Ttu/1.87\, r^2 \qquad (16)$$

When u is very small, that is, less than say 0.01, e^u approaches unity, and accordingly (14) becomes

$$F(u) = W(u)/2.3 \qquad (17)$$

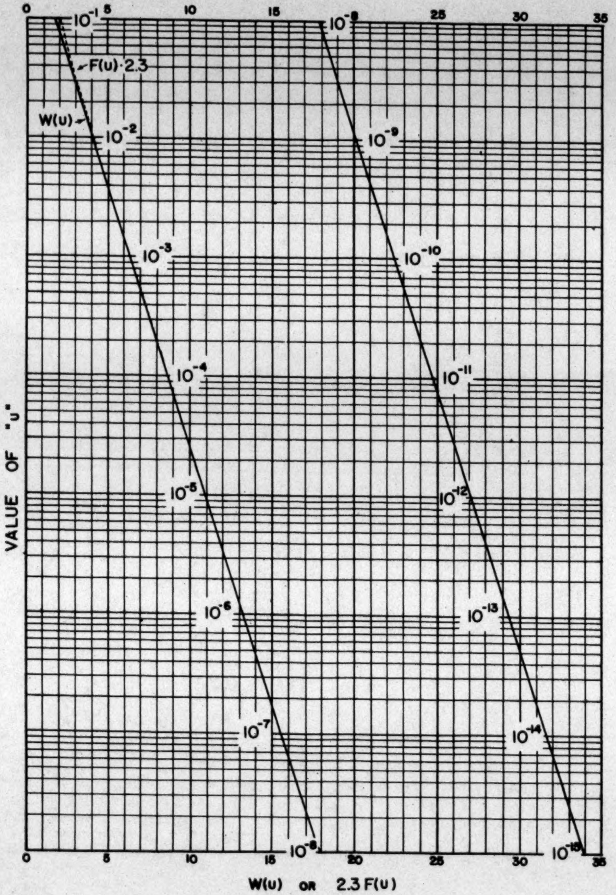

Fig. 1--Relation between W(u) or F(u) and u for small values of u

and (15) becomes

$$T = 264\ Q\ F(u)/s$$

or, from (13)

$$T = 264\ Q/(\ \delta s/\delta \log_{10} t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

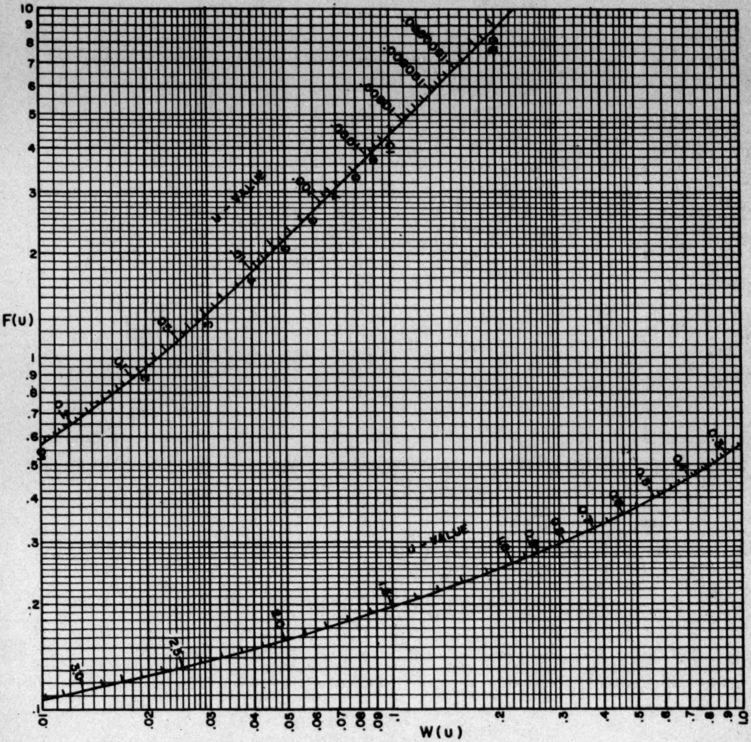

Fig. 2--Relation between F(u) and W(u) for large u-values

As mentioned previously, when u is very small, a plot of s against $\log_{10}$ t would approximate a straight line. The slope of this straight line is $\delta s/\delta \log_{10} t = \Delta s$. the drawdown difference per log cycle of time. Thus, (18) becomes (4).

Furthermore, when u is very small (less than 0.01), (2) approaches

$$W(u) = -0.577216 - \log_e u \quad \ldots\ldots \quad (19)$$

Substituting (17) in (19) and transposing

$$u = e^{-0.577216 - 2.3F(u)} \quad \ldots\ldots \quad (20)$$

Substituting (20) in (16) and transposing,

$$S = Tte^{-0.577216 - 2.3F(u)}/1.87\ r^2 \quad \ldots\ldots \quad (21)$$

in which T is computed by (18) or (4).

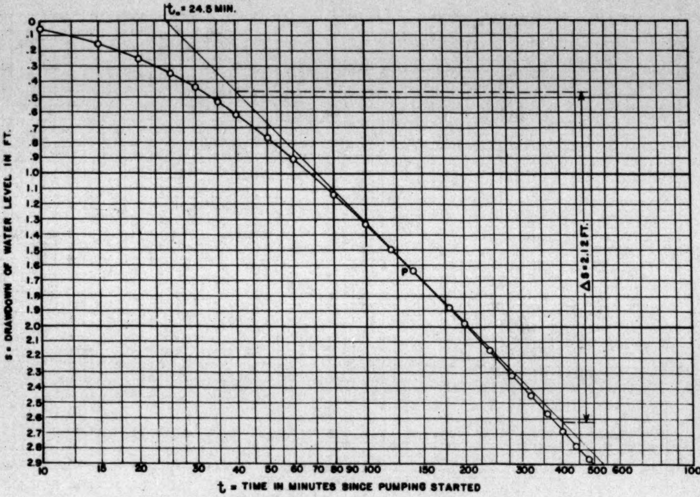

Fig. 3--Relation between drawdown and duration of pumping

From the straight-line assumption, or u being very small, when $s = 0$, $t = t_0$, which is the time intercept on zero-drawdown axis as shown by Figure 3, and from (18), $F(u) = 0$. Substituting these in (21) and simplifying, (21) becomes (5).

Equations (4) and (5), hence (18) and (21), are employed in Jacob's method of approximation. It can be proved that the relation between t_0 and t is

$$t_0/t = e^{-2.3F(u)} \qquad (22)$$

and the relation between the approximate T computed by (4) and the T value computed by (15) is

$$T_{approx}/T = e^{u} \qquad (23)$$

Consequently, the errors involved in the approximation for computing the values of T and S by (4) and (5) can be evaluated by means of (15), (16), (18), (21), (22), and (23) as follows

$$\text{The overestimated error in } T = e^{u} - 1 \qquad (24)$$

$$\text{The underestimated error in } S = 1 - 0.561\, e^{u-2.3F(u)}/u \qquad (25)$$

For given values of F (u) or u, the percentages of error involved in the computed T and S can be determined by (24) and (25) respectively. They are plotted against F (u) as shown by Figure 4. It is interesting to note that for an error of less than five per cent the values of F (u) should be restricted as follows: For an error of less than five per cent in T; $F(u) > 1.13$. For an error of less than five per cent in S; $F(u) > 1.62$ and $0.23 > F(u) > 0.21$. In other words, if an error of five per cent in transmissibility is tolerable, as generally accepted in such field methods, the method of approximation should be applied only when $F(u) > 1.13$.

Through the study and analysis of numerous pumping test data, the author has found that in most cases long-term test data are not available because the collection of such data is either uneconomical or beyond the interest of the well driller or the well owner. As a result, the value of F (u) derived from the available data is usually less than 1.13. Long-term test data could probably be secured only if it were the objective of a comprehensive investigation.

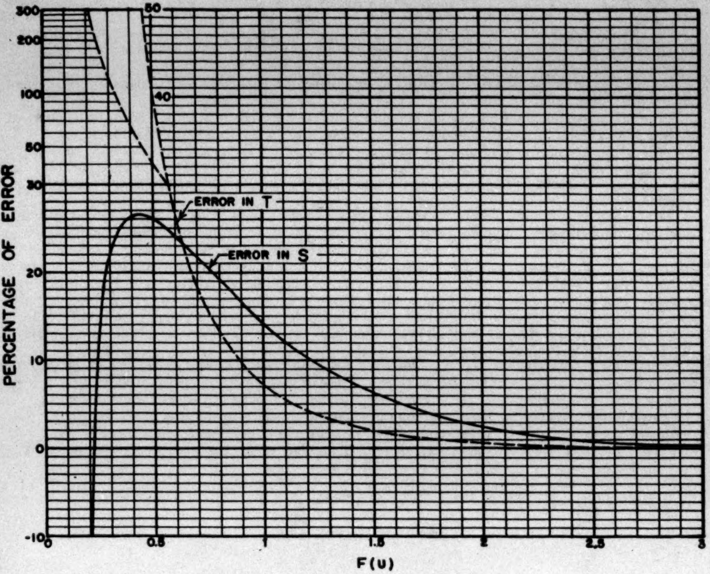

Fig. 4--Variation of per cent of error with F(u)

Procedure--The procedure for the application of the above theory to the determination of formation constants from pumping test data may be outlined as follows:

(1) Plot the observational data for a well on semi-logarithmic coordinate paper in which the drawdown, s, in feet is in linear scale and the time, t, in minutes since pumping started, is in logarithmic scale.

(2) On the plotted curve, choose an arbitrary point and note the coordinate values, t and s.

(3) Draw a tangent to the curve at the chosen point and measure the slope of the tangent which is equal to $\delta s/\delta \log_{10} t$. For convenience, this slope may be taken as the drawdown difference per log cycle of time, or Δs, in feet. To construct a proper tangent may seem to be not easy for a beginner. However, with a little practice, the result should be a perfect job.

(4) The Computation of T and S can be performed in either of the following two methods:

(A) Compute F(u) by (13) and find the corresponding values of W(u) and u from Figure 1 or 2. Compute T and S by (15) and (16) respectively.

(B) Note the time intercept, t_0, on zero-drawdown axis, obtained by extending the tangent. Compute F(u) by (13) and find the percentages of error in T and S from Figure 4. Compute the approximate values of T and S by (4) and (5) respectively and correct the values for errors.

Example--A numerical example is given to illustrate the method of computation. The rate of pumping is 750 gpm. The distance of the observation well from the pumped well is 804 ft. As shown in Figure 3, the drawdown, s, is plotted against the logarithm of time, $\log_{10} t$. A point P is chosen on the curve. The coordinates of this point are found to be: t = 140 min = 0.0972 day; s = 1.64 ft. A tangent to the curve at point P is constructed. The slope of this tangent expressed in drawdown difference per log cycle of time is Δs = 2.12 ft. The intercept of the tangent at zero-drawdown axis is t_0 = 24.5 min = 0.0170 day.

Method A--By (13), F (u) = 1.64/2.12 = 0.775. From Figure 2, W (u) = 1.55 and u = 0.135. Hence, by (15), T = 114.6 × 750 × 1.55/1.64 = 81,300 gpd/ft. By (16), S = 81,300 × 0.0972 × 0.135/1.87 × 804 × 804 = 8.83×10^{-4}.

Method B--By (13), F (u) = 0.775. From Figure 4, the overestimated error in T = 14.4 per cent and the underestimated error in S = 20.0 pct. The approximate T = 264 × 750/2.12 = 93,400 gpd/ft. The corrected T = 93,400/1.144 = 81,600 gpd/ft. The approximate S = 0.3 × 93,400 × 0.0170/804 × 804 = 7.38×10^{-4}. The corrected S = $7.38 \times 10^{-4} \times 1.2 = 8.86 \times 10^{-4}$.

Summary--The method for determining the formation constants as described in this paper has several significant features.

The computation of the formation constants can be performed in either of the two ways, Method A and Method B. The results of one method may be easily checked by the other.

The same s - $\log_{10}$ t plot used in the present procedure is readily applicable to the study of hydrologic boundaries of the ground-water reservoir [CHOW, BUSHMAN, and HUDSON, 1950], so that a more consistent and systematic procedure for the analysis of ground-water problems could be achieved.

A number of arbitrary points on the s - $\log_{10}$ t curve can be taken at various t. The values of T and S thus computed may be plotted against $\log_{10}$ t on the same coordinate paper resulting in T - $\log_{10}$ t and S - $\log_{10}$ t curves. As the non-equilibrium theory is based on the assumption of constant T and S, the theoretical T - $\log_{10}$ t and S - $\log_{10}$ t curves should appear to be horizontal straight lines. However, as often happens in practical problems, the values of T and S may be gradually increasing or decreasing, depending on the nature of the aquifer, during the pumping test. In such cases, the computed values are referred to as apparent values, the plot of which values against $\log_{10}$ t would give a clear picture showing the variation of aquifer characteristics with respect to the time of pumping.

For F (u) greater than 1.74, or u less than 0.01, the method of approximation gives practically the exact theoretical values of T and S. For F (u) less than 1.13, or u greater than 0.05, the method of approximation should not be employed without tolerating an error of greater than five per cent.

The method of superposition involves the personal error from making trial solutions. Not uncommonly, a satisfactory fit between the observed curve and the type curve cannot be easily obtained, particularly when the apparent values of T and S are varying with the time of pumping at a remarkable degree.

Acknowledgments--The author is greatly indebted to J. J. Doland for his encouragement in pursuing this study, and to Engineering Subdivision, Illinois State Water Survey, for affording opportunities to investigate practical ground-water problems. The head of the Engineering Subdivision is H. E. Hudson, Jr.

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(Communicated manuscript received June 14, 1951; open
for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

SOME EFFECTS OF FIRE AND ASH ON THE INFILTRATION CAPACITY OF SOILS

R. H. Burgy and V. H. Scott

Abstract--This paper is a preliminary report on the effects of certain physical and chemical relationships that may influence the movement of water into soils and the infiltration rates of some typical brushland soils. Factors studied were the effects of the presence of ash, the application of heat, the burning of brush, and the chemical constituents of the brush ash and their ramifications. Results tend to indicate that infiltration rates of soils may be influenced by burning brush and the presence of ash on the surface of soils. On the soils tested no decrease in infiltration rates occurred.

Introduction--A large portion of the lands in the foothill areas of the southwestern United States are covered with brush, commonly referred to as chaparral. Conversion of certain portions of these areas to grazing lands by the use of fire is considered one of the most expeditious means. The effects of burning brush on the soil surface and the resulting influence on the infiltration rate of the soil have been studied by a few investigators. In an effort to obtain further knowledge on this subject, the authors have undertaken a series of experiments both in the field and laboratory. These experiments were designed specifically to determine what effects fire and ash have on the water-intake characteristics of soils.

Review of the literature--Many investigations have been reported on the factors that influence infiltration. Some literature is available concerning the effects of fire on the physical and chemical characteristics of soil which control the intake and movement of water in soils.

WAHLENBERG [1935] reports, "At McNeil, Mississippi, after fire had been excluded for seven years, the surface soil was found to be less dense and more porous" and the exclusion "increased the capacity to absorb water from one to five times." Using a technique of measuring infiltration by direct measurement on one plot not burned for many years and on an adjacent area where burning had been practiced at intervals, terminating with a burn before the sampling, AUTEN [1933] found that the average infiltration rate was 2.91 times greater on the unburned area as on the burned area.

In some studies of the influence of woodland chaparral on one range soil, ROWE [1941] reports, "Changes in the environmental factors, brought about by burning the vegetation, have resulted in a reduction of 90 to 95 pct in the infiltration capacity of the soil on the 1/40-acre annually burned plots."

From records of soil-moisture conditions on paired plots, one of which was denuded annually by cutting and burning and the other left undisturbed, VEIHMEYER and JOHNSTON [1944] report, "Denudation by burning did not prevent the soil from becoming wet throughout its full depth as soon as that in the unburned plots. . . . . The soil-moisture records indicate that the infiltration capacity of the soil on the burned plots was not impaired."

Veihmeyer's studies include both paired plots and small watersheds, which were selected to give a variety of soil type, cover, topography, precipitation, and elevation typical of so-called chaparral land.

Experimental procedure on field studies

Soil-moisture studies, Test 1--The initial study undertaken was to determine what effect the presence of ash has on the ability of moisture produced by natural rainfall to move into the soil. The soil type of the test area was Yolo loam. Eight plots, seven by eight feet, were laid out in two rows. The two end plots in each row were designated as controls, with the four center plots receiving the treatment.

Dense brush was cut from a carefully measured area and weighed to determine the density of cover. The brush was cut in the late summer and was extremely dry. It consisted of two species, namely, chamise (*Andenostoma fasciculatum*) and buckbrush ceanothus (*Ceanothus cuneatus*), in the approximate proportions of 60 and 40 pct, respectively. The field density of the cover was 20.3 tons per acre when cut. Brush of this density was concentrated on the area of the test plots, increasing the density to 72.5 tons per acre.

Application of this brush was made by piling it horizontally on the plots as uniformly as possible. By the nature of the application of this dense brush, the fire was concentrated much closer to the soil than usually occurs in natural fires. A complete burn resulted, leaving only a heavy blanket of white ash on the surface of the plots, averaging approximately one inch in depth. The average ash yield was determined to be approximately 2.5 pct of the air-dry weight of the brush.

Prior to treating the plots, the moisture content of the soil was sampled by means of a soil tube to a depth of four feet.

In the following period, as the natural rainfall added moisture to the test area, the moisture front was followed down through the soil column. All sampling was done from low scaffolding so as not to trample or compact the surface of the soil, and the sample holes were refilled. The soil was allowed to drain for a minimum of four days after the end of rainfall before sampling.

Rainfall was measured by means of a standard eight-inch rain gage adjacent to the site. The precipitation was so distributed over the period that it was possible to take only two sets of soil samples between storms, after which the additional precipitation was of such large amount as to cause the moisture front to pass beyond the sampling depth of four feet.

Fire-temperature studies, Test 2--Temperature data for fires in brush areas have been secured and a wide range of values reported. To determine what range of temperature distribution and duration might be expected in the soil from a brush fire, a test plot was established from which these data could be obtained. The plot consisted of a two-foot square area of Yolo clay loam soil which was covered with a light stand of grass. The soil was nearly air dry in the top two inches.

A ditch was opened around the plot to a depth of two feet. Using extreme care, a series of thermocouple leads and remote-reading thermometers were placed at several levels in the plot. Thermocouple leads were placed at four locations in the 1/4- to 1/2-inch depth, and four at the 1/2- to one-inch depth uniformly distributed over the area. Long-stemmed thermometers were placed in the soil at four points 1 1/2 inches below the surface and at four points two inches below the surface. This arrangement of temperature-measuring devices provided a network of 16 points in the top two inches of the four-square-foot plot without disturbing the surface of the soil. All thermocouple leads were covered to minimize interference due to heat conductance.

To obtain the most intense fire possible, it was necessary to apply the maximum possible density of brush that could be found in the field. On a 0.001-acre field area the maximum cover density that could be obtained was 111 tons per acre of air-dry brush, whereas on large areas the density was much less as mentioned previously. This amount of brush was applied to the test plot.

Initial soil temperature, air temperature, and relative humidity were measured prior to the fire. During the fire, the temperatures of the soil were read at the end of five minutes and at five- and ten-minute intervals thereafter until the peak temperatures were reached. The temperatures of the thermocouples were read directly on a potentiometer. Ambient temperatures and relative humidity were recorded simultaneously. The recession of temperatures was obtained at intervals until the soil had returned to normal.

Experimental procedure on laboratory studies

By supplementing the field studies with laboratory experiments designed to provide control over certain variables not possible in the field, the authors studied the effect of ash on the penetration of water into soils and the effects of various treatments on the infiltration rate of one agricultural and some typical brushland soils.

Penetration studies, Test 3--A series of carefully hand-packed soil columns were made up in glass graduates using Yolo loam soil. At the center of each graduate, a glass tube to provide for escape of air was placed so that the lower end opened into a fine gravel layer and the upper end extended above the soil surface. Compensation for differences in diameter was made for the volume of water required for specified depths of application.

The treatments of the surfaces of the cylinders consisted of four replications of the following: (1) controls; (2) two tons of ash per acre; and (5) eight tons of ash per acre (see Test 4 for treatments (3) and (4)). The ash of these treatments was obtained by burning a combined sample of the chamise and ceanothus over an open grate. Distilled water was slowly added in 1/2-inch increments to the surfaces of these cylinders, and the depth of penetration was measured from the top surface of the soil. The soil was allowed to drain until no appreciable movement could be detected, at which time a second 1/2-inch increment of water was added and the process repeated. The third increment consisted of a one-inch addition.

Infiltration studies--Infiltration-rate determinations for five soils using nine treatments were made following a laboratory technique developed for this purpose. The soils to be studied were prepared by sieving them through a two-mm screen. They were in an air-dry condition when packed. The containers, four-inch-diameter cylinders, six inches tall, with a 5/8-inch hole punched in the bottom, were prepared by placing a fine screen over the hole and covering the inside bottom of the cylinder with a 1/8- to 1/4-inch layer of fine gravel. The soil was weighed out and hand-packed uniformly into the cylinders to obtain a homogeneous mass. The soils were packed to densities approaching those found in the field. Precautions were taken to prevent any disturbance of the soil surface. A constant head rack was used to supply water. Volumes were measured by catching the percolated water.

The term infiltration rate as used in this paper is defined as the rate per unit of time at which water enters a soil when there is standing water on the soil surface. The rate indicated for any given hour is the average rate for the time interval preceding that hour. In all of the laboratory tests the infiltration rates are relative rates only, influenced by many phenomena associated with this type of laboratory technique.

Distilled water was used on all but two tests. The cylinders were calibrated initially (designated Run A) and grouped according to similarity of infiltration rates. Pairings of the cylinders were then made so as to give both high and low rates in each treatment (designated Run B).

Ash treatment, Test 4--Yolo loam soil was used in the cylinders to study the effect of various ash treatments on infiltration rates. Again the ash was obtained as in Test 3. Treatments consisted of placing dry ash at rates of two and eight tons per acre in several combinations on the surface of the soil in the cylinders. A head of one to 1 1/2 inches of water was maintained on the surface. There were four replications of each of the five treatments which were: (1) controls (no treatment); (2) chamise ash at two tons per acre; (3) buckbrush ceanothus ash at two tons per acre; (4) combined ash at two tons per acre; (5) combined ash at eight tons per acre.

Application of heat alone to soil surface, Test 5--A new series of cylinders were packed using Yolo loam soil. Initially calibrated cylinders were matched in the controls and treatments.

The treatments of Test 4 were repeated in part, supplemented by burning the soil surface with an oxygen-gas torch. The torch was so arranged that the tip of the flame touched the surface of the soil. The burned cylinders were treated for a period of ten minutes which caused the soil surface to become heated to a cherry-red color. The temperature of the flame was over 1200°C.

The replicated treatments were as follows: (1) controls; (4) combined ash at two tons per acre; (5) combined ash at eight tons per acre; (6) wet surface burned; (7) treatment 4 plus burning; and (8) treatment 5 plus burning. The ash treatments were applied after burning and presoaking.

Brush burned in place on cylinders, Tests 6, 7, 8, 9, 10--It was found that the best laboratory procedure that could be made to approach an intense natural fire was obtained by burning brush in place on the surface of the soil in the cylinders. The technique developed to facilitate this operation was to pack the soil to the desired density and precalibrate the cylinders until a stable rate was reached. The moisture was allowed to drain from the cylinders. All of the cylinders were then placed in a drying oven held at approximately 220°F for a period of 12 hours which was sufficient to dry a considerable portion of the soil mass.

Cylinders were then paired on the basis of the initial calibration and half of the cylinders were treated. The treatment was accomplished by piling and burning the brush on the soil surface. A density of 111 tons of brush per acre was used in all tests. Due to the difficulty of obtaining a draft inside of the tops of the cylinders, it was necessary to supply some outside heat in the form of a Bunsen burner flame to burn the material completely. The brush burned rapidly down to the level of the cylinder walls and then was reduced to a white ash by the combination of the flame and an artificially induced draft under an exhaust hood. When the burning was completed, the cylinders were allowed to cool and were then placed in a water bath to wet the soil column from the bottom, without allowing the surface to become submerged. This priming was necessary because, in drying, the soils tended to shrink from the sides of the cylinders. Priming restored the seal between soil and sides of cylinders, tending to eliminate the possibility of channeling when water was applied.

A series of five soil types was studied using the above-outlined technique. Each soil was replicated eight times in controls and treatment, with the exception of the last soil where six replications were used. The soils studied were Aiken, Corning, Hugo, Holland, and Yolo, designated as Tests 6 through 10, respectively.

Aggregation studies, Test 11--On cylinders of the Yolo, Corning, Hugo, and Aiken soils, a water-stable aggregate analysis was performed. This analysis incorporates a standard laboratory technique developed by TUILIN [1933] and YODER [1936]. These analyses were made on composite samples of the first and second 1/4-inch layers of soil taken from both the control and burned cylinders.

Chemical studies, Test 12--Chemical studies were made in two phases of the work on infiltration in an effort to determine the kinds and concentrations of the various elements that might influence infiltration, namely, calcium, sodium, potassium, and magnesium.

(A) Samples of the combined brush ash were prepared and subjected to two analyses: water-soluble and acid-soluble cations. The water-soluble cation analysis is believed to be more applicable to the natural condition of leaching of soluble salts from the ash.

(B) Analyses were made of the leachate from tests on Aiken and Yolo soil. Samples were taken as follows: (1) the first 250 ml of leachate in Run A; (2) the last 250 ml from Run A; (3) the first 250 ml from Run B; (4) last 250 ml through cylinders.

In addition, at the time of each reading of the infiltration rates, electrical conductivity measurements were made on the leachate. These data were used as indicators to denote any unusual changes in the leachate composition which might have occurred during a run, as a relative measure of the total salt concentrations, and for possible correlation with infiltration rates.

Data and discussion

Field trials--A summary of the various densities of brush cover and the equivalent amounts of ash, based on a 2.5 pct ash yield, is found in Table 1. The treatments in the various tests were designed to equal or exceed these rates of application.

Table 1--Summary of brush densities and relative ash yields for chamise and buckbrush ceanothus

Nature of cover[a]	Brush density	Ash density[b]	Equivalent depth of ash
	tons/acre	tons/acre	in
Average brush density in field	20	0.5	0.08
Maximum density on 0.001 acre in field	111	2.78	0.30

[a] Relative proportions: 60 pct chamise; 40 pct buckbrush ceanothus.
[b] Based on a 2.5 pct ash yield.

Figure 1 is a graph of the soil-moisture content in inches depth of water at various sampling depths in the plots of Test 1 at the three sampling dates, October 16, November 4, and November 24, 1950. The solid and dotted lines, used to designate the natural and ash-covered plots, respectively, merely join the points of soil-moisture content for the dates sampled. Values of the moisture equivalent are indicated by the horizontal lines identified by the letters ME. Daily amounts of precipitation are shown. There appears to be no appreciable reduction in the amount of moisture which penetrated to the various depths in the ash-covered and natural-covered plots. The difference between the moisture content in the zero to six-inch depth might be explained on the basis of increased evaporation from the blackened, ash-covered surfaces of the burned plots. However, in the two- to three-foot depth there is a higher moisture content in the ash-covered plots which may be due to the greater net precipitation reaching the soil because of the lack of vegetative interception.

Information on changes in soil temperature resulting from fires in various types of vegetative cover have been reported. SAMPSON [1944] found maximum temperatures of 630°F and 410°F at a depth of 1.5 inches in the mineral soil on two fires in chaparral. In another location the temperature reached 410°F at a 0.5-inch depth. In studying the soil changes associated with forest fires occurring in longleaf pine, HEYWARD [1936] found temperatures of 210°F infrequent at the 1/8-inch depth and a rise of only a few degrees over normal soil temperatures at the 1/4-inch depth. HOFMANN [1924] found in a slash fire in Douglas fir in the Pacific Northwest that a rise of 15°F over the mineral-soil temperature of 60°F occurred at the one-inch depth.

Figure 2 depicts the soil temperatures obtained in Test 2. Note that the brush density used in this test was over five times greater than the usual field condition. Piling the air-dried brush on the soil obviously caused a greater concentration of heat nearer to the soil. Approximately 20 minutes after the fire was ignited, a preliminary peak of temperatures in all the depths to one inch was reached. Following this, there was a slight decline and then a continued increase occurred. This phenomenon is explained by the collapse of the burning brush to the soil surface and the consequent increased heat conductance from the burning embers. The time lag in the movement of the heat front from the upper to lower depths can be observed from the figure. Proper contact between soil and thermocouple leads and insulation of the leads are critical factors in measuring soil temperatures under fire.

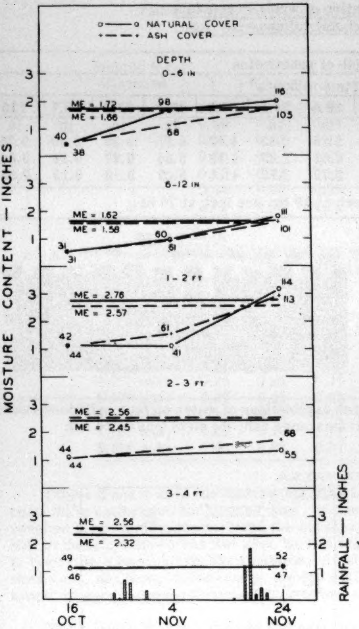

Fig. 1--Soil-moisture content in inches depth of water for various sampling depths and for three sampling dates in natural and burned plots; moisture-equivalent values are in terms of inches of water; numbers at each point are the ratio, in per cent, of field-moisture content to moisture equivalent

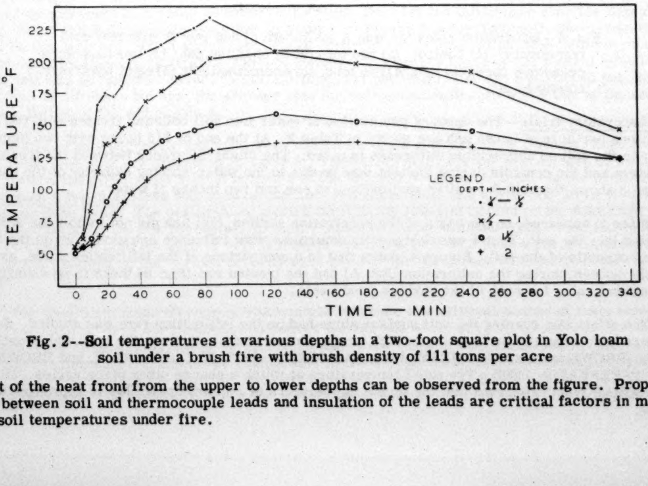

Fig. 2--Soil temperatures at various depths in a two-foot square plot in Yolo loam soil under a brush fire with brush density of 111 tons per acre

Table 2--Depth of penetration of water in controls and ash-covered soil columns

Treatments	Depth of penetration											
	Time in hours[a]											
	0.25	0.50	1.00	1.50	2.50	18.0	21.3	23.3	42.3	52.3	90.7	115
	in	in	in	in	in	in	in	in	in	in	in	in
Controls	1.75	1.91	2.02	2.08	2.19	2.83	3.83	4.12	5.05	5.28	9.30	9.75
Treatment 4	1.56	1.76	1.98	2.07	2.19	2.82	3.82	4.12	5.05	5.27	9.24	9.84
Treatment 5	1.44	1.63	1.99	1.98	2.09	2.72	3.70	4.11	5.02	5.19	9.13	9.66

[a] Water application: 1/2 inch at 00 hr; 1/2 inch at 19 hr; one inch at 76 hr.

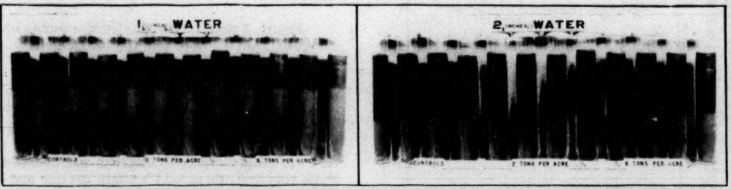

Fig. 3--Depths of penetration of one- and two-inch applications of water on four cylinders each, treated as follows: (1) controls; (2) two tons/acre ash; (5) eight tons/acre ash

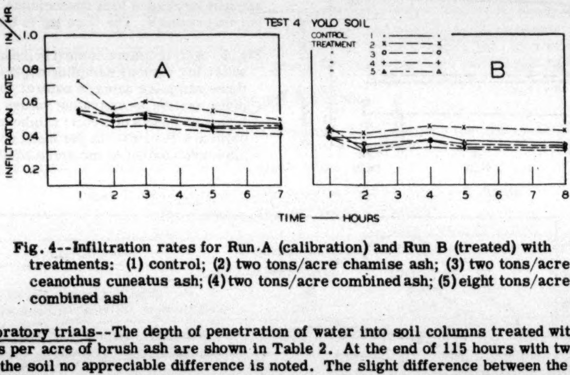

Fig. 4--Infiltration rates for Run A (calibration) and Run B (treated) with treatments: (1) control; (2) two tons/acre chamise ash; (3) two tons/acre ceanothus cuneatus ash; (4) two tons/acre combined ash; (5) eight tons/acre combined ash

Laboratory trials--The depth of penetration of water into soil columns treated with two and eight tons per acre of brush ash are shown in Table 2. At the end of 115 hours with two inches of water in the soil no appreciable difference is noted. The slight difference between the treated cylinders and the controls during the test may be due to the water-holding capacity of the ash. Figure 3 shows the cylinders after applications of one and two inches of water.

Since it appeared, on the basis of the penetration studies, that ash did not inhibit the movement of water into the soil, Test 4 was designed to determine what influence ash alone had on the infiltration capacity of the soil. Figure 4 shows that in a comparison of the infiltration rates, as previously defined, during the calibration (Run A) and the treated run (Run B) there is no change in the relative rates for each of the various treatments.

The effect that burning the soil surface alone had on the infiltration rate was studied. Several workers have reported that heating to high temperatures tends to destroy the colloidal properties of soil [BROWN and MONTGOMERY, 1913; KAMOSHITA, 1937; KELLEY, DORE, and BROWN, 1931; STEENKAMP, 1928]. The exact temperature at which a change takes place varies. PURI and ASGHAR [1940] reported that the fusion temperature of colloidal particles is approximately 400°C.

Table 3--Infiltration rates on ash-covered, torch-burned wet soil surfaces in calibration runs, no treatment

Sample for treatment	Time, hours							
	1	2	3	4	5	6	11	22
	in/hr	in/hr	in/hr	in/hr	in/hr	in/hr	in/hr	in/hr
1	0.60	0.55	0.52	0.50	0.45	0.49	0.41	0.35
4	0.47	0.45	0.42	0.45	0.35	0.38	0.30	0.28
5	0.60	0.54	0.53	0.50	0.43	0.45	0.38	0.34
6	0.50	0.53	0.52	0.48	0.42	0.42	0.37	0.32
7	0.64	0.60	0.56	0.47	0.45	0.46	0.40	0.34
8	0.61	0.56	0.52	0.53	0.46	0.48	0.41	0.36

Table 4--Infiltration rates on ash-covered, torch-burned wet soil surfaces in treated runs

Treatment	Time, hours							
	1	2	5	6	17	20	26	41
	in/hr	in/hr	in/hr	in/hr	in/hr	in/hr	in/hr	in/hr
1 (control)	0.31	0.27	0.28	0.24	0.24	0.21	0.22	0.20
4 (2T ash)	0.28	0.26	0.27	0.24	0.25	0.22	0.22	0.20
5 (8T ash)	0.27	0.26	0.23	0.21	0.20	0.17	0.17	0.17
6 (no ash)	0.18	0.16	0.15	0.10	0.09	0.06	0.06	0.05
7 (2T ash)	0.19	0.16	0.13	0.11	0.09	0.08	0.07	0.06
8 (8T ash)	0.08	0.08	0.07	0.06	0.05	0.04	0.04	0.04

Tables 3 and 4 show the infiltration rates for the ash-covered, torch-burned wet surfaces and controls in calibration and treated runs. It was noted that a marked decrease in infiltration rate resulted from burning the soil with the oxygen-gas torch at temperatures exceeding 1200°C. The soil in these cylinders was wet when burned which greatly affected the surface of the soil, tending to bake it to a hard, brick-like condition. A set of air-dry cylinders was burned under similar conditions, and it was noted that on the dry surfaces the soil particles tended to aggregate, making larger particles without forming the crustlike surface noted in the wet-burned soils.

On the basis of previous tests and experiences it seemed desirable to simulate field conditions of burning as closely as possible. This was accomplished by burning cut brush on the surfaces of the cylinders. As previously mentioned, it was necessary to add additional heat resulting in a higher temperature than would be found in the field. Data for the infiltration runs for Aiken, Corning, Hugo, and Holland soils are given in Figure 5. Figure 6 is for Yolo soil. Note from these figures that in every case a significant change occurred in the infiltration rates after burning. The percentage increase in infiltration rates of the burned cylinders based on the control cylinders for each soil was as follows: Aiken, 64 pct; Corning, 131 pct; Hugo, 103 pct; Holland, 48 pct; Yolo, 385 pct. In all cases but one, the average rate for the control cylinders was lower at the beginning of Run B than at the end of Run A.

Water-stable aggregate analyses were run on the soils of the tests on Corning, Hugo, Aiken, and Yolo. The data are presented in Table 5. The values given are the percentages of material retained on the given sized screen in terms of the original weight of sample. In the preliminary screening a two-mm sieve was used on the Yolo and Aiken soils and a five-mm sieve on the Corning and Hugo soils. The preliminary sieving through the two-mm screen on the Aiken and Yolo soils accounts for the low percentage of larger-sized particles in these soils. Note that in 13 of the 24 cases, for both the zero to 1/4-inch depth and the 1/4- to 1/2-inch depth, the percentages of sample retained on the various screens were greater for the burned samples than for the controls.

The final tests made on factors which might influence the infiltration rate of soils subjected to brush burning were the chemical analysis of the ash and of the percolated waters. Table 6 presents the mineral analysis of the combined ash of the chamise and buckbrush ceanothus. Data for the calcium concentrations are comparable to those reported by Sampson [1944]. However, the data for potassium are several times greater than he reported. HEYWARD and BARNETTE [1934] comment that on burned areas there is a higher calcium content in soil, attributable to the

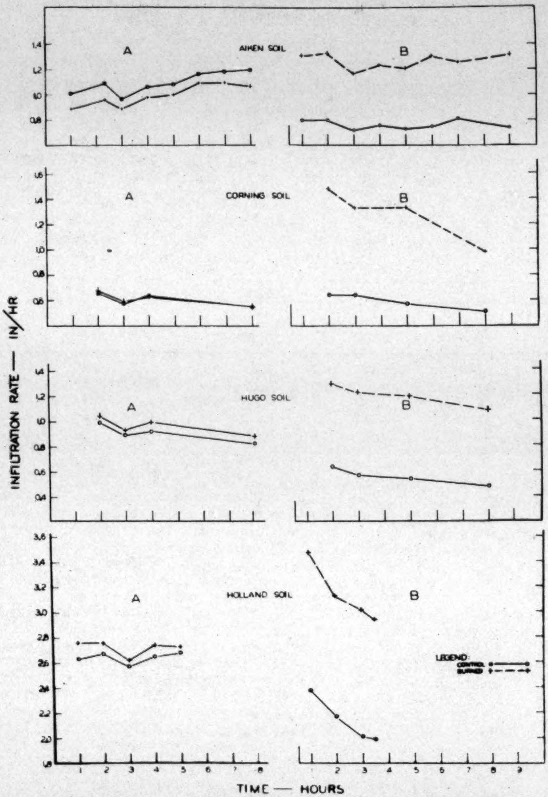

Fig. 5--Infiltration rates for controls and treated cylinders in Run A (calibration) and Run B (treated) for typical brushland soils; treatment was burning of 111 tons/acre of brush on soil surfaces

ash, and it is probable that quantities of magnesium, potassium, and other constituents are also increased by fire.

Analyses of water leachate constituents in the samples taken at the times listed in the procedure are shown in Table 7. It will be noted that in a comparison of the concentrations of total salts, there is a significant increase between burned samples 2 and 3 for both soils. This increase includes substantial rises in calcium, magnesium, and particularly in potassium. For the Yolo soil, it is known that application of waters containing approximately eight milliequivalents of total salts tends to maintain infiltration rates, whereas distilled water will leach cations (electrolytes) from the soil solution and cause the colloids to be dispersed (L. D. Doneen, unpublished data), [BODMAN, 1937]. Dispersion will render the soil less pervious.

Table 5--Water-stable aggregate analysis of burned and unburned soils by depths

Soil	Treatment	Depth	Standard screen mesh						Total
			9	16	32	60	115	250	
		in	pct	pct	pct	pct	pct	pct	pct
Corning[a]	Control	0 - 1/4	9.2	13.6	15.0	16.2	10.8	11.8	76.6
	Burned		11.0	12.2	15.6	16.2	13.2	12.4	77.6
	Control	1/4 - 1/2	6.0	11.3	26.4	14.0	13.2	8.2	89.1
	Burned		7.8	17.2	21.2	15.4	10.8	9.4	82.0
Hugo[a]	Control	0 - 1/4	4.4	15.2	27.0	14.2	8.4	8.6	77.8
	Burned		10.6	15.8	21.8	13.8	8.0	7.4	77.4
	Control	1/4 - 1/2	4.2	19.2	27.0	13.6	7.2	7.0	78.2
	Burned		4.0	16.6	23.8	13.8	6.8	7.2	72.2
Aiken[b]	Control	0 - 1/4	0.4	24.8	27.6	19.6	9.8	6.8	89.0
	Burned		0.2	13.4	25.2	22.0	14.6	10.2	85.6
	Control	1/4 - 1/2	0.4	22.6	30.0	17.2	10.8	7.4	88.4
	Burned		0.0	18.2	25.6	21.8	12.6	9.4	87.6
Yolo[b]	Control	0 - 1/4	0.0	15.0	11.4	9.4	14.0	18.8	68.6
	Burned		0.0	8.8	12.6	12.2	18.0	20.0	71.6
	Control	1/4 - 1/2	0.0	7.8	8.2	6.6	11.6	21.0	55.2
	Burned		0.0	6.2	10.2	9.4	13.6	19.8	59.2

[a] Preliminary screening through a five-mm sieve.

[b] Preliminary screening through a two-mm sieve.

Table 6--Brush ash analysis of chamise and buckbrush ceanothus

Cations	Acid soluble	Water soluble	Equivalent rate based on water soluble constituents
	pct	pct	lb/acre
Na	0.213	0.107	5.7
Ca	6.464	0.670	36.6
Mg	2.365	0.035	1.83
K	5.588	4.158	222.0
Total	14.630	4.970	266.13

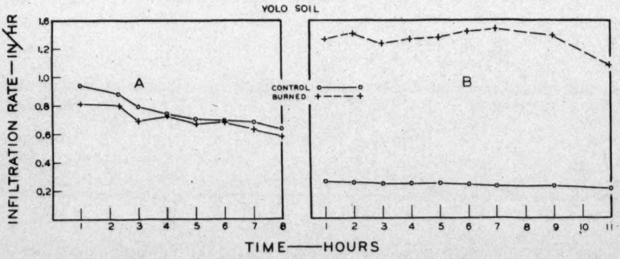

Fig. 6--Infiltration rates for control and treated cylinders in Run A and Run B; treatment was burning of 111 tons/acre of brush on soil surface

Table 7--Comparison of chemical analysis of water leachate before and after brush burning on control and burned cylinders (me/L = milliequivalents per liter)

Group	Treatment	Sample	Cation concentration									
			Yolo soil					Aiken soil				
			Na	Ca	Mg	K	Total	Na	Ca	Mg	K	Total
			me/L	me/L	me/L	me/L	me/L	me/L	me/L	me/L	me/L	me/L
					Calibration run							
A	None	1[a]	0.81	4.94	10.31	1.01	17.08	0.92	8.86	10.31	1.18	21.27
B	None		0.59	4.26	7.89	0.99	13.73	0.85	8.36	9.17	1.15	19.53
A	None	2[b]	0.63	0.38	1.67	0.34	3.02	0.14	0.35	1.01	0.19	1.69
B	None		0.22	0.34	1.67	0.34	2.55	0.12	0.27	0.56	0.13	1.08
					Treated run							
A	Control	3[c]	0.22	0.81	1.48	0.60	3.11	0.14	0.33	0.72	0.30	1.49
B	Burned		0.31	2.12	2.81	2.45	7.69	0.19	0.78	0.84	1.06	2.87
A	Control	4[d]	0.17	0.78	1.48	0.56	2.99	0.09	0.19	0.44	0.13	0.85
B	Burned		0.15	0.67	1.83	0.58	3.23	0.10	0.33	0.63	0.27	1.33

[a] Beginning of calibration. [b] End of calibration. [c] Beginning of treatment. [d] End of treatment.

Electrical conductance values taken throughout the treated infiltration runs showed that as the cation concentration decreased the infiltration rate decreased. This phenomenon was reported by BODMAN [1937] for Aiken clay loam and Yolo silt loam.

To confirm that the presence of certain salts have a direct effect on infiltration rates, one additional test was performed on Yolo soil. Exactly the same procedure as in tests 6 to 10, that is, calibration and oven-drying, was followed. Replicated cylinders were then treated with a solution of salts equivalent to that concentration contained in the ash following the burning of brush at a density of 111 tons per acre. The infiltration rate data are presented in Figure 7. Table 8 gives the chemical analysis from samplings of the leachate at the beginnings and ends of Run A and Run B. It should be noted that an increase in the infiltration rate occurred in the treated cylinders. The control cylinders, however, continued on at a much lower rate, after the first hour, and continued to drop, while the treated cylinders maintained a steady rate. A drop in the rate of the treated cylinders did not occur until the 11th hour when the leachate also began to show a marked decrease in total salt concentration. The difference in infiltration rates of the treated and control cylinders is significantly less than that recorded in Test 10 on the same soil, the only difference in treatment being the lack of heat from the burning process.

As previously discussed, distilled water was used in all but the tests on Corning and Hugo soils where laboratory tapwater having a total salt concentration of eight milliequivalents per liter and a calcium-magnesium concentration of six milliequivalents per liter was used. The increase in rate at the beginning of Run B over the rate at the end of Run A on Corning soil may be attributed to the maintenance of electrolytes in the soil solution by using the tapwater with high salt concentration. For Corning and Hugo soils, the greater percentage increase in infiltration rates of the burned

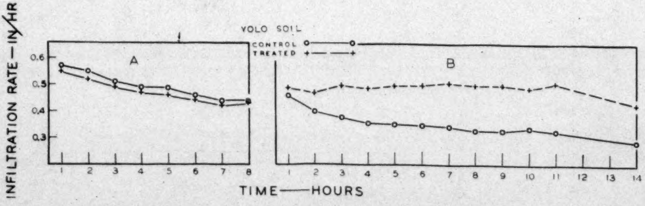

Fig. 7--Infiltration rates for control and chemical-treated cylinders in Run A and Run B; treatment was addition of chemical constituents equivalent to those liberated by burning of 111 tons/acre of brush

Table 8--Comparison of chemical analysis of water leachate for calibration and treated runs of control and chemically treated cylinders, Yolo soil (me/L = milliequivalents per liter)

Group	Treatment	Sample	Cation concentration				
			Na	Ca	Mg	K	Total
			me/L	me/L	me/L	me/L	me/L
				Calibration run			
A	None	1[a]	1.06	0.64	2.89	4.53	9.12
B	None		1.01	0.57	3.00	4.53	9.11
A	None	2[b]	0.46	0.26	0.27	1.13	2.12
B	None		0.46	0.22	0.31	1.13	2.12
				Treated run			
A	Control	3[c]	0.67	0.28	0.81	1.25	3.01
B	Treated		2.66	2.47	13.92	10.92	29.97
A	Control	4[d]	0.69	0.29	1.25	1.70	3.93
B	Treated		2.26	1.23	7.72	8.33	19.54
A	Control	5[e]	0.51	0.25	0.63	1.00	2.39
B	Treated		1.46	0.84	3.30	5.00	10.60
A	Control	6[f]	0.42	0.15	0.35	0.69	1.61
B	Treated		0.90	0.55	1.15	1.54	4.14

[a] Beginning of Run A. [b] End of Run A. [c] Beginning of Run B.
[d] At 8 hours of Run B. [e] At 11 hours of Run B. [f] End of Run B.

cylinders based on the control may also then be attributed to the use of water of high salt content. Both Aiken and Holland soils showed considerably less percentage increases.

Summary

On the basis of the field and laboratory studies of soil moisture and penetration, it is concluded that ash on the surface of the soil did not render the soil impervious and that the soil in the burned, ash-covered plots was wetted as readily as that in natural-covered plots.

Ash, alone, on the soil surface, did not alter the infiltration rate of the laboratory test cylinders. However, intense burning of a wet Yolo soil surface did cause a reduction in the infiltration rate.

On the basis of the data obtained from the method employed in the water-stable aggregate analysis, there appeared to be a slight tendency toward a change to a larger particle size distribution due to burning.

In every case, using a laboratory technique that most nearly approached field-burning conditions, yet with brush-density applications exceeding five times the usual field density, and with a complete and more intense burn, infiltration rates were not reduced.

It is further concluded, on the basis of the preliminary work reported herein, that infiltration rates of soils may be influenced by the burning of certain brush species and the presence of ash on the surface of some brushland soils in two ways, namely, aggregation of soil particles and chemical changes within the soil.

Acknowledgments--Grateful acknowledgment is made to L. D. Doneen and D. W. Henderson, University of California, Davis, California, for valuable assistance and for making available data concerning certain chemical relationships influencing infiltration currently in preparation for publication.

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(Communicated manuscript received July 2, 1951; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union   Volume 33, Number 3   June 1952

FRESH WATER FROM SALT WATER

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Abstract--This paper surveys the several proposals for refining sea water, including compression distillation, multiple-effect distillation, the electrolytic process, ionic exchange, freezing, solar distillation, and the thermal-difference power plant. Cost estimates and design information lead to the conclusion that none of the methods using fuel as a source of heat or power is feasible at the present time. Consideration of the use of waste heat from power plants for the distillation of sea water gives hope that such developments could be used to avoid drought conditions by supplementing existing water supplies.

Introduction--The recommendations of President Truman's Water Resources Policy Commission include the following statements [vol. 1, p. 17, sec. 68]: The Government should accept the responsibility for large investment in broad research programs designed to expand the available supplies of water. Such research programs will be directed toward: (a) Exploring and developing the necessary techniques for utilizing the full possibilities of the Nation's ground-water resources; (b) establishment of a sound national policy for control of artificial rain-making activities and the application of science to devising methods which can be utilized in the public interest; and (c) exploration and experimentation, including pilot-plant operation, in the general field of conversion of sea water to fresh water on an economical basis. In terms of the Nation's future, such research and experimentation may well be found to rank with the development of atomic-energy utilization.

Work on ground-water resources in California is being carried on by Federal and State agencies. Artificial rain-making is being attempted by several private organizations. There is considerable interest in sea-water purification but costs of accepted methods appear to be prohibitive so that presently there is no prototype installation here nor is there any pilot-plant work being done. It is the purpose of this paper to review the present status of purification methods and to suggest avenues of experimentation which may be productive.

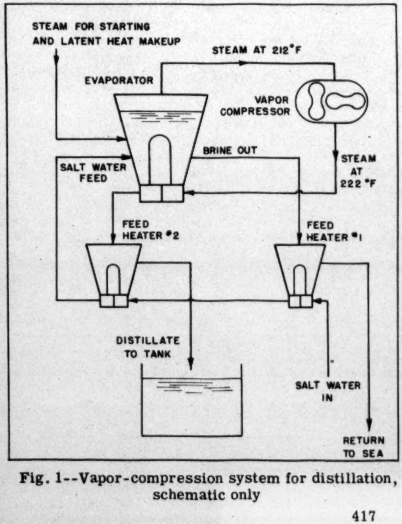

Fig. 1--Vapor-compression system for distillation, schematic only

Methods of purification--Several methods of salt-water purification have been suggested, among which are the following:

(1) Compression distillation, using the general scheme of the Kleinschmidt Still developed for use in the U. S. Navy during World War II. The vapor-compression still is shown schematically in Figure 1. The distinguishing feature of this equipment is the compressor, by means of which the temperature of the water vapor is increased enough so that its latent heat, released by condensation, may be used to evaporate more water. A brine cooler serves to decrease the heat loss in the waste brine. An external heating unit serves to supply to the evaporator the difference between the latent heats at the two temperatures as well as the heat losses from compressor and evaporator. Technical problems in the development of equipment of this type include those relating to compressor design as well as the important one of salt deposition. W. F. Langelier of the University of California [LANGELIER, CALDWELL, and LAWRENCE, 1950] has made important progress

toward the control of salt deposition. This type of equipment, through its use of the heat-pump principle, has the best economy of any scheme for distillation and produces somewhat more than 200 lb of distillate for each pound of fuel.

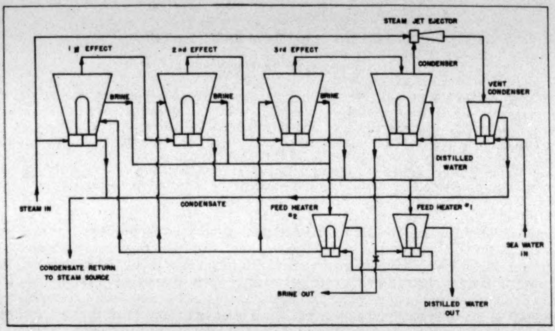

Fig. 2--Triple-effect distillation unit, schematic only

(2) Multiple-effect distillation, using the familiar shipboard-type equipment. Figure 2 shows a triple-effect still. Steam is supplied to the first-effect evaporator, causing the salt water therein to be partially vaporized. The vapor from this effect is piped to the second-effect evaporator where it condenses, furnishing heat to evaporate additional sea water. Vapor from the second effect is used to furnish heat to vaporize additional sea water in the third effect. The enriched brine from each effect is piped back to the waste through heat exchangers which serve to reduce the heat lost from the system. If only one effect were used, it will be clear on reflection that each pound of distillate would require about one pound of steam supply. With two effects the steam required is reduced to approximately half the weight of distillate, with three effects approximately one-third, and so on. Although the figure shows only three effects, plants are built which employ as many as six effects and have correspondingly greater yields of distillate for each pound of steam input. Since each effect added complicates the problems of maintenance and operation, practice in the United States has adopted three effects as standard. British practice goes as high as six effects, probably due to different relative costs of fuel and labor. Triple-effect units producing 100,000 gal/day have been produced for use in Fahaheel, Kuwait, on the Persian Gulf [ANONYMOUS, 1949]. Fuel cost in this case is no barrier to the use of this equipment as the oil-production developments located there would be impossible without an artificial fresh-water supply.

(3) Electrolytic process involves the separation of the ionized solutes from the water by passage of an electric current through the sea water [STREICHER, 1946]. A porous diaphragm or membrane makes possible separation of the two samples of water in contact. No large installation exists at this time, but experiments performed so far indicate that extrapolation to large sizes present no serious problems. This method of water recovery has been evaluated by AULTMAN [1949], who estimates the cost for power alone to be at least $239 per acre foot. He also calls attention to the pumping costs, since the sea water which must be circulated through the equipment is about four times the quantity of fresh water produced.

(4) The general principles involved in ionic exchange are those used in the familiar softeners of the Zeolite or Permutit design. This scheme involves exchange of unwanted ionized chemicals for less offensive ions coated on the surface of a coarse filter medium. Periodically, the filter coating is renewed by flushing with an appropriate mineralized fluid. Various materials have been tried, but so far the large quantity of water required for flushing as well as the cost of the regenerating chemicals does not give much promise of success in large-scale operation.

(5) Freezing of sea water in the form of small crystals gives an ice relatively free from salt. Since the latent heat of fusion is much less than the latent heat of evaporation, this method properly applied should give better economy than a corresponding distillation process. Some develop-

ments of this method have occurred in Italy[VACINO and VISINTIN, 1945], but so far the counterpart of the multiple-effect still has not yet been produced. Costs of producing fresh water by freezing have been estimated at $400 per acre foot. Serious technical problems exist, including the separation of salt crystals from the ice, recovery of heat from the melting ice, and design of heat-transfer units for application during the melting of ice.

(6) A solar still for use on the Virgin Islands has been proposed by Maria Telkes of Massachusetts Institute of Technology. Her proposal is to use a series of trays four by eight feet, insulated on the bottom and covered over with panes of glass which slant up from the two sides and meet along the ridge. The ends are also to be closed in with glass and the whole structure made so that little leakage of air will occur. Sea water is to be placed in the tray to a depth of less than one inch and should evaporate at a rate of about eight gallons per day. The vapor is condensed by contact with the glass enclosure and is to be collected in a trough along the bottom of the glass. A single unit was constructed at the Virgin Islands [HOLLINGSWORTH, 1948], but no experimental data have yet been published concerning its performance. However, since the original presentation of this paper, TELKES [1951] has published a description of a unit 4 × 50 ft which has been installed at Cohasset, Massachusetts, and yields approximately 50 gal/day.

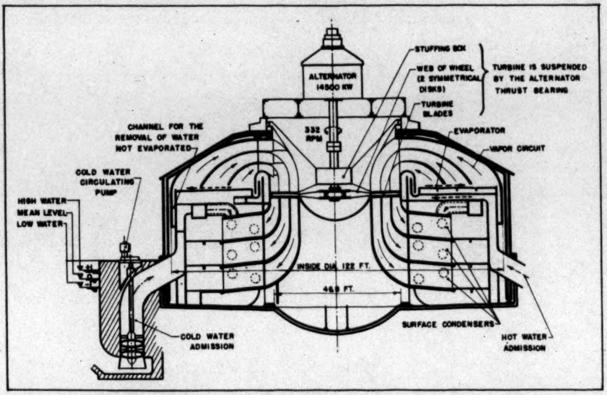

Fig. 3--Schematic diagram of power plant constructed to produce 10,000 kw of power accompanied by 150,000 gal/day of fresh water at full load

(7) Thermal difference power plant proposed by CLAUDE [1930] involves a power plant to be operated by the heat in the ocean. Cold water from great depths was to be used for condensing the vapor from warmer surface water. A simple turbine wheel is interposed between the source of vapor and condenser. A small pilot plant was built in Cuba in the late 1920's, but ocean-bottom conditions proved unsatisfactory so the prototype was not built. Recently it has been pointed out that Claude's turbine unit produced fresh water from the sea as well as electric power and could therefore be regarded as a source of fresh water. A plant of this type is now being constructed at Abidjian on the Ivory Coast of French West Africa, and is expected to produce 10,000 kw of power accompanied by 150,000 gal/day of fresh water (at full load). Figure 3 is a schematic diagram of a unit of this type and size. Others have suggested that plants of this type could be built to utilize the heat in the cooling water discharged from steam power plants. If this were feasible, some of the heat energy now wasted would be salvaged.

Costs--Cost estimates for the various methods have been published by several writers, the most recent summary having appeared in the Engineering News-Record [ANONYMOUS, 1950]. Figures published by different writers vary, largely because of the different fuel costs and unit efficiencies quoted. However, all summaries so far published agree that distilled water is far more expensive than water developed through conservation measures. For example, compression distillation would result in an approximate cost of $400 per acre foot as compared with natural water supplies available at a cost of from $30 to $50 per acre foot.

The matter of depletion of fuel reserves has been advanced as an argument against distillation. An overall idea of the importance of this factor may be gained from the following figures: The average daily domestic water use per capita is 127 gallons and the average daily consumption of electric power (from fuels only) is 4.27 kw hr. Fuel consumption in power plants averages about one pound of oil per kw hr of power delivered to the consumer so that one could say that about 4.3 pounds of fuel per day per person must be burned to provide the present fuel-generated power used. Distillation of 127 gallons of water, using the compression-distillation scheme, would require about five pounds of fuel. Thus, if all domestic water were to be produced by distillation, the fuel required would more than equal the total fuel now burned in power plants. From this it may be concluded that any distillation system using fuel as a source of heat or power is not presently feasible.

Use of waste heat--Since the major use of heat energy is for the generation of power and, further, since there is a considerable wastage of heat accompanying any power-generation scheme, the question may well be raised as to the possibility of using the waste heat from power plants for the distillation of sea water. Two methods have been suggested for arranging plants of this kind, one pertaining to diesel plants and the other to steam plants.

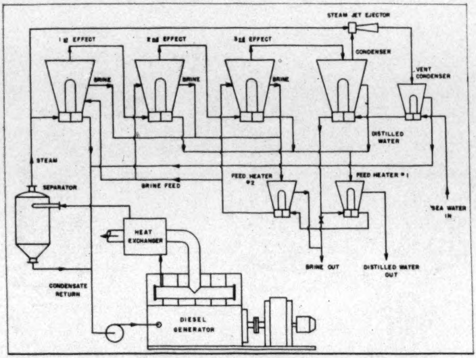

Fig. 4--Combination distiller and diesel power plant, schematic only

The first method utilizes the waste heat from a diesel electric-power plant to supply steam to a multi-effect still, a schematic diagram being shown in Figure 4. Using accepted values for heat recovery from diesel engines and steam consumption of a triple-effect still, it would appear that distilled water may be produced at the rate of about eight pounds per kw hr or say one gallon per kw hr. Thus, where diesel plants are used for supplying electric power to communities, a fraction of the water needs could be gained as a by-product of power generation. For example, a 20,000-kw base-load electric generating plant would produce roughly 480,000 gallons of distilled water per day. Distillation of water at this rate the year round would be used to augment underground or surface supplies and therefore conserve annually enough water to supply the communities for an appreciable period of time, probably longer than the average duration of droughts. Under conditions approaching drought, the water consumption could be limited to less than the per capita average of 127 gal/day, thereby stretching the potential supply over a period of nearly a month. In summary, a waste-heat distilling plant connected with a power plant should be regarded as auxiliary to a water supply of the normal conservation type.

The second method suggested utilizes a once-through, forced-circulation, steam-generating unit. The proponent of this method, Mark Benson, visualized its use in connection with the Benson boiler, best known for the generation of steam at the critical pressure. Recent experimental work done by Benson [ANONYMOUS, 1946] produced encouraging results. Figure 5 gives schematically the nature of the proposal. Sea water is first degassed by heating to 328°F and then pumped at a pressure greater than the critical through an economizer and radiant heater, the temperature rising to the critical value. After leaving the furnace wall the sea water passes through a reducing

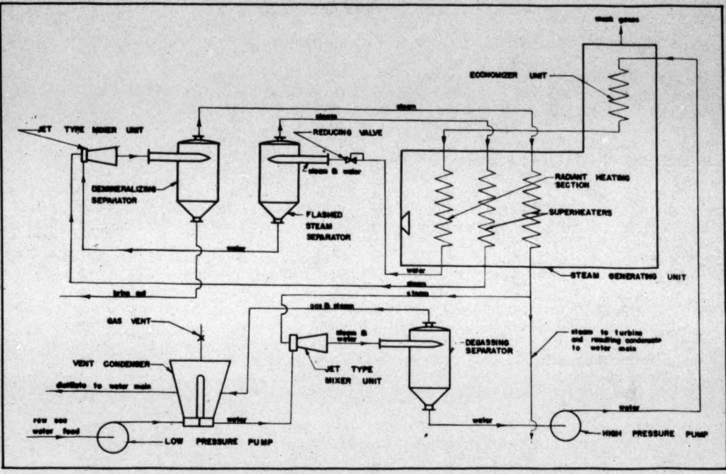
stack gases
ECONOMIZER UNIT
steam
steam
JET TYPE MIXER UNIT
REDUCING VALVE
steam & water
DEMINERALIZING SEPARATOR
RADIANT HEATING SECTION
SUPERHEATERS
FLASHED STEAM SEPARATOR
water
water
STEAM GENERATING UNIT
steam
brine out
steam
gas & steam
GAS VENT
steam & water
DEGASSING SEPARATOR
steam to turbine and resulting condensate to water main
VENT CONDENSER
JET TYPE MIXER UNIT
distillate to water main
water
raw sea water feed
water
water
LOW PRESSURE PUMP
HIGH PRESSURE PUMP

Fig. 5--Critical pressure system with sea-water feed, schematic only

valve and approximately half of it flashes into steam at 3100 pounds per square inch, absolute. This steam is then superheated to 850°F and mixed with the remaining sea water at 2800 pounds per square inch, absolute, causing the evaporation of most of the remaining sea water. The very rich brine amounting to about two per cent is then piped to waste. Preliminary tests were made [ANONYMOUS, 1946] on the degassing and de-mineralizing systems only.

This scheme would use sea water as the only source of feed water and would discharge condensate (distilled water) from the power plant to the city mains at a rate dependent upon the plant arrangement but probably lying between seven and ten pounds per kilowatt hour. The net result, therefore, is approximately the same as for the waste-heat units proposed for use with diesel plants, namely, one gallon of distillate for each kilowatt-hour output. The cost in fuel would be slight and equal to only the heat losses occasioned by the added equipments.

Conclusions--The foregoing remarks should serve to delineate the present situation with respect to sea-water refining. The conclusions of the author with respect to the future may be stated as follows:

(1) None of the methods described herein could be used to refine sea water for irrigation, which is the major consumptive use of water.

(2) Distillation of sea water (or other polluted water) using waste heat could provide some relief from drought conditions in small urban communities. Pilot-plant studies and economic designs for this type of situation are warranted.

(3) Distillation of sea water (or other polluted water) using a steam power plant as a still seems to present some promise and merits further investigation.

(4) Utilization of solar energy, either directly as in solar stills or indirectly as in the thermal-difference power plant, merits investigation and development. However, since the techniques of utilizing solar energy are not yet developed, it should be expected that water-refining plants using this type of energy will be somewhat delayed in development.

In concluding this presentation the author suggests that sea-water purification be regarded as supplementary to the natural water supply resulting from the normal conservation measures. Given proper development, water refining can serve to alleviate drought conditions in urban communities. In 1950-51 many parts of California have received ample rain. It is hoped that this occurrence will not reduce the support and interest in research to provide for water-refining plants which will be badly needed in dry years.

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(Manuscript received July 2, 1951, and, as revised, December 17, 1951; presented at the Twenty-Sixth Meeting, Fresno, California, February 10, 1951; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

A RIGOROUS, SIMPLE METHOD OF MEASURING AND RECORDING PARTICLE-SIZE DISTRIBUTION IN DISPERSED MATERIAL

M. Rim

Abstract--In all conventional methods for particle-size distribution analysis, measuring gear is introduced or permitted to be present in a suspension of settling particles. This inevitably causes inhomogeneity and gravitational instability in the suspension with the result that the most interesting and relevant features of the particle-size distribution curve are wiped out.

Here an apparatus is described where only external measurements (buoyancy) have to be taken on a column of settling particles, and stability is rigorously maintained by the employment of an additional liquid. The design is simple enough to make this method not only suitable for reference purposes but also equipment- and effort-saving in routine determinations. It was combined with a bead-recorder, necessitating no photographic work and almost no mechanical parts.

Introduction--The particle-size distribution in granular material of subsieve size, such as soils, sediments, precipitates, ground materials, cement powders, and slurries, is usually of fundamental interest. The size-distribution curve is determined by dispersing the material in a neutral liquid and recording the rate of sedimentation. To cover all particles down to the limit of microscopic visibility a 24- to 48-hour sedimentation record is necessary under usual conditions. Extensive application in the many fields of interest, however, has been hampered by basic difficulties of measuring the sedimentation and by technical difficulties of designing continuously measuring and self-recording apparatus.

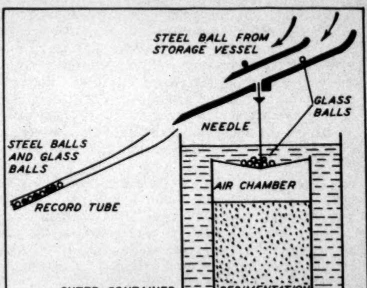
STEEL BALL FROM STORAGE VESSEL
GLASS BALLS
NEEDLE
STEEL BALLS AND GLASS BALLS
RECORD TUBE
AIR CHAMBER
OUTER CONTAINER FILLED WITH CLEAR LIQUID (0.5 PER CENT NaOH)
SEDIMENTATION TUBE FILLED WITH SUSPENSION IN LIGHTER LIQUID (0.2 PER CENT NaOH)
HONEYCOMB RECEPTACLE
PARAFFIN

Fig. 1--Sketch of recording sedimentation balance

The measuring device--Sedimentation can be followed in either of two ways: (1) by measuring the accumulation of deposit on the bottom of the containing vessel; (2) by measuring the decrease of concentration in the suspension. However, the introduction of any instrument into the settling suspension for that purpose, such as a balance pan, a plummet, or a hydrometer, unavoidably intercepts settling particles. This invariably causes density-inversion in the sedimentating column which in turn creates currents in the fluid and upsets the measurements. Similar instabilities are caused by external piezometrically measuring devices (such as the Kelly tube), as the manometer fluid tends to enter the suspension. Only KNAPP's [1934] "microneter" avoids these intrinsic errors by use of extremely delicate metallic bellows at the liquid junction.

In the apparatus described here, two liquids of different density were employed to maintain gravitational stability. As most of the substances to be dispersed and analyzed were soils and silts, the two liquids were conveniently chosen to be 0.2 and 0.5 pct NaOH solutions. The more dilute solution, that is, the specifically lighter liquid, invariably serves as the dispersing medium for the material. This suspension occupies the whole of the sedimentation tube as shown in Figure 1. This tube is a few inches in diameter and approximately 12 inches high, and is contained in a large cylinder filled with the clear, heavier solution. It floats in an absolutely vertical position by virtue of a sealed air compartment in its topmost section. Just enough small glass beads are placed on its inward slanting roof to keep the float from rising. The center of gravity of the float is arranged to be close to its lower end, since otherwise it will swing sideways when the weights are applied.

A short while after establishing equilibrium, the float tends to rise again. It can be shown that the additional weights necessary to reestablish equilibrium must equal the apparent weight of the particles which had meanwhile settled out of the tube. A necessary condition for the validity of the above assertion is that fluid currents due to the float's motion be prevented from resuspending the deposited particles. This is accomplished by placing a honeycomb receptacle on the big container's bottom. The honeycomb can be made by sticking sharp-edged rectangular pieces of glass (microscope cover glasses) upright into a cake of paraffin in such a way that they form narrow prismatic cells.

Absence of mixing within the tube, which is another necessary condition, was demonstrated by using colored solutions. The side effect of molecular diffusion across the junction line between the two liquids was found to be negligible. (The magnitude of this side effect can be determined in a blank experiment and a correction can be made if necessary.) The same experiment showed that temperature must be kept constant throughout, which can be well enough achieved by making the outer container as large as possible and by wrapping it with heat-insulating material.

Self-balancing and self-recording device--Under the described conditions the weights on top of the float, minus those initially present, represent at any time the apparent weight of the accrued bottom deposit. Readings at distinct time intervals give the particle-size frequency within broad, predefined classes. However, by confining oneself to such classes defined by convention, characteristic and interesting points will be missed and the resulting data render only a general description of the tested material. This marks the necessity of obtaining continuous records of complete sedimentation curves.

The present apparatus, which is essentially a sedimentation balance, was so designed as to record the time intervals which correspond to the deposition of each one-half per cent of the total material. Such, approximately, was the uniform and well-determined weight of each one of more than a thousand tiny glass beads (8.55 mg). At definite time intervals, every two or three minutes, one bead was allowed to roll over an open narrow rail, which bridges the mouth of the outer container. Somewhat off center in the rail's groove a narrow hole allows a fine needle to move, without much friction, up and down. The needle is affixed to the float and its height arranged in such a manner that the tip of the needle is flush with the rail surface when the float is in equilibrium position. As soon as the float rises, however, the needle tip emerges and derails the next glass bead that comes down the rail. This bead falls onto the float and weighs it down again.

The periodic release of the glass beads onto that rail from their storage tube is effected by means of a gate mechanism operated by clockwork (see Appendix). Together with each glass bead, a small steel ball is dropped from another storage tube onto the rail, but beyond the range of the needle. All balls come to rest eventually in a long, narrow glass tube - the record tube. However, when the float rises and a glass bead is intercepted, two steel balls will successively line up in the record tube. The position of the successive pair of steel balls marks the time the glass bead fell onto the float.

The glass beads falling into the liquid were well wetted by it, due to the presence of NaOH, so that no difficulties were created by air bubbles adhering to their surface. (Otherwise the beads would have had to be collected above the solution-surface on a screen fixed to the needle.) Small air bubbles always manage to creep into the sedimentation tube as, at the beginning of the run, it has to be inserted quickly into the container, but they do not interfere as long as there are not too many. If the outer container is very large, the sedimentation tube can be inserted quite easily with the help of a piece of cardboard or glass pressed against the open end of the tube. In the actual setup a topless four-gallon petrol tin served as outer container, able to house four simultaneously operating floats. The float holds close to one liter of suspension containing not more than two grams of solids.

For compactness, the record tube was given the shape of a coil. When it was full, or the run finished, the balls could be quickly sorted by means of a magnet and re-fed into their respective storage vessels. This nonphotographic recording method, apart from acting as an automatic balancing device, gave immediate results without the necessity of darkroom work, a definite advantage in survey and field work. The main usefulness of the apparatus was in determining the degree of uniformity of near-uniform material which could not be checked by the existing methods as they do not employ rigorous principles. The results corresponded well, however, with the display of certain properties specific for uniform granular material (maintaining a smooth surface when exposed to wind, while rippling occurs on material not completely uniform).

Acknowledgments--The measuring parts of the apparatus were worked out at the Hebrew University in Jerusalem, Israel, and the recording device added later at a field laboratory. For the critical reading of the manuscript and for many helpful suggestions in matters of presentation and language, I should like to thank V. A. Vanoni and R. W. Davies.

APPENDIX--A BEAD-FEEDING DEVICE TO SERVE A BATTERY OF BEAD RECORDERS

The device, shown schematically on Figure 2, consists essentially of a bead-dropping unit and a timing device. Its main feature is a steeply slanting board on which a sledge, strung to the upper

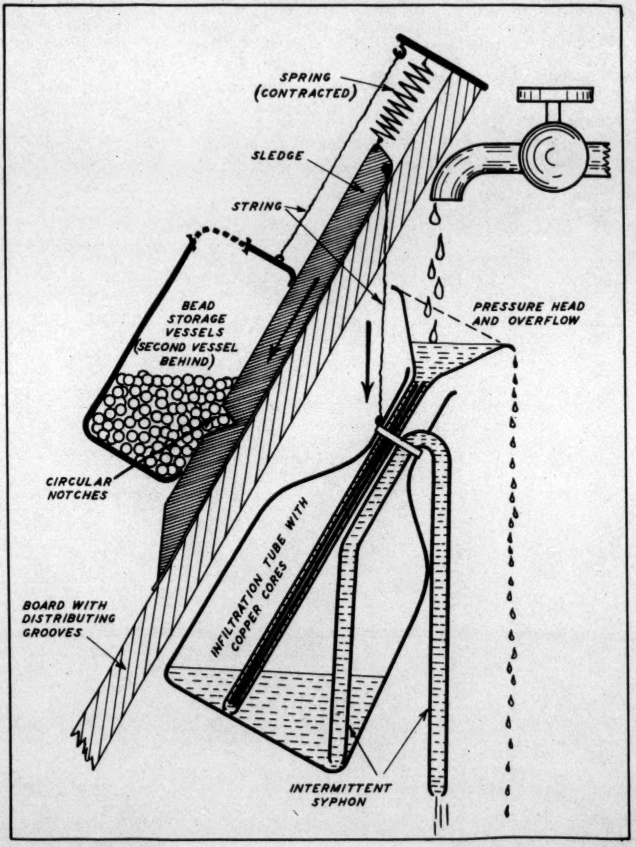
SPRING (CONTRACTED)
SLEDGE
STRING
PRESSURE HEAD AND OVERFLOW
BEAD STORAGE VESSELS (SECOND VESSEL BEHIND)
CIRCULAR NOTCHES
INFILTRATION TUBE WITH COPPER CORES
BOARD WITH DISTRIBUTING GROOVES
INTERMITTENT SYPHON

Fig. 2--Bead dropper and timing device

edge of the board by a spring, slides up and down. Also strung to the upper edge of the board, but not capable of sharing the up-and-down movements of the sledge, are the twin storage vessels for the glass and steel beads. They are mounted side by side, of box-like, bottomless shape and rest loosely with sharp scoop-like edges on the upper surface of the sledge. Into this surface, in contact with the beads, a series of notches is drilled, each just large enough to accommodate a single bead. When the sledge is pulled forward, the row of notches emerges from underneath the storage vessels and all the beads promptly roll out, downwards into grooves in the board that distribute them to their respective recorders. On retraction of the sledge the notches become occupied by another crop of beads.

The periodic pulling down of the sledge against the action of the spring is accomplished by a weight-changing bottle hung onto the sledge. This bottle is being filled with tap water at a steady slow rate, letting the water infiltrate through a narrow tube under a pressure head maintained at a defined level by an overflow. The tube, made of glass, is filled with thick copper or constantan wire, which makes the infiltration rate independent from temperature. When the bottle is approximately half full, the spring gives way and the sledge slides down. Each time it becomes filled to a certain level in its neck, it is emptied of all its contents by a self-enacting syphon and the spring is able to pull the sledge upwards again.

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(Communicated manuscript received September 13, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

WAVES FROM BLASTS RECORDED IN SOUTHERN CALIFORNIA

B. Gutenberg

(Contribution No. 587, California Institute of Technology,
Division of the Geological Sciences)

Abstract--Records of five atomic explosions in Nevada and of a quarry blast of about 120,000 pounds of Dupont Nitramon at Corona, California, are investigated. The resulting travel times through the ground are combined with similar data reported previously. At a given station, records of different explosions near the same locality show great similarity. The following average velocities of longitudinal waves in Southern California are found: 5.9 km/sec below the sediments; a gradual or sudden increase to about 6.5 km/sec near a depth of 6 km, and to about 6.9 km/sec near 10 km; about 8.2 km/sec below the Mohorovicic discontinuity at a depth of 35 to 40 km. A "low-velocity layer" somewhere in the depth interval between 15 and 30 km has been inferred from earthquake data. This may prevent the finding of velocities from blast records for this depth range. No clear phase corresponding to $\overline{P}$ in earthquake records is observed in blast records. This is to be expected if $\overline{P}$ is a wave traveling along a low-velocity channel approximately at the depth of foci of shallow earthquakes. The velocity of shear waves below the sediments is about 3.6 km/sec. Waves with such an apparent velocity recorded at distances between about 150 and over 400 km are possibly surface waves. Shear waves are usually relatively small in explosions.

The recorded waves through the air include the direct sound waves, various types of waves with their highest points near 40 or 50 km, and waves with considerably longer travel times which have reached or passed through the low-velocity layer near 80-km elevation.

Materials used--According to newspaper reports, five atomic explosions took place near Frenchman Flat, about 100 km northwest of Las Vegas, Nevada, on the mornings of January 27, 28, February 1, 2, and 6, 1951. Records of these explosions were obtained at the Pasadena group of stations, at stations of the U. S. Coast and Geodetic Survey, and at stations of the Berkeley group. The author was kindly furnished copies of records from the latter two organizations. The locations and origin times were first calculated by the method used in locating earthquakes. Later, locations and origin times were kindly furnished to the nearest minute of longitude and latitude and to the nearest tenth of a second respectively by the Los Alamos Scientific Laboratory of the University of California. The "error" in location was less than five kilometers. The problem of the origin times is discussed later.

On March 27, 1951, the Seismological Laboratory was informed by the Minnesota Mining and Manufacturing Company that on March 31 a tunnel shot of about 120,000 pounds (about 55 metric tons) of Dupont Nitramon similar to that of August 6, 1949 [GUTENBERG, 1951a], was to be fired in their Corona quarry. F. Lehner of the Seismological Laboratory arranged recording of the explosion on a portable Benioff vertical seismograph at a point roughly 150 meters from the charge (between the two water tanks in Fig. 1). The 1951 blast originated about 200 ft from that in 1949. We are very grateful for the continued cordial cooperation from all concerned. The blast was

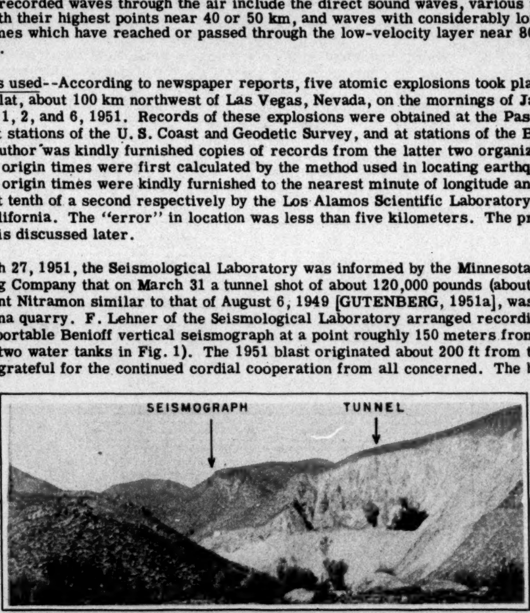

Fig. 1--Site of Corona blast, March 31, 1951, immediately after explosion; photo by F. Lehner recorded by the portable instrument at 15h 30m 33.5s Pacific Standard Time, and the origin time was assumed to be 0.1 sec earlier, that is, at 15h 30m 33.4s. This can be assumed to be correct within 0.2 sec.

Waves through the ground--The 1951 Corona blast was recorded at all stations of the Pasadena group and at the three stations of the Lake Mead group (U. S. Coast and Geodetic Survey) for which copies were kindly furnished by Frank Werner. Data for the stations may be found in GUTENBERG [1951a, p. 6]. The stations at Pomona, Crestline, and El Cajon, which recorded the 1949 blast, were no longer in operation in 1951, but excellent records (Fig. 2) were obtained this time at China Lake (lat. 35° 49.0′ N, long. 117° 35.8′ W, h = 766 km, distance Δ = 218.6 km) with a travel time of 33.8 sec for the first impulse, and later phases, for example, after 36.4, 37.4, 56.4, 55.9, 63.6 sec, and at Dalton (lat. 34° 10.2′ N, long. 117° 48.6′ W, h = 523 m, Δ = 45.2 km) with travel times of 7.9 and 13.4 sec. Practically all other stations recorded the first impulse with travel times within 0.1 sec of those found for the 1949 explosion; many of the later phases were observed with travel times not deviating by more than 0.2 sec from those published for the blast in 1949 [GUTENBERG, 1951a, Table 2, p. 6]. Practically all outstanding phases are reproduced although the source was not exactly at the same place. There is also great similarity of the records obtained at a given station from the various Nevada explosions. The beginning of a given phase can be determined more accurately by comparing series of records of the type reproduced in Figure 2 than from a single record.

Travel times of waves from various surface sources are reproduced in Figure 3. For the P-phases, Δ/6 has been subtracted from the observed times to permit the use of a larger scale, and similarly Δ/3.6 from the times of the S-phases. Older data are from GUTENBERG [1951a], TUVE and TATEL [1950], WOOD and RICHTER [1933], and from unpublished readings by C. F. Richter for a blast near Colton on November 3, 1949.

Time differences for the five Nevada explosions were calculated from the times of the first motion where this was definite. The following are such time differences in seconds (leaving out the numbers of days, hours, and minutes) for one pair of explosions at ten stations: 2.9, 3.0, 3.2, 2.8, 3±, 2.9, 3.5, 2.8, 3.0, 3.0. In general, the probable error of the resulting average time difference is about 1/4 sec. Unfortunately, time differences calculated from the reported times of the various explosions differed from those found from the records by amounts which varied from case to case and exceeded one second. For this reason it was preferred to calculate the origin times on the assumption that Pn-waves from Nevada to Southern California have the same average travel-time curves as those from Corona to Nevada. This supposes that there is no appreciable dip in the Mohorovicic discontinuity between the two regions. Travel times plotted in Figure 3 for the Nevada explosions are all based on these origin times. For each station, Figure 3 contains average travel times for corresponding phases in blast records from a given source. At distances between 100

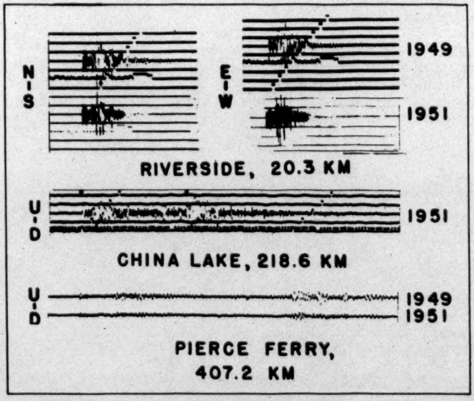

Fig. 2--Records of Corona blasts, 1949 and 1951; arranged by J. M. Nordquist

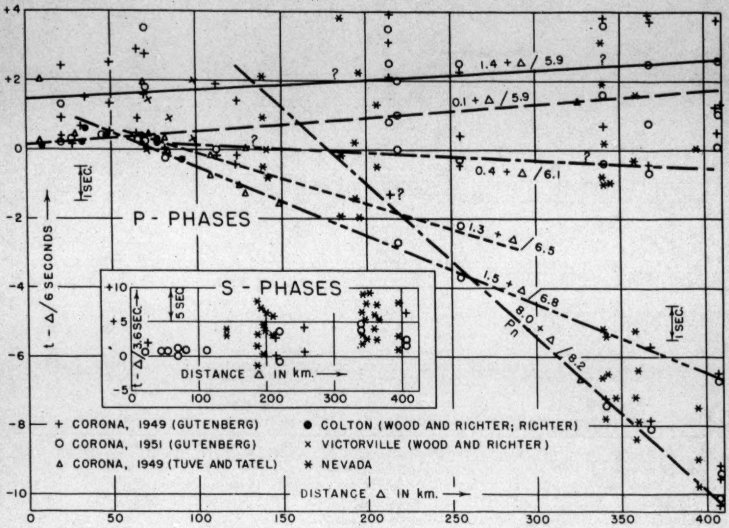

Fig. 3--Travel times of longitudinal waves after deduction of Δ/6 (Δ = distance in km); insert: travel times of transverse waves after deduction of Δ/3.6; drafted by J. M. Nordquist

Table 1--Velocities of longitudinal waves

Depth	Velocity
km	km/sec
1	5.9
5	6.5
10	6.8-6.9
40	8.2

and 250 km the first phase is then on the average about 1/2 sec later in the Nevada than in the California explosions. This may be a result of a systematic error in the origin time (travel time of Pn) by such an amount or of a slightly deeper boundary of the layer with a velocity of 6.8 to 6.9 km/sec near the Nevada source, or it could be caused by somewhat variable conditions in all the quantities involved, as is to be expected in a region with a radius of a few hundred kilometers.

The velocities of longitudinal waves calculated from Figure 3 are about as shown in Table 1. There is no definite indication to what extent the increase in velocity in the upper ten km is gradual or sudden. There are no clear phases from which the velocity at depths between about 15 and 35 km can be found. The evidence from earthquake records [GUTENBERG, 1951b] is thus far in favor of a layer with a lower velocity, probably gradually decreasing below a depth of roughly 15 km. Such a layer could not be found from refracted waves in blast records, since waves which have passed through a layer with a given velocity are bent downward into a deeper layer with lower velocity and cannot have their deepest point before they have reached a layer with a velocity greater than the maximum in the layers above. On the other hand, relatively large channel waves ($\bar{P}$) could be produced by earthquakes with their focus in or near the low-velocity layer while such waves would be much smaller if the source is in the surface, 15 or more kilometers above the channel.

The insert in Figure 3 shows the travel times of "shear" waves after deduction of Δ/3.6. The observed times as a function of distance do not exhibit any clear indication of a sequence of more or less straight lines similar to those representing the travel times of P and corresponding to an increase of velocity with depth. All earliest points of "S" in Figure 3 are near an almost horizontal line which corresponds to an apparent velocity of about 3.6 km/sec. While for the shorter distances this may well indicate the velocity of transverse waves (corresponding to the longitudinal wave velocity of about 5.9 km/sec), it is possible that the observations at greater distances refer to surface waves. Shear waves can be expected to be relatively small in explosions in which an

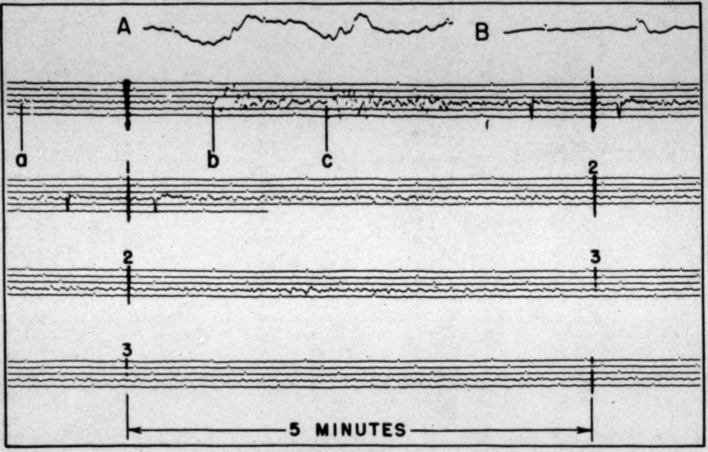

Fig. 4--Record of Benioff microbarograph at Pasadena after one of the Nevada explosions; consecutive sections (1 to 3) overlap with corresponding times (vertical lines) marked by the same figures; A is record of Benioff strain seismograph at the time corresponding to the barograph record below it (the background of irregular long waves is due to a disturbance, possibly from air currents); B is similar record from New Mexico test, July 16, 1945; arranged by R. Gilman

appreciably smaller ratio of shear to change in volume is to be expected than in earthquakes; in rockbursts this ratio is probably between that in blasts and that in earthquakes. Rockbursts are therefore more suitable for the study of shear waves from near-surface sources than blasts.

Waves through the air--Most Benioff short-period vertical seismographs in the region and some of the short-period horizontal seismographs recorded sound waves through the air after the Nevada explosions. Good records were also obtained from the Benioff microbarograph at Pasadena, and we are indebted to other organizations for copies of their microbarograph records. Several waves were recorded on the Benioff strain seismograph at Pasadena (Fig. 4A) at times when the barograph recorded relatively large pressure waves (at b and c respectively in Fig. 4). The corresponding strain seismograph record obtained from the New Mexico explosion on July 15, 1946 (Δ = 1083 km), is shown on Figure 4 at B for comparison of the wave form. These strain records indicate compressions (upward on the record) and dilatations of the ground near the Laboratory by the air waves (N-S component).

All sound records are very complex; waves of relatively short periods (order of 0.1 sec) and longer waves (period up to about ten seconds) are frequently intermixed. In several instances the total duration of these wave trains exceeded ten minutes. At any given station the details changed considerably from day to day. At some stations good sound records were obtained on one day, none or very small ones on another without obvious correlation between the stations. Detailed investigation of these changes requires more information on the meteorological elements than is available to the author. In general, the wind in the troposphere was westerly to northerly (more northerly on February 1 and 2), increasing with height, and the temperature near the ground increased toward the coast. The wind situation (but also to some extent the temperature distribution) is favorable to and probably responsible for recording of the direct sound waves (traveling near the ground) in the southeast sector to distances of a few hundred miles while these waves were not recorded at similar distances in other azimuths. There is also good evidence that the waves with their highest point near 40 or 50 km (direct stratosphere waves) were recorded at about the same stations, and corresponding waves reflected once or twice at the ground at even greater distances. The travel times of the direct stratosphere waves were one to 1 1/2 minutes greater than those of the direct waves traveling near the ground.

Many records written at stations in southerly directions indicate a phase with a velocity of slightly over 300 m/sec. This corresponds possibly to the similar phases which were observed in the New Mexico blast [GUTENBERG, 1946] and to the waves which circled the Earth repeatedly after the Krakatoa explosion in 1883. Possibly all of these waves were conditioned by the low-velocity layer near the tropopause.

There are many additional relatively late phases, occasionally even larger than the preceding, but usually not recorded in all azimuths. There is especially no indication of the late phases in easterly directions, but they are usually abundant to the west and south. This may be a consequence of easterly winds at elevations above at least 50 km. The extrapolated travel-time curves of some of these late phases seem to have an intercept time of several minutes with the time axis, indicating the possibility of an appreciable vertical component of the path. Among them is possibly the wave suspected by COX [1949] to be refracted above the 80-km low-velocity layer.

The problem of interpreting the results from sound waves which have traveled higher than about 25 km is complicated by the fact that the velocity of sound seems to increase as the pressure decreases below about ten cm of mercury. MAULARD [1949] found such an increase by about 1 1/2 per cent at a pressure of ten cm and by about four per cent at a pressure of about four cm of mercury (corresponding to roughly ten m/sec at an elevation of 25 km). Similar results have been published by ABBEY and BARLOW [1948], who also are investigating the effect of frequency on sound velocity in rarefied gases. A few small but clearly recorded phases seem to line up approximately along a line given roughly by $t = 660 + \Delta/3.5$ (t = time in seconds, Δ = distance in km). This may correspond to a sound wave which has traveled as a channel wave in the low-velocity layer near 70 or 80 km. These waves were recorded with periods of from one to five seconds; it is not certain how far such a wave (wave length about one km) can travel at the pressure of the order of 0.1 millibar (mean free path of the molecules of the order of one mm) without losing too much energy. There may be other ways of explaining a sound wave arriving at a distance of 200 km from the source with a travel time of about 22 minutes.

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(Communicated manuscript received October 4, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

THE GREAT LAKES, A SOURCE OF TWO-SECOND FRONTAL MICROSEISMS

Joseph Lynch

Abstract--In the fall of 1950, a new tripartite station was set up at Poughkeepsie, New York. Results of the station gave a velocity of a half mile per second for these microseismic waves and also indicated a frequent source west of Poughkeepsie. Since the Hudson River is the nearest sizable body of water west of Poughkeepsie, it was thought advisable to explore the possibility of the River being the source. A second tripartite group was set up opposite Poughkeepsie on the west bank of the Hudson. Results from this station eliminate the Hudson as the source and indicate that the Great Lakes are probably the source. All the work has been done with the aid of a grant from the Office of Naval Research.

In the fall of 1950 a new tripartite station was begun on the grounds of the College of St. Andrew-on-Hudson, some three miles north of Poughkeepsie and adjoining the Roosevelt estate. Three concrete boxes were set up on bed rock for the housing of the instruments. These boxes are at the apices of a triangle of sides 2000, 2600, and 3200 ft. This is a right triangle with the 2600-ft side running due north-south and the 2000-ft side due east-west. This orientation was not by design but determined by the position of suitable outcrops of rock and suitable avenues of trees along which to string the two-conductor cable connecting the boxes to the recording room in the basement of the College. The 2600-ft side roughly parallels the Hudson River about 1000 ft inland from the east bank. The 2000-ft side crosses the Albany Post Road at right angles and roughly parallels Dorsey's Lane. The instruments are all horizontal, of the Sprengnether type, two-second period, about 2000-ohm coil resistance. They are wired up to matching galvanometers in the recording room. Recording is done on a Sprengnether triple drum run at a speed of 600 mm/min. The drum is run for two minutes every half hour. Haloid paper of the standard size, 36 by 11 inches, is used and time marks are put on every 15 seconds by rotating contacts.

The first records obtained showed clearly that the ground motion was faithfully recorded by all three instruments with practically the same magnification.

The records are read by superimposing them in pairs over an illuminated box, the time lag between peaks being measured by a traveling microscope. Gilmore's formula $\tan \alpha = \sin \epsilon/(k-\cos \epsilon)$ is used.

On February 3, 1951, the tripartite network indicated a storm moving from 57° S of E at midnight to about 26° S of E at 7h 30m February 4. A very pronounced low was moving in the same general direction around the same time, but it is not being suggested that the low was necessarily the cause of the microseismic storm.

On January 26, 1951, around noon, the tripartite network indicated a storm originating 32° S of E (of Poughkeepsie). Weather maps indicate a pronounced low southeast of New York at this time and moving northwestward. A second low branched off from this and moved due west. Meanwhile a cold front was moving out to sea and was southeast of New York around noon on January 26; again, it is not suggested that either the low or the cold front was necessarily the cause of the microseisms; they are merely suspicious loiterers.

On February 25, 1951, around 21h 00m, the tripartite network indicated a microseismic source two degrees south of due west of Poughkeepsie. This was right in line with the two stations 2000 ft apart. The difference in time of arrival at the two stations was very closely one second (9.50 mm on our time scale of 600 mm/min). This indicates a velocity of somewhat less than a half mile per second for these microseismic waves. Weather maps for this date show the following meteorological happenings. A very pronounced low was moving up the Atlantic Coast but well out to sea. A high of 1021 mb was moving southward on land about a hundred miles inland. This passed due west of Poughkeepsie. These two probable had little to do with the microseisms. The most likely cause was a polar air mass cPk which moved northeastward from a position (west of Poughkeepsie) on parallel 40 degrees at 01h 30m on February 25 to parallel 43 degrees at 1h 30m on February 26.

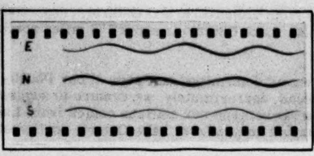

Fig. 1--Records of a microseismic wave at Poughkeepsie tripartite station; wave arrives simultaneously at north and south instruments and almost a second later at east instrument indicating a velocity of almost a half mile per second and a source from the west

On March 2, 1951, around 06h 00m, the tripartite group indicated a microseismic storm coming from a direction six degrees south of west. Weather maps indicate a cold front moving southeastward from the Great Lakes and out to sea. This seemed the only likely cause of the storm. On March 5, 1951, the tripartite group showed a storm coming from due west. Weather maps indicated a cold front which had passed nearly 24 hours earlier and was now well out to sea. There seemed nothing on the maps that might be taken as the cause of the storm.

The fact that on several days a source of microseisms from slightly south of west is indicated (see Fig. 1) presents a serious problem. Present-day theory calls for the proximity of a body of water to explain the mechanism of these microseisms. M. S. Longuet-Higgins of Cambridge University [Philosophical Trans. R. Soc., v. 243 A, p. 857] attributes the microseisms to the fluctuating pressure on the ocean bottom beneath a train of standing waves, the frequency of this pressure variation being twice the fundamental frequency of the waves. J. Darbyshire [Proc. R. Soc., April 1950] from a study of the Kew Observatory records concludes that microseisms are produced in a region of interference between sea wave trains near the coast or in deep water. L. Don Leet had previously suggested the presence of water as a necessary factor in the generation of microseisms. "The cold front may or may not be a participant in the development of a coastal storm as it hits the ocean, but microseisms do not start until it has reached the water" [Bull. Seis. Soc. Amer., v. 38, p. 178]. A technical report on frontal microseisms by the Lamont Geological Observatory [Tech. Rep. 7 by William L. Donn] seems to take for granted throughout that a body of water is essential to the generation of microseisms, one of the final observations being "the abruptness with which microseisms commence after a front or strong winds disturb the sea."

The nearest sizable body of water west of Poughkeepsie is the Hudson River which is quite wide at this point. To explore the possibility of the Hudson River as a possible source, a further extension of grant from the Office of Naval Research was requested and given. A second station was set up on the west bank of the Hudson opposite Hyde Park. It was felt necessary to install only two instruments on an east-west line since we merely desired to determine whether the microseisms originated in the bed of the Hudson or farther west, that is, whether they originated east or west of the instruments set up.

The authorities of Mother Cabrini's School for Orphans at West Park generously allowed us to use their property. A concrete box was built a quarter of a mile up from the River on the east side of Highway 9W. A small chapel built on bed rock about a half mile farther west across the road and up the hill housed the second instrument. The existence of a tunnel under the highway made it unnecessary to string wires over the main highway. A dark recording room was set up in the basement of a frame house near the main highway, and two-conductor cables were strung out thence to the two, two-second horizontal pendula in the concrete box and the chapel respectively. The recording drum was run at a speed of 600 mm/min.

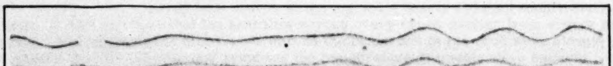

Fig. 2--Records of a microseismic wave at West Park; top record (west) has been displaced to right so that wave peaks instead of time breaks are in line; wave arrives almost one second later at east instrument (bottom)

Results so far obtained indicate a persistent source of microseisms west of the Hudson River (see Fig. 2), thus eliminating the River as the source. Over a period of a month, two predominant wave groups have been recorded always from the west with time intervals of ten and six mm between the west and east instruments (ten mm = one second) (see Fig. 2). It is unlikely that these represent waves of different velocities. It is more likely that they represent waves coming from two different directions. If, as suggested by Longuet-Higgins and Darbyshire, microseisms are due to standing waves in a large body of water, it seems reasonable to suppose that the two predominant groups of two-second microseisms come to us from Lakes Erie and Ontario, respectively. Lake Erie is some 350 miles west of our instruments. Lake Ontario is about 175 miles northwest.

On the basis of the velocity of these microseismic waves determined last year at our Poughkeepsie tripartite station, namely, one-half mile per second, approximately, we venture to suggest that the group of waves with a ten-mm (one-second) interval between our stations comes from Lake Erie to the west of us while the group with a six-mm interval comes from Lake Ontario northwest of us.

If our surmise is correct, microseisms should be picked up with equal intensity at equal concentric distances from the lakes. This suggests the advisability of setting up a tripartite station 350 miles south of Lake Erie to see if microseisms are picked up there from the north while we at West Park are picking them up from the west. This we are now planning to do.

I should like to add in gratitude that this work would not have been possible without the generous voluntary aid of the Jesuit students at St. Andrews in stringing cables both at Poughkeepsie and West Park.

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(Manuscript received August 28, 1951, and, as revised, December 18, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., April 30, 1951; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

HEAT FLOW AT KIRKLAND LAKE

T. H. Leith

Abstract--This is the first of a series of papers on heat flow in the Canadian Shield. The flow depends upon thermal conductivity and temperature gradient. The latter has been measured for the Kirkland Lake region by Misener who obtained a value of 0.0130°C/meter from many measurements in three local mines, extending over a length of two miles and depths of 7500 ft. The conductivities of specimens from the same points have been measured in the laboratory with an average value of $76.7 \pm 1.0 \times 10^{-4}$ cal/cm sec °C, resulting in a surface heat flow of 1.00 ± 0.02 cal/cm^2 sec. One measurement of conductivity was made in situ and reasonable agreement obtained with a laboratory determination on the same rock.

The heat flow was corrected for the effects of an assumed glacial sequence. The same value was obtained because differences from the present mean annual surface temperature are small.

The data have been correlated with other geophysical measurements made in the same area. Qualitative estimates of crustal features and the distribution of radioactivity show that perhaps less than 50 pct of the heat flow can be explained as radioactive in origin.

It is suggested that radioactivity and heat flow, on the average, may increase toward continental margins. The implications of this on continental history are considered.

Introduction--The temperature at the Earth's surface is very nearly the mean air temperature, but underground the temperature is greater and increases at depth at a variable rate resulting in a flow of heat upwards. A large percentage of this heat is attributed to radioactivity, derived largely from granitic type rocks within a few tens of kilometers of the surface. The remainder is assumed to be either residual heat from a cooling Earth or to some other heating effect in the Earth. In regions where structure and radioactivity distribution are known it should be possible to estimate what portion of the heat flow is the result of radioactivity. The Kirkland Lake area of northern Ontario lends itself well to this type of study.

In very few areas has the temperature gradient been so well determined over a large area as it has at Kirkland Lake. Also, the ease and reliability with which heat-flow determinations over an area of some size could be obtained holds practically nowhere but in a mining region in a Shield area. At Kirkland Lake three large mines extend to considerable depths and cover a considerable area, making possible the determination of rock temperatures for a large volume of rock. Furthermore, the heat flow can be determined at the surface around these mines.

Procedure--Because heat flow is the product of the thermal conductivity and the normal temperature gradient, measurements of conductivity were made for many rock samples from the three mines. (For the remainder of the paper, "conductivity" will be assumed to mean "thermal conductivity.") The apparatus consisted essentially of two cylindrical brass bars held vertically, with a disc of rock of nearly the same diameter placed between them. The vertical column so formed was supplied with a steady heat source at the top and a nearly constant temperature sink at the bottom. It was assumed that the heat flow across every cross section from source to sink was the same. The temperature differences across the brass and in the rock were always small enough to ensure that for all practical purposes the temperature gradients were uniform. A full account of a similar apparatus, its calibration, and the procedure used is given in a paper by BENFIELD [1939] and will not be treated here. The following, however, should be noted.

In preparing the faces, it was necessary to have the faces accurately flat in order to minimize the thermal resistance at the brass-rock interface. For good measurements, this is a value of less than one-tenth of the temperature drop across the thickest sample. Flatness was achieved on these rocks by trepanning and flattening with corundum. The core used gave the samples a smooth wall and also very nearly the correct diameter. A tolerance of 0.0004 inch was allowed the averages in thickness of six evenly spaced points around the periphery and the center.

To attempt to duplicate the moisture conditions under which the rock existed in situ, air was removed from the samples in a small vacuum chamber for 15 minutes followed by soaking in water in the vacuum for another 30 minutes. This was assumed to saturate the samples, allowing

measurements to be made under much the same moisture conditions as underground. However, the rocks tested were very nearly impervious even after long heating at 105°C and then soaking.

Whenever sufficient core could be obtained (in many cases the rocks were badly fractured), three samples of different thickness were prepared from each specimen. The results obtained for each of the three samples were plotted on graphs of thermal resistance, R′, of the specimen plus contacts against the thickness. The plots of the thickest and the thinnest samples were joined by a line and the distance from this to the intermediate point gave the accuracy of the results. The resistance of the contacts was found from the intercept of the line with the R′ axis. The intercept is a measure of the resistance of the contacts compared to brass and depends largely on the material used to make contact and the flatness and grain size of the sample. The intercepts and the differences between the observed result for the sample of intermediate thickness and that given by the straight line are given in Table 1.

All laboratory measurements were made at 16 to 25°C, but in situ the temperatures of the same rocks ranged from 4 to 34°C. The conductivity of quartz decreases approximately one per cent per four-degree Centigrade rise [BIRCH and CLARK, 1940] and that of igneous rocks about a half per cent for the types found in the mines. Thus the maximum error may be about two per cent. On the average, it was less and no correction was made. Pressure effects on conductivity were assumed negligible.

Table 1--Data on the tests

Hole	Mine	Depth	Rock type[a]	Density	Mean specimen temp.	Conductivity	Intercept	Obs. minus calc.	Thermal resistivity	Remarks[b]
		ft		gm/cc	°C	$\frac{10^4 \text{ cal}}{\text{cm sec °C}}$			$\frac{\text{cm sec °C}}{10^4 \text{ cal}}$	
3260	Lakeshore	800	SP	2.72	20.1	75.7	0.38	0.16	1.32	
4325	Lakeshore	800	SP	2.70	21.3	74.3	0.22	0.12	1.35	
3958	Lakeshore	800	SP	2.71	18.5	75.2	0.35	0.18	1.33	Slightly silicified
2489	Lakeshore	1600	SP	2.70	21.9	74.5	0.23	-0.11	1.34	
4931	Lakeshore	1600	SP	2.71	18.1	64.0	0.25	-0.01	1.56	HO
3193	Lakeshore	1800	SP	2.68	20.6	71.4	assumed	. . .	1.40	
2870	Lakeshore	1800	SP	2.53	19.8	77.7	0.47	0.08	1.29	
4359	Lakeshore	2825	AS	2.69	19.6	67.0	0.37	-0.24	1.49	
1716	Lakeshore	3950	SP	2.73	19.2	73.7	0.38	0.11	1.36	
4338	Lakeshore	4200	Silicified SP	2.70	17.8	79.9	0.15	0.10	1.25	HO
4274	Lakeshore	4200	Silicified SP	2.66	16.8	143.3	0.05	0.00	0.70	HO
3206	Lakeshore	4825	SP	2.69	17.5	94.8	0.85	0.03	1.06	Quartz stringer
3588	Lakeshore	4825	SP	2.67	19.9	71.5	0.23	0.25	1.40	
5327	Lakeshore	5075	SP	2.72	17.8	74.3	0.36	-0.75	1.35	HO
4815	Lakeshore	5825	SP	2.72	19.4	78.2	0.40	0.03	1.28	
5326	Lakeshore	6075	Sheared SP	2.75	16.2	67.8	0.09	-0.22	1.48	HO
4257	Lakeshore	6825	SP	2.69	19.3	85.0	0.80	. . .	1.18	
5237	Lakeshore	6893	SP	2.68	18.0	80.3	0.17	0.35	1.25	HO
2300	Leck-Hughes	728	S	2.78	16.4	75.4	0.29	0.14	1.33	HO
2969	Leck-Hughes	857	P	2.69	24.6	63.2	0.01	-0.07	1.58	
3000	Leck-Hughes	977	S	2.69	20.7	75.4	0.15	0.38	1.33	
2993	Leck-Hughes	1605	P	2.69	22.3	86.1	0.30	0.10	1.16	
280	Leck-Hughes	1981	L	2.76	20.0	82.2	0.19	0.10	1.22	
3177	Leck-Hughes	3978	SP	2.73	22.2	80.7	0.08	-0.26	1.24	
2602	Leck-Hughes	3978	SP	2.80	16.5	80.2	0.23	0.17	1.25	HO
2667	Leck-Hughes	4477	SP	2.68	17.2	76.9	0.34	0.00	1.30	HO
1183	Leck-Hughes	4477	SP	2.69	22.2	78.0	0.30	0.02	1.28	
2633	Leck-Hughes	4477	SP	2.67	17.0	77.5	0.54	0.17	1.29	HO
1629	Leck-Hughes	5476	SP	2.67	21.7	95.0	0.62	0.56	1.05	
236	Wright-Hargreaves	2000	SP	2.72	21.1	75.1	0.24	0.15	1.33	
2787	Wright-Hargreaves	2850	SP	2.70	18.0	76.0	0.18	0.00	1.32	HO
3607	Wright-Hargreaves	3000	SP	2.68	17.0	75.5	0.16	. . .	1.33	HO
4050	Wright-Hargreaves	4050	SP	2.70	16.0	84.0	0.08	-0.19	1.19	HO
1451	Wright-Hargreaves	5100	SP	2.69	19.8	77.0	0.47	-0.41	1.30	
2448	Wright-Hargreaves	5100	SP	2.72	18.5	78.8	0.17	-0.45	1.27	HO
1402	Wright-Hargreaves	5500	SP	2.75	17.6	74.5	0.20	-0.08	1.34	HO
1327	Wright-Hargreaves	5850	SP	2.70	16.1	69.3	0.26	0.36	1.44	HO
Test	Wright-Hargreaves	6600	SP	2.67	16.6	74.9	0.24	. . .	1.34	HO
3631	Wright-Hargreaves	7200	SP	2.70	18.8	73.7	0.16	0.00	1.36	HO

[a] Abbreviations: SP, syenite porphyry; S, syenite; P, porphyry; AS, augite syenite; L, lamprophyre.

[b] HO indicates those samples on which heavy oil was used in place of the motor oil.

Table 2--Differences between observed and calculated thermal resistance for 34 samples

Differences in thermal resistance, observed less calculated		No. of samples
From	To	
-0.45	-0.35	2
-0.35	-0.25	1
-0.25	-0.15	4
-0.15	-0.05	3
-0.05	+0.05	8
+0.05	+0.15	8
+0.15	+0.25	5
+0.25	+0.35	1
+0.35	+0.45	2

Table 1 gives a record of all measurements. All samples were wet, and the contact material was No. 30 motor oil unless otherwise indicated.

In order to determine the resistance of the contact materials, runs were made using a brass disc with the three contact materials used in the laboratory. Calculations showed the No. 30 motor oil had a resistance of 0.47, the heavy gear oil a resistance of 0.29, and the silicone grease a resistance of 1.23. The above results are in the ratio of 1:0.6:2.6. Thus the heavy gear oil was preferable as it considerably reduced the temperature drops at the interfaces, and the rocks from the mines were sufficiently impervious not to absorb it.

Specimen LS 1716 (syenite porphyry) was cut in three mutually perpendicular directions and the conductivity measured for samples in each direction. The effect of direction was negligible and thermal isotropy was therefore assumed.

Conductivity measurements were made for 37 samples of core. The results for 34 samples are given in Table 2 as the difference between observed and calculated thermal resistance.

Three specimens gave differences outside this range and were neglected. These were perhaps due to internal cracks or more likely to one of the specimens having a different conductivity from the other two. In four more cases only two specimens could be cut from the core samples, but it was assumed that two specimens would give a reliable value of conductivity.

The mean of the above is 0.024 ± 0.033°C sec/cal with a standard deviation of 0.19. The resistance of the thickest sample is usually about 12°C sec/cal so that, as there is less than a one per cent curvature in the resistance-temperature relation, it would appear that the measured conductivity is independent of thickness, and the method of allowing for contact effects is satisfactory.

Conductivity measurements underground--In order to obtain the relationship between the conductivities obtained in the laboratory and those in situ, a method of measuring the conductivities in place in the mine was developed. The work was done on the 1600-ft level at the Lakeshore Mine, Kirkland Lake.

Two holes were drilled on each side of test hole No. 2489 at distances of approximately one and two feet. Surveys showed the holes to be very nearly parallel. A long source of heat was placed in the center hole and measurements of temperature changes with time were taken in the outside holes by means of platinum resistance thermometers.

Three Canadian Chromalox heaters with an effective heated length of 21.75 inches and a diameter of about 5/16 inch were used as the heat source. These were placed end to end, porcelain cement insulating each heater from the next and also keeping the three heaters rigid and straight. Fifteen discs of transite were cemented at equal intervals along each heater, with holes to carry lead wires. A section of the heat source is shown in Figure 1 for the two end heaters placed farthest from the face of the drift. The total source length was approximately 7 ft 7 1/8 inches.

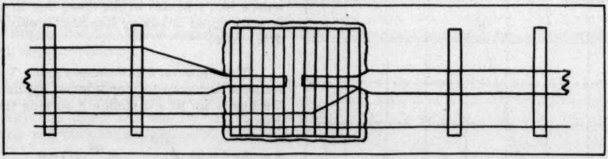

Fig. 1--Sketch showing a section of the heat source

For the heat source, three platinum resistance thermometers with copper jackets were used. The changes of resistance with temperature were measured by comparing the emf across a standard resistance with that across any thermometer. A Weston cell was used as a standard. The standard resistance was a precision wire-wound resistor, which was compared with a Leeds

and Northrop standard both before and after the experiment. Both the cell and standard resistance were placed in a can filled with transformer oil to prevent leakage in the 100 pct humidity underground.

A portable Leeds and Northrop potentiometer was used; this, together with the rest of the circuits for the thermometers and heaters, was mounted on a large board.

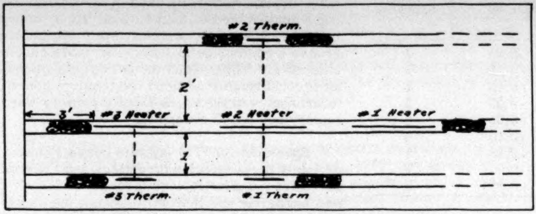

Fig. 2--Array of heaters and thermometers

The array of heaters and thermometers is shown in place in the holes in Figure 2. A trial experiment showed that a thermometer placed in hole No. 3 opposite the No. 1 heater gave effectively the same results as No. 1 thermometer; therefore, only one thermometer was placed in No. 3 hole. The platinum coils were located as accurately as possible opposite the centers of the heaters. The thermometers and the heaters were insulated from one another and from the rest of the hole by steel wool to prevent air circulating in the holes.

Since electric fields in the mine did not allow thermojunctions to be used for measuring the heater temperature, an attempt was made to keep the same known, steady power input in each heater. Variations in mine load caused fluctuations about a mean and in future work a device should be used to keep the input at one value. The mean input was about 350 watts.

An interesting check on the theory of a cooling hole [BULLARD, 1947] was obtained as the lower hole was drilled only about 4 1/2 days before the readings were taken. Since the hole could not be used until it was certain that normal conditions were very nearly attained after drilling, readings of temperature were taken for a day prior to turning on the heaters. According to Bullard, about three days should have sufficed for the hole to return to within one per cent of equilibrium. The temperature was steady when measured.

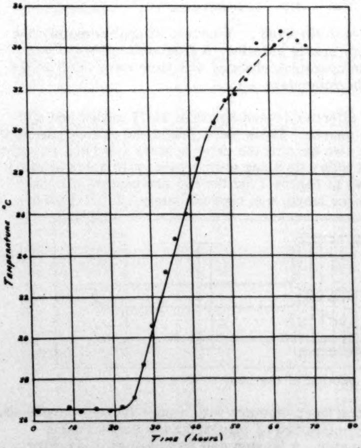

Fig. 3--Graph of data obtained from thermometer No. 1

The change of temperature with time using the averages of three sets of readings is shown in Figure 3. Readings were taken every 45 minutes once the heat was turned on, and required about five minutes to secure. At 43.33 hours No. 1 heater broke down due to oxidation of the copper leads by the acidic water in the drill hole.

The relation for a temperature T due to an infinite line source (considered applicable here) for any point at a distance r outside the line is given by

$$T = (Q/4\pi a)\int_{r^2/4at}^{\infty} (e^{-\mu}/\mu)\, d\mu$$

$$= (-Q/4\pi a)\, E_i\, (-r^2/4at) \qquad (1)$$

where $-E_i(-x) = \int_x^{\infty} (e^{-\mu}/\mu)\, d\mu$, $Q =$

strength of the source, a = diffusivity = $k/c\rho$, where k = conductivity, ρ = density, c = specific heat, and t = time in seconds [CARSLAW and JAEGER, 1948].

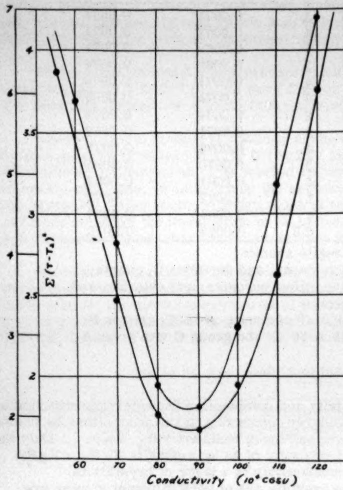

Fig. 4--Relationship of conductivity to $T_{obs} - T_{calc}$ for two different ranges

In the analysis of the curve for No. 1 thermometer, r = 33.0 cm, and the nearly straight portion of the graph was used. The value of specific heat used was obtained from the method of mixtures. (In this regard it is felt that another method should be used for obtaining specific heats, since the method of mixtures only gives approximate results. A method such as that described by VASILEFF [1949] is much better.) Q was obtained approximately from the values of power input read every hour. The average value in cal/sec cm could be estimated knowing the heater resistance.

A least-squares approximation using values of conductivity between 60 and 120 cgs units at intervals of ten was made. Values of T were obtained for different values of t over the steep portion of the curve, and compared with the experimental points. Eight values of $T_{obs} - T_{calc}$ were secured for each conductivity. These were squared and added, and a curve drawn of $\Sigma (T_{obs} - T_{calc})^2$ against conductivity. Two such curves were obtained using different ranges, and the results are shown in Figure 4. The parabolas have a minimum at approximately 87×10^{-4} cal/cm sec °C.

The analysis for the curve for No. 2 thermometer was difficult because the values fell in a smaller range with consequent inaccuracy. The result obtained was very high, probably due to a fracture crossing the drill hole. The value was neglected as this would provide a channel for heat and erroneous results would obtain.

This technique of measuring conductivities underground is in reasonable agreement with laboratory results considering that this was the initial attempt at such measurements under differing underground conditions. Experimental errors were also involved and several recommendations are made for future work.

(1) The experiment should be carried out for a longer period until the curves can be approximated by a logarithmic relation. Comparisons can then be made between the two outside holes neglecting the accurate values of the heat given off by the source.

(2) Humidity underground makes accurate experimentation difficult, and all switch terminals and measuring devices should be enclosed in moisture-proof containers.

(3) Non-oxidizing leads should be used and a higher input utilized in order to keep the holes dry. It should not, however, be high enough to change the rock properties.

(4) Some method, such as a point source, would allow rapid measurements underground once the above difficulties have been overcome, and should eliminate some, at least, of the need for laboratory determinations.

The effect of composition on conductivity--It follows that the conductivity of a disc thinner than the grain size may differ from that of a thicker one. If the matrix is an insulator, the former will conduct while the latter will not; that is, if a specimen is thicker than its grain size, the measured conductivity and the contact temperature drop are too small.

It is known that the measured results appear to depend on the principal constituents of a rock and on their form, but at present calculations of conductivity based on exact composition and form are not possible. A certain amount of advance has been made, however, and is summarized by BIRCH and CLARK [1940].

Commencing with homogeneous crystal aggregates of known conductivity, Birch and Clark

Table 3--Volume percentages of minerals from thin section

Rock type	Number	A	B	C	D	E	Theoretical	Experimental
SP	LS 3260	23	58	2	13	4	0.0071	0.0076
SP	LS 3193	28	62	0.5	6	14	0.0067	0.0071
SP	LS 3588	9	65	2	23	1	0.0069	0.0071
SP	LS 2489	21	63	1	12	3	0.0068	0.0074
SP	LS 4815	9	59	2	26	4	0.0071	0.0078
SP	LS 3206	11	60	8	16	5	0.0074	0.0095[a]
SP	WH 3631	8	63	6	22	2	0.0074	0.0074
SP	WH 1327	27	60	2	9	2	0.0071	0.0069
SP	WH 1402	12	59	6	20	4	0.0075	0.0075
SP	TH 2993	10	68	5	14	3	0.0070	0.0086[b]
SP	TH 2969	4	70	3	19	4	0.0064	0.0063
SP	TH 2633	16	63	5	15	1	0.0072	0.0077
AS	LS 4359	11	70	3	12	4	0.0064	0.0067
S	TH 3000	10	68	6	16	0.5	0.0072	0.0075

SP, syenite porphyry; S, syenite; AS, augite syenite.

Groupings: A, carbonate; B, sericite, feldspar, and perthite; C, quartz, magnetite, and pyrite; D, hornblende, olivine, clino-pyroxine, and chlorite; and E, accessories (for example, epidote, apatite).

Groups A and D were given values of 0.0110 cgs units at 20°C; groups B and E were given values of 0.0045 cgs units at 20°C; and group C was given a value of 0.0190 cgs units at 20°C.

[a] Quartz stringer. [b] Quartz stringer not in slide.

established a working correlation between conductivity and composition for heterogeneous aggregates. Several crystal types A, B, C, . . . are considered, occurring in the proportions by volume X_A, X_B, . . ., having mean conductivity K_A, K_B, . . ., and mean resistivity R_A, R_B, . . . Only the results are given here (see Table 3). If the mean resistivity of an aggregate is $X_AR_A + X_BR_B$ . . ., it is called a "series" arrangement. If the mean conductivity of a layer of crystals is $X_AK_A + X_BK_B$ . . ., it is termed "parallel." If the crystals are of very different size or are regularly oriented, the terms would not apply. In the rocks of the Kirkland Lake area there is probably little departure from random orientation.

The percentage of each mineral type was obtained from the study of thin sections for 14 of the samples. Since the conductivities of the minor constituents are unknown, they are lumped with the major ones of probable like conductivity.

Both the series and parallel possibilities were tried for these rocks to see which simplification gave a result nearer to the experimentally determined value. The parallel was found to be more accurate because the specimen thickness is of the order of the grain size.

It is apparent that, except for the two cases with the large quartz stringers, the approximate theory gives surprisingly good results. Most values are low, however, and since the phenocrysts in all cases are not of the order of the specimen thickness used in the conductivity measurements, it is felt that either some assumption regarding the conductivities of one or more constituents has been underestimated, or that some mechanical effect is influencing the answers.

More data are needed on conductivities of ordinary minerals with due regard to the different variables. Here all choices of the groupings are the writer's, but there is probably a certain amount of cancellation of errors. Too, the analyses as obtained from examination of thin section may not represent the composition of any particular sample accurately if it is coarse-grained, but agreement is probably close.

The heat flow--During the summers of the last few years, temperature surveys have been carried out on an unprecedented scale in the Kirkland Lake mines [MISENER, 1949, 1950]. Locations were selected where workings extended to depth over a large area. At various levels, drill holes extending into the country rock were chosen and resistance thermometers were inserted far enough to give the true rock temperature. A large number of readings near one level gave a representative idea of temperature conditions. It is stated that projection of these temperatures on two perpendicular vertical planes allowed isothermal levels, and hence temperature variations, to be shown in a somewhat three-dimensional picture.

Samples of the rock type at the thermometer in the temperature holes were secured and conductivities of many measured. The geology is complex [TODD, 1928], but the conductivities were not appreciably different on the average with depth. Thus by taking the mean of all the conductivities obtained, an average conductivity could be determined for general application. Neglecting only the one abnormally high result (due to large amounts of quartz), the average conductivity was 76.7 ± 1.0 cgs units, with any specimen having a dispersion of 4.2 cgs units.

Misener has obtained a temperature gradient of 1°C per 252 ft or 0.0130°C/meter. This multiplied by 76.7×10^{-4} cal/cm sec °C gives a heat flow at the surface of 1.0×10^{-6} cal/cm^2 sec. The heat flow depends on several factors, however.

BIRCH's [1948] method is used for evaluating the effect of geological climate. The deepest temperature secured was at 7575 ft (≈ 2.36 km) and presumably the Pliocene climate had an effect on observed temperatures. The method required that definite temperatures be given to all time intervals until then. It is assumed the area underwent the same larger fluctuations in climate as did areas 300 miles south, although much of the record at Kirkland Lake has been eroded away and does not allow the assumption to be tested. Table 4 presents a chronology which gives fairly good agreement with tables drawn up farther south [BIRCH, 1948].

Birch states that in the Ice Ages the temperature under the ice would be around 0°C, while the present mean annual surface temperature around Kirkland Lake is approximately 2°C. To be conservative, the mean temperatures of the long interglacial periods are taken to be no different than at present. The mean temperature for the Tertiary is taken as 4°C; probably slightly low [BIRCH, 1948].

Table 4--Assumed deviations of temperature from the present

Interval of time before the present	Deviations in temperature	Interval of time before the present	Deviations in temperature
years	°C		°C
0- 5000	0	100000- 200000	2
5000- 10000	2	200000- 300000	-2
10000- 20000	-1	300000- 600000	2
20000- 30000	-2	600000- 700000	-2
30000- 40000	0	700000- 900000	2
40000- 60000	-2	900000-1000000	-2
60000- 80000	0	all time before	2
80000-100000	-2		

Using Birch's tables, a table of "recollections" is set up. A sample calculation is shown in Table 5. Similarly the correction at 1.5 km is 0.09°C, at 1.0 km is 0.13°C, and at 0.5 km is 0.12°C. Thus the corrections to be added are -0.1°C at all levels. The diffusivity chosen seems best for these rocks, particularly near 0°C. It is impossible to tell whether the assumptions regarding temperatures are correct by examining the temperature - depth curves. The relation shows no uniform curvature and the irregularities are due to lack of thermal uniformity, geological structure, and erosion and uplift (probably too slow for marked effect). Thus it seems reasonable to assume that the gradient can remain as a straight line.

Table 5--Sample calculations of "recollections," $k = 0.02$ cm^2/sec

Interval	Temperature deviation	Depth 2 km	
		Recollection	Temp. change
	°C		°C
All glacial time	-2	0.310	-0.62
10000-20000 yrs	-1	0.132	-0.13
Interglacial, etc.	2	0.415	0.83
Remainder	0	0.143	0
		Total	0.08

The area around the mines is now quite flat compared with profiles liable to affect temperature gradients [BULLARD, 1938]. It is generally agreed that this area was once mountainous, but the effect of erosion and deposition is unknown. It seems likely that although erosion has been great [SMITH and PEACH, 1949], its effect is small for the observed gradients at deeper levels, although its effect on the heat flow may be noticeable. Misener pointed out that below 3500 ft the isothermal levels are nearly horizontal, but above this, irregularities appear. These are probably due to structural effects, erosion, and uplift.

Part, at least, of the effect of radioactive decay can be estimated from values obtained from SLACK [1949]. Correcting his values to new constants in the heat-generation equation, the heat produced by the various rock types present becomes: for Timiskaming sediments, $(3.66 \pm 1.80) \times 10^{-13}$ cal/cc sec; for granitoid rocks, $(1.36 \pm 0.36) \times 10^{-13}$ cal/cc sec; and for basaltic rocks, $(0.74 \pm 0.09) \times 10^{-13}$ cal/cc sec; with a probable error on the average of at least 30 pct, due to (1) the error in the measured results, (2) the fact that the last two values may be still lower at depth although the best values obtained were used (an average value instead of the value used by Slack was given to the granitic rocks in order to be on the safe side), and (3) the effect which potassium has on the heat generation.

Slack's assumptions regarding the thickness of the rock types (for example, sediments, five km; granitic rocks, 12 km; and basaltic rocks, 19 km) gives a heat flow at the surface of 0.49×10^{-6} cal/sec cm^2. This is approximately half the observed flow. However, there are some factors which could change this greatly. It is likely that the granitic rocks could be assumed a little thicker and the basaltic rocks a similar amount thinner, since the mines are on the north limb of a large syncline formed by the old mountain roots. There is a likelihood that these rocks should have a much lower radioactivity at depth, also. The thermal history of the Earth is the greatest unknown factor, since a cold origin would necessitate a much larger portion of the heat flow being of radioactive origin. These, together with the probable errors in Slack's values, make any such estimates of the effects very qualitative but are suggestive.

Some suggestions--The heat flows determined up to the present reveal an apparent lack of correlation between heat flow and elevation except in mountainous regions [BIRCH, 1950]. The writer presents the following, based on the ideas of growing continents, as a possible solution to the difficulty.

During geological history, the continents as we now know them, have undergone a complex sequence of erosion and deposition causing changes in heat flow from place to place and time to time. WILSON [1949] has suggested that there might be a definite pattern to this, the continents having grown out from nuclei by a series of marginal orogenic belts. During the formation of these belts [HOLMES, 1946] the rocks would probably increase in radioactivity as solutions carried hyperfusibles upward from depth. The result would be an increase in average radioactivity, and hence of regional heat flow, of the rocks with distance from the original nuclei. Thus the heat flow in elevated regions can depend on lithology and history as well as compensation.

The interrelation may be capable of some resolution, however. Mountain systems become younger toward continental margins and, as younger mountains have deeper roots, the resultant larger heat flow, added to the probable increase in average radioactivity away from continental nuclei, should lead us to expect increased regional heat flow outward also. A possible consequence would be an increase in frequency and intensity of orogenies as time goes on.

Acknowledgments--In conclusion, the writer thanks A. D. Misener and Lloyd Thompson for making available the results of their temperature work in these mines. He would also like to thank the staff of Lakeshore Mine for their many kindnesses. The help of E. R. Niblett and D. Ramsey during the work is more than appreciated, particularly the latter who obtained all the conductivities in the laboratory.

Throughout the work, J. Tuzo Wilson has freely given his advice and suggestions for which I would like to express my gratitude. Finally, I would like to acknowledge the help of the Ontario Research Council in providing the funds for the work.

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(Communicated manuscript received February 2, 1951, and, as revised, November 5, 1951; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

OROGENESIS AS THE FUNDAMENTAL GEOLOGICAL PROCESS

J. Tuzo Wilson

Abstract--A comparison is made between the different approaches to the study of the Earth by the field geologist and by the physicist interested in broad terrestrial problems. On one hand, the rapid increase in geological knowledge is giving an increasingly clear picture of the progress of geological history and the nature of the processes which have been involved. On the other hand, the application of new and rapid methods of obtaining measurements of physical properties of the Earth are serving to limit and direct physical speculation about the nature of the Earth's mechanisms. These different approaches must lead to the same conclusion. When reached, it will be a history of the Earth and an explanation of its processes satisfying both to the geologist and to the physicist.

Progress is such that some preliminary agreement upon the most basic processes should soon be possible. It seems that the forces causing orogeny are fundamental, and some conclusions can already be drawn from the rates at which certain processes have been operating.

Estimates of the limits in rate of erosion since Mesozoic time are made, as well as estimates of the present rate of volcanism. It is suggested that these rates are such as to suggest strongly that the continents themselves, like the atmosphere and oceans, have grown in situ during the course of geological time.

Geology is often divided into two major parts, physical geology in which the materials and processes of the Earth or of its crust are described and historical geology in which an account is reconstructed of what has happened to the materials under the repeated operation of these processes during the whole of geological time. Stated in the simplest terms, the two divisions deal respectively with causes and with results.

It must be admitted that at the present state of development of geological science much more is known about the results than about the causes. Remarkable progress has been made in geological mapping of the Earth and in elucidating the geological history especially of the fossiliferous rocks. Minerals, ore deposits, and the varieties of rocks are now well described.

There is less certainty about the causes of geological phenomena. Partial answers can often be given, but the fundamental and essential parts of geological processes are largely hidden. Thus some particular ore deposit or area of metamorphism may be linked with and partially explained by the intrusion of an igneous body, but is the source of the heat that formed the intrusive often known or why it was intruded at that particular time and place? Or, again, the stratigraphical history of an area can often be illuminated by indicating which areas were uplifted and eroded and at what times, but can an explanation often be given as to why uplift has affected particular areas which are so widely scattered in place and time?

A better understanding of the fundamental geological processes would surely be of great practical value to geologists. Such an understanding would link geology more closely to physics and chemistry. Indeed, it seems likely that the recent progress in geophysics and geochemistry added to the accumulated store of geological knowledge may shortly make the discovery possible of the nature of the fundamental processes that have controlled geological history. In this paper it is proposed to discuss, not the nature of the fundamental process, but the ways in which it can be sought and also some evidence bearing upon the rates at which it has operated throughout geological time.

From the immediate practical point of view, not too much attention has been given to the search for causes because it has seemed that in economic geology and in geological mapping it is history rather than process which is important. Only a few types of mineral deposits are accumulating today at rates which are fast enough to be economically significant.

Old deposits which accumulated slowly in past ages are those which are important. In many cases, geologists are not precisely certain how these deposits were accumulated and, indeed, the evidence which would enable the process to be defined exactly may have vanished. On the other hand, the age of the deposit and of its enclosing rocks, the dating of structures which controlled the accumulation, and the existence and time of subsequent disturbances which enriched or modified the deposits are important aspects of economic studies. So it is with geological mapping, which is an interpretation of the sum total of the results of repeated operations of the geological processes. The original surface of the Earth, whatever it was, has vanished and the present continents are made up of the scars of repeated cycles of geological processes acting over thousands of millions of years.

The geological map is therefore essentially a very complex thing, and it is not to be expected that it can be approximated to any simple form directly capable of physical interpretation, nor can it be reduced to any form which can be stated in precise mathematical terms.

If geology is to be interpreted in terms of a relatively simple physical theory, it will be the primary processes which provide the interpretation and not the resulting maps. However, once we begin to understand the processes which have operated, we may be able to pick out parts of the geological map for physical interpretation. For example, we may be able to connect fault patterns and movements with their contemporary and causative stresses.

How can the primary process or processes be found, identified, and stated in fundamental terms? From a geological point of view, the primary processes are those which are independent and causative, whereas the secondary processes are those which are dependent upon others. Thus erosion and subsequent deposition are among the more important geological phenomena, but they are certainly not primary processes; both are the result of uplift or down-warping and would long since have ceased if there had not been repeated uplift of parts of the Earth. Uplift therefore precedes and is more fundamental than erosion. That type of argument appears to be the best way to determine what is a fundamental process from a geological point of view. Many geologists have already come to the conclusion that orogenesis is the most fundamental process acting within the Earth.

We may also examine the Earth from a physical or chemical point of view. In physics and chemistry there are a few well-established laws which have been found to hold in all cases which have been examined. Such laws include those of gravitation, radioactive decay, heat transfer, electromagnetic theory, hydrodynamics, failure of materials in physics, and those of thermodynamics and chemical combination in chemistry. Until there is evidence to the contrary, these laws can be expected to hold in the Earth and they place definite limitations upon the processes which can operate in the Earth.

Any data obtained about the physical properties of the Earth can be interpreted in terms of these laws to state some limits to the possible ways in which the Earth might behave. For a long time, there was so little known about the physical properties of the Earth that only the most general arguments could be applied. These generalizations were too broad to have much bearing upon the details of geology. For that reason, the value of the papers of G. H. Darwin and the earlier work of such men as Jeffreys and Gutenberg was not fully understood by geologists. Recently improved instruments, more measurements, and better interpretations have vastly increased the scope of geophysical data. The distribution of deep and shallow earthquakes, of gravity anomalies, the pattern of the Earth's magnetic field, and the quantities of radioactive elements and of heat flow in the crust have recently been measured fairly widely and provide a better basis upon which to apply the physical laws. As a result, much more precise statements can now be made about the processes which can operate within the Earth.

Now if, as is believed to be the case, geologists accept the assumption that the general laws of physics and chemistry apply to the Earth and that no other exclusively geological laws are necessary, then the fundamental processes arrived at from physical reasoning must be the same as those arrived at from geological reasoning. The quest for a mechanism which will suit both kinds of data is the present gap between geology and geophysics (or physics of the Earth) which both sides must try to bridge.

When the cause of orogenesis can be stated in precise physical terms and when the result of repeated application of the fundamental orogenetic process can be shown to be adequate to produce the complexities of geology, then geology and physics of the Earth will have merged. This has not yet been achieved, but there appears to be a reasonable expectation that it can be achieved and that fairly soon.

Consider briefly the present operation of the orogenetic process. At present and in recent time, mountain building has been concentrated in two belts about the Earth. One extends from the Alpine and Atlas mountains across southern Asia to New Zealand, and the other branches from it at Indonesia and stretches around the Pacific Ocean.

The existence and continuity of these belts have been recognized by geologists since the time of Suess half a century ago. Not only are they the site of recent mountain building, but also they are the loci of all deep-focus earthquakes, and a majority of the shallow earthquakes and of the world's most numerous type of volcanoes, the andesitic type. There is folding, uplift or downwarping of sections of the land along these belts. There is great release of energy in earthquakes and evidence from them of actual fracture occurring to depths as great as 450 mi within the Earth. There is release of gases, lava, ash, and heat from within the Earth. These phenomena appear to have characteristics such as might be associated with a mechanical process.

Many geologists and physicists have analyzed the phenomena of these belts, and from their general shape, from the shape of the arcs and ranges that constitute them, and from studies of the earthquakes, folding, gravity anomalies, and other features, have tried to arrive at an explanation of the belts in terms of some physical cause.

The chief theories advanced may be divided into three groups, those involving continental drift, those depending upon convection currents, and those due to contraction of a cooling Earth. There is now considerable doubt as to whether any hypothesis involving continental drift is a satisfactory explanation. With more data it may soon be possible to decide upon the precise part played by contraction and by convection currents.

When such a mechanism has been found, it must be shown to be physically valid. It must also be shown that if it has operated throughout geological time it must have been capable of having produced the older mountain chains such as the Appalachians and those hidden in the Precambrian Shields. When that can be done, geology will have been placed upon a sound physical basis.

The establishment of a general and fundamental geological process would mean that any subsidiary theories such as those concerning ore deposition or the origin of various types of igneous rocks would have to be fitted into the general theory. If they did not fit, they would have to be modified or else the primary theory would have to be reconstructed. This is the type of control which the general physical and chemical laws exercise over speculation in those fields. We, as geologists, should look forward to obtaining a similar if less strong control over geological speculation in the not too distant future.

It is not the object of this paper to consider the nature of the fundamental geological process. Its nature is not yet agreed upon, although much data about it can be obtained from the writings of such geologists and geophysicists as Benioff, Bucher, Daly, Gilluly, Griggs, Gutenberg, Holmes, Jeffreys, Lake, Umbgrove, and Vening Meinesz. Scheidegger is at present preparing a lengthy review of the present limits of our knowledge of the subject.

We have assumed orogenesis to be the fundamental geological process acting within the Earth and that it, together with the external radiation from the Sun, is the cause of geological change. Without waiting to discover its exact mechanics, two things can be said about its rate of operation during geological time.

From recently published data about the material in the Gulf Coast shelf of the United States, the conclusions can be drawn that all old mountain ranges may have been formed out of former shelves which in turn have been folded and "granitized" by the orogenetic process. From a consideration of the rate of volcanism, we can conclude that the acid rocks required to make the continental blocks could perhaps have been extruded from volcanoes during geological time.

The rates of erosion and of deposition--It was estimated by DOLE and STABLER [1909] that erosion reduces the surface of the United States by one foot in 9000 years. If this process had continued at the same rate throughout the length of geological time, a layer about 17 km thick would have been removed since the beginning of Paleozoic time (5×10^8 yr) and 110 km thick since the probable time of origin of the Earth (33×10^8 yr). For comparison, the thickness of the whole crustal layer above the Mohorovicic discontinuity is only about 30 to 40 km.

It has been suggested that the present rate is high. A better average measure is therefore provided by figures recently published by MURRAY [1950], giving the volume of sediments deposited on the Gulf Coast in Mesozoic and Cenozoic time. Murray's data from results of drilling and reasonable extrapolation is that the emerged and offshore portions of shelf underlying the Gulf Coastal Plain contain at least 500,000 cu mi of Cretaceous and later sediments. This was accumulated during 130 million years from a drainage area of 1 1/2 million square miles. It follows that the rate of erosion would have been about one foot per 85,000 years, but this takes no account of the sediments swept out to the deep ocean and the floor of the Gulf of Mexico. KUENEN [1941] has shown that the volume of deep-sea deposits may be three times as great as the corresponding volume of continental shelf sediments.

We need not discuss this in more detail but can safely conclude that the rate of erosion is such that it has removed a foot in each few tens of thousands of years off continents similar in state to North America. The quantity of sediment deposited in geological time might easily equal the total volume of the continental blocks.

The important point is that if erosion at all comparable to that of Mesozoic and Cenozoic time had taken place in Paleozoic and Precambrian time, then there would exist shelves of those earlier ages around North America many times as extensive as those of the Gulf Coast. Of course, no such shelves exist. Except for the Atlantic and Gulf Coast shelves, there are virtually no shelves at all except in the Arctic where the Coppermine Series might be regarded as a very small example. To suggest that the Atlantic and Gulf Coast shelves have only a veneer of young sediments over older rocks is to contradict the evidence from deep wells quoted by Murray. To maintain that no similar Precambrian and Paleozoic shelves ever existed is to deny the existence of an average rate of erosion only a fraction of that of today.

It seems quite untenable to suggest that there was so little erosion before Mesozoic time. It is also quite unnecessary because KAY [1948] and EARDLEY [1947] have pointed out that the Appalachian and Cordilleran Mountains were formed out of precisely such shelves as are now forming on the Gulf and Atlantic Coasts. This seems to be an entirely satisfactory explanation. The Cretaceous age of the oldest rocks known in these shelves suggest that they only started to form after the Appalachians had been built. LAWSON [1932] has even suggested that the mountain-building process is due soon to be repeated on the Gulf Coast. On the Pacific Coast there has not yet been time to form a shelf since the Cordillera was built.

This suggests that the mechanism of continental growth is already known. Indeed, it is widely accepted for recent time, but it is often coupled with the idea that there was throughout Precambrian time continental blocks formed in some other undefined manner. Perhaps this idea is widely held because even after the idea had been accepted that continents are growing it was felt necessary to have a pre-existing block to supply the sediments by which growth proceeds. But is that so? A shelf must be formed only by erosion, but once failure has occurred volcanism supplies abundant lava, ash, and intrusive rocks from the interior of the Earth to add to the continent.

The shelf along the whole east coast of Asia is bounded by a series of island arcs, all volcanic and all adding abundant material at the margin of the continent. This material is distinguished from the lava of the volcanoes not in orogenic belts by its higher silica content which is precisely the kind of material needed to extend continents.

Rate of volcanic extrusion--Something can be said about the rate of volcanic extrusion also. The surface of the continental blocks is approximately 50,000,000 sq mi and they are about 20 mi thick. If the Earth is somewhat over three billion years old, then all that is required in order to build the continents is that an average of 1/3 of a cubic mile of new sialic rock should have been extruded each year.

It would be difficult, if not impossible, to estimate precisely the rate at which such contributions have been made, but it will perhaps suffice to show that the known contributions are such as to make the rate appear to be a reasonable one. There are three kinds of igneous rock which are known to be quantitatively the most important. These are granites, andesites, and basalts. They will be discussed in turn.

The rate and method by which granite is formed cannot be observed in recent rocks as it takes place at depth. Much granite and more granite gneiss has been formed in the past and it is increasingly abundant in the older rocks. But it is doubtful how much of the contributions of granite should be included in this calculation. Admittedly, in areas of young rocks like the United States and central Europe, there is evidence of the intrusion of granite magma, but its source is a matter of debate. If it came from the depths as granite, the answer to our problem is easy, but erosion should expose more granite in the older rocks. The old shield areas are not composed of granite, but of well-foliated granite gneiss showing abundant evidence of containing great amounts of metamorphosed sediments as has been maintained by geologists in every shield area. The argument based upon the disappearance of the shelves rather than indeterminate petrological arguments forces one to believe that the roots of mountains are formed of sediments to which only small additions of materials have been made. These additions have chiefly been water and heat which are supplied during primary mountain building along the great arcuate fractures from the depths. The contributions of new continental material in the granites and gneisses may thus be largely illusory and will not be included, but to take the opposite view would increase the source of continental material.

The rocks which are added along the island arcs are predominantly andesites, dacites, and latites so that they have an average composition similar to granodiorite. At present there are about 500 volcanoes listed as active. To form 1/3 cubic mile of new rock each year each volcano would have to emit a sheet of lava a square mile in area and about four feet thick. Probably most volcanoes do not contribute this much each year, but the most active like Paricutin extrude much more. FENNER [1923] has estimated that the tuff in the Katmai Valley of Ten Thousand Smokes is more than one cubic mile in volume and represents one principal ash flow. That would equal the contribution necessary from the whole world for three years and it is not the largest single eruption known.

Turning to basalts and pyroxene andesites, the quantities known to have been formed are very great. Von Tillo [DALY, 1940] estimated that 2,000,000 sq mi on continents and islands were covered by "young" (Tertiary?) volcanic masses. Great volumes have been formed under the oceans, for example, 500,000 cu mi in the Hawaiian Islands alone [ZIMMERMAN, 1951]. Of course, oceanic eruptions contribute nothing to continents today, but the continents may include rocks poured out in the ocean. Lawson has argued that the action of sea water in breaking down basalt leaves a more siliceous residue which would be of sialic composition.

These figures do not establish the rate of volcanic outpouring, and it is not known whether adequate data to establish that rate exists, but they perhaps suffice to show that the formation of continents by extrusion of volcanic rocks and to a greater or lesser extent by the intrusion of plutonic rocks is a reasonable proposition. Naturally, most of the rocks now exposed have been reworked through succeeding cycles of erosion, deposition, and metamorphism so that they are no longer in their original form.

Conclusion--This paper has suggested that it may shortly be possible to discover a fundamental mechanism operating within the Earth which would both be explicable as a natural consequence of the physical state of the Earth and which would serve to explain the geological behavior of the Earth. The discovery and establishment of such a process would be of great practical value in geology and give it for the first time a unified basic theory.

The only serious suggestions as to the nature of such a process are either that the Earth is cooling and contracting or that it has convection currents within the mantle. Both would arise from the fact that the Earth is a heat engine. In either case it has been supposed that these movements give rise to orogenetic movements which are the fundamental cause of geological change.

These theories are not discussed further in this paper, but two arguments are advanced about the rate of geological changes and particularly about the rate of growth of continents. It is shown that the rate of erosion and deposition is such that it should have given rise to great shelves about the continents in past time. The shelves that survive are inadequate, so that it can be argued that orogenesis has generally occurred in and destroyed former marginal shelves and caused the growth of the continental blocks.

It is also argued that the source of material with which the continents have ultimately been built is the andesitic volcanoes which form along the continental margins during the orogenetic process. The present rate of volcanism would probably suffice to have built the continents during geological time.

I am indebted to J. W. Greig for drawing my attention to the work by C. N. Fenner, and to Dorothy Perryman for typing this paper.

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(Communicated manuscript received October 15, 1951;
open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

REPORT OF THE COMMITTEE ON LAND EROSION, 1950-1951

H. V. Peterson, Chairman

Paul C. Benedict
Gunnar M. Brune
E. A. Colman
J. E. Fletcher
M. R. Huberty
E. N. Kavanagh
Luna B. Leopold
Jesse D. Sinclair
Austin W. Zingg

Activities of the Committee on Land Erosion, since the present Committee was appointed in September, 1950, have been confined mainly to correspondence among members as a means of becoming better acquainted and of deciding on a suitable program for consideration of the Committee. Erosion being such a far-flung and complex phenomenon defies any small group to consider all of its many facets. It is obvious, therefore, that greatest practical results would be obtained if Committee activities were confined in the main to those investigations in which members are currently engaged.

Colman and Sinclair state that one of their chief current interests is the study of possibilities for decreasing creep on steep mountain slopes in Southern California. They have reason to believe that a considerable part (the precise amount being unknown) of the debris issuing from the canyons starts its way to channels as dry sliding or other type of soil movement that takes place during the dry seasons although the actual damage occurs during wet periods. Initially they hope to obtain more information on channel accumulations and to learn more about where the material comes from and how and when it moves to the channel. With this information they expect to be in a better position to study means of decreasing debris movement. The importance of debris movement into the densely populated valleys of Southern California is of such an order that additional planning along these lines can be well justified.

Fletcher, whose area of activity encompasses the deserts of the Southwest, reports that he is currently conducting work on the erodibility of soils as expressed in their susceptibility to sheet erosion. He is particularly interested in the development of badlands on valley floor areas having slopes as low as ten feet per mile. To one who is acquainted with the deserts of the Southwest and who has observed the deterioration of great acreages of former highly productive valley range lands into badland wastes completely devoid of vegetation, there can be no doubt about the importance of this form of erosion and the need to initiate studies directed at developing some practical method for abatement of erosion and for restoration of the lands to their former productive state.

Brune, working in the Midwest, calls attention to the high rates of erosion taking place in the loess hills in the Missouri Basin. On the basis of surveys of 30 farm reservoirs, he shows that gross sediment production from drainage areas ranging in size from 0.038 to 41.3 sq mi varies from 2.6 to 68.6 ac ft/sq mi of drainage area per year. The maximum rates measured in this area are far in excess of any found to date in even the worst eroding tracts to the arid West. In contrast to the findings of Fletcher in the desert areas of the Southwest, Brune attributes the greatest portion of sediment production in this locality to sheet erosion with only minor amounts coming from gully advancement and widening. He shows that the highest rates can be traced to areas using straight-row cultivation on steep slopes with a high average length of slope. Obviously, farming by these methods on this type of terrain should be discouraged. The areas considered by Fletcher, on the other hand, are not under cultivation but have been extensively used for grazing with a high probability that the gullied areas have been overgrazed.

The findings of Brune, as high-lighted above, are described in detail in a recently issued report titled Sediment Design Criteria for the Missouri Basin Loess Hills authored by himself and L. C. Gottschalk. As the objective of the study was to work up criteria for design of corrective measures for this type of erosion, the report is particularly commended to those interested in this aspect of the erosion problem.

Brune also reports several other investigations on which he is active. A recently completed paper titled Sediment Records in Midwestern United States summarizes 192 sediment records and lists rates of sediment production from 16 major physiographic sources in the Midwest. The factors which affect rates of sediment production are discussed. Another paper in course of preparation titled Acceleration of Rates of Sediment Production in Modern Times presents a comparison of suspended sediment records from three stations on the upper Mississippi, Ohio, and Missouri Rivers from various periods 1878-1948. Although data gathered thus far are not conclusive, they indicate that the rate of sediment has been accelerated 750 pct in modern times from the upper Mississippi Basin, 290 pct from the Ohio Basin, and 17 pct from the Missouri Basin. If these figures prove to be true, it will constitute convincing evidence of the sorry spectacle that man is making of some of our most fertile land.

Zingg, working in the Kansas area, has been concentrating on wind erosion which he describes as a combination of engineering, soil science, and agronomy. Studies of the basic causes of wind erosion and the relations between the rate of wind erosion and the basic factors affecting it have been in progress at the High Plains Wind Erosion Laboratory at Manhattan, Kansas, for 3 1/2 years. A portable wind tunnel has been developed to evaluate the erodibility of field surfaces. It has been used on plots in Texas, Kansas, and Nebraska. From these data, relationships for estimating soil removal in terms of wind force, surface residue, surface roughness, and the per cent of erodible soil material at the soil surface have been developed. Intensity-frequency curves for wind movement have been developed for periods varying from five minutes to 30 days for the western Kansas portion of the High Plains.

The characteristics of soil- and sand-particle movement by wind have been investigated utilizing methods employing ultra-high-speed photography. Studies of soil properties, namely, apparent density, dry aggregate structure, and surface roughness, as they influence erodibility by wind have been carried out in a laboratory wind tunnel.

With the specter of the black dust clouds, so prevalent during the mid 30's, again looming as a possibility because of severe droughts in parts of the Midwest, the urgency for more knowledge in designing defenses against wind erosion is apparent.

Benedict reports several studies under way in the Missouri Basin. Two severely gullied areas in Wyoming, the Five Mile and Muddy Creek channels in the Wind River Basin and the Alkali Creek channel in the Shoshone River Basin, are being intensively investigated to ascertain the rate of sediment production from channel sources and from other sources and as a basis for appraising the results of remedial measures being installed in parts of the channel. Preparations are also in progress for setting up aggradation and degradation studies in the Bighorn River below Boysen Reservoir in anticipation of stream-flow regulation that will begin late in 1951 on completion of the Boysen Dam. The Powder River and Medicine Creek Basins are being studied to determine the important sources of fluviatile sediments with the objective of assembling data needed in the design of land treatment measures aimed at erosion abatement.

Studies in the upper part of the Cheyenne Basin above Angostura Dam in South Dakota to determine the effect of stock-water reservoirs on runoff and sediment production from the drainage area are reported by Peterson. Examination of a five per cent sample of the area chosen on a random basis indicates that the 9000 sq mi drainage area contains nearly 9500 stock reservoirs having an aggregate storage capacity of about 52,000 acre feet. Nearly two-thirds of the drainage area is controlled in some measure by the reservoirs. The effect on both runoff and sediment movement from upland areas to the main channel is described in two reports now being reviewed preparatory to publication.

The results of measurements of sediment deposition in a number of typical reservoirs show that the gross rate of sediment movement from contributing areas underlain by shales and sandstone of Cretaceous and Tertiary age and having low to moderate relief varies from one-half to 4 1/2 acre feet per square mile per year. A part of this is intercepted by the reservoirs while another portion is deposited in channels and along the flood plains of tributary streams so that the percentage of the total reaching the main Cheyenne channel is relatively small. Several areas showing exceptional high rates of aggradation on flood plains were found during the course of the investigation. In one example a 3 1/2 mile reach of a formerly gullied, small tributary valley has been aggraded to a depth ranging from one to four feet in the past 25 to 30 years as indicated by deposition along fenced lines. This represents a deposition of 960 acre feet originating from a drainage area of slightly less than eight square miles, or an average annual rate of 4 to 4.5 acre feet per square mile of drainage area. Several other tributary streams in the basin show similar rates of aggradation along their flood plains.

The practical aspects of these findings become apparent when it is realized that unless deposition of this type continues, the material now being deposited on valley floors will be transported downstream and eventually find its way into the Angostura Reservoir. Considering the fact that the aggradation has taken place during a period when there has been no particular change in land use and, so far as can be seen, no great acceleration in erosion, the findings point to several interesting possibilities in relation to the formation of terraces and other erosion features commonly found in the western Great Plains areas. A paper giving full details of the findings is under preparation.

Huberty has posed an important question concerning the development of adequate data for the evaluation of flood damage and the benefits resulting from flood-control measures. Although realizing that detailed surveys are very expensive and time-consuming, he expresses a doubt that the small sample method of evaluation now being generally used is adequate for a complete analysis and appraisal of anticipated benefits as presented in flood-control reports. As this question has been raised in several other quarters an effort will be made by the Committee to give it further consideration.

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(Manuscript received May 8, 1951; presented at the Thirty-Second Annual Meeting, Washington, D. C., May 1, 1952; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

GRAVITY MEASUREMENTS AT SEA, 1948 AND 1949

J. L. Worzel and Maurice Ewing

(Lamont Geological Observatory Contribution No. 58)

Abstract--The long-range program of gravity measurements at sea has been continued on board United States Navy submarines. Improvements in the Vening Meinesz apparatus have included improved pendulum lifts, self-aligning gimbal bearings, electric motor drive of photographic paper, and 100 eclipse markings per second on the pendulum records. During 1948 and 1949, gravity observations were made as follows: 180 on U.S.S. Argonaut in the Bahama Island area, 118 on U.S.S. Medregal along the east coast of the United States between Key West and Cape Hatteras, 260 on U.S.S. Capitaine on a route from San Diego, California, to Brisbane, Australia, to Japan, to Hawaii, 103 on U.S.S. Bergall between Hawaii, Australia, and Guam, 72 on U.S.S. Chopper along the west coast of Mexico, and 142 on U.S.S. Baya along the west coast of Canada, south coast of Alaska, across the Aleutian Islands chain, and in the Bering and Chukchee Seas.

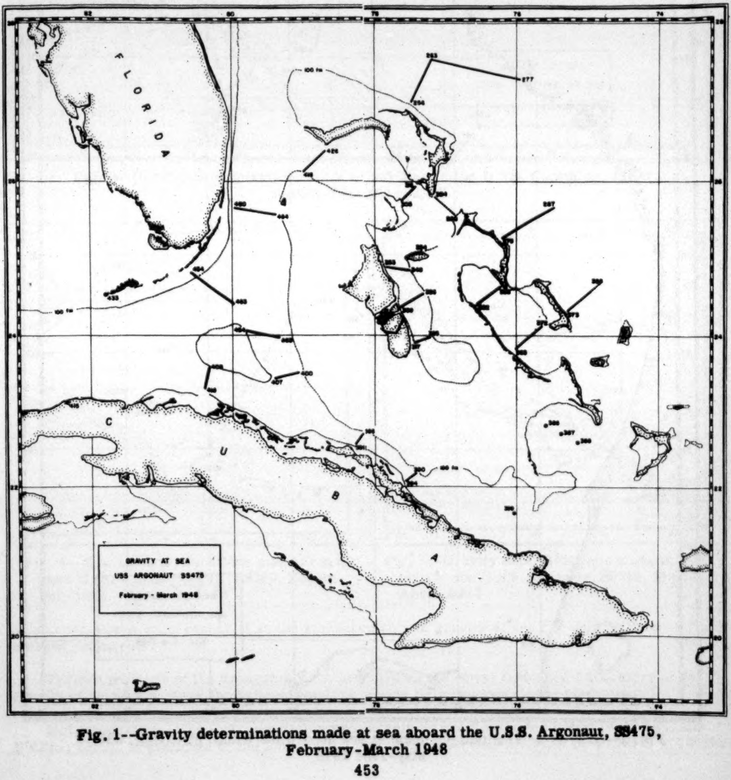
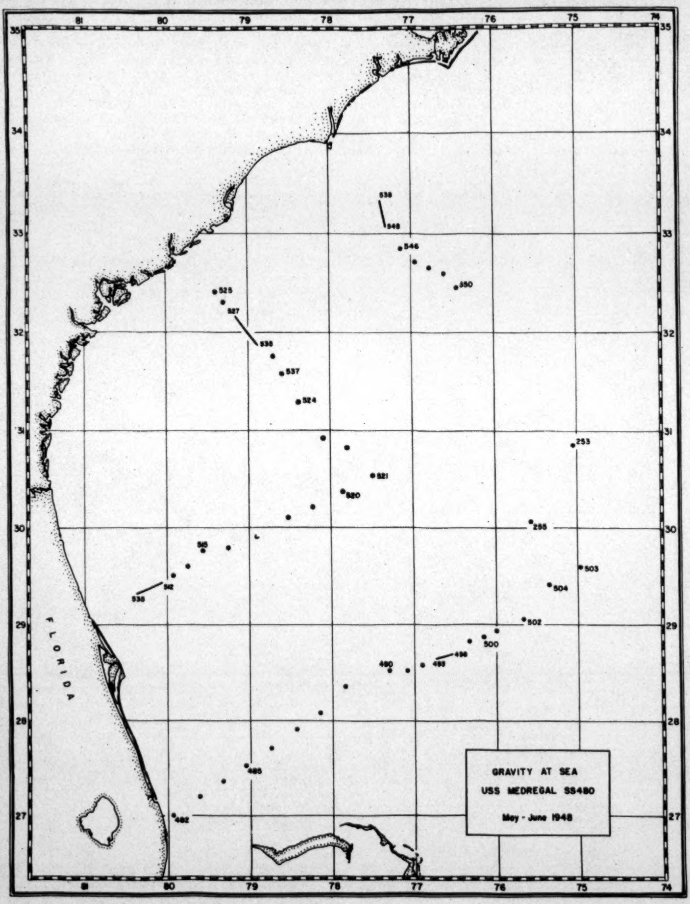


Fig. 1--Gravity determinations made at sea aboard the U.S.S. Argonaut, SS475, February-March 1948

Introduction--The program of gravity measurements at sea aboard United States Navy submarines begun in 1947 [WORZEL and EWING, 1950] were continued in 1948 and 1949 by graduate students of the Geophysical Laboratory, Department of Geology, Columbia University. This work was carried out under the auspices of the Office of Naval Research, Navy Department.

The arrangements for the cruises were made with the assistance of Commander, Submarine Force, U. S. Atlantic Fleet, Commander, Submarine Force, U. S. Pacific Fleet, and the Chief of Naval Operations. The officers and men of U.S S. Argonaut, Medregal, Capitaine, Bergall, Chopper, and Baya have all fully cooperated to make the work a success. The services provided by the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FLORIDA
GRAVITY AT SEA
USS MEDREGAL SS480
May - June 1948

Fig. 2--Gravity determinations made at sea aboard the U.S.S. Medregal, SS480, May-June 1948

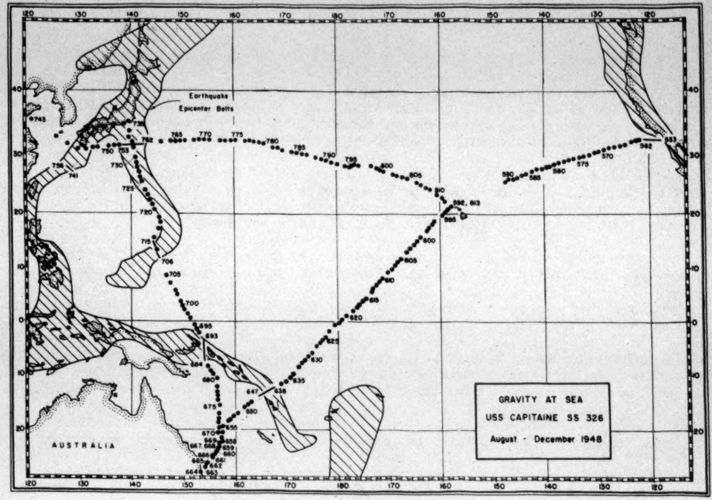

Earthquake
Epicenter Belts
AUSTRALIA
GRAVITY AT SEA
USS CAPITAINE SS 326
August - December 1948

Fig. 3--Gravity determinations made at sea aboard the U.S.S. Capitaine, SS326, August-December, 1948

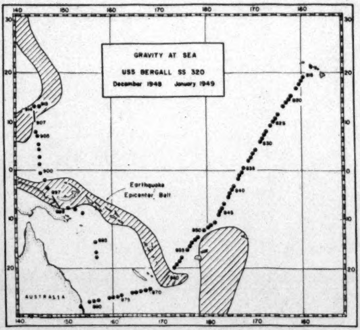

GRAVITY AT SEA
USS BERGALL SS 320
December 1948 January 1949
Earthquake
Epicenter Belt
AUSTRALIA

Fig. 4--Gravity determinations made at sea, aboard the U.S.S. Bergall, SS320, December, 1948 - January, 1949

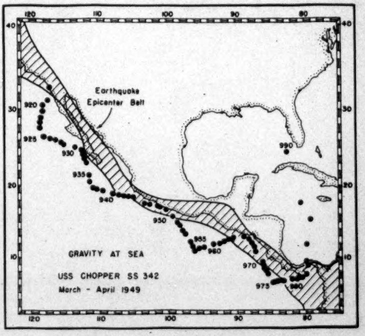

Earthquake
Epicenter Belt
GRAVITY AT SEA
USS CHOPPER SS 342
March - April 1949

Fig. 5--Gravity determinations made at sea aboard the U.S.S. Chopper, SS342, March-April, 1949

Submarine Force were essential as the measurements of gravity at sea can only be done on a submerged submarine.

Various portions of the apparatus were provided by the Naval Ordnance Laboratory of the Bureau of Ordnance, Navy Department, and the Woods Hole Oceanographic Institution.

Instrumentation--The instrumentation was largely as previously described [WORZEL and EWING, 1950]. A duplicate of the long-period pendulum apparatus designed by VENING MEINESZ

[1941] was constructed, and the original apparatus generously loaned by Vening Meinesz for the early work was returned to him.

At the beginning of 1948 the lifting devices for the main pendulums were revised. The old lifts had not been operating satisfactorily and had caused some movement of the pendulums each time the lifts were operated. An attempt to replace the spring-wire portion of the fast lifts showed the desirability of providing adjustments for each lift individually. The old lifts were marred in numerous places, showing that the pendulums had been lifted and set down from different parts of the flats on which they swing. The slow lifts were so badly damaged that it was necessary to cut them off and make end inserts to replace the old V's. Two adjustments were put on both the slow and the fast lifts: one to adjust elevation, and one to adjust horizontally in a direction perpendicular to the knife edge. With these new adjustments it is possible to insert a set of pendulums and adjust them to be laid down simultaneously, parallel, and in the same place on repeated operations, without the assistance of an instrument maker.

In 1947 it had been necessary to harden and grind the knife edges on the gimbal frame as excessive wear was apparent. However, examination of the records showed that there was still excessive friction at these knife edges. It was decided that this friction was caused by a lack of collinearity of the two opposite sides of the gimbal frame. All attempts to make these knife edges collinear were unsuccessful as the gimbal frame would not maintain its shape. This difficulty was finally eliminated by replacing the gimbal knife edges with self-aligning bearings, which permit free motion even though the bearing shafts are not collinear. The effectiveness of these bearings is attested by the lack of any alternating motion on the β trace since their installation. In fact, it is often necessary to artificially disturb the apparatus to convince the observer that the β trace is operating properly.

In the spring of 1949 the spring-wound motor which moves the photographic paper was replaced with a synchronous motor. This furnishes a more reliable drive, simplifies the operation of the equipment, and allows a complete record to be taken for any desired length of time. We had abandoned the fast-slow-fast paper-drive system used by Vening Meinesz from our first observations in 1947 in favor of a continuous record at fast speed. This is advantageous to observe all the correction traces in full detail throughout the observation, but had crowded the record somewhat due to the limited power available on one winding. With the electric motor the crowding has been eliminated and the records are still of a suitable length (about eight feet) to be easily developed after each observation.

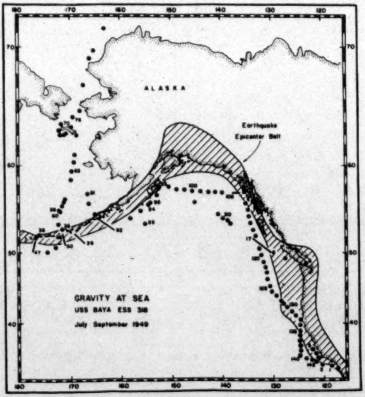


Fig. 6--Gravity determinations made at sea aboard the U.S.S. Baya, ESS318, July-September, 1949

In late 1949 the synchronous motor operated by the crystal chronometer had to be rebuilt as the bearings had begun to show signs of serious wear. It was decided that the number of interruptions of the light beam would be increased from 20/sec to 100/sec and that each interruption would be made by the same spoke of a wheel rotating 100 cycles/sec to eliminate any abnormalities of construction. A second wheel interrupts the light beam for a slightly longer interval each time ten marks have been inserted by the first wheel. Phasing is accomplished by dimming the light beam with a shutter for 1/2 sec out of each second, which is controlled by the synchronous motor, as is total interruption of the light beam for one second each minute. This latter operation permits the determination of the number of swings at any point along the record.

Cruises--Aboard U.S.S. Argonaut, 180 gravity observations were made in the deep-water passages in the Bahamas and around the margins of the Bahamas (Fig. 1) by J. L. Worzel, M. Ewing, and C. L. Drake. These observations were made transverse to the linear features evident in this area. On the traverses across the deep-water passages, one observation was commenced as quickly after the termination of a previous one as possible.

Oil companies have made numerous gravity observations in the shoal water areas, and several have promised to make a grid of observations available to supplement our observations in the deep water. Some of these have already been received. The gravity coverage of this area should make the interpretation of this remarkable topography more significant and should indicate whether this is only superficial evidence of more deep-seated and fundamental structure.

Aboard U.S.S. Medregal, 118 observations were made along the coast of the southeastern United States (Fig. 2) from Key West to Cape Hatteras by C. L. Drake. These observations, coupled with those on land of G. P. Woollard, the sea observations of 1947 along the northeast coast of the United States, and the observations in the Bahama Islands, make this the most thoroughly surveyed coastal area in the world. It is anticipated that these observations will provide fundamental information on the structure of the Blake Plateau and the relation of this type of continental margin to the ocean basins.

Aboard U.S.S. Capitaine, 260 observations were made over a large part of the Pacific Ocean (Fig. 3) by C. L. Drake. This network provided six crossings or partial crossings of the earthquake epicenter belts. The epicenter belts were sketched using the figures of "Seismicity of the Earth" [GUTENBERG and RICHTER, 1949] as a guide and enclosing the majority of the epicenters. Topographic troughs exist at most of these crossings, and it is expected that these observations will show the degree of similarity of these features to those of the Dutch East Indies so thoroughly studied by VENING MEINESZ and Others [1934]. The remaining observations were made for a broad coverage of the oceans as a part of the network required for a precise determination of the figure of the Earth.

Aboard U.S.S. Bergall, 103 gravity observations were made over another part of the Pacific Ocean (Fig. 4) by J. G. Heacock, Jr. Three more crossings or partial crossings of earthquake epicenter belts were made as well as further broad oceanic coverage for studies of the figure of the Earth. It is hoped that these last two cruises may shed some light on the nature of the discontinuity known as the Andesite Line.

Aboard U.S.S. Chopper, 72 gravity observations were made along the west coast of Mexico (Fig. 5) by J. G. Heacock, Jr. Limited time reduced the observation program somewhat and caused the oblique coverage of the earthquake epicenter belts. Nevertheless, five crossings or partial crossings of this belt were accomplished, which may serve to evaluate the theoretical computations for this area [GUNN, 1947].

Aboard U.S.S. Baya, 142 gravity observations were made along the west coast of Alaska, across the Aleutian Island chain, in the Bering Sea, and in the Chukchee Sea (Fig. 6) by C. L. Drake. Seven crossings or partial crossings of the earthquake epicenter belt were made with additional broad coverages for studies of the figure of the Earth. These data will determine whether a gravity minimum belt exists in association with the island arc, topographic trough, and earthquake epicenter belt here as it does in the Dutch East Indies [VENING MEINESZ and Others, 1934] and in the West Indies [EWING, 1937].

Figure 7 shows the overall coverage of gravity in the Pacific Ocean by this group. A large part of the circum-Pacific earthquake belt has been covered in great detail in some places and to a lesser degree in others. Some broad coverage of the Pacific Ocean has been achieved.

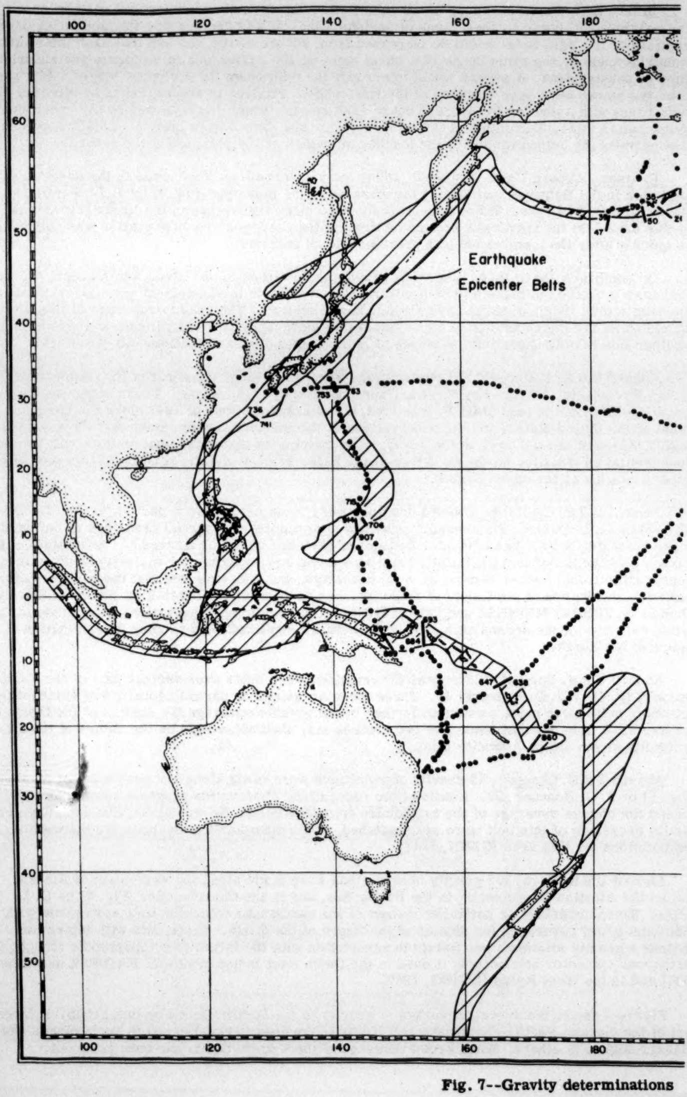
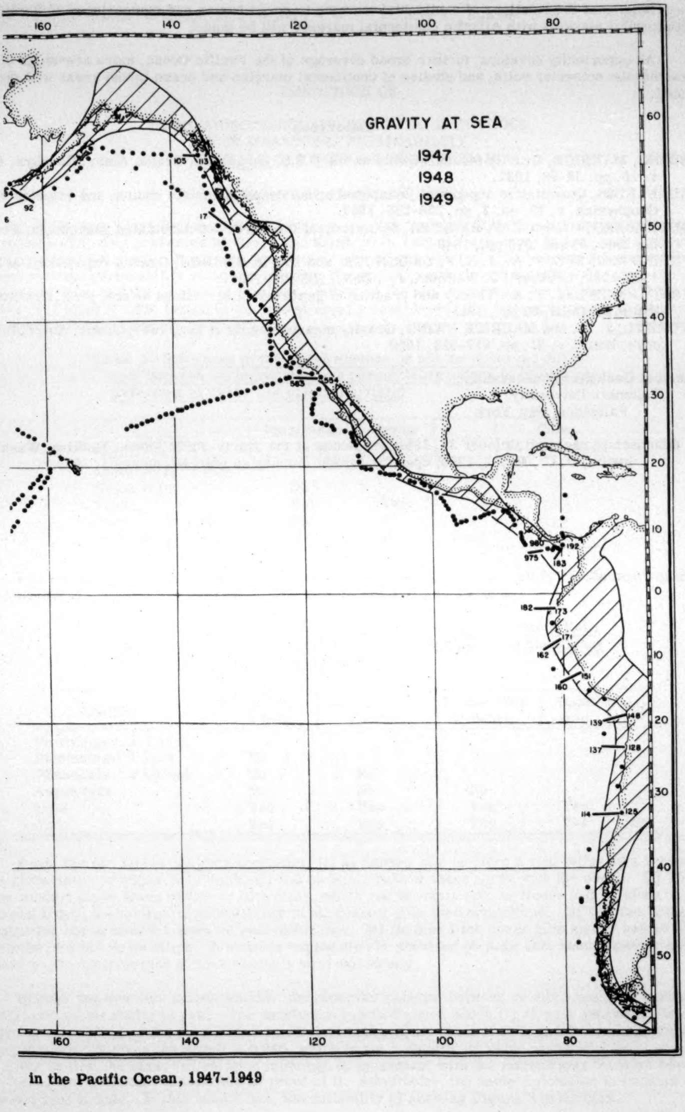

100
120
140
160
180
60
50
40
30
20
10
0
10
20
30
40
50
Earthquake
Epicenter Belts
753
63
736
741
715
704
907
697
693
684
689
647
638
650
659
39
35
30
50
21
47
40

Fig. 7--Gravity determinations


GRAVITY AT SEA
1947
1948
1949
160
140
120
100
80
60
50
40
30
20
10
0
10
20
30
40
50
105
113
92
6
17
563
554
980
975
192
183
182
173
171
162
151
160
139
148
137
128
114
125

in the Pacific Ocean, 1947-1949

Studies of the relations of continental margins to ocean basins and comparisons of Pacific continental margins with Atlantic continental margins will be made.

As opportunity develops, further broad coverage of the Pacific Ocean, more crossings of the earthquake epicenter belts, and studies of continental margins and ocean island areas will proceed.

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(Manuscript received October 17, 1951; presented at the Thirty-First Annual Meeting, Washington, D. C , May 1, 1950; open for formal discussion until November 1, 1952.)

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

DISCUSSION OF

"SOIL ANISOTROPY AND SOME FIELD METHODS FOR MEASURING PERMEABILITY

by RONALD C. REEVE and DON KIRKHAM

[Trans., v. 32, pp. 582-590, 1951]

C. H. M. van Bavel (Department of Agronomy, North Carolina State College, Raleigh, North Carolina)--The data presented by Reeve and Kirkham in Tables 1 and 2 can be analyzed further with statistical methods. First, the question may be raised which of the observed differences between average permeability values as found by the various methods are significant in view of the stated variance and the number of samples. In testing the null hypothesis for the data of Table 1 at the 19:1 level of odds (which is the lowest usually considered), the following result is obtained for each of the ten possible comparisons.

Table 3--Statement to the effect whether or not the observed difference between measured average permeability values is significant (referring to Table 1 of original article)

Method	Piezometer 1 inch	Auger hole	Tube	Core 14-16 inches
Piezometer, 1 inch				
Auger hole	No			
Tube	Yes	Yes		
Core, 14-16 inches	No	No	Yes	
Core, 20-22 inches	No	No	Yes	No

For the data presented in Table 2 by Reeve and Kirkham, a similar study can be made of the 15 possible comparisons of averages. The results are given in Table 4.

Table 4--Statement to the effect whether or not the observed difference between measured average permeability values is significant (referring to Table 2 of original article)

Method	Piezometer 1/2 inch	Piezometer 1 inch	Piezometer 2 inches	Auger hole	Tube
Piezometer, 1/2 inch					
Piezometer, 1 inch	No				
Piezometer, 2 inches	No	No			
Auger hole	No	No	No		
Tube	Yes	Yes	Yes	Yes	
Core	Yes	Yes	Yes	Yes	No

From the two tables one may conclude: (1) At neither site is there a real difference between the piezometer or auger-hole methods, and on Site 1 both of these agree with the cores. (2) The tube method gives lower values at both sites, which can be explained, as Reeve and Kirkham do, by anisotropy, the horizontal permeability being greater than the vertical one. (3) The two sampling depths for cores at Site 1 show no real difference. (4) On Site 1 the cores give higher values than the tube, but not so on Site 2. A suitable explanation is provided on page 588, second paragraph, based on the interference of root channels with anisotropy.

Second, the question arises whether the observed relation between relative length of cavity and measured permeability is real. The conclusion reached above under (1) already throws doubt on this. A calculation of the correlation coefficient between the value of R/L and the average value for the permeability gives the result − 0.840, which is not a significant value at the 19:1 level of odds. In other words, the observed relation, although in agreement with the meritorious theorem advanced by Reeve and Kirkham, constitutes no proof of it. Admittedly, the same conclusion is implied in the original article. In this connection, the suitability of showing Figure 3 is doubted.

It would seem that the discussion presented by Reeve and Kirkham would be more pointed and that the data from the two experimental sites would seem to be in better harmony if only statistically significant effects were being considered.

It has been clearly brought out by Reeve and Kirkham that the core method has a much greater inherent variability, and this agrees with their discussion of the "sample" size. The inference might have been to use a larger number of samples in the core method for intercomparison at different depths and for comparison with the other methods. Efficient design of experiments should strive to obtain nearly equal experimental errors for treatments that are to be compared.

Ronald C. Reeve and Don Kirkham (U. S. Salinity Laboratory, Riverside, California, and Departments of Agronomy and Physics, Iowa State College, Ames, Iowa, Authors' closure)--The authors are pleased by the reviewer's interest in this paper. The statistical analysis of the data and attendant discussion provide a view of the subject from a different angle than our own, which helps to clarify certain points. Most of the questions raised and conclusions stated by the reviewer, however, are already in our paper. One conclusion relating to experimental proof of a theorem, which many may infer to be in our paper and which is subjected to criticism by the reviewer, actually is not in our paper at all.

In our paper, emphasis is placed on physical concepts and the effect of factors which influence permeability measurements in soils. Our paper was partly theoretical, partly experimental. We did not feel our data extensive enough (see first statement of our conclusion, p. 589) to stress a statistical viewpoint, though we did supply pertinent data for those interested in further statistical analysis. On page 586, in the second line of the discussion, we said ". . . the piezometer and core methods yield essentially equal values"; and on page 585, line 6, "The auger-hole value 3.99 is approximately the same as the average piezometer value . . .," etc. The reviewer's tables put it this way: These values are not significantly different at the 19:1 level of odds.

Although we knew certain of our comparisons were not "statistically" significant, and, at least when considered alone, should be discounted, we saw no reason for discounting certain other comparisons (see, for example, p. 586, line 10, of the Discussion). Taken together, we felt our data showed that soil anisotropy was a primary physical factor operating to cause differences between several methods for measuring soil permeability. The reviewer concludes on the basis of his tables that the soil was anisotropic.

Inasmuch as the question of statistical significance has been raised, it seems worthwhile to point out that the "yes" differences in Table 4 are significant not only at the 19 to 1 level of odds, which, quoting him, "are the lowest usually considered." In addition, the "yes" differences in Table 4 are also all highly significant at the 99 to 1 level of odds, the highest usually considered. There is an error in the reviewer's discussion in his third-to-last paragraph. With 16 degrees of freedom a correlation coefficient of -0.84 is highly significant. Actually the correlation coefficient in this case should be -0.25, which is in agreement with the rest of the reviewer's sentence.

A point in the reviewer's discussion which concerns us relates to our use of Figure 3. Many readers, we fear, may infer from the reviewer's remarks, that we have concluded Figure 3 to be proof for a theorem we presented.

We thought it was made clear why we presented the curve of Figure 3. In the first place, we said (first paragraph, p. 588), "This (apparent linear relation of Fig. 3) in view of the scatter of the data must be considered fortuitous." We went on to say ". . . the curve is presented just as it fell for it pointed up to the writers, and illustrates an important theoretical consideration (theorem). This consideration . . . depends on Darcy's law but is independent of any of the present experimental data, . . ." We said that, theoretically, the curve of Figure 3 should have a negative slope, but that it was not necessarily a straight line. In the title of our Figure 3 we designated the linearity "apparent." It would seem from the above that our use of Figure 3 was not intended as a proof of the theorem.

With regard to the reviewer's last paragraph, we agree that it is desirable to keep the variations attributable to experimental error at a minimum in order to evaluate differences between treatments being tested. This is especially important where differences are not large. But we point out that the experimental determinations must first be made before statistical theory can be used to calculate how many determinations might best have been made.

Transactions, American Geophysical Union Volume 33, Number 3 June 1952

REVIEWS, ABSTRACTS, AND BIBLIOGRAPHY

REVIEWS AND ABSTRACTS

KING, PHILIP B., The Tectonics of Middle North America, Princeton University Press, Princeton, N. J., 203 pp., 52 illus. in text, 1951.

Whether he is engaged in prospecting or in the theoretical aspects of the physics of the Earth, the geophysicist must concern himself with the realities of geological structure. The prospecting geophysicist, whose immediate task refers to local detail of structure, must see it in the perspective of the larger regional picture to do it justice. For basic theory, a realistic comprehension of the facts on which the geologist's generalizations are based is fundamental. Unfortunately, the systematic synthesis of regional geological knowledge requires a combination of temperament, training, and freedom of action which is rare among geologists. Hence, critical syntheses lag far behind the growth of factual knowledge. The publication of a new competent summary such as Philip B. King's, The Tectonics of Middle North America, is therefore an important event.

Several features make this work especially valuable to the geophysicist. (1) It places, ahead of all others, references to basic summary maps -- geologic and tectonic -- which form the foundation on which the whole text rests. The reader who does not compare the cross sections and text constantly with the map data builds a house without foundation. (2) All text figures have been redrafted from the original sources, omitting distracting detail and supplying new data where necessary. This makes especially the cross sections directly comparable. For the regions of low relief (Interior Lowland, Coastal Plain), all sections are drawn on the same scale and with the same vertical exaggeration. In regions of higher relief, the sections are drawn without vertical exaggeration, with scales indicated. Thus the purpose of the work, to provide a basis for "comparative tectonics," is effectively fulfilled. (3) The text is completely up-to-date, with each chapter documented with references to all pertinent recent literature. (4) The text is delightfully concise -- setting a new standard in American regional geological literature.

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NOTES AND PERSONALIA

NOTES

Geophysical meeting in Hamburg, Germany--The German Geophysical Association (Deutsche Geophysikalische Gesellschaft) and the Meteorological Association of Hamburg will hold a joint meeting in Hamburg, August 24 to 30, 1952. The first three days will be devoted to sessions in meteorology. August 27 will include a morning session on the theme Geophysikalische Wissenschaften im Dienst der Schiffahrt with A. Defant of Innsbruck as moderator. A tour of Hamburg harbor and the lower Elbe will be made in the afternoon. The Geophysical Association will hold its meetings from August 28 to 30, 1952.

Fellowships of the National Science Foundation--The National Science Foundation announces the awarding of 569 predoctoral and 55 postdoctoral fellowships for the academic year 1952-1953. Of the 569 predoctoral fellowships, 169 were awarded to first-year graduate students. There were about 3000 applications. Announcements concerning fellowships for 1953-1954 will be made about October 1, 1952. Application forms will not be available until that time.

Seventeenth International Geographical Congress to be held in Washington--The International Geographical Union will convene in Washington, D. C., August 8-15, 1952. It is anticipated that representatives of more than 60 countries will be in attendance. Two pre- and three post-congress excursions are being planned. Interested persons should write to the Chairman, National Committee of the United States, IGU, National Research Council, 2101 Constitution Avenue, Washington 25, D. C.

New publication on opportunities for employment of geophysicists--A new publication entitled Employment Outlook for Earth Scientists has recently been issued by the Bureau of Labor Statistics. This is a 38-page booklet describing various fields of geophysics and other of the Earth sciences, and the opportunities for work in these fields, and is intended chiefly for vocational guidance counsellors and high school students. Copies are available through the Superintendent of Documents, Government Printing Office, Washington 25, D. C., for 30 cents.

Four Corners Geological Society to publish symposium--The Four Corners Geological Society, embracing Arizona, Colorado, New Mexico, and Utah, plans to publish a symposium of the structure and geology of the Four Corners region in the fall of 1952. Sherman A. Wengerd, Associate Professor of Geology at the University of New Mexico, is editor. Inquiries should be addressed to Silas Brown, President of the Society, in Durango, Colorado.

Meeting of the International Association for Hydraulic Research--The International Association for Hydraulic Research will hold a meeting in Minneapolis, Minnesota, during the week beginning August 30, 1953. The Association will be joined in this meeting with the Section of Hydrology of the AGU and the Hydraulics Division of the American Society of Civil Engineers. Interested persons should write to Lorenz G. Straub, who is the President of the Association, at St. Anthony Falls Hydraulic Laboratory, Minneapolis 14, Minnesota.

Research grant and fellowship program of the Shell Oil Company--The program of research grants and graduate fellowships of the Shell Oil Company is being continued and augmented for the academic year 1952-53. A total of 12 grants of $5000 each has been made to eight universities to assist them in basic research in the physical sciences. Forty-five graduate fellowships are being awarded, with preference given to those in the final year of doctorate study. Interested persons should write to the Shell Fellowship Committee, Shell Oil Company, 50 West 50th St., New York 20, N. Y.

New journal in statistics--A new journal, Applied Statistics, sponsored by the Royal Statistical Society and edited by L. H. C. Tippett, is being published three times a year in Great Britain, the first number appearing in March, 1952. Interested persons should get in touch with Oliver and Boyd, Lt., Tweedale Court, Hight Street, Edinburgh 1, Scotland.

Anthracite Conference held at Lehigh University--The Tenth Annual Anthracite Conference was held at Lehigh University, May 8-9, 1952, with over 200 in attendance. The Conference dealt with problems of smoke abatement, new uses of anthracite such as the drying of alfalfa, the use of anthracite ash, and other problems of the industry.

PERSONALIA

Carl I. Aslakson, Commander, U. S. Coast and Geodetic Survey, on special assignment with the Air Force on electronic measurement of distances, has been awarded the Exceptional Service Gold Medal of the Department of Commerce.

Henry B. Bigelow, Curator of Oceanography, Museum of Comparative Zoology, Harvard University, was recently awarded the Daniel Giraud Elliot Medal by the National Academy of Sciences. This Medal is awarded for meritorious work in zoology or paleontology.

Mills E. Bunger has taken up duties as Director of Water Resource Development for the Hashemite Kingdom of Jordan at Amman.

Horace R. Byers, Professor of Meteorology at the University of Chicago and Past President, Section of Meteorology, AGU, has been elected President of the American Meteorological Society. He was also recently elected to membership in the National Academy of Sciences.

Edward A. Colman, Chief of the Division of Forest and Range Influences of the California Experiment Station of the U. S. Forest Service, has undertaken an assignment with the Conservation Foundation to prepare a book on land use and water yield.

Carroll E. Dobbin, Geologist of the U. S. Geological Survey with headquarters in Denver, was recently given an honorary degree of Doctor of Engineering by Colorado School of Mines.

David T. Griggs, Chief Scientist with the U. S. Air Force, and Professor of Geophysics, Institute of Geophysics, University of California at Los Angeles, was recently elected to membership in the National Academy of Sciences.

Henry W. Hemple, Captain, U. S. Coast and Geodetic Survey, serving as Chief of Division of Geodesy of the Survey, has been awarded the Exceptional Service Gold Medal of the Department of Commerce.

Harry H. Hess, Professor of Geology at Princeton University and President of the Section of Geodesy, AGU, has recently been elected to membership in the National Academy of Sciences.

Richard David Hughes, formerly with Socony-Vacuum Exploration Co., is forming a geological consulting firm in association with D. Bruce Bullock. The firm will specialize in the surface and subsurface geology of northeastern British Columbia.

Charles R. Hursh of the Southeastern Forest Experiment Station is now in British East Africa on a teaching and research mission under the Fulbright Act.

Ade E. Jaskar has accepted a position as soils engineer with Knappen, Tippetts, Abbett, and McCarthy, Consulting Engineers, and is en route for duty in Libya.

Robert T. Knapp, Professor of Hydraulic Engineering at California Institute of Technology, has been invited to deliver the annual James Clayton Lecture before the Institution of Mechanical

Engineers in London. The topic of his address is Cavitation Mechanics and its Relation to the Design of Hydraulic Equipment. He will visit various hydraulic laboratories in Great Britain and the Continent before returning.

Charles J. Kraebel of the U. S. Forest Service, with headquarters in Berkeley, California, is on special assignment with the Military Hydrology Section of the U. S. Geological Survey.

R. M. Leggette and M. L. Brashears have joined in a partnership under the name of Leggette and Brashears, in New York City, for consulting work in connection with ground-water problems.

Howard W. Lull is now in charge of the project of the Forest Service, cooperating with the Corps of Engineers, on problems of soil-moisture prediction, with headquarters at Vicksburg, Mississippi. This was formerly under the direction of Edward J. Dortignac, who is now in charge of Upper Rio Grande Research Center of the Southwestern Forest and Range Experiment Station with headquarters at Albuquerque.

John Lyman has been appointed Director of the Division of Oceanography of the U. S. Navy Hydrographic Office.

Harry A. Marmer, Physical Science Administrator and Assistant Chief of the Division of Tides and Currents, U. S. Coast and Geodetic Survey, was recently awarded the Alexander Agassiz Medal by the National Academy of Sciences. This is awarded for outstanding original contributions in oceanography.

Frank A. Melton, Professor of Geology and Aerogeology, University of Oklahoma, was recently given the Talbert Abrams Award by the American Society of Photogrammetry for his paper, The Geomorphology and Photo-Geological Study of the Flat-Lands.

Alf Nyberg will be Visiting Professor of Meteorology at Florida State University for the academic year 1952-53. He will be on leave from his post as Chief of the Aerology Section of the Swedish Meteorological and Hydrological Institute, and from his position on the faculty of the University of Stockholm.

A. E. Pallister has accepted a position as geophysicist with the Sub-Surface Exploration Ltd. with headquarters in Calgary, Alberta.

Chaim Pekeris, member of the Institute for Advanced Study at Princeton, currently on work in Israel, has recently been elected to membership in the National Academy of Sciences.

Aaron L. Shalowitz, Special Assistant to the Director, U. S. Coast and Geodetic Survey, has been awarded the Exceptional Service Gold Medal of the Department of Commerce.

Lyman Spitzer, Jr., Director of the Princeton University Observatory, has recently been elected to membership in the National Academy of Sciences.

Harald U. Sverdrup, Director of the Norwegian Polar Institute, was recently elected a Foreign Associate of the National Academy of Sciences.

George D. Whitmore of the U. S. Geological Survey has been elected President of the American Society of Photogrammetry.

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THIRTY-THIRD ANNUAL MEETING (1952)

MINUTES, GENERAL ASSEMBLY AND SECTION MEETINGS

EVENING SESSION

The Evening Session of the Thirty-Third Annual Meeting was called to order by President Walter H. Bucher at 20h 05m, Monday, May 5, 1952, in the Lecture Room of the National Academy of Sciences, 2101 Constitution Avenue, Washington, D. C. About 200 were in attendance. After preliminary and welcoming remarks, President Bucher introduced John Putnam Marble, Chairman of the Committee on Meetings. He introduced R. A. Alpher and R. C. Herman of the Applied Physics Laboratory of Johns Hopkins University, who spoke on The Origin of the Elements [see abstract in Program, p. 313].

Following these addresses, President Bucher thanked the speakers and called on Walter D. Lambert, Chairman of the Committee on the Award of the William Bowie Medal, to present the citation of Harold Jeffreys of the Cambridge University, England, as the Fourteenth Recipient of the Medal. President Bucher then made the presentation to J. Roger Carter, First Secretary and Education Officer of the British Embassy, who received the Medal on behalf of Professor Jeffreys and presented his acceptance. These proceedings are set forth in the opening pages [pp. 337-340] of this issue.

The session was then adjourned, and was followed by an informal social hour, with refreshments, in the Auditorium.

SMOKER

A Smoker was held in the Auditorium of the Cosmos Club on Tuesday, May 6, 1952, from 20h 00m to 22h 30m. About 175 were in attendance. This was informal. There was no program, but there was opportunity to renew old acquaintances and friendships and make new ones. Full advantage was taken of these opportunities, and there were many lively discussions.

GENERAL ASSEMBLY, LECTURE AND BUSINESS MEETING

The General Assembly was called to order by President Bucher at 14h 05m, Wednesday, May 7, 1952, in the Auditorium of the National Academy of Sciences. About 125 were in attendance. The President introduced Ellis A. Johnson, Director, Operations Research Office, Johns Hopkins University, who addressed the Assembly on the subject, Experimental and Theoretical Operations Research, Its Applications in Combat, Industry, and Science [see Program, p. 313, for abstract].

Following the address, there were questions and considerable discussion from the floor. The President then thanked the speaker and called the Business Session to order. The minutes of the last meeting [see Trans., v. 32, pp. 487-489] were approved.

Report of the General Secretary--General Secretary Beij summarized the activities of the Union since the Thirty-Second Annual Meeting. These are reported in connection with the Minutes of the Meetings of the Executive Committee of October 15, 1951 [see Trans., Dec., 1951, pp. 934-940] and May 7, 1952 [to be published in the August, 1952, issue of the Transactions].

A recapitulation of the changes in membership since April 20, 1951, is as follows:

Membership, April 20, 1951	4740	
New members, April 20, 1951, to April 30, 1952	416	
Reactivated	5	5161
Resignations	184	
Deceased	17	
Inactive	14	
Dropped	171	386
Membership, April 30, 1952		4775

Reports of Standing Committees

Meetings--Chairman J. Putnam Marble of the Committee on Meetings announced the dates for the Thirty-Fourth Annual Meeting as May 4-6, 1953.

Reports of other standing committees were not presented, and will be published in the Transactions.

Reports of Special Committees

Resolutions--Chairman Mary C. Rabbitt presented seven resolutions, which were unanimously adopted as follows:

(1) RESOLUTION OF THANKS FOR THE USE OF FACILITIES

WHEREAS, The National Academy of Sciences and the United States Public Buildings Administration have provided the use of meeting rooms and other services for the Thirty-Third Annual Meeting of the American Geophysical Union without cost to the Union, therefore be it

RESOLVED, That the American Geophysical Union express its great appreciation of this cooperation, extended over many years, and instructs the General Secretary to express the thanks of the Union to the appropriate officials.

(2) RESOLUTION OF THANKS TO THE CALIFORNIA INSTITUTE OF TECHNOLOGY

WHEREAS, The California Institute of Technology generously provided space and other facilities which greatly contributed to the success of the Twenty-Seventh Regional Meeting at Pasadena, California, February 8-9, 1952, therefore be it

RESOLVED, That the thanks of the American Geophysical Union be extended to the California Institute of Technology, and instructs the General Secretary to forward a copy of this resolution to Dr. L. A. DuBridge, President of the Institute.

(3) RESOLUTION OF THANKS TO THE CARNEGIE INSTITUTION OF WASHINGTON

WHEREAS, The Carnegie Institution of Washington has for many years provided office space for the American Geophysical Union, without which the Union could not adequately function, therefore be it

RESOLVED, That the American Geophysical Union express its deep appreciation of this continued assistance and instructs the General Secretary to express its thanks to the appropriate officers of the Carnegie Institution of Washington.

(4) RESOLUTION ON COMMUNICATIONS SERVICES

WHEREAS, The Secretary of Commerce, in cooperation with the Foreign Service Office of the United States, the officers of the Foreign Service, and the Foreign Service Operations Staff of the Office of International Trade, have made available their special telegraphic facilities for transmitting teleseismic data to the United States Coast and Geodetic Survey, and

WHEREAS, The data have greatly accelerated and expanded the earthquake location program of that Survey, and

WHEREAS, The data have been made available by the Survey to seismologists throughout the world to aid in the more thorough analysis of their records, therefore be it

RESOLVED, That the American Geophysical Union express its appreciation for this valuable contribution to seismological research, and be it further

RESOLVED, That copies of this resolution be sent to the Secretary of Commerce, the Foreign Service Office of the United States, the Foreign Service Operations Staff of the Office of International Trade, and officers of the Foreign Service at the following stations: Djakorta, Java; Bombay, India; Brisbane, Australia; Copenhagen, Denmark; Guatemala City, Guatemala; Lima, Peru; Malaga, Spain; Mexico City, Mexico; Perth, Australia; Rome, Italy; Santiago, Chile; Strasbourg, France; Sydney, Australia; Tananarive, Madagascar; Pretoria, South Africa; Wellington, New Zealand; Pretoria, South Africa; and Dublin, Ireland.

(5) RESOLUTION ON TIME-SIGNAL SERVICE

WHEREAS, There is a vital need in all seismological work for accurate time service, and

WHEREAS, The time service maintained by the United States Naval Observatory, the Naval Communications Service, and the National Bureau of Standards has been of great use in this work, therefore be it

RESOLVED, That the American Geophysical Union express its appreciation of the valuable scientific cooperation and reiterate its desire for the continuance of this time-signal service, and be it further

RESOLVED, That copies of this resolution be sent to the Secretary of the Navy, the Director of Naval Communications, the Superintendent of the Naval Observatory, the Secretary of Commerce, and the Director of the National Bureau of Standards.

(6) RESOLUTION ON THE DEATH OF THOMAS WAYLAND VAUGHAN

WHEREAS, In the death of Thomas Wayland Vaughan, one of the great pioneers of oceanography in the United States and one of the world's leading authorities on interrelationships between marine organisms and their environment has been lost, and

WHEREAS, Dr. Vaughan was an active and well-beloved member of the Section of Oceanography of the American Geophysical Union, which he joined in 1921, served as Chairman in 1926-28, and represented as Chairman of the International Commission of the Pacific, Pacific Science Association, from 1926 to 1935, therefore be it

RESOLVED, That the American Geophysical Union hereby express its appreciation of the great and lasting contributions made by the late Thomas Wayland Vaughan to the development of the science of Oceanography and extend its deepest sympathy to his family in their loss.

(7) RESOLUTION ON FINANCIAL SUPPORT OF INTERNATIONAL UNIONS

WHEREAS, Membership in the international scientific unions provides an excellent means for world-wide collaboration among scientists, and

WHEREAS, Such membership and collaboration bring to the United States scientific data and information of great value, and

WHEREAS, The membership dues of these unions (including International Council of Scientific Unions (ICSU) and its associated unions) are in general based on prewar or immediate postwar levels, and

WHEREAS, A statutory limitation of $9000 per year is placed on membership dues to ICSU and its associated unions, therefore be it

RESOLVED, That the American Geophysical Union strongly urge increased financial support to scientific international unions, including ICSU and its affiliated unions, and be it further

RESOLVED, That the American Geophysical Union urge that the statutory limit of $9000 be raised to a more realistic value, $50,000, for example.

Resolution (7) was discussed at length. It was pointed out that this is just an initial step to secure an increase in the income of the various international unions, and, further, that dues of the United States are the same as for France and for Great Britain, and are only about five per cent of the total dues received. The payment of the United States is a much higher percentage for most international bodies.

The meeting adjourned at 16h 00m.

K. HILDING BEIJ, General Secretary
WALDO E. SMITH, Executive Secretary

APPENDIX A--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF GEODESY, AMERICAN GEOPHYSICAL UNION

President Hess called the meeting to order in the Lecture Room of the National Academy of Sciences at 14h 00m on May 6, 1952.

There being no objection from the floor, and no corrections received, the President declared the minutes of the 1951 meeting approved as published in the Transactions [v. 32, pp. 489-490, 1951].

The President announced the appointment by the Executive Committee of a Committee on Nominations for the forthcoming election of officers. Members of this Committee are H. W. Hemple, Chairman; Floyd W. Hough; and George P. Woollard.

W. D. Lambert, C. A. Whitten, and J. A. Duerksen gave informal reports on the General Assembly of the International Union of Geodesy and Geophysics in Brussels in 1951.

There being no further business to transact, the scientific session proceeded as published in the Program [p. 307]. After the fourth paper, the meeting recessed until 09h 10m on the next day.

Papers by J. E. Lilly and P. Brunavs, Arthur S. G. Grant, and Manuel Medina, who could not be present, were read by B. K. Meade, D. A. Rice, and F. L. Culley, respectively.

Approximately 80 members and guests attended the meeting on May 6 and approximately 75 on May 7.

The meeting adjourned at 11h 08m, May 7, 1952.

FRANK L. CULLEY, Secretary

APPENDIX B--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF SEISMOLOGY, AMERICAN GEOPHYSICAL UNION

The Thirty-Third Annual Meeting of the Section of Seismology was held May 6, 1952, in the Lecture Room, National Academy of Sciences, Washington, D. C. The session was called to order at 09h 10m by President Byerly. The minutes of the 1951 meeting were approved as published in the Transactions [v. 32, p. 490, 1951].

President Byerly appointed a Committee on Resolutions consisting of L. M. Murphy (Chairman) and J. B. Macelwane. President Byerly also appointed a Committee on Nominations consisting of V. J. Blum (Chairman), L. M. Murphy, and Frank Press. As there was no further business at this time, the Business Session was adjourned and followed by the Scientific Session shown on the Program [see p. 307].

Following the Scientific Session, L. M. Murphy presented two resolutions for action by the Section. These were approved and forwarded to the Union [see Resolutions (4) and (5) in the Minutes of the Business Meeting of the Union]. The session was adjourned at 11h 30m.

JAMES T. WILSON, Secretary

APPENDIX C--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF METEOROLOGY, AMERICAN GEOPHYSICAL UNION

The Thirty-Third Annual Meeting of the Section of Meteorology was held May 6, 1952, in the Auditoria of the National Academy of Sciences and of the General Services Building. Slightly more than 300 members and guests attended the two sessions of the Section.

The opening business session was held from 09h 00m to 09h 15m, May 6, 1952, with Secretary Woodrow C. Jacobs presiding in the absence of President Benjamin Holzman. Since no corrections were received, the Chairman declared the minutes of the Thirty-Second Annual Meeting of the Section of Meteorology approved as published in the Transactions [v. 32, p. 490, 1951]. The Chairman then appointed a Committee on Resolutions (Conrad P. Mook, Chairman; William Molo; P. R. Stinchfield).

A brief report was given by the Meteorological Editor, Charles S. Gilman. The reports of the Committee on Cloud Physics, Atmospheric Diffusion, Problems of the Upper Atmosphere, and Climatology were read by title.

The Chairman also announced the membership of the Committee on Nominations which had been appointed by the President to serve prior to the elections in 1953. The composition of the Committee is as follows: R. Hanson Weightman, Chairman; Charles F. Brooks; Helmut E. Landsberg.

As there was no other business, the Business Meeting was adjourned and immediately followed by the schedule of scientific papers as shown in the Program [pp. 307-308].

WOODROW C. JACOBS, Secretary

APPENDIX D--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF TERRESTRIAL MAGNETISM AND ELECTRICITY, AMERICAN GEOPHYSICAL UNION

The Thirty-Third Annual Meeting of the Section of Terrestrial Magnetism and Electricity of the American Geophysical Union was held on May 5 and 6, 1952. The first session was called to order at 09h 13m on Monday, May 5, in the Auditorium of the National Academy of Sciences by President J. Wallace Joyce. There were approximately 120 members and guests in attendance. The minutes of the previous meeting were approved as published in the Transactions [v. 32, p. 491, June, 1951].

President Joyce announced that the following had been selected as members of the Committee on Nominations by the Executive Committee: Irwin Roman, Chairman; E. B. Roberts; Chas. R. Burrows; E. H. Vestine; and Joseph Kaplan.

President Joyce appointed the following as members of the Committee on Resolutions to act during this session if the need should arise: A. H. Shapley, Chairman; E. O. Schonstedt; and R. E. Gebhardt.

After routine announcements, the scientific program was begun in accordance with the Program [see pp. 307-309]. The session adjourned at 12h 25m.

The second session was convened at 14h 00m on May 5 in the Auditorium of the General Services Building. After routine announcements the technical program was given as published. This session was adjourned at 16h 30m.

The third session which was held during the morning of May 6 in the Auditorium of the National Academy of Sciences was held jointly with the Section of Meteorology of the American Geophysical Union and the American Meteorological Society. The Section of Terrestrial Magnetism and Electricity obtained D. H. Menzel as a guest speaker for this session. He presented a 40-minute discussion of The Origin and Physical State of the Aurora Borealis. The Section also contributed two other papers to this joint session.

L. R. ALLDREDGE, Secretary

APPENDIX E--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF OCEANOGRAPHY, AMERICAN GEOPHYSICAL UNION

The Thirty-Third Annual Meeting of the Section of Oceanography was held in Washington, May 5 and 6, 1952. The first session convened at 09 00m in the Auditorium of the General Services Building and adjourned at 12h 00m. The second session met at 14h 00m in the Lecture Room of the National Academy of Sciences and adjourned at 17h 00m. President A. C. Redfield presided at both sessions.

In addition to the two formal sessions, a Round Table on Problems of the Gulf Stream System was held in the Reading Room of the National Academy of Sciences on May 6 at 14h 00m. C. O'D. Iselin presided.

As no corrections were offered to the minutes of the previous Annual Meeting published in the Transactions [v. 32, p. 491, June, 1951], they were approved.

The Chairman announced the appointment of the following members to the Committee on Nominations: Thomas S. Austin, Chairman; Henry Stommel; Dayton E. Carritt; Robert O. Reed; and Robert S. Arthur.

A resolution was presented recognizing the death of Thomas Wayland Vaughan, one of the founding members of the Section [see Resolution (6) in the Minutes of the Business Meeting of the Union].

The scientific program of the two sessions consisted of the presentation and discussion of the papers listed in the Program [see pp. 309-310]. Changes in the program were as follows: Papers (1) and (2) were read by the Secretary. During the afternoon meeting an additional paper was presented by Walter H. Munk and Roger R. Revelle entitled "Sea Level and Earth Rotation." Additions to the Round-Table program are as follows:

(19a) JUNE PATTULLO: A 200-meter temperature chart of the western North Pacific
(21a) H. J. McLELLEN, L. LAUZIER, and W. B. BAILEY: Studies of the slope water off the Scotian Shelf
(21b) ILMO HELA: Florida current studies at the University of Miami
(22a) MELVIN STERN: Variation of velocity with depth in oceanic jet streams

RICHARD H. FLEMING, Secretary

APPENDIX F--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF VOLCANOLOGY, AMERICAN GEOPHYSICAL UNION

The Business Meeting of the Section of Volcanology was called to order by President Larsen at 09h 05m, May 5, 1952. The Secretary described the present status of the move to change the Section's name. Despite unanimous approval of the name "Volcanology and Igneous Petrology" by the previous business meeting, everyone he had spoken to after that meeting had expressed strong disapproval. Most of these people had attended the meeting and must have voted in favor of the resolution. J. F. Schairer, the Section's representative in the reorganization discussions at Brussels, later reported that the parent organization was disinterested in any name change for itself, whether this involved addition of either "petrology" or "geochemistry" to its title, and was strongly opposed to deletion of the term "volcanology." This information was presented to the Executive Committee of the Union at the October, 1951, meeting, and the Committee voted that no change in the Section name be made at present.

The members showed no inclination to continue the discussion and there was no further business. The Executive Committee of the Section solicited suggestions for candidates for office and members of a Committee on Nominations.

The technical program, listed elsewhere in this volume [see p. 310], began at 09h 30m and adjourned at 12h 45m.

Since our meeting, President Larsen has named the following to the Committee on Nominations for the Section: Harry H. Hess, Chairman; George T. Faust; Howard A. Powers; Howel Williams; and Hatten S. Yoder, Jr.

FELIX CHAYES, Secretary

APPENDIX G--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF HYDROLOGY, AMERICAN GEOPHYSICAL UNION

President Frank J. Veihmeyer called the meeting to order at 14h 00m, May 5, 1952, and adjourned at 16h 40m. The session conformed with the program [pp. 310-311] except as follows:

(2) Discussion of the Warnick paper by A. T. Lenz was presented by Walter B. Langbein.
(2) Discussion of the Warnick paper by W. T. Wilson was presented by Ralph F. Kresge.
(4) Discussion of the Fry paper by H. G. Wilm was presented by Marvin D. Hoover.
(5) Discussion of the Smallshaw paper by T. W. Kleinsasser was presented by Vance A. Myers.
(5) Discussion of the Smallshaw paper by W. T. Wilson was presented by Ralph F. Kresge.
(6) Paper by S. D. Resnick was presented by Franklin F. Snyder.

President Veihmeyer called the business session to order at 09h 00m on May 6, 1952, and gave a brief report on the Ninth General Assembly of the International Union of Geodesy and Geophysics held in Brussels in 1951. The President discussed the various Sections of the IUGG and their organizations and programs.

Carl F. Izzard, Chairman of the Committee on Awards, announced that the Committee was of the opinion that no paper published in the Transactions during 1950 was sufficiently outstanding to be recommended for an award. The Committee is now reviewing the 1951 papers and will forward appropriate recommendations to the Executive Committee of the Section at a later date. Announcement of the findings will be published in subsequent Transactions.

Vice President Walter B. Langbein discussed Regionalization of the Union. A Chapter has been organized in Hawaii under the Pacific Southwest Region. Surveys have been made in the Northwest, North-Central, New England, and Southeast areas to determine the interest in these localities regarding Regionalization. It is expected that additional Regions of the Union will be established in some of these areas in the near future.

Announcement was made that the lag in publication of papers is decreasing. Papers on Oceanography, Meteorology, and Hydrology are now grouped together in the Transactions. The fact was emphasized that there is no limit on the length of qualified individual papers.

Attention was called to the Centennial of Engineering to be held in Chicago in September, 1952, and to the meeting of the International Association for Hydraulic Research to be held in Minneapolis in August, 1953.

The appointment of William C. Ackermann as Chairman of the Committee on Nominations for the Section of Hydrology was announced. The Committee is composed of Chairmen of the Permanent Research Committees of the Section of Hydrology.

A report was made on the status of the Annotated Bibliography on Hydrology covering American literature during the period 1941-1950. The cost of the Bibliography was underwritten by the Subcommittee on Hydrology of the Federal Inter-Agency River Basin Committee. The material was compiled by Ray K. Linsley and edited by Waldo E. Smith, and the report is to be printed by the Government Printing Office in the near future. Copies will be available for sale to the public through the Superintendent of Documents.

There being no further business, the President turned the meeting over to Walter B. Langbein as Chairman for the scientific session which followed. The session conformed with the Program except as noted below:

(R3) Report by Rhodes was presented by title.
(11) Paper by Nelson, Wilm, and Work was presented by Marvin O. Hoover.
(R4) Report by Peterson was presented by R. F. Hadley.

During the discussion of the Bean paper (10), Bean predicted that the maximum flow of the Columbia River at The Dalles during 1952 would be approximately 700,000 cfs.

The session adjourned at 12h 15m.

The afternoon session of May 6 was called to order by Chairman Albert S. Fry at 14h 00m. The Program was followed with the exceptions noted below:

(16) Paper by Durum was presented by J. G. Connor.
(R6) Report by Croft was presented by title.
(R8) Report by Johnson was presented by title.

The session adjourned at 16h 55m.

Chairman William E. Hiatt called the final session at 9h 20m. The session conformed with the Program except as follows:

(R9) Report by Poland was presented by title.
(24) Paper by Hantush was presented by title.
(25) Paper by Nelson and Thomas was presented by Thomas E. Eakin.
(R10) Report by Harrold was presented by title.
(27) Paper by Schiff was presented by Vaughn E. Hansen.
(R11) Report by Bodman was presented by title.
(28) Paper by Irmay was presented by title.
(R12) Report by Hough was presented by title.
(29) The paper, The Effect of Earthquake Waves on an Artesian Aquifer, by George B. Bradshaw and William W. Donnan, added to the program, was presented by title.

The meeting adjourned at 11h 20m.

RALPH N. WILSON, Acting Secretary

APPENDIX H--MINUTES OF THE ANNUAL MEETING OF 1952, SECTION OF TECTONOPHYSICS, AMERICAN GEOPHYSICAL UNION

The Annual Meeting of the Section of Tectonophysics was called to order by President Robert Balk at 09h 10m, May 7, 1952, in the Lecture Room of the National Academy of Sciences. The President announced the appointment of the Committee on Nominations for officers of the Section for the next triennium consisting of the following members: M. P. Billings, Chairman; Eleanora B. Knopf; A. Nadai; Philip B. King; and Perry Byerly.

There being no further business, the technical session was begun. This conformed to the Program [see p. 312] except as follows: Papers (2) and (3) by Byron K. Thomas and Chester R. Longwell, respectively, were read by title only, the authors having sent their regrets on being unable to attend the meeting. A paper by W. H. Munk and R. R. Revelle, offered for presentation after close of the program printing date was presented by W. H. Munk. The title and abstract of this paper are given herewith.

(6) W. H. Munk and R. R. Revelle (Scripps Institution of Oceanography, La Jolla, Calif.) On the Geophysical Interpretation of Slight Irregularities in the Rotation of the Earth--Astronomic observations have indicated several marked changes in the angular velocity of the Earth during the last century. There is also some indication of changes in the position of the mean annual pole of rotation with respect to the surface of the Earth. The relative magnitudes of these effects indicates that the causes must be nearly symmetrical with regard to the Earth's axis of rotation, for otherwise the shift in the pole would be much larger than has been observed. It is shown that the atmosphere or oceans cannot be responsible for the observed variations. Movements of the Earth's crust and mantle might have an appreciable effect, but it is unlikely that this should be the major cause. Movements in the core could account for the observed changes.

Papers (4) and (5) were then presented and the meeting adjourned at 11h 45m.

WILBUR S. BURBANK, Secretary

CONFERENCE OF THE U. S. NATIONAL COMMISSION FOR UNESCO

The Third National Conference of the U. S. National Commission for UNESCO met in New York from January 27-31, 1952. The American Geophysical Union was represented by Frank Press of Columbia University and Gordon G. Darkenwald of Hunter College. Frank Press reported as follows:

The theme of the Conference was The Citizen and the United Nations, and its purpose was to consider ways of improving our understanding and participation in international affairs through the agencies of the United Nations. The Conference was organized around section meetings and panels, each delegate attending the special section and panel of his choice. At plenary sessions the conclusions of the separate sections and panels were reviewed. I attended the session of the section on Communications in the World Community. I was disappointed at not being able to obtain tickets for the work group on Opportunities for Scientists and Engineers to Contribute to Peace through the United Nations.

The main thoughts I carried away from the meeting were:

(1) The role of the United Nations in preserving peace, freedom, and justice is not fully appreciated by most of us. Lack of factual information about the United Nations and its activities leads to apathy, discouragement, and results in a failure to provide the support of an informed public without which the United Nations cannot succeed.

(2) Foreign peoples have obtained a distorted picture of what Americans, their customs and way of life, are really like. This arises partly from the Hollywood Films shown abroad, partly from the falsification in Russian propaganda, and from press reports of activities contrary to democratic principles which occasionally still occur in this country. The United Nations, as an excellent and trustworthy source of information about its member nations, helps remove some of these distortions. It is up to us, through support of United Nations and through our own contacts with foreigners, to convey to them a true picture of our basic attitudes, ideals, and hopes for a more enlightened world.

SPECIAL ANNOUNCEMENTS

Regional Meeting in Chicago, Illinois, September 11-12, 1952--Plans are progressing well for the Twenty-Eighth Regional Meeting to be held in Chicago, September 11-12, 1952, in connection with the celebration of the Centennial of Engineering. Details regarding this may be secured by writing to the Chairman of the General Committee for the Meeting, W. S. Hamilton, Department of Civil Engineering, Northwestern University, Evanston, Illinois.

Meeting in St. Louis--The American Association for the Advancement of Science with which AGU is now formally affiliated will hold its Annual Meeting of 1952 in St. Louis between Christmas and New Year's Day. In connection with this, a program in geophysics is being developed of which the AGU is co-sponsor. Watch SPECIAL ANNOUNCEMENTS for further information.

Thirty-Fourth Annual Meeting to be held May 4-6, 1953--The Thirty-Fourth Annual Meeting of AGU will be held in Washington, D. C., May 4-6 (Monday-Wednesday), 1953. Mark these dates on your calendar. This is also a first call for titles and abstracts of papers. Forward titles and abstracts as soon as your plans permit. Please send any suggestions for symposia, panel discussions, and other features to Chairman, Committee on Meetings, the appropriate Section Secretary, or the office of AGU. Watch SPECIAL ANNOUNCEMENTS for further information.

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Volume 33 AUGUST 1952 Number 4

PROCESSES OF EMISSION OF THE MOLECULAR AND ATOMIC SPECTRA OF NITROGEN OBSERVED IN THE LIGHT OF THE POLAR AURORA

Renée Herman, Louis Herman, and Georges Hepner

Abstract--New experiments have been performed on the discharge and afterglow spectrum of a nitrogen-helium mixture. Their spectral composition is very different: The apparent excitation of this spectrum is much higher in the afterglow than in the discharge. The recombination of the N_2^+ ions and electrons is followed, not by the emission of the triplet band systems, but by that of the singlet band systems. These bands are to be expected in the near ultraviolet of the spectrum of the aurora. The paper closes with a discussion of possible emission processes that might account for the spectrum of the aurora.

The spectrum of the polar aurora in the wavelength region 3150 A to 9000 A is known to be predominantly that of atomic and molecular nitrogen and oxygen, and atomic hydrogen. There have been numerous laboratory investigations of the spectrum of molecular nitrogen stimulated by the desire to find the physical mechanisms of excitation common to the laboratory and the aurora. Much attention has been given in the laboratory investigations to the study of the radiation emitted after the end of the discharge proper -- the so-called "afterglow" spectrum. For example, KAPLAN [1932, 1938] has made extensive studies of afterglow which persist for intervals of the order of magnitude of one second after the end of the discharge. The present authors have, on the other hand, been engaged for some time [HERMAN and HERMAN, 1949] in the investigation of the afterglow spectrum immediately after the end of the discharge (10^{-4} to 10^{-2} sec), which we call the short-duration afterglow. The present investigation is concerned with an extension of our short-

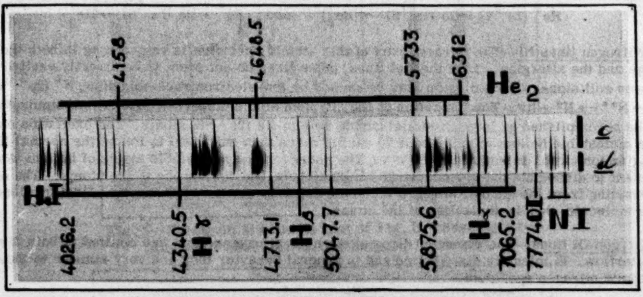

Fig. 1--Emission spectrum of helium containing traces of nitrogen, oxygen, and hydrogen: d, discharge; c, afterglow

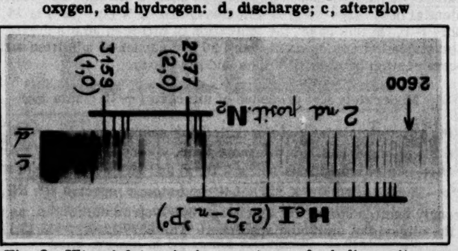

Fig. 2--Ultraviolet emission spectrum of a helium-nitrogen gas mixture: d, discharge; c, afterglow duration afterglow studies. They show how different bands and lines may be excited and may give some information on the possible processes of emission in the upper atmosphere during and after the auroral displays.

We shall describe now the experimental results obtained in a mixture of helium and nitrogen, with traces of oxygen and hydrogen as impurities. The partial pressure of helium was kept constant at five mm Hg, and we studied the change in the emission spectrum when the proportion of nitrogen was varied.

Very low pressure of nitrogen (10^{-5} to 10^{-3} mm Hg)

The emission observed contains spectra of He_2, HeI, NI, NII, HI, and OI. The spectrum, Figure 1, shows the discharge spectrum (d) and the afterglow spectrum (c). It is seen that the intensity of the emission spectrum does not diminish rapidly when the electric current through the lamp ceases to flow. This insensitivity of the intensity to the speed of the exciting electrons is more important for the atomic lines, NI, OI, than for the He_2 bands. The main process of excitation at low pressures of nitrogen is the ionization in He^+, N^+, H^+, and O^+ rather than the electronic excitation of normal He, N, H, and O atoms. Thus, the main process of emission could be the radiative ion-electron recombination. The ultraviolet spectrum observed with a higher partial pressure of nitrogen (Fig. 2) lends strong support to this hypothesis. It shows the series of lines ($2\ ^3S$ - $n\ ^3P^o$) of HeI having the limit of convergence at 2600 A. Here again the helium lines do not change in intensity when the discharge stops. The high relative intensity of the helium lines, near the limit of convergence, is a well-known feature of the recombination spectrum. The behavior of the atomic HeI spectrum may be compared with that of the second positive system of N_2 which is emitted, mainly during the discharge, by electron collisions.

At low pressures of nitrogen, the molecular He_2 spectrum is also emitted by a recombination process and its intensity does not change much by interrupting the discharge current through the emission lamp (see Fig. 1). This gives no definite information about the process of formation of the molecular He_2^+ ions or excited He_2^* molecules. Different processes have already been proposed, such as

$$He^+\ (1s\ ^2S) + He\ (1s\ ^1S) \rightarrow (He_2^+)^* \quad \text{and} \quad He^* + He\ (1s\ ^1S) \rightarrow He_2^*$$

Nitrogen lines NI--The infrared part of this atomic spectrum is very strong in both the discharge and the afterglow. Like the HeI lines, these lines do not seem to be directly excited by electron collisions. The emission may be caused by ion-electron recombination, $N^+\ (2p^2\ ^3P_0) + e \rightarrow N^{**} \rightarrow N^* + h\nu$. The ionization of the nitrogen atoms dispersed in a helium atmosphere may be accomplished by long-lived metastable helium He ($2s\ ^{1,3}S$) atoms. The excitation energy of the metastable helium atoms (about 20 e-v) is more than sufficient to ionize the normal N atoms having an ionization potential of 14.5 e-v. The energy of the metastable atoms of helium is not sufficient to dissociate the normal N_2 ($X\ ^1\Sigma$) molecole and ionize one of the atoms. The excitation starting from the normal nitrogen molecule can only occur in two steps -- first by dissociation of the molecule, then by ionization of the atoms.

Oxygen OI lines--The strongest lines of the neutral oxygen atom are emitted in both discharge and afterflow. In intensity distribution and in general behavior they are very similar to those of the atomic nitrogen spectrum.

Nitrogen NII lines--A number of lines of ionized atomic nitrogen are emitted at low pressure. The ionization potential of N^+ ($2p^2\ ^3P_0$) is 29.60 e-v, that of N ($2p^2\ ^4S^o$) is 14.54 e-v, making a total energy of 44.14 e-v. In this case, the ionization of nitrogen atoms during the afterglow by collisions with metastable helium atoms of about 20 e-v cannot be admitted and the only likely process is the recombination between N^{++} ions and electrons.

Moderately low pressure of nitrogen ($\sim 10^{-2}$ mm Hg)

As expected, the relative intensity of the molecular spectrum of N_2 and N_2^+ increases with the partial pressure of the nitrogen gas. At the same time, a very striking difference appears in the behavior of the triplet-triplet transitions ($C\ ^3\pi$ - $B\ ^3\pi$) and ($B\ ^3\pi$ - $A\ ^3\Sigma$) of N_2 and the negative band systems ($A\ ^2\pi$ - $X\ ^2\Sigma$), ($B\ ^2\Sigma$ - $X\ ^2\Sigma$) of N_2^+ and atomic spectra NI, NII, and OI. The positive bands are strongly excited during the discharge by electron collisions, as may be seen in Figure 2 showing the ultraviolet emission spectrum. The 2977 (2,0), 2962 (3,1), 2953 (4,2) of the

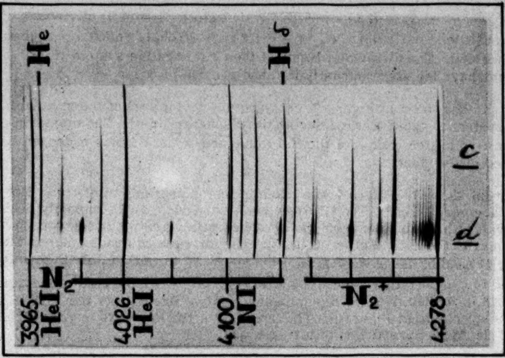

Fig. 3--Emission spectrum of a helium-nitrogen gas mixture in the wavelength region 3965 - 4278 A: d, discharge; c, afterglow

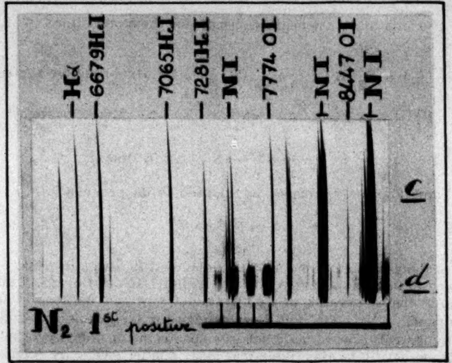

Fig. 4--Spectrum similar to that of Fig. 3, wavelength region 6400 - 8900 A: d, discharge; c, afterglow

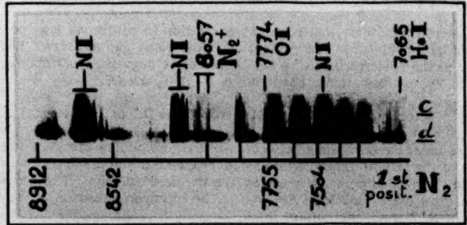

Fig. 5--Emission spectrum in the wavelength region 7000 - 8900 A obtained with a grating spectrograph, showing the 8057 (3.1) band of the (A $^2\pi$ - X $^2\Sigma$) band system of N_2^+ in the afterglow: d, discharge; c, afterglow v′ - v″ = 2 sequence and 3159 (1,0), 3136 (2,1), 3117 (3,2), 3104 (4,3) of the v′ - v″ = 1 sequence of the second positive band system ($C\,^3\pi$ - $B\,^3\pi$) are strongly emitted in d (discharge) and very weak in c (afterglow). The ultraviolet bands of the first negative system ($B\,^2\Sigma$ - $X\,^2\Sigma$) of N_2^+ and the HeI lines have the same intensity in d and c.

A similar picture, seen in Figure 3, concerns the violet part of the spectrum. The second positive band system is again emitted during the discharge (in d). The emission of the main part of the first negative system of N_2^+, of the HeI lines, and of the Balmer hydrogen lines is not sensitive to the speed of the electrons.

The spectrum shown in Figure 4 was taken in the photographic infrared. Here the infrared nitrogen lines are very strong. The OI lines are also present as an impurity of the helium gas. This same wavelength region has been photographed with a grating spectrograph having a dispersion of 200 A/mm in the second order (Fig. 5). The nitrogen content is higher there and the Meinel 8057 (3,1) band of the ($A\,^2\pi$ - $X\,^2\Sigma$) system of N_2^+ is well visible in the afterglow spectrum. It is interesting to compare this laboratory afterglow with that observed in the auroral spectrum. The main feature in the latter is the much higher relative intensity of the 7828 (2,0) band in comparison with the spectrum in Figure 5. The intensity of the HeI lines is very strong in d and weak in c because of the higher proportion of nitrogen gas.

In the visible region, numerous NI lines are emitted in the afterglow, as is shown in Figure 6. Here again the bands of the first positive system of N_2 belonging to the sequence v′ = v″ = 3 and 4 are very weak in c.

The process of excitation of the negative bands and the atomic lines is probably an ion-electron recombination.

$$N_2^{++} + e \rightarrow N_2^+ (B\,^2\Sigma) \rightarrow N_2^+ (X\,^2\Sigma) + h\nu \text{ (first negative system)} \ldots\ldots\ldots\ldots (1)$$

$$N_2^{++} + e \rightarrow N_2^+ (A\,^2\pi) \rightarrow N_2^+ (X\,^2\Sigma) + h\nu \text{ (Meinel system)} \ldots\ldots\ldots\ldots (2)$$

$$N^+ + e \rightarrow N^{**} \rightarrow N^* + h\nu \text{ (NI lines)} \ldots\ldots\ldots\ldots (3)$$

The processes (1) and (2) must, of course, be followed by the process

$$N_2^+ + e \rightarrow N_2^{**} \rightarrow N_2^* + h\nu_1 \ldots\ldots\ldots\ldots (4)$$

and the emission of the spectrum of the neutral molecule. It seems that this process gives almost entirely singlet-singlet systems and very little of triplet-triplet systems. This hypothesis is strongly supported by the observation of the ultraviolet spectrum between 2300 and 2600 A. In this region the Lyman bands,($a\,^1\pi$ - $X\,^1\Sigma$) of N_2 are emitted in the afterglow. Like the negative bands, they are practically not enhanced in the discharge spectrum. It seems probable that they are emitted by the process (4). It is also likely that other singlet levels are formed by the recombination (4). Such levels are well known from the study of the absorption spectrum of Hopfield's Rydberg series with the upper limit N_2^+ ($B\,^2\Sigma_u^+$) at 151240 cm^{-1} and Worley-Jenkins' Rydberg series with the upper limit N_2^+ ($X\,^2\Sigma_g^+$) at 125666 cm^{-1}.

Thus, the spectrum obtained in nitrogen by excitation in collisions may be entirely different from that obtained by ion-electron recombination in the same gas mixture. The former process gives rise chiefly to the positive bands -- the latter to the negative band systems, the singlet-singlet systems of N_2, and the atomic spectrum of NI, NII, and OI.

Intermediate pressure of nitrogen (~ 0.1 to 1.0 mm Hg)

At these pressures, the collisions between nitrogen molecules are very important, and the relative intensity of the triplet-triplet systems increases. In the discharge as well as in the after glow, the first and the second positive systems are the most intense. The spectrum of the discharge d and afterglow c are visible in Figure 7. The infrared emission in the afterglow shows the Meinel bands of N_2^+ but not the helium lines.

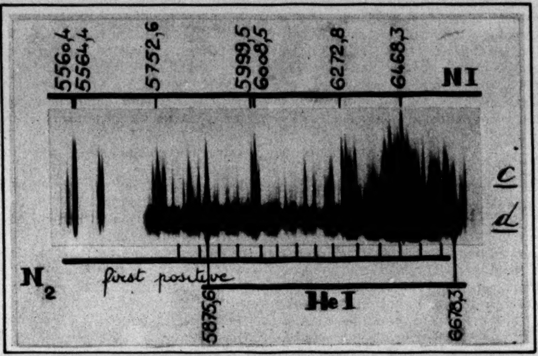

Fig. 6--Emission spectrum in the wavelength region 5500 - 6700 A showing the afterglow emission of numerous NI lines: d, discharge; c, afterglow

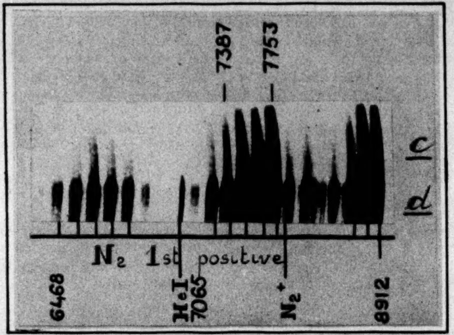

Fig. 7--Emission spectrum of a helium containing a high amount of nitrogen: d, discharge; c, afterglow

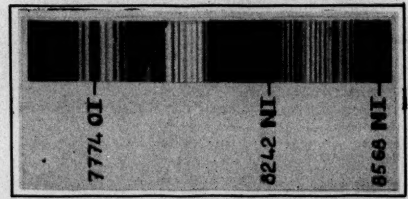

Fig. 8--Rotational structure of the 8912 (1.0) band of the ($B\ ^3\pi$ - $A\ ^3\Sigma$) system of N_2

Very high pressure of nitrogen (one to ten atmospheres)

It is interesting to compare the above observations obtained in a mixture of helium and nitrogen and those obtained in pure nitrogen at a pressure of a few atmospheres. An example of the latter is seen in Figure 8. Almost all the visible and near infrared bands of the first positive system have disappeared; instead, very strong NI lines are emitted together with a strong band having a very open structure. This structure is actually the rotational structure of the 8911,6 (1,0) band of the first positive system. (M. W. Feast, in a private communication, has observed a similar spectrum using a shock-wave excitation.) The comparison of the rotational and vibrational temperatures at low and high pressures brings out a very odd behavior. At low pressure, the rotational temperature is low and the vibrational temperature very high, whereas at high pressure the rotational temperature may be very high while the vibrational temperature is very low.

Excitation processes in auroral displays

Since the discovery of Balmer hydrogen lines in the spectrum of the aurora by VEGARD [1950], Störmer, GARTLEIN [1950], and MEINEL [1950], it has been generally accepted that fast incoming protons in the upper atmosphere are the primary excitation source of the aurora. The auroral spectrum may be partially emitted by direct proton collisions, but our experiments seem to indicate that an important part is emitted after the passage of the exciting particles and is actually an afterglow spectrum. This is especially true for the negative bands and the nitrogen and oxygen lines. It is interesting to notice that, at least in our experiments, the high excitation by direct electron and atomic collisions gives only a spectrum having the appearance of a weakly excited gas. On the contrary, when the excitation is stopped, an emission may appear which gives a spectrum of highly excited levels.

If our interpretation is correct, one should expect, in the spectrum of solar-type aurorae of high altitude and in the twilight, an enhancement of the singlet-singlet band systems of the N_2 molecule. In aurorae of this type the negative band systems are very strong, and the recombination of N_2^+ ions should result in a strong emission of the spectrum of the neutral molecule. Unfortunately, these bands are emitted mainly in the ultraviolet; but a few band systems are known at the ultraviolet limit of the auroral spectrum where their presence could be reasonably expected. They are (1) the Gaydon and Herman $(c\,{}^1\Sigma - a\,{}^1\pi)$ system at 3118 A, 2967 A; (2) the $(c' - a\,{}^1\pi)$ system at 3241 A, 3080 A, 2932 A; and (3) the $(q' - a\,{}^1\pi)$ system at 3175 A, 3020 A.

Other processes have been proposed to explain the excitation of the negative bands in the auroral spectrum. In a recent paper SHKLOVSKIĬ [1951] discussed the following excitation process by protons

$$H^+ + N_2 \rightarrow H + N_2^+ (B\,{}^2\Sigma) - 5.15 \text{ e-v}$$

In this process the energy of 5.15 e-v is supposed to come from the kinetic energy of the incoming protons. This energy has been found by Meinel to be as high as 350 e-v and the ionization of the N_2 molecules in N_2^{++} has thus a very high probability. In this case the recombination of these ions with electrons could explain partially the emission of the negative bands.

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(Communicated manuscript received May 14, 1951, and, as revised, May 13, 1952;
open for formal discussion until January 1, 1953.)

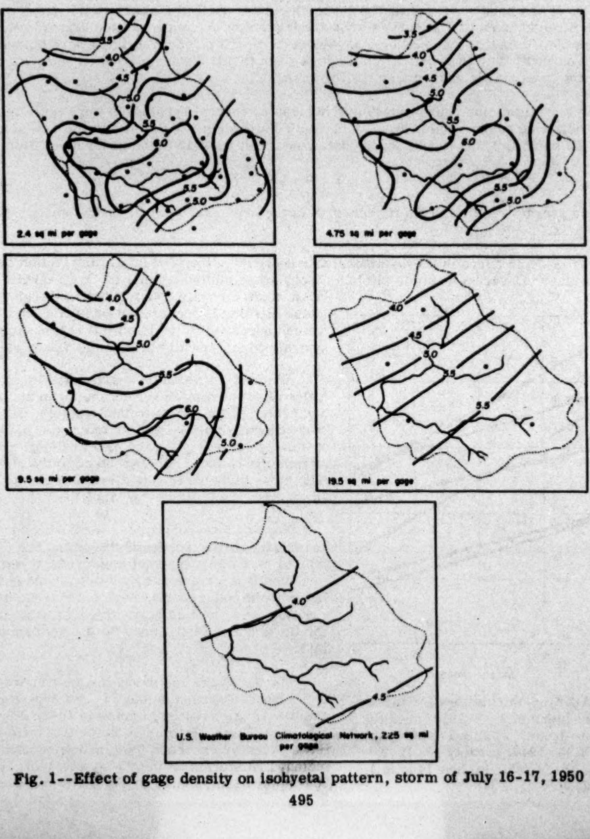
Transactions, American Geophysical Union Volume 33, Number 4 August 1952

AREA-DEPTH STUDIES FOR THUNDERSTORM RAINFALL IN ILLINOIS

F. A. Huff and G. E. Stout

Abstract--The areal distribution of individual-thunderstorm rainfall on three small watersheds in central Illinois was investigated. Curves were fitted (for individual storms) for the maximum rainfall over certain sizes of area versus the size. These curves generally show maximum rainfall data to have a linear relationship to the square root of area.

Introduction--For many engineering purposes, a detailed knowledge of area-depth relations for small areas is essential. Few detailed data for areal units under 300 sq mi have been pub-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5.5
5.5
19.5 sq mi per gage
4.0
4.5
U.S. Weather Bureau Climatological Network, 225 sq mi per gage

Fig. 1--Effect of gage density on isohyetal pattern, storm of July 16-17, 1950

lished. Rain-gage networks of density sufficient to define area-depth relations accurately from isohyetal maps for such small areas are uncommon. While publications of the MIAMI CONSERVANCY DISTRICT [1936] and the U. S. CORPS OF ENGINEERS [1945] include data on small watersheds, the data are limited due to the relatively sparse rain-gage spacing employed. The concentrated networks employed in this study were designed for collecting more precise data.

In an effort to obtain concrete information on the characteristics of the area-depth curve for small watersheds, thunderstorm-rainfall data collected on three densely-gaged networks [HUDSON, STOUT, and HUFF, 1952] in central Illinois during 1948-50 were analyzed. These three networks consisted of 7, 20, and 46 gages on basins of 5.2, 95, and 280 sq mi, respectively. Prior to the thunderstorm season of 1950, the 95-sq mi network was increased to 40 gages. The necessity for concentrated networks to obtain detailed isohyetal patterns and thereby minimize interpolation errors in the construction of area-depth curves is illustrated by the series of isohyetal maps in Figure 1. These maps were drawn for the same storm on the 95-sq mi basin using the various gage densities indicated.

Results of analysis--Most previous investigations of area-depth relations have been concerned with relatively large watersheds, employing much sparser networks than those utilized in this study. It has been observed that area-depth curves derived from isohyetal maps sometimes approach straight lines or flat curves when the logarithm of area is plotted against average rainfall. This often results from a liberal envelopment of points. Curved lines were obtained with the Illinois area-depth data in practically all cases when the data were plotted on semi-log paper with depth on the linear scale and area on the log scale.

Since a straight-line representation is desirable, several hypotheses were investigated in an effort to obtain an easily computed straight-line relationship. Within the limits tested for the three small basins, it was found that the data conformed closely to the general equation

$$Y = a + bX^{1/2}$$

where Y is average rainfall depth in inches, X is the area enveloped in square miles, and a and b are constants.

The area-depth curve is a two-dimensional representation of rainfall distribution over area, in which values of average rainfall within an isohyet are plotted against the area enveloped. The area-depth curve is constructed as though the highest value of rainfall occurred at one point, and lower values appeared (in an isohyetal representation) roughly concentrically around the higher values.

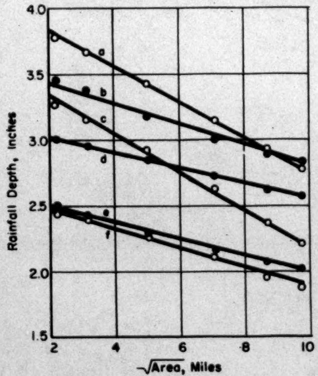

Fig. 2--Area-depth relations for the six heaviest storms, 1948-1950, over the 95-sq mi basin; (a) July 17, 1950; (b) June 13-14, 1949; (c) July 16, 1949; (d) June 18-19, 1950; (e) June 14 (p.m.), 1949; (f) July 25, 1948

Rainfall may also be represented by plotting the value of each isopleth versus the mean distance from the center of the storm to that isopleth. If this is done for uniform rainfall-isopleth intervals, the slope of the resulting curve depicts the mean rainfall gradient across the isopleth corresponding to the abscissa. The mean distance is represented by the square root of the area enclosed by the isopleths.

For analysis purposes, values below one, five, and ten square miles, respectively, on the 5.2-, 95-, and 280-sq mi basins were not employed. It was felt that sufficiently accurate interpolations could not be made below these values. It is possible that the developed relation does not hold below these values, particularly for the single-celled, short-duration storms discussed later.

The data were tested on storms ranging from one to 18 hours' duration on the 95- and 280-sq mi basins, and for durations of 30 minutes to several hours on the 5.2-sq mi basin. Storms in which the basin mean rainfall equaled or exceeded 0.50 inch were used. These included 28 storms on the 95-sq mi basin, 19 on the 280-sq mi basin, and 18 on the 5.2-sq mi basin. Re-

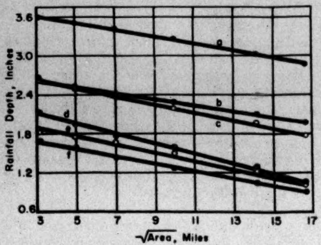

Fig. 3--Area-depth relations for the six heaviest storms, 1948-1950, over the 280-sq mi basin; (a) June 13-14, 1949; (b) June 14 (p.m.), 1949; (c) July 25, 1948; (d) July 21, 1949; (e) June 26, 1948; (f) June 23, 1948

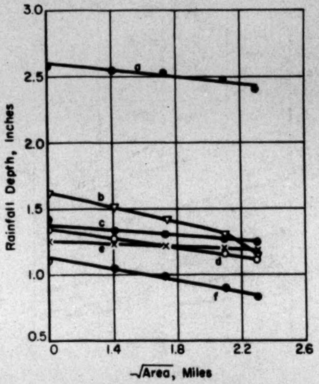

Fig. 4--Area-depth relations for the six heaviest storms, 1948-1950, over the 5.2-sq mi basin; (a) July 17, 1950; (b) July 6, 1949; (c) Oct. 21, 1949; (d) June 28, 1949; (e) May 19, 1949; (f) Aug. 17, 1949

sults for the six heaviest storms in each basin are shown in Figures 2-4.

Storms were classified according to duration and location for analysis purposes. Duration classifications used were: less than six hours, 6-12 hours, and over 12 hours. The storms were then classified according to whether the location of the storm core or area of heaviest rainfall was near the center or the boundary of the basin. Figure 5 shows the relationship between mean percentage of area and depth derived for storms with durations under six hours and with area of heaviest rainfall near the center of the basin. Figure 6 illustrates the same relation for each basin but for storms in which the heaviest rainfall occurred near the boundary of the basin.

Since most of the storms analyzed on the 95- and 280-sq mi basins were of several hours' duration, they were multicellular in nature. The multicellular storm has by definition a tendency toward greater areal uniformity than the single-cell storm. BYERS and Collaborators [1949] have shown that the single-celled storm is usually rather weak compared to the multicellular storm, and generally dissipates within less than one hour. However, exceptionally strong rainfall gradients may exist in these cells. Some limited tests of the single-cell type of storm were made for the 95-sq mi basin. Satisfactory agreement with the general equation was obtained in most cases. Results for a number of these storms chosen at random are shown in Figure 7.

Tests made on published area-depth data for the 8000-sq mi Muskingum Basin [U. S. WEATHER BUREAU, 1947] indicated the developed relation is not applicable to large basins.

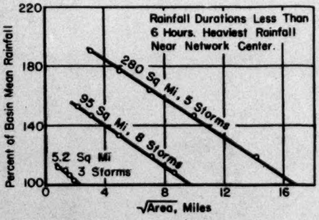

Fig. 5--Mean percentage area-depth relations based on three Illinois networks; storm data from 1948-1950

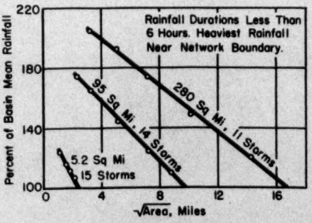

Fig. 6--Mean percentage area-depth relations based on three Illinois networks; storm data from 1948-1950

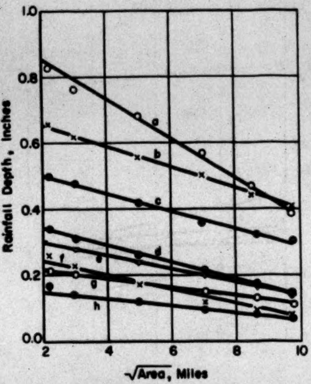

Fig. 7--Examples of area-depth relations for a single-cell type of storm over 95-sq mi basin; (a) June 16, 1950; (b) May 21, 1950; (c) June 28, 1950; (d) May 4, 1950; (e) June 2, 1950; (f) July 31, 1950; (g) July 3, 1950; (h) June 12, 1950

Summary--Analysis of three years of data on three small watersheds in central Illinois indicated that area-depth curves for thunderstorm rainfall approach a straight-line relationship when depth is plotted against the square root of the area. While the relationship apparently does not hold for large watersheds, it does appear applicable to basin areas up to 300 sq mi in the region studied. Further testing of the relationship for small basins is contemplated as more data are obtained from the several concentrated rain-gage networks operated by the Illinois State Water Survey.

Acknowledgment--The authors are indebted to H. E. Hudson, Jr., Head, Engineering Sub-Division, for guidance in this study.

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(Communicated manuscript received September 17, 1951, and, as revised, March 14, 1952; open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

ON A BAROMETRIC DISTURBANCE RECORDED ON A VERTICAL LONG-PERIOD SEISMOGRAPH

A. P. Crary and Maurice Ewing

On February 1, 1951, between 01h 00m and 02h 00m EST, large barometric fluctuations were observed on a recording microbarograph at the Air Force laboratories at Watertown, Mass. This disturbance occurred with prefrontal activities of a warm front which passed through the area from the south at about 07h 30m EST on that date. Periods of fluctuations were about six minutes and peak-to-peak amplitudes about 1.0 mb. This barometric oscillation was not noted on nearby Weather Bureau barographs operating at drum speeds of 0.25 cm/hr, but was indicated on a barograph with a drum speed of about 1.1 cm/hr at the Blue Hill Observatory in Milton, Mass., about 11 miles south of Watertown. Here the disturbance has amplitudes of about 0.7 mb and periods of six to eight minutes. Tide records along the New England coasts did not show abnormal seiche activity, but such resonance phenomena are usually of longer periods than six minutes.

At the Weston College Observatory, located eight miles west of Watertown, oscillations of a similar period were recorded at about the same times on the Benioff vertical seismometer with 60-sec recording galvanometer, with no corresponding movements on the horizontal instruments. The vertical seismograph recording from Weston Observatory is shown in Figure 1, and a reduction of part of this seismogram is shown in Figure 2 together with a reduced recording of the microbarometric trace taken at Watertown. The next nearest seismograph with a long-period (60 sec) galvanometer is located at the Lamont Geological Observatory at Palisades, N. Y., and no oscillations were observed there.

Calculations of the effect of atmospheric pressure fluctuations upon the inertia mass of the seismograph show that changes in atmospheric buoyancy effects can satisfactorily explain the results observed. The acceleration, A, of the seismograph mass due to the barometric fluctuations can be represented by the relation

$$A = (\rho_0 g/\rho_m)\,(C/p_o)\,\sin \omega_a t = 1.68 \times 10^{-4}\, C \sin \omega_a t$$

where ρ_0 is the atmospheric density (0.00136 gm/cm^3), ρ_m is the density of the inertia mass (7.8 gm/cm^3), p_o is the barometric pressure (1019 mb), ω_a is the angular frequency of the barometric fluctuations, or $2\pi/T_a$ (0.0175), and C is the amplitude of the pressure change, assumed to be isothermal, in mb. This acceleration A replaced the $\partial^2\xi/\partial t^2$ term in the standard seismograph equations, where $\xi = a \sin \omega_e t$, the ground motion. Consequently, the amplification term Q, as given by BENIOFF [1932] for steady-state solution, neglecting the reaction of the gal-

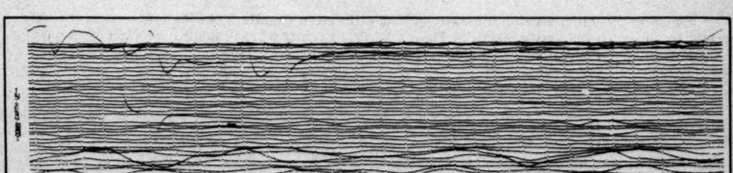

Fig. 1--Seismogram from long-period vertical seismograph, Weston Observatory, Feb. 1, 1951

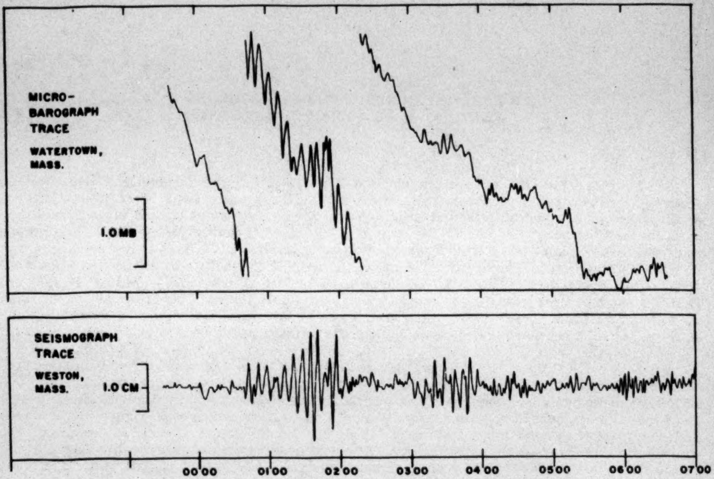

Fig. 2--Comparison of barometric and seismic disturbances

vanometer on the seismograph, and assuming damping constants are equal to unity, becomes $\omega_a/(\omega_o{}^2 + \omega_a{}^2)\,(\omega_g{}^2 + \omega_a{}^2)$ instead of $\omega_e{}^3/(\omega_o{}^3 + \omega_e{}^3)\,(\omega_g{}^2 + \omega_e{}^2)$, where ω_o and ω_g refer to $2\pi/T_o$ and $2\pi/T_g$, the angular undamped frequencies of the seismometer and galvanometer respectively. The peak response of the system to these oscillations then corresponds to ω_g, assuming ω_g is less than ω_o.

The approximate solution of the seismograph and galvanometer equations giving the trace amplitude a (t), corresponding to the pressure amplitude C, then becomes

$$a\,(t) = (2\ell\sigma)\,(\rho_o g/\rho_m)\,(C/p_o)\,[\omega_a/(\omega_o{}^2 + \omega_a{}^2)\,(\omega_g{}^2 + \omega_a{}^2)]\,\sin\,(\omega_a t + \Delta)$$

where $2\ell\sigma$ depends on various instrumental constants.

In order to obtain a value for $2\ell\sigma$ we can utilize the results from the test weight (10^{-6}M) release shown on the seismogram in Figure 1. Substituting a trace amplitude a′(t) of 11 cm for a time t of 15 sec after the release into the given equation for given accelerations, we obtain

$$2\ell\sigma = a'(t)(\omega_g - \omega_o)^3/10^{-6}g\,\left[\left\{(\omega_g - \omega_o)\,t - 2\right\}e^{-\omega_o t} + \left\{(\omega_g - \omega_o)\,t + 2\right\}e^{-\omega_g t}\right] = 14.0 \times 10^4$$

and the equation for a (t) above reduces to

$$a\,(t)_{max} = 0.92\,C$$

showing comparable values of trace amplitude in centimeters and barometric pressure amplitude in millibars for six-minute periods. This is in accord with the results in Figure 2, assuming that the barometric pressures at Weston and at Watertown were of the same order.

Theoretical analysis of atmospheric oscillations have been made by numerous observers, among them HELMHOLTZ [1889], BRUNT [1927], SOLBERG [1936], and MARTYN [1950]. It has been shown that these oscillations may consist of mainly vertical displacements of the air particles, with periods dependent on the lapse rate, varying between about three and five minutes, or

of oscillations with appreciable horizontal movement where certain atmospheric conditions cause vertical bounding. These periods may vary from a few minutes to a few hours. The numerous observations by JOHNSON [1929], KOHNE [1940], NAMEKAWA [1934], and others indicate a maximum occurrence between about ten and 15 minutes. The study of recordings of these long-period disturbances on seismograms will aid considerably in outlining the areal extent of such cellular structures and the atmospheric conditions that may cause them.

Acknowledgments--Acknowledgments are due to Daniel Linehan and F. J. Donohoe of the Weston Observatory for their interest and assistance in this problem.

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(Communicated manuscript received October 29, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

THE LARGE-SCALE VERTICAL EDDY STRESS IN THE FREE ATMOSPHERE

Robert M. White and Duane S. Cooley

Abstract--An attempt is made to determine the order of magnitude of the large-scale vertical eddy flux of angular momentum in the free atmosphere for the middle latitude region 30°N to 60°n. It is suggested that a significant portion of the vertical flux may be transported downward by vertical eddies associated with migratory cyclones and anticyclones in middle latitudes.

Introduction--The maintenance of the belt of mid-latitude westerlies against the frictional torques at the Earth's surface requires a flux of eastward angular momentum into this region within the atmosphere. The mechanisms whereby the atmosphere transfers this momentum horizontally have been studied by a large number of investigators [WIDGER, 1949; MINTZ, 1951; STARR and WHITE, 1951; PRIESTLEY, 1951; and PALMEN, 1951]. Although there exists a considerable difference of opinion among these investigators, it is agreed that the principal flux of angular momentum occurs at levels in the atmosphere near the tropopause. Since this momentum can only be abstracted from or supplied to the atmosphere at the Earth-atmosphere boundary, the atmospheric circulations must be organized in such a fashion that there is a vertical upward flux of angular momentum in the regions of the surface easterlies and a vertical downward flux in the regions of the surface westerlies.

In a previous study, the first author of the present paper [WHITE, 1950] attempted to determine the sign of the large-scale vertical eddy flux in middle latitudes in the free atmosphere by using the presence of precipitation as a crude indicator of vertical motion. This paper presents another method whereby this eddy flux may be determined on a hemispherical basis from pressure data alone.

The vertical flux of momentum--The total vertical transport of absolute angular momentum through a horizontal surface in the atmosphere may be thought of as the sum of the transport of relative angular momentum and the transport of angular momentum due to the Earth's rotation. Over long periods of time there can be no net mass flux through a given horizontal surface which extends over the entire Earth. Therefore, any net vertical flux of angular momentum through this surface can only be a result of the exchange of air with differing momentum across this surface neglecting true molecular viscosity. Because the angular momentum due to the Earth's rotation is a function of latitude, there may be a contribution to the vertical flux of absolute angular momentum if there exists a latitudinal correlation between the mass flux and this component of the angular momentum. The determination of the nature of this correlation is not possible without an accurate and complete knowledge of the vertical velocity field over the entire globe. The scope of the investigation reported below deals only with the vertical flux associated with motions relative to the Earth.

The total vertical flux of relative angular momentum, W, through a horizontal surface in the atmosphere within a unit latitudinal strip over a time interval, t_1, extending around a complete latitude circle may be expressed as

$$W = r \cos \phi \int_0^{t_1} \oint \rho \, uwdxdt \qquad (1)$$

where u is the eastward and w the upward component of velocity, ρ is density, r is the mean radius of the Earth, x is linear distance longitudinally, ϕ is latitude, and t is time.

If the horizontal variations in density are neglected

$$W = r \bar{\rho} \cos \phi \, [\overline{uw}] \, L \, t_1 \qquad (2)$$

where $\bar{\rho}$ is an average density for the given level. The square brackets indicate a space average over the length L of the complete latitude circle, the bar denotes a time average over the interval t_1. It is seen that the vertical flux of momentum is proportional to the product $[\overline{uw}]$.

Considerable information about the nature of the atmospheric circulations which accomplish this vertical momentum flux may be obtained by expanding (2) in terms of mean quantities and deviations from these means.

The velocities u and w may be expressed in terms of their means taken with respect to longitude [u], [w], and the deviations from these means u′, w′ so that

$$[uw] = [u]\,[w] + [u'w'] \qquad (3)$$

where the brackets indicate a longitudinal mean and the primed quantities are deviations from this mean. Over a given time interval t_1

$$\overline{[uw]} = \overline{[u]}\,\overline{[w]} + \overline{[u'w']} \qquad (4)$$

where the bar indicates a time mean.

The first term on the right-hand side of (4) represents the contribution to the vertical flux due to a non-zero mean rate of vertical velocity [w]. This is the flux which may be associated with large-scale organized vertical convective circulations operating in meridional planes. The second term on the right is the flux due to the existence of a systematic correlation along a latitude circle between the vertical and zonal components of motion. This we will call the vertical eddy flux.

Contributions to the vertical eddy-flux term may occur from eddies of all sizes. In practice the method of evaluation of this term determines the scale of the contributing eddies that can be measured. In the current investigation the eddy size which is taken into account is determined by the spacing of the grid at which the velocities are measured and also the manner in which the velocities at these grid points are determined

In the present investigation a grid of 72 evenly spaced points around a given latitude is used. The geostrophic approximation is used to measure the velocity components at the grid points. The geostrophic velocities are deduced from pressure differences over ten degrees of latitude and longitude, and are then averaged over 24-hour time intervals by taking the linear average of the velocities at the beginning and end of each period. The computations of W are therefore proportional to the average over ten-degree latitude belts. It is evident that only the very large-scale eddies are sampled, that is, eddies whose size is that of migratory cyclones and anticyclones of middle latitudes.

Procedure--The key problem involved in this investigation reduces to one of obtaining suitable representations of the vertical wind velocity in the free atmosphere over a sufficiently broad area of the northern hemisphere. Numerous studies of vertical motion and associated weather phenomena have been made by PANOFSKY [1946], FLEAGLE [1947], MILLER [1948], and others. The advective technique reported by Panofsky was used in this investigation. This technique rests upon the basic assumption of the conservation of potential temperature. It is easily applied and can be modified so that the field of vertical velocity may be obtained from the pressure distribution alone permitting determinations over the entire northern hemisphere.

According to this technique, the vertical velocity may be expressed as

$$w = (\partial T/\partial t + \mathbf{V} \cdot \nabla T)/(\gamma - \gamma_d) \qquad (5)$$

where $\partial T/\partial t$ is the local temperature change at a point in the free atmosphere, $\mathbf{V} \cdot \nabla T$ represents the horizontal advection of temperature, and $\gamma - \gamma_d$ is the difference between the actual γ and dry adiabatic γ_d lapse rate. The assumption is that any local temperature change not accounted for by advection must be due to the adiabatic heating or cooling which accompanies the vertical motion.

Because temperature data were not available over the entire hemisphere, (5) was modified so that the vertical velocities could be computed from the pressure or contour height data. To illustrate the required modification, a mean of (5) may be taken with respect to pressure in the vertical. If the factor $\gamma - \gamma_d$ is constant, then

$$\left\{w\right\} = (\left\{\partial T/\partial t\right\} + \left\{\mathbf{V} \cdot \nabla T\right\})/(\gamma - \gamma_d) \qquad (6)$$

where the braces denote a mean with respect to pressure. The vertical velocity thus computed is the mean between two constant-pressure levels. The temperatures entering into the formula will be approximated by using the mean virtual temperatures within the given layer (surface to 500 mb).

For ease in computation the following simplifications were introduced.

(a) The stability factor ($\gamma - \gamma_d$) was assumed to be constant. A value of γ equal to that given by the U. S. Standard Atmosphere was used. Because the mean lapse rate between the surface and 500 mb is rarely if ever superadiabatic, this simplification cannot change the sign of the vertical velocity, but only its magnitude.

(b) A linear average of the geostrophic wind at the surface and 500 mb is used to approximate the mean geostrophic wind within the layer in the computation of the $\{\mathbf{V} \cdot \nabla T\}$ term.

(c) The local 24-hour change of temperature is used in conjunction with the average of the two values of advection computed at the beginning and end of the 24-hour period.

(d) The computation of the average advection within the layer surface to 500 mb involves the following approximation

$$\{\mathbf{V} \cdot \nabla T\} = \{u\, \partial T/\partial x\} + \{v\, \partial T/\partial y\} \approx \{u\}\{\partial T/\partial x\} + \{v\}\{\partial T/\partial y\}$$

(e) The sea-level pressure was converted to the height of the 1013-mb surface assuming the U. S. Standard Atmosphere. This permits easy calculation of the mean virtual temperature in the layer between the 1013-mb and 500-mb surface since then the mean virtual temperature is proportional to the height difference between these two pressure levels.

(f) To obtain the vertical velocity at the 500-mb level a linear change in vertical velocity from the surface, where it is assumed to vanish, is used.

Before proceeding with this modified form of the advective formula to compute the vertical velocity, a set of comparisons was made over eastern United States between the vertical velocities at the 500-mb level computed by the modified techniques presented here and the standard use of (5). The standard use of (5) involves the computation of the temperature change and advection directly from the contour and isotherm patterns at a given level. A linear correlation coefficient of +0.58 was obtained for 50 pairs of values. The tetrachoric correlation coefficient for the same 50 pairs was +0.74. These facts seem to indicate that the two methods agree better in sign than in magnitude.

The vertical eddy flux of relative angular momentum through the 500-mb level was computed from 24-hour averages of the zonal and vertical components of velocity at 72 points around a given latitude circle.

Results--The results of the computations for the period January 15-25, 1946, at latitudes 35°N, 45°N, and 55°N are presented in Table 1 and Figure 1. These results were obtained from 2376 computations of w and u. The time required to make these calculations precluded the extension of the study beyond a ten-day period. The important results which are suggested by these data are as follows:

Table 1--Numerical analysis of the data entering into the computation of the vertical eddy flux of angular momentum[a] through the 500-mb surface for the period January 15-25, 1946

Latitude belt (1)	$\overline{[w]}$ (2)	$\overline{[u]}$ (3)	$\overline{[uw]}$ (4)	$\overline{[u]}\,\overline{[w]}$ (5)	$\overline{[u'w']}$ (6)	$K\,\overline{[u'w']}$ (7)
°N	cm/sec	m/sec	cm^2/sec^2	cm^2/sec^2	cm^2/sec^2	gm cm^2/sec^2
50 - 60	+0.25	+ 8.6	+ 76	+ 298	-222	-1.48×10^{25}
40 - 50	-0.79	+19.6	-2342	-1539	-803	-7.98×10^{25}
30 - 40	-0.43	+21.1	-1341	- 917	-423	-5.47×10^{25}
Average (30 - 60)	-0.32	+16.4	-1202	- 719	-483	....
					Total	-14.93×10^{25}

[a] Denoted by $K\,\overline{[u'w']}$ where $K = r\,\bar{\rho}\cos\phi\, L\,\Delta y$.

(1) There appears to exist a downward eddy flux of angular momentum through the middle-latitude regions from 30°N to 60°N. The average magnitude of this flux for the ten-day period January 15-25, 1946 (column 7, Table 1), is about 15×10^{25} gm cm^2 sec^{-2}. This is of the same

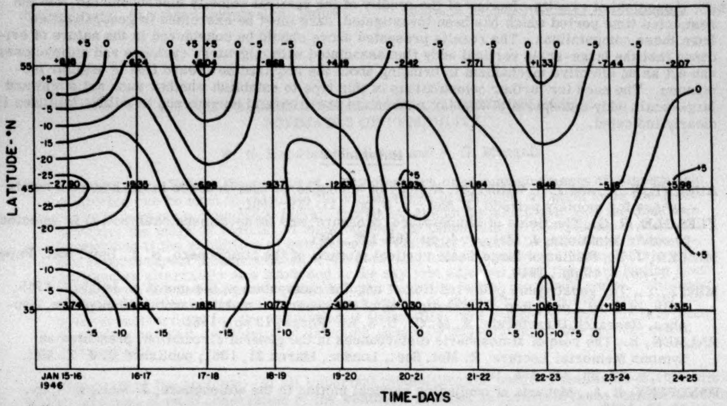

Fig. 1--The space-time distribution of the daily rate of vertical eddy transport of relative angular momentum through the 500-mb surface per degree of latitude in units of 10^{24} gm cm^{2} sec^{-2}

order although smaller than the horizontal eddy flux of angular momentum computed by other investigators [MINTZ, 1951; STARR and WHITE, 1951] across latitude 30°N. It is of the same order and smaller than the surface frictional drain of angular momentum in this latitude belt computed for this same period by WIDGER [1949].

(2) It may be noted that in this study the computations show a mean downard vertical velocity for the ten-day period over the belt extending from 30°N to 60°N. The authors do not intend to imply that such strong mean motions actually occur. The sum of the advection ($\mathbf{V} \cdot \nabla T$) and local temperature change ($\partial T/\partial t$) as computed above reflect non-adiabatic heating as well as vertical motion. Thus the magnitudes of the vertical velocities obtained are probably overestimates due to the operation of non-adiabatic effects. On the basis of the assumption that the average vertical motion over the entire belt of latitude for the ten-day period is zero, the sum of these two terms represents a rate of non-adiabatic heating equal to 3.79×10^{-6} cal sec^{-1} gm^{-1}. For a column of air of unit cross section from sea level to three km and a mean density of 1.12×10^{-3} gm cm^{-3} this represents a heating of 110 cal cm^{-2} day^{-1}. This is about the same order of magnitude as the heating obtained by AUBERT and WINSTON [1951] in the same layer for January, 1945.

(3) It is apparent from Figure 1 that a region of maximum downward transport appears nearly every day. This region of maximum downward flux is variable from day to day both in latitude and magnitude. For the ten-day period considered here it has an average position of 46°N.

(4) In a sample of data of this type, it is of interest to know whether random fluctuations could produce the results which have been obtained. It is not clear just what statistical procedures should be used for this purpose. Some indication may be gained from the application of Student's t-test which is a means of determining the probability that a certain deviation from a nul hypothesis could have occurred by chance. It should be kept in mind, however, that this test was derived for independent data. The degree of independence in these data is indicated in (c) below.

(a) If the daily values of the vertical eddy flux for the three latitude belts are taken as a single sample (30 values), application of Student's t-test indicates that the mean value is significantly different from zero. The level of significance is 0.005 (t = 3.23), indicating a probability of one in 200 that such a mean could have occurred by chance.

(b) If the daily values for each latitude belt are treated as three separate samples of ten variates each, then the mean downward eddy flux is significantly different from zero in the belt 30° - 40°N, and 40° - 50°N at the three and two per cent level respectively (t = 2.24 and 2.41). The mean in the latitude belt 50° - 60°N is not significantly different from zero.

(c) Because of the serial and spatial correlation in these data the significance will be somewhat reduced. For example, the serial correlation of the ten daily values in the latitude belt 30°N - 40°N at a one-day lag is 0.42 but falls off to -0.05 at two days.

Appraisal of results--In view of the crudity of the vertical velocity measurements, and the restricted time period which has been investigated, care must be exercised in generalization from these computations. The results presented above should be considered in the nature of evidence that the large-scale vertical eddy flux associated with migratory cyclones and anticyclones can act as an effective mechanism in bringing about the required downward flux of angular momentum. The need for further computations of this type to establish whether such net downward large-scale eddy transports of angular momentum are a normal occurrence in middle latitudes is clearly indicated.

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(Communicated manuscript received November 19, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

ON THE PSYCHOPHYSICAL BASIS OF METEOROLOGICAL ESTIMATES OF "VISIBILITY"

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Abstract--Before the results of telephotometry or of measurements of atmospheric scattering can be used to calculate the visual range in daylight, we must know the least contrast between a "visibility mark" and its surroundings which will lead the observer to report the mark as visible. A knowledge of the dispersion of this quanity is also of importance if the reliability of estimates of "visibility" has to be calculated. This paper reports direct measurements of the contrast, made by pointing a photoelectric telephotometer alternately at a mark and at the sky just adjacent to it, the mark having been chosen by a trained meteorological observer as being just at the visual range. The results of 1000 observations show a range of values of about 20:1. It is concluded that the present instructions for the estimation of "visibility" are chiefly to blame for this state of affairs, and that a criterion of detection should be substituted for the present one of recognition.

Statement of the problem--At all important meteorological stations estimates of "the visibility" are part of each observation of the weather. In order to facilitate these estimates during the hours of daylight, a number of marks are chosen, preferably appearing against the horizon sky and dark in color, at various distances but subtending, as far as possible, similar angles of the order of one degree in each dimension. There will necessarily be a good deal of variation in the sizes and shapes of the natural objects visible from any station, and the use of very extensive (>10°) and very small (<0.1°) objects is discouraged. These several requirements are set out officially in Resolution 147 of the Conference of Directors, Washington, 1947 [O.M.I., 1948, p.145]. To this resolution is appended two notes, which are quoted in full because of their importance in this problem.

"Note 1--There has been a difference in the instructions in different countries in regard to daylight visibility. In some countries daylight visibility has been determined by the distance at which the outline of objects seen against the sky disappears. In other countries the instruction has been that visibility is the distance at which an object such, for example, as a tree can be recognized as a tree.

"Note 2--Thus the instructions of Meteorological Services should be 'the distinguishing of objects as such.'"

While these statements are far from clear, there is no doubt that they are intended to impose on the observer a criterion of recognition, not merely of detection. These instructions made official what was already the practice in most countries.

It has been recognized for a long time that such estimates of "visibility" leave much to be desired, and as early as 1921 attempts at instrumental measurement began [MIDDLETON, 1941, chap. 7] to be made. The first of these were empirical, but after the theoretical investigations of KOSCHMIEDER [1924] attention centered on the measurement of the extinction coefficient σ of the atmosphere. It may be shown that the visual range of a black object rising against the horizon sky is given by the formula [MIDDLETON, 1941, p. 50]

$$V = (1/\sigma) \log_e 1/\epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In this equation ϵ represents the least contrast that the eye can appreciate under the prevailing conditions of (1) background luminance and (2) angular size of object. The contrast of such an object-background field is defined as

$$C = (B - B')/B' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where B is the luminance of the object and B' is that of the surroundings, namely, the horizon sky. Equation (1) is not seriously in error for any dark objects [DUNTLEY, 1948].

Very extensive investigations of the dependence of ϵ on the two variables mentioned above were made during the Second World War at the Tiffany Foundation and reported on by BLACKWELL

[1946]. When simultaneous measurements of σ and estimates of V were used to calculate ϵ by means of (1), however [HOUGHTON, 1939; DOUGLAS and YOUNG, 1945], it was found that the values obtained ranged from five to ten times the values given by the Tiffany experiments. It was not immediately obvious why this should be so, and a great deal of rather speculative discussion took place. The difficulties of this sort of indirect measurement finally led one of us to attempt to make direct measurements of the contrast between the mark and its surroundings at the moment when a meteorological observer identifies it as being at the visual range. This was done with a specially constructed photoelectric telephotometer.

Apparatus--The telephotometer need not be described in much detail. It consists of a well-baffled telescope having an objective 4.5 cm in diameter and 75 cm in focal length, with an aperture in the focal plane giving a field of view two minutes of arc in angular diameter. A field lens immediately behind this aperture forms an image of the objective on the cathode of a photomultiplier tube, type 1P22. Filters were provided to correct the spectral sensitivity of the 1P22 to an adequate approximation to the luminosity curve of the normal eye.

Immediately in front of the objective is a prism of small angle, which produces a deviation of 4.5 minutes of arc. This can be rotated in one plane through an angle of 180°, so that if the telescope is directed towards the edge of a visibility mark, fields just inside and just outside the mark can be presented alternately to the photomultiplier tube (Fig. 1). A finder telescope (a standard telescopic rifle sight) is mounted on the main telescope for aiming it accurately.

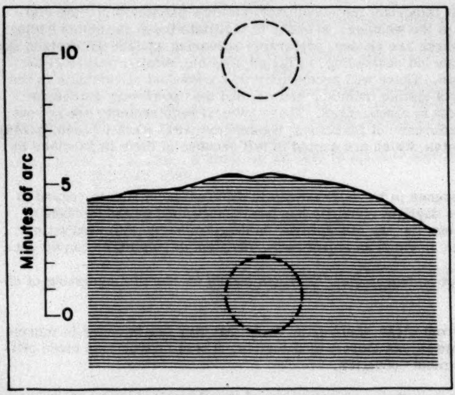

Fig. 1--Fields selected by rotating the prism

In order to obtain the maximum speed of reading, the photomultiplier tube is connected through a cathode-follower circuit to a high-speed recording potentiometer. The prism being moved through 180° in each direction at intervals of about two seconds, 20 or 30 readings are recorded in such a way that the mean difference B - B′ may easily be obtained by estimation. If the deflection is kept at about 25 cm by adjustment of the voltage on the photomultiplier-tube dynodes, the contrast can be measured to one or two parts in a thousand, depending on the steadiness of the atmospheric conditions.

The linearity of the instrument was carefully checked and found more than adequate. In addition, a very severe test for stray light was applied. The instrument was directed at the center of a white disk subtending ten minutes of arc, supported on a thin rod so as to appear against the sky. A large black screen was then placed behind the disk and the luminance of the disk measured. The screen was then removed and the apparent increase in luminance due to scattered light found to be only 1.1 pct. It was therefore concluded that since, under the conditions of observation, there are no large contrasts near the direction of view, the effect of stray light in the instrument might be neglected.

The program of observations--A short series of observations was made in the summer of 1950, mainly by physicists [MIDDLETON and CHAPMAN, 1950]. It was then decided to install the apparatus at an airport meteorological station, and use the observations made by the regular meteorological observers. Through the kindness of Andrew Thomson, Controller of Meteorological Services, and of the Commanding Officer of Rockcliffe Air Station, the instruments were installed outside the meteorological office at that station, which is near Ottawa.

While these arrangements were being made, the available visibility marks were surveyed and a plan prepared. Forty-two objects were finally chosen at distances ranging from 340 m to 26.3 km. Twelve of these were trees or groups of trees; seven were buildings or other structures; the rest were the crests of hills, all the marks at distances of more than eight km being of this kind. The distances of the more distant marks were obtained by triangulation, the nearer ones with a range finder.

The program of observations was conducted by Mungall during the hours 08 30m to 16h 30m on most days when the visual range was less than about 25 km, except that observations had to be ended earlier during December and January. In clearer air, no useful measurements were possible. The observations covered the period November 21, 1950, to September 6, 1951, and were discontinued when 1000 observations, considered to be reliable, had been accumulated. Twelve observers took part, but the estimates of one were felt to be so erratic that they were not used.

The observers were aircraftsmen and non-commissioned officers of the Royal Canadian Air Force who had had varying amounts of experience as meteorological observers before the start of the experiment, ranging up to 11 years. Six had had more than three years' experience. Only one observer was nearly without experience. It is felt that the group was representative of what might be found at any military station.

Care was taken to disturb the routine of observation as little as possible. When the experimenter saw that an estimate of the visual range was being made, he would ask the observer, "Which mark did you consider to be at the visual range just now?" He would then immediately train the telephotometer on the mark and make the measurement of contrast. The electronic circuits were kept in operation, so that only a few seconds elapsed between the observer's estimate and the instrumental reading.

The natural curiosity of the observers about the experiment was satisfied by explaining that it was designed to investigate the possibility of improving the techniques of observation. It is highly probable that the effect of the experiment was to make the observers somewhat more careful than they would otherwise have been, so that the disturbing results presented in the next paragraph may well be more favorable than they ought to be.

Results--Since the contrast was measured to three significant figures, the 1000 observations were analyzed into classes 0 - 0.0095, 0.0095 - 0.0195, etc., with the result shown in Table 1. As will be seen by the frequency diagram, Figure 2, the distribution is strongly skewed, the mean of 0.042 contrasting with a mode of 0.024 and a median of 0.030.

Table 1--Frequency of observations of contrast

Contrast	Frequency	Contrast	Frequency
0 - 0.009	20	0.100 - 0.109	8
0.010 - 0.019	152	0.110 - 0.119	18
0.020 - 0.029	218	0.120 - 0.129	5
0.030 - 0.039	206	0.130 - 0.139	8
0.040 - 0.049	125	0.140 - 0.149	7
0.050 - 0.059	81	0.150 - 0.159	5
0.060 - 0.069	65	0.160 - 0.169	1
0.070 - 0.079	37	0.170 - 0.179	1
0.080 - 0.089	22	0.180 - 0.189	1
0.090 - 0.099	19	0.190 - 0.199	1

The range of values of contrast included in Figure 2 covers a range of values of V of more than three to one for any given state of the atmosphere, calculated by (1). It is surely not necessary to make any great display of statistical treatment of such an alarming result. It is our duty, however, to anticipate the possible question whether such a result could have arisen from one or two bad observers. The answer is that the 74 observations of contrast greater than 0.0895 were distributed among all the observers but one. From a practical standpoint it would also seem to be of importance that one observation in 14 is in this category.

A second objection might be that some anomalous marks were being used. It is a partial answer to say that the marks were chosen in the manner usual at meteorological stations. None of the marks actually used was less than about a half degree in one dimension, and if some of them were smaller than might be desired it was for the usual reason that no better mark was available. An examination of Table 2, however, shows no obvious relation between the size of the mark and the number of observations of high contrast, as may be emphasized by comparing the pairs 25 and 27, and 26 and 37. In the first pair the longer object gave the lesser contrast, but in the second

Table 2--Median contrast for each mark for which N > 9; also number of observations of high contrast.

Mark No.	Type	Angular dimension		N	Median	No. of times C > 0.0895
		Vertical	Horizontal			
		°	°			
24	Building	1.5	0.5	20	0.040	2
26	Evergreen trees	0.4	1.5	58	0.035	3
33		0.8	0.4	10	0.040	1
37		0.4	2.6	27	0.065	8
2	Deciduous trees	0.4	8.	18	0.030	1
d		0.8	>10.	17	0.026	2
K		0.5	>10.	12	0.026	1
e		0.4	>10.	29	0.030	0
4	Hill top	0.6	1.4	22	0.022	0
5		0.3	1.4	18	0.024	0
9		0.6	7.2	51	0.026	2
10		0.2	2.5	11	0.027	0
11		0.2	2.0	23	0.029	1
13		0.2	7.	34	0.031	0
16		0.2	3.4	63	0.031	2
25		0.1	0.7	57	0.045	6
27		0.1	1.4	23	0.043	1
29		0.2	4.	54	0.037	1
34		0.2	1.	29	0.030	3
36		0.2	2.6	17	0.032	1

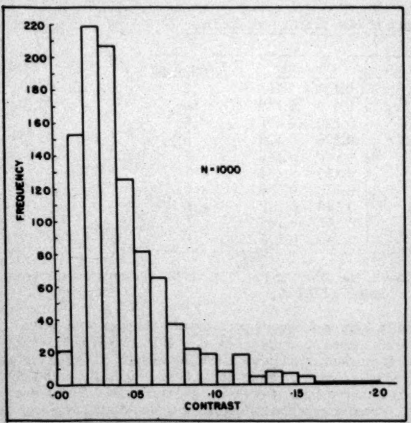

Fig. 2--Frequency distribution of measured values of contrast of an object at visual range, Rockcliffe, Ontario, November, 1950, to September, 1951

pair the shorter object gave the smaller value of C. Mark 37 ought to have been a good one, but had an exceptionally high median contrast as well as a bad record of aberrant observations. Marks 4 and 9 form another instructive pair. Even the marks with neither dimension less than 0.4° offer quite enough observations of high contrast to cause concern about the whole procedure.

Comparison with earlier measurements--The present frequency distribution looks remarkably like that which may be deduced from the observations taken on Mount Washington in 1943 and 1944 [VAN LEAR, 1945]. This is shown in Figure 3. It was obtained in an entirely different manner, namely, from the result of combining (by Eq. (1)) telephotometer observations of σ with direct estimates of V. Although the present technique is much more direct, and though the Mount Washington observations were all made in cloud, the general similarity is striking.

Discussion--The wide range of values of C at which a mark is adjudged to be at the visual range (or rather at the "visibility distance") is, in the opinion of the authors, primarily the result of the instructions to the observers. These instructions demand the recognition of an object rather than the detection of a boundary. Instead of a simple visual task, they require a complex process about which very little is at present known.

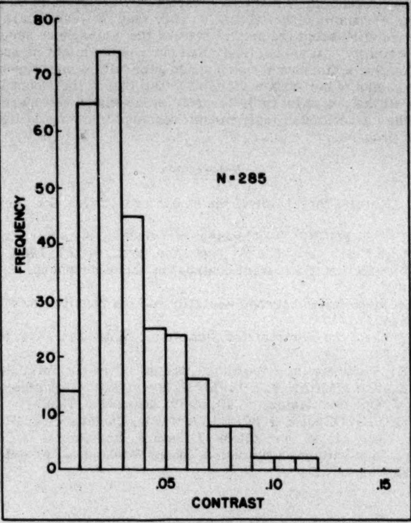

Fig. 3--Frequency distribution of values of contrast of an object at the visual range calculated from the Mount Washington observations, 1943-1944

The difficulty of this process results in undesirable attitudes. In the present investigation the experimenter detected a tendency for the observers to make up their minds about the "visibility" from a general glance at the landscape and then to choose a mark to justify this estimate. One of the authors can confirm this impression from the results of much earlier experience as an inspector of meteorological stations.

We have every confidence that instrumental errors cannot have contributed materially to the large spread of values. The error of a single measurement (which is really the average of ten or 15 readings) may be estimated at ± 0.002.

It is evident that we must abandon the attempt to choose a value of ϵ by means of which the current estimates of "visibility" can be converted into information about the optical properties of the atmosphere. It is no wonder that the synoptic use of this element has found little favor.

What is needed is, of course, good instrumental measurements of σ. In the absence of these, the writers suggest that the official instructions be changed to require the use of a criterion of detection rather than of recognition. By the use of the Tiffany data, or more probably some similar data soon to be obtained (private communication from H. Richard Blackwell), it should then be possible to derive a reasonably good value of σ and consequently of the sighting range of objects of any area and contrast.

It is not hoped that such a drastic change in official procedure will be easy to bring about. To make its adoption easier, the authors wish to suggest an operational test, as follows.

At some well-staffed and well-equipped airport (preferably furnished with instruments for measuring σ), several additional observers should be set to make estimates of the visual range, using a criterion of detection according to carefully worded instructions. These observers should not be aware of the estimates or instrumental measurements made simultaneously by the regular staff. At the completion of a sufficient number of observations, the two (or more) sets of data could be intercompared.

The results of the present experiments are essentially negative, in that they show the unreliability of the present estimates of "visibility." They may, however, have a positive value. While no reliable information about the optical state of the atmosphere seems to be obtainable from estimates of "visibility," it seems likely that our results might be applied to instrumental measurements of σ to give a distance V which would generally seem reasonable to the working meteorologist. The median of our observations is 0.030; that of the Mount Washington observations, so different in all their details, is 0.024. May we conclude, therefore, by suggesting ϵ = 0.030 as a value for the conversion of instrumental readings into "visibility," but not on any account for the reverse process.

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(Communicated manuscript received November 26, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

DAMPING OF WATER WAVES BY VERTICAL CIRCULAR CYLINDERS

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Abstract--When considering the wave-height transmission capacity of a dense pile structure, the spacing between piles transverse to the wave front is of considerable importance, and it is upon this characteristic length that studies should be based, thereby permitting a comparison with the effects of longitudinal spacing of piles. The results of this series of model experiments indicate that the relative depth parameter, d/L, may be neglected in the comparison of various transmission capacities. A maximum damping of waves resulting in a wave transmission of 42 pct of the incident wave height was obtained with a relatively dense configuration of 48 rows of 3/8-inch-diameter cylinders rectangularly arrayed and spaced one inch center-to-center. It was noted, within the limits of experimentation, that increasing the number of rows by 100 pct resulted in an average decrease in wave transmission of only 18 pct, irrespective of the density and configuration of the cylinders. Progressive wave-height measurements through a dense cylinder configuration (rectangular array with one-inch center-to-center spacing) showed that between 40 and 80 pct of the total decrease in wave transmission occurs within the first 1/5 of the length of the structure. More generally, it may be stated that approximately 50 pct of the total decrease in wave transmission occurs within the structure a distance, from the seaward face, of less than 1/4 of the wave length. The overall results of the experiments show rather conclusively that a moderately dense piled structure is highly selective in its capacity to reduce wave action. Its ultimate transmission capacity for a given test section depends upon the magnitude of the incident wave steepness.

Introduction--The question often arises as to what extent a piled structure will interfere with the normal propagation of waves. Interference is present in the form of reflections from, and transmission through, the structure with a corresponding loss of wave energy. This loss of energy is the result of eddies within the structure due to the flow disturbance created by the piles. The purpose of this study was to discuss the interference effects of piled structures on wave propagation from the results of model tests by observing and measuring the effects on wave transmission of banks of vertical circular cylinders arrayed in various configurations.

A similar situation arises when one considers such obstructions in streams as bridge piers, the spacing of piers being all-important in minimizing the energy losses and establishing optimum flow conditions in the restricted section. However, a more analogous problem in uniform flow is that of the effect of tube spacing and density on the pressure drop across various arrays of tube banks in pipe stills, heat exchangers, and similar apparatus. Among the significant results obtained in the various heat-transfer tests [ALLEN, 1917; MONRAD, 1932; REIHER, 1926] and pertinent to this paper for the purpose of qualitative comparison were those of pressure drop across staggered and rectangularly spaced tubes. The results indicated that the head loss across uniformly spaced banks of tubing was greater for a staggered array than for rectangularly spaced tubes, the former generally exceeding the latter by approximately 4 1/2 times. Furthermore, the head loss illustrated the effectiveness of the initial rows of tubes in a uniformly spaced configuration, staggered or rectangular, as the number of rows was increased. The paper of MONRAD [1932] presented a summary of work by REIHER [1926], RIETSCHEL [1910], and himself in which he noted the dependence of pressure drop on the increased number of rows as the number increased from two to ten after which the effect of the first five rows was negligible. This effect in the first few rows was found to be in agreement with the data obtained by REIHER [1926] and others on gas flow past a bank of tubes in this manner. This was explained on the basis that the gas did not become completely turbulent for some distance down the bank. In addition, no effect of tube spacing, other than that of rectangular or staggered, was noted. It should be realized at this point that most material in the references refers primarily to heat transfer, but, since the heat transfer is proportional to the pressure drop, the two terms were interchanged in order to restrict this discussion to that of a hydraulic nature.

With the above results in mind combined with interest of oscillatory head loss represented by a decrease in wave energy or height, the model tests of wave transmission through vertical circular cylinders were initiated.

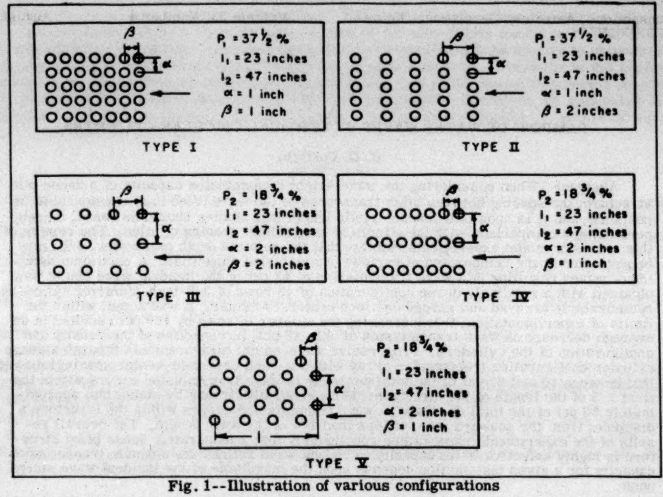
β
α
P₁ = 37 1/2 %
l₁ = 23 inches
l₂ = 47 inches
α = 1 inch
β = 1 inch
TYPE I
P₁ = 37 1/2 %
l₁ = 23 inches
l₂ = 47 inches
α = 1 inch
β = 2 inches
TYPE II
P₂ = 18 3/4 %
l₁ = 23 inches
l₂ = 47 inches
α = 2 inches
β = 2 inches
TYPE III
P₂ = 18 3/4 %
l₁ = 23 inches
l₂ = 47 inches
α = 2 inches
β = 1 inch
TYPE IV
P₂ = 18 3/4 %
l₁ = 23 inches
l₂ = 47 inches
α = 2 inches
β = 1 inch
l
TYPE V

Fig. 1--Illustration of various configurations

Experimental equipment and procedure--The experimental work was performed over a horizontal bottom in a wave channel 60 × 1 × 3 ft, at one end of which was situated a conventional flap-type wave machine with an adjustment for varying the height and period of the waves. At the opposite end of the channel was a gravel beach for the absorption of wave energy.

The waves were allowed to pass through a rigid structure which consisted of 3/8-inch circular cylinders (wooden dowels) uniformly spaced center-to-center perpendicular to the direction of the generated waves (Fig. 1). The cylinders extended well above the maximum wave height. All side effects were minimized by extending the test section across the full width of the channel, thus simulating the condition of an infinitely wide structure.

The wave characteristics were measured by resistance elements of the parallel wire type [MORISON, 1949] and recorded with an oscillograph. Standard static calibration curves were obtained for each run in order to interpret the wave records. The measurements taken from the wave records were based on the average values obtained from the first four fully developed waves measured 3 1/2 ft shoreward of the test section. This point of measurement was sufficiently distant from the test section to permit the recording of fully developed waves subject only to minor distortions occurring owing to transmission through the test section. To obtain information on the variation of wave height through a test section, a set of height measurements at closely spaced points was made with a four-foot section of cylinders with the Type I arrangement (Fig. 1).

In all except three tests, the still-water level was held at a constant depth of 1 1/2 ft. For each arrangement of vertical cylinders shown in Figure 1, the test range of incident wave characteristics was essentially the same; that is, $0.010 < H_i/L < 0.120$, and $0.200 < d/L < 0.600$.

The wave characteristics and their corresponding dimensionless parameters were obtained as follows: d/L was held constant by maintaining a constant wave length while adjusting the height of the incident wave; that is, constant d/L over a range of H_i/L, thereby leading to a relation between H_e/H_i and H_i/L with d/L as a parameter.

Symbols--The following terminology is used:

A	Cross-sectional area of vertical cylinders based on one square foot of test section in a horizontal plane (that is, plan view of test section), and is equal to the cross-sectional area of one cylinder times the number of cylinders in one square foot of horizontal cross-section of test structure
a	Dimensionless exponent based on the plan-view area and the length and width of the test section.
B	Total area in one square foot of test section (horizontal plane), 144 sq in
D	Nominal diameter of vertical cylinders, inches
d	Still-water depth, ft
H	Wave height, ft
L	Wave length, ft
ℓ	Total length of test section, inches
T	Wave period, sec
w	Width of channel, inches
x	A horizontal distance in the direction of wave propagation
α	Transverse center-to-center distance between cylinders, inches
β	Longitudinal center-to-center distance between cylinders, inches

Subscript e	Characteristic of the transmitted wave shoreward of the test section
Subscript i	Characteristic of incident wave at a large distance seaward of the test section
Subscript j	Any particular point in a horizontal direction
Subscript 1,2	Refers to length of test section

Experimental results--The following variables were observed: H_e, H_i, d, L, D, x, ℓ, α, β, and w. From a consideration of these variables and the usual parameters of wave action the following dimensionless parameters can be stated

$$H_e/H_i = f\,(H_i/L,\ d/L,\ \ell/\beta,\ \alpha/w) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which the terms H_e/H_i, H_i/L, and d/L are referred to as the transmission coefficient, wave steepness, and relative depth, respectively. The last two terms determine the geometry of the system and are referred to as the row effect and column effect, respectively. For each of the arrangements of cylinders shown in Figure 1, tests were made in which the transmission coefficient, H_e/H_i, was determined for various values of H_i/L and d/L. The results of these experiments are shown in Figures 2 to 4.

In order to determine the effect of the parameter d/L on wave transmission, experiments were performed on a test section of Type I (ℓ_1 of 24 rows) over a relatively large range of values of d/L and H_i/L. From Figure 2 it will be noted that the distribution of experimental points about an average curve shows a deviation of approximately plus or minus five per cent, which is within the

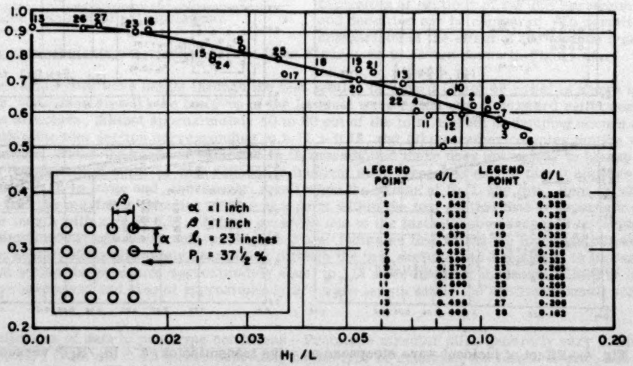

Fig. 2--Relationship between transmission coefficient, wave steepness, and relative depth

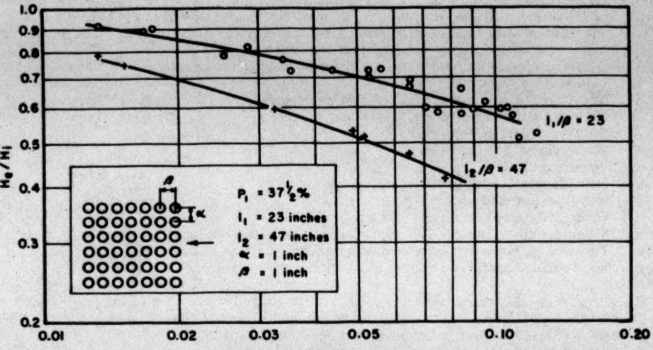

Fig. 3--Relationship between transmission coefficient and wave steepness for two lengths of structures

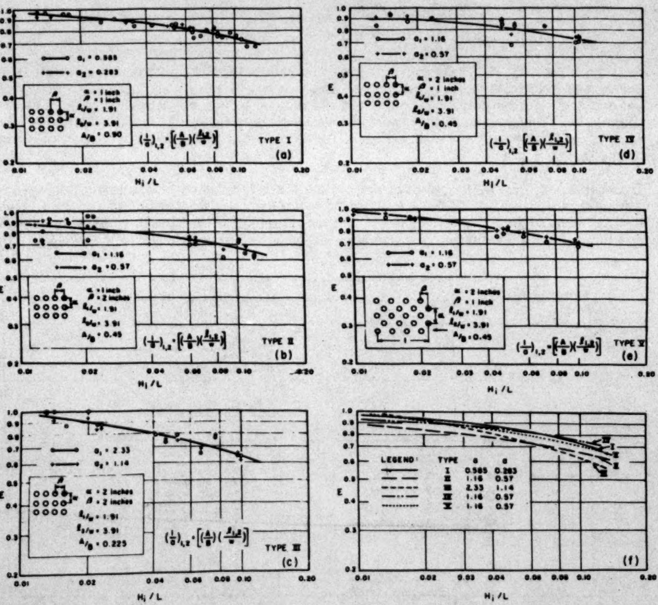

Fig. 4--Effect of incident wave steepness on wave transmission, $E = (H_e/H_i)^a$ versus H_i/L

limits of experimental error. These data indicate, therefore, that the effect of d/L on wave transmission may be neglected in all practical applications.

Figure 3 shows the effect of incident wave steepness on wave transmission for the Type I arrangement of cylinders shown in Figure 2 but with two lengths of the test section. The two curves (denoting section lengths of 23 and 47 inches) diverge as the value of wave steepness becomes greater. This implies that with increasing steepness the increased length of test section (greater number of rows) becomes progressively more effective in dissipating energy. Within the limits of the experiments the change in transverse channel constriction (varying number of columns) while retaining a constant row parameter has essentially the same effect as changing the number of rows. In presenting a general picture, the energy loss due to eddies or reflections is determined by the density of the structure and its per cent transverse constriction of the channel. With wide transverse spacing the energy loss will be a result of the extent to which eddies are permitted to form, while for a dense configuration the loss in wave height will be apparent in the magnitude of reflections from the section in question. As an example, the greatest transmission was obtained through a test section of Type IV, a result which is attributed to the wide cross-channel spacing with respect to the cylinder diameter and the close longitudinal spacing, thereby decreasing the total reflecting surface and reducing the formation of eddies to a minimum, thus approaching a condition which would be analogous to solid longitudinal walls with artificial roughness.

Table 1--Comparative summary of the effect of varying the number of rows of cylinders while retaining a constant transverse spacing, ($H_i/L = 0.100$)

Type	Number of rows	H_e/H_i	Difference
		pct	pct
I	24	57	
	48	38[a]	19
II	12	70	
	24	51	19
III	12	82	
	24	65	17
IV	24	77	
	48	59	18
V	24	72	
	48	56	16

[a] Value obtained by extrapolation.

A summary of results of the effect of the number of rows on wave transmission at a given per cent transverse constriction of the channel is shown in Table 1. This comparison, based on an incident wave steepness of $H_i/L = 0.100$, shows that irrespective of the array of vertical cylinders in a test section, the increase in number of rows from 24 to 48 (that is, doubling the length of the section) results in an average decrease in transmission of approximately 18 pct.

In order to demonstrate the effects of actual cylinder arrangement on wave transmission, the parameter of $(H_e/H_i)^a$ which includes the transmission and the density of the test section was adapted from energy considerations [COSTELLO, 1951]. The exponent a is defined as: $1/a = [(A/B)\ (\ell/w)]$. Plots of $(H_e/H_i)^a$ versus H_i/L for various data are shown in Figure 4. The advantage of these plots is that the transmission capacities of sections of various arrangements and densities can be compared. To permit a comparison of the effect of the various types of cylinder arrangements the average curves from Figures 4a to 4e are shown in Figure 4f.

The variation of wave height through the test section of Type I (ℓ_2 of 48 rows) is shown in Figure 5, and illustrates (1) the build-up of the incident wave height due to seaward reflections from the structure, (2) that approximately 40 to 80 pct of the total amount of damping occurs at a point within the test section corresponding to $x_1/L = 0.16$, and (3) the rather complex motion within the structure. From the range of values of H_i/L investigated there does not appear to be any trend in the amount of build-up with respect to incident wave steepness except that it appears to be large when H_i/L is large and, conversely, small when the values of H_i/L are low. Item (2) above implies that the maximum damping occurs at a point within the test section that is dependent on the wave length with $x_1/L \cong 0.16$. This is probably due to the instantaneous decrease in incident wave steepness through the structure. Figure 5 also indicates that within the investigated range the wave height, while ultimately decreasing through the test section, has a tendency to increase at a point within the structure approximately equal to 1/3 wave length at moderate values of incident wave steepness and also at approximately 2/3 wave length within the section at small values of H_i/L.

Application of data to prototype conditions--Prototype circular piles generally vary in diameter anywhere from six to 22 inches depending on the type of material, the length of pile, and the design load. The spacing and configuration of piling is the greatest variable, varying from zero

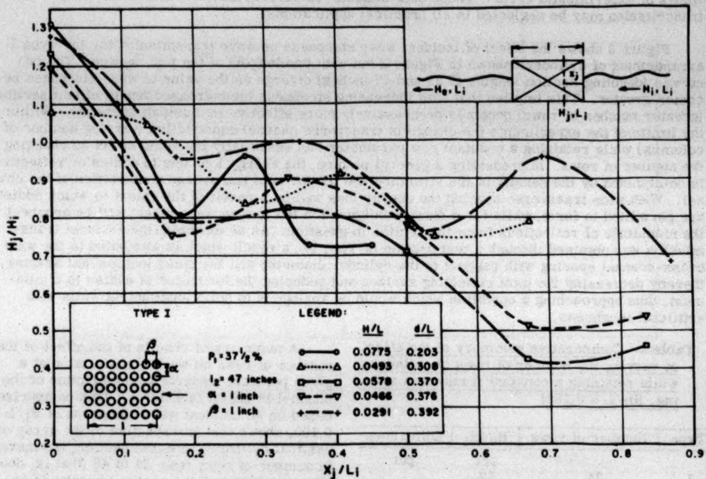

Fig. 5--Transmission coefficients at various relative positions within a pile structure (Type I arrangement)

to 15 ft or more depending on the particular installation. When considering the transmission capacity of a dense pile structure, either from a model or prototype point of view, the spacing between piles transverse to the wave front is of considerable importance, and it is upon this characteristic length that model studies should be based. This recommendation is on the assumption that there is an upper limit to the diameter of the piles, whereas their installation may vary anywhere from a very dense to a widely spaced configuration, the former acting in the capacity of an almost solid barrier to a wave front in which transmission is negligible while the latter may be such as to permit almost complete transmission of waves of moderate steepness.

Since the decrease in transmission through rows of vertical cylinders can be, to a great extent, attributed to viscous damping, the Reynolds modulus is of importance when comparing model results and possible model-prototype applications. When considering individual piles on a prototype scale the order of magnitude of Reynolds number is 10^6 when based on the pile diameter (single pile) and the maximum particle velocity, while the magnitude of Reynolds modulus in these model tests was of the order of 10^3 based on the cylinder diameter (single cylinder) of 3/8 inch. More specifically, the range of Reynolds number encountered was from 400 to 7000 depending on the incident wave characteristics.

Conclusions--The following conclusions may be drawn:

(1) When considering the wave transmission as a function of incident wave steepness, the parameter of d/L may be neglected since any deviation of results that could possibly be attributed to a d/L effect is within the limit of experimental error.

(2) The relatively dense configuration of Type I yields the greatest amount of damping with a transmission of 58 pct at $H_i/L = 0.100$ (24 rows) and 42 pct at $H_i/L = 0.080$ (48 rows).

(3) By reducing the number of cylinders by 50 pct and varying the array of cylinders (Types II, IV, and V) the most efficient configuration of uniformly spaced cylinders is Type II inasmuch as the reduction of 50 pct results in an increased transmission of only 15 pct at H_i/L of 0.100 (12 rows) and 25 pct at $H_i/L = 0.080$ (24 rows). Inasmuch as the center-to-center spacing in both directions (both perpendicular and in the direction of flow) is large, Type III offers little damping; H_e/H_i = 81 pct at $H_i/L = 0.100$ (12 rows), while H_e/H_i = 65 pct when $H_i/L = 0.100$ (24 rows).

(4) Owing to the range of Reynolds numbers involved in the laboratory tests, care must be taken in any attempt to compare with prototype observations, since prototype Reynolds numbers are generally of the order of 10^6 as compared with the model value of approximately 10^4, depending

upon the selection of a characteristic length. This range of Reynolds modulus, however, is over the range of constant coefficient of drag.

(5) For a given total length ℓ of a test section the types in order of increasing damping effect are: Types I, II, V, IV, III.

(6) Regardless of the pile arrangements involved, as long as the spacing is uniform in each direction (no x/β or x/α gradient), increasing the number of rows by 100 pct results in an average decrease in wave transmission of only 18 pct.

(7) Between 40 and 80 pct of the total decrease in wave transmission for Type I (ℓ_2 of 48 rows) occurs within the structure at a relative distance of approximately $x_j/L = 0.16$. More generally it may be stated that, for a test section of cylinder arrangement similar in density to that of Type I (ℓ_2 of 48 rows), approximately 50 pct of the total decrease in wave transmission occurs within the structure a distance, from the seaward face, of less than L/4.

(8) The test sections investigated are highly selective in their ability to reduce wave action, inasmuch as they become effective only with increasing incident wave steepness.

Acknowledgment--The work summarized in this paper was part of an extensive investigation on waves and surf made at the University of California in a project sponsored by the Office of Naval Research.

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(Communicated manuscript received November 8, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

SALINITY CURRENTS IN ESTUARIES

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Abstract--In connection with studies of the mouth of the Columbia River, extended current measurements were made in the estuary; also included in the observations were several sets of salinity measurements between the ocean and the upper limits of salinity. This paper summarizes these data and presents an approximate method for separating the effects of tidal, fresh-water, and salinity currents.

At various times during the year 1932, current measurements were made in the estuary of the Columbia River between Clatsop Spit and Sand Island (see Fig. 1) along a line perpendicular to the local direction of flow. Five boats were anchored at equal spacing across the channel, and measurements were made from each boat at five depths every 20 min continuously over several days. The current meters were not equipped with direction indicators. One series of measurements, made June 20-22 when the fresh-water runoff amounted to 580,000 cfs, was selected for analysis.

The simultaneous curves of velocity in vertical sections obtained in the June measurements showed a marked tendency for the reversal of current from flood to ebb to occur first at the surface, while the change from ebb to flood started near the bottom. Furthermore, at maximum strength of current, the ebb velocities increased almost linearly from bottom to surface while the flood velocities showed a marked depression of the level of maximum velocity. These peculiarities were at first attributed to tidal influences alone and to irregularities in the bottom. However, there is no reason to expect that the tide alone should produce such different flood and ebb velocity distributions, when combined with the currents caused by fresh-water flow. The June measurements showed this effect more clearly than the others and were selected for analysis.

Isolation of the salinity currents was accomplished in two steps. First, the tidal currents were eliminated by obtaining the net outflow at five depths at each of five boat stations over an interval embracing several whole tidal cycles, counting from times at which the tidal elevations were the same at the beginning and end of the interval. In this manner, the net outflow velocity or velocity attributable to fresh-water flow plus salinity was obtained as a set of five vertical velocity curves, one for each boat station. The next step was to construct the vertical velocity curves which would have existed at the same mean velocity, total depth, and roughness in a homogeneous liquid. The difference between the measured net outflow velocities at each point in each vertical section and the velocity which would have occurred had there been no salinity gradient gave, approximately at least, the magnitude of the velocities caused by salinity alone. The general effect of salinity alone is to produce an inward velocity along the bottom and a seaward velocity at the surface.

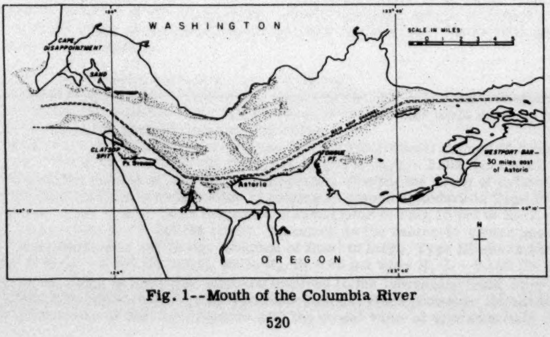

Fig. 1--Mouth of the Columbia River

The time interval chosen extended from 17.4 hr on June 20 to 5.5 hr on June 22. The current measurements were made at intervals of 20 min, and it was only necessary to sum up the flood and ebb velocities separately, take the difference in these sums, and divide by the total number of observations. The average outward velocity in the vertical section at each boat station was then obtained by averaging graphically. The average fresh-water flow obtained from the average velocity at the five stations was 530,000 cfs, as compared with 580,000 cfs indicated by the river stage some miles upstream.

The curves of net outward flow at each boat station included the effect of both the fresh-water outflow and the salinity current. The method of separating these two effects was to estimate the velocity distribution which would have existed for the same average velocity in a homogeneous fluid, using as the basis of comparison velocity measurements made in the Columbia River at Westport Bar where only fresh-water flow occurred. These generalized coordinates of the velocity curve used in the computation are shown in Table 1.

Table 1--Coordinates of vertical velocity curve

Depth	Ratio to mean velocity
pct	
Surface	1.09
80	1.07
60	1.05
40	1.02
35	1.00
20	0.94
Bottom	0.75

The results of the segregation of the salinity currents appear in Table 2. Column (3) shows the average outward flow at each boat station during the whole period, including the effect of both fresh-water flow and salinity gradient. Columns (4) to (7), inclusive, apply to the salinity currents alone, obtained in the manner previously described. Column (4) gives the depth below mean sea level to the surface of separation between the outward surface current and the and the inward bottom current. Columns (5) and (6) give the surface and bottom currents, respectively. All of the curves of salinity currents had the same shape.

Table 2--Segregation of salinity currents

Station (1)	Depth (2)	Velocity of fresh-water flow (3)	Salinity current				Remarks (8)
			Depth to U = 0 (4)	Surface velocity outward (5)	Bottom velocity inward		
					Graphical (6)	Computed (7)	
	ft	ft/sec	ft	ft/sec	ft/sec	ft/sec	
A	25	0.93	12	0.7	0.6	0.9	North side
B	41	1.88	14	1.8	1.4	1.4	
C	31	1.36	11	0.8	0.7	1.1	
D	39	1.01	15	0.8	0.6	1.4	
E	37	1.90	16	0.7	1.2	1.3	South side

At the time of these current measurements, no salinity samples were taken, and it was necessary to determine later what salinity distribution existed by salinity surveys made before, during, and after the freshet season. Lines of salinity samples were taken along the main ship channel and at scattered points throughout the estuary between the ends of the jetties and Tongue Point. From two to six samples were taken in each vertical section with an average of three per station. Samples also were taken throughout one tidal cycle in the main channel near Fort Stevens. Salinity surveys were made before, during, and after the freshet season. Specific gravities were measured by a calibrated hydrometer without correction for temperature.

The results of the three surveys were represented by computing the average salinity in vertical sections to a depth of 40 ft and plotting against the distance along the ship channel. The position of each sample was corrected to the time of high-water slack at Fort Stevens by means of measurements in a scale model [JOHNSON, 1948]. The corrected curves were remarkably regular, with the salinity surveys before and after the freshet coinciding, and that taken at the peak of the freshet being similar but displaced about six miles seaward. The distance within which the specific gravity changed from that of fresh to sea water was roughly 16 mi for all three surveys. The water depth below MLLW along the main channel is approximately 35 ft. The rate of change of specific gravity per mile was 0.0021 in April and August, 1932; and 0.0017 in June, 1932. The current-meter measurements used in segregating the salinity currents were taken at a section which would, at both freshet and low-water stages, lie in the region of the greatest rate of change of specific gravity, and it appears reasonable to ascribe the secondary currents to salinity effects. If the tidal currents and fresh-water flow were temporarily eliminated, and if the salinity gradient represented the only force causing flow, the inward salinity current at the bottom would be resisted by bottom friction acting seaward against the greater density. The balance of forces would then be represented by

$$w\Delta S\, D^2/2 = kX\rho U^2$$

Here w is the unit weight, ρ is the density of fresh water, D is the average depth, ΔS is the difference in specific gravity in distance X, measured along the channel, U is the velocity near the bottom, and k is a friction coefficient. Solving for U

$$U = D\sqrt{g\Delta S/2kX}$$

Using D = 35 ft, $\Delta S/X = 0.0017$ for June conditions, and k = 0.004, one obtains the bottom velocities shown in Column (7) of Table 2. The agreement between the computed and measured salinity currents at the bottom is not perfect, but it is sufficiently good to justify the method employed to segregate the salinity currents.

It is obvious that the friction forces and the bottom velocities are related and, because friction depends upon the square of the velocity, that the velocities due to tides, fresh-water runoff, and salinity cannot be separated quantitatively and with complete accuracy on the assumption of linear superposition made in the preceding analysis and calculation. However, more complete analysis indicates that, when the bottom current due to fresh-water runoff alone is equal approximately to the bottom current due to salinity alone, the error in the computed salinity current is of the same order of magnitude as the uncertainty over the friction coefficient; the change in the bottom velocity caused by salinity may be computed within a factor of 2.

Velocity and current measurements in a large estuary are both time-consuming and expensive. The salinity and velocity study reported here was made because of the peculiarities noticed in the vertical velocity curves, and the work was not continued beyond the stage necessary to demonstrate that the anomalies were caused by the salinity gradient. The bottom friction and other characteristics of estuaries vary greatly, and there appears to be little to be gained by quoting details of the observations underlying these notes. However, the method of separating tidal, fresh-water, and salinity currents presented may be generally applicable. So far as is known, no efforts to separate these effects have been made previously, and it is believed that this omission has obscured important relationships between tides and tidal currents in estuaries. During periods of small fresh-water flow, each tidal cycle includes phases during which the salinity currents are of the same magnitude as the tidal currents, and hence these currents are not negligible. Particularly in a deep estuary or bay, salinity currents may play an important role as regards the silting and scouring of channels.

Acknowledgments--The observations discussed in this paper were obtained by the Corps of Engineers in a study of factors affecting sand movement and beach erosion at the mouth of the Columbia River.

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(Communicated manuscript received April 2, 1951, and, as revised, October 16, 1951; open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

THE REFRACTION OF GROUPS AND OF THE WAVES WHICH THEY GENERATE IN SHALLOW WATER

E. Allan Williams and John D. Isaacs

(Contribution from the Scripps Institution of Oceanography, New Series, No. 580)

Abstract--A process is described which may be of importance in beach and near-shore development and which may account for surging in the surf zone as well as other phenomena. It is shown how a group front may be refracted according to one law as it approaches a coast and then, after reflection, proceed seaward under quite a different law. This permits the outgoing wave to move in a direction that it could not ordinarily attain, and causes "trapping" under many conditions. Stoneley's theory of refraction of energy fronts is further developed to permit quantitative investigation of group refraction on various types of shores. On the basis of the assumed hypothesis, the outgoing surf beat may be totally reflected from deep water, then reflected from the beach, a process which can occur repeatedly and result in newly arriving groups with variable phase relationships. Theory and techniques are developed for the investigation of the surf-beat path. The refraction of the reflected groups is shown to display regions of high convergence, divergence, and reinforcement.

Introduction--The variability of the swell and wind waves upon a coast imparts a complexity to their refraction and to the resultant processes of shore and offshore transport. Further investigation and analysis will be required before this complexity is fully understood.

A typical train of waves consists of a number of crests which are relatively consistent in period but variable in height. The variation in height takes the form of groups of higher waves arriving at intervals of one to four minutes. As such a train enters shallow water, the individual phase crests are refracted, converging on ridges and shoals, and diverging over basins and canyons. The refraction of these crests has been much studied, but all such studies have considered only the phase refraction [U. S. HYDROGRAPHIC OFFICE, 1944; MUNK and TRAYLOR, 1947; JOHNSON, O'BRIEN, and ISAACS, 1948].

The refraction of the group, however, is quite different from that of the individual phase crests. Whereas the phase velocity decreases continuously as the water shoals, the group velocity first increases, reaching a maximum at a depth of h/L_o (or d/L_o) equal to 0.157, and then decreases continuously for water shoaler than this. As a consequence of these velocity relationships, the group front in the initial phase of refraction increases its angle to the contour, while the phase wave-front angle diminishes. In the final stage of the group refraction the angle to the contour decreases but remains considerably greater than the corresponding phase wave angle. Therefore, when this refraction takes place over a gradually sloping bottom, the group or energy front will be curved so that it crosses several phase wave crests. Each phase wave will partake of the higher energy associated with the mid-portion of the group at the point of crossing, which will cause it to have greater than normal height for some distance along its crest. Thus an echelon pattern should result as discussed by STONELEY [1935] and ISAACS, WILLIAMS, and ECKART [1951]. This may frequently account for the early short crests of waves approaching the shore.

As the waves of a group break at the shore, they cause the water level to vary in height with a period of one to four minutes. Outgoing waves generated by this fluctuation in height have been designated "surf beat" by MUNK [1949]. In many instances, as a result of group refraction, the energy front approaches the shore at an angle much greater than that of the phase wave. The fluctuation of water height will then be expected to progress along the beach in step with the arrival of the energy front. Under these conditions the surf-beat wave would proceed, not directly outward from shore, but along a curved ray path which is dependent upon the direction and characteristics of the incoming waves, and upon the refraction which they have experienced. Graphical constructions and analysis indicate that in most cases at a straight beach with parallel contours, and in some other situations as well, the surf-beat ray curves, becomes parallel to the shore at some maximum depth, and then returns to shore. Moreover, this phenomenon of total reflection from deep water [ISAACS, WILLIAMS, and ECKART, 1951] may be accompanied by reflection from the shore, with the result that a wave experiencing such alternate multiple reflection can be propagated great distances along a coast line.

The surf beat which is totally reflected from deep water can return to the shore either in or out of phase with the energy front of an incoming group. This interference will cause added variation in the water level at the shore line both above and below normal. The exaggerated surging noted in open bays may be the result of a condition of resonance between the incoming group, which provides the force of excitation, and the natural frequency of the surf-beat oscillation in its travel from the region of generation, reflection from deep water, and return to the shore.

The study of the mechanisms associated with the phenomena described above is presented in two parts; first, the treatment of the incoming group refraction by application of principles developed by Stoneley, and second, the development of techniques and theory for the investigation of the surf-beat path.

Group refraction of surface gravity waves--The general theory of the refraction of a wave group as developed by STONELEY [1935] can be applied to the graphical construction of group refraction diagrams and, where bottom topography permits, to analytical determinations. The energy between two phase-wave orthogonals is presumed to remain constant and to travel with the group velocity, C_G. Accordingly, an element of the energy front advances with group velocity, not necessarily normal to the energy front, but normal to the phase-wave front through the element in question; that is, along the phase orthogonal through the element.

If the water is deep or of constant depth, the energy front advances so that lines representing its successive positions are parallel. The distance between two successive positions in this case will be proportional to $C_G \cos \psi$ (as shown at the right of Fig. 1), where ψ is the angle which the energy front makes with the phase-wave crest, while the distance which an element moves along a phase-wave orthogonal is proportional to C_G. In general, the angle α between the phase-wave front and the refracting boundary may be either larger or smaller than ϕ, but for deep-water waves which have entered shallow water ϕ exceeds α. In deep or constant depth water the angle ψ between the two fronts will remain constant. When phase waves are refracted, however, the energy or amplitude front likewise is refracted. Suppose the orthogonals of a phase-wave refraction diagram to have been drawn and the initial position of an energy front determined. The change in position of a given element of the energy front can be established by advancing that element a distance proportional to the group velocity, C_G, along the phase-wave orthogonal through the element. The positions of the elements thus obtained are joined to form the advanced energy front. Energy fronts so obtained cannot be thought of as a representation of simultaneous positions of different group crests as in the case of phase-wave crests, but only as representing the advance with time of an individual energy front. Techniques similar to those used in construction of the usual phase-wave refraction diagrams can also be applied in the case of group refraction.

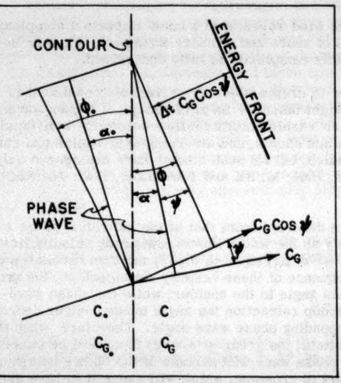

Fig. 1--Refraction of phase wave and energy front; advance of energy front in water of constant depth

The special case of refraction of a group over a straight beach with parallel contours has been examined by analytical methods. The expression for the angle between the energy front and contour line for this condition can be obtained from the relation given by Stoneley for refraction of the energy front. His expression, which becomes

$$\sin\phi_0 / C_{G_0} \cos(\phi_0 - \alpha_0) = \sin\phi / C_G \cos(\phi - \alpha) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

for progression from deep to shallow water, gives the general relationship between energy fronts before and after refraction (see Fig. 1). Using this equation and Snell's law for phase-wave refraction and taking $\phi_0 = \alpha_0$ one obtains the equation

$$\phi = \cot^{-1}\left\{[1 - 2\,(C/C_0)\,(C_G/C_0)\,\sin^2\alpha_0]/2\,(C_G/C_0)\,(\sin\alpha_0)\,(\sqrt{1 - (C/C_0)^2\sin^2\alpha_0})\right\}\;.\;.\;.\;(2)$$

where ϕ is the angle which the energy front makes with the contour, C is the phase velocity and C_G the group velocity at the contour, C_0 is the deep-water phase velocity, and α_0 is the initial deep-water angle between the wave front and the contour.

Diagrams have been constructed for a straight beach with uniform slope to determine the refraction of the energy front. This requires that phase-wave orthogonals or rays first be drawn. Energy fronts and phase-wave rays are indicated in Figure 2 for 14-sec phase waves, for which $\alpha_0 = 30°$, approaching a straight beach of 4.25 pct slope. The time interval of advance for the energy front is twice the phase-wave period. The angle between the phase-wave crest and the contour continuously decreases as the wave progresses from deep to shallow water. However, the corresponding angle for the energy front increases from its initial value of 30° to about 36° at a depth of 25 to 30 fm and then decreases rapidly to approximately 11° at one fathom. The corresponding values of α for the phase waves are 25° and 5 1/2° respectively. In the advance into shallow water the group velocity exceeds 1/2 the phase velocity and is even greater in this region than in deep water; hence an initial increased energy-front angle to the contour is obtained. The line (AA′) is a refracted energy front constructed by use of data from (2). In view of the generally accepted inaccuracies of graphical refraction diagrams, the agreement between the graphical and analytical constructions is remarkable.

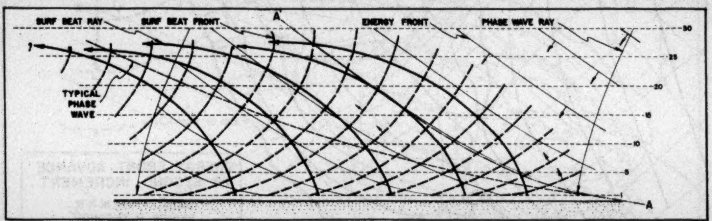

Fig. 2--Refraction of waves generated by incoming wave groups in the surf zone along a hypothetical straight beach of uniform slope; waves are totally reflected by deep water; a representative phase wave is shown; the group front AA′ was constructed from analytical data

In Figure 3 is shown a group refraction diagram for the irregular bottom north of La Jolla. The phase-wave refraction diagram, designated as Figure 8 of MUNK and TRAYLOR [1947] for 14 sec from WNW, was made available and has been used as a basis for the construction. The difference in the refraction patterns results in the formation of phase and group fronts of opposite curvature as evidenced in the region of the three representative phase crests shown in the figure. At La Jolla Cove, where the phase orthogonals converge, the group fronts approach a parallel relation to the orthogonals. This is apparently the manner in which the group-refraction diagram indicates regions of convergence. North of the canyon the phase waves and group fronts approach the shore practically parallel to each other. It is seen that, far from simplifying the problem of refraction, this application shows it to be even more complex than phase-wave refraction diagrams have indicated.

Refraction of long-period waves--MUNK [1949] in his investigation concluded that the shore line acts as a radiating line source which returns approximately one per cent of the incoming wave energy in the form of long-period (two-to-three minute) waves termed surf beat. He correlated the height oscillations with the variability of water transport into the surf zone by studying the advance of groups of waves over wave and tsunami recorders at Scripps Pier. TUCKER [1950] has measured long sea waves with periods of two to three minutes, shown them to be the result of the varying height of groups of waves breaking on the shore, and discussed the mechanism by which they are produced. Such a study at a point along the shore could be carried out without consideration of group refraction.

The investigation of the path of a surf-beat wave, however, assuming it to be generated at the shore by an incoming group, requires (1) the determination of the effect of group refraction upon the incoming energy front, and (2) the effect of refraction upon the surf beat itself. The advance of

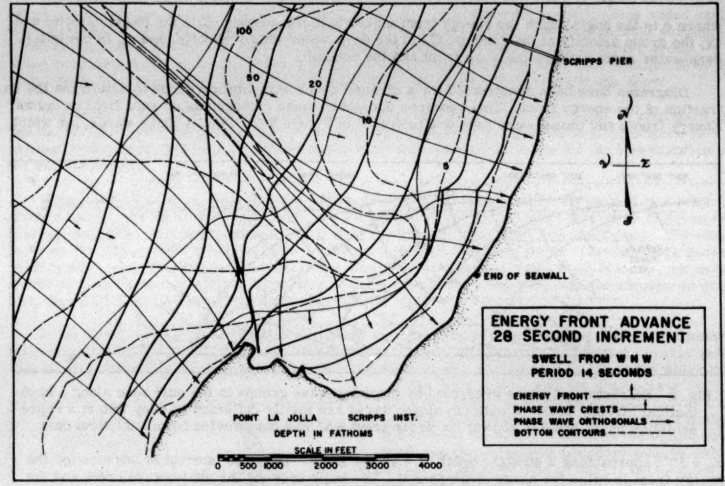

Fig. 3--Graphical energy-front refraction construction for the Point La Jolla - Scripps Institution region

the generating source along the beach is determined by the character and initial direction of the incoming waves, and the effects of bottom topography upon the refraction of the group. The initial direction of the surf beat is determined by these conditions also. The seaward return of the long-period waves generated in this manner at the shore and the resultant total reflection from deep water is considered below. Graphical techniques have been used for the most part, although the special case of a straight beach with uniform slope has been treated analytically and found to be a source of generation of long-period waves, most of which exhibit the interesting phenomenon of total reflection from deep water.

The outgoing wave-front positions were established from the application of wave generation in accordance with Huygen's principle, assuming the one-fathom contour to be the line of generation. Thereafter, a given wave front was formed by joining points obtained through the advance of each element a distance, based upon $\sqrt{gh}$ as the velocity of propagation, along the direction of the normal to the wave front. If this velocity is compared with the velocity of the incoming group near shore, it is found that the two velocities approach equality for values of h/L_0 less than 0.006. Accordingly, an energy front may be considered as partially reflected, in accordance with the laws of regular reflection, at some convenient depth within this limitation. This has been the assumption in the analytical approach. However, there must also be a practical lower limit of h/L_0 at which either method may be applied, since the laws upon which the refraction is based may not hold after the waves break.

The graphical determination of the surf-beat path for one set of conditions in the region of Scripps Institution is shown in Figure 4. For convenience in studying the diagram, the positions of the wave fronts have been marked by letters. The energy-front positions are designated by small letters, and those of the outgoing wave by capitals. Simultaneous locations are indicated by the same letter. The letter designations begin with a and A as the point at which the energy front first reaches the one-fathom mark. The time increment is 28 sec except where complexity requires advance by intermediate steps, such as E′ and E″ between E and F at the head of Scripps Canyon.

The surf beat below the Point, as shown by the front advance A to H, returns seaward to deep water. North of Scripps Pier at the head of Scripps Canyon the refraction, though complex, can

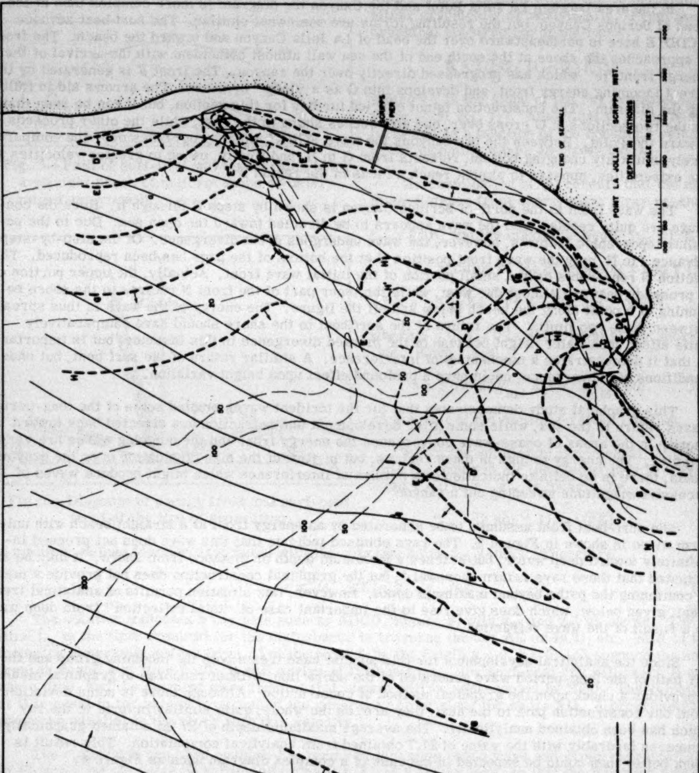
SCRIPPS PIER
END OF SEAWALL
POINT LA JOLLA SCRIPPS INST.
DEPTH IN FATHOMS
SCALE IN FEET
SURF BEAT FRONT
ENERGY FRONT
BOTTOM CONTOURS
E
E'
E''
F
G
H
10
20
50
100
200

Fig. 4--Refraction of waves generated in the surf zone by wave groups, Point La Jolla - Scripps Institution

still be followed with some measure of certainty. Beginning at the wave front E, a 14-sec intermediate step (E'-E') is shown. Arrows indicate that the wave is moving to cross the canyon from each side. Seven seconds later (E''-E'') the two segments have almost met, while at the end of the full 28-sec increment (F-F) the "loop" over the head of the canyon has been completed. The wave fronts advancing from each side have crossed the canyon and are continuing as indicated. Since this is a region of high energy, there must be considerable dissipation in this action. The advance down the canyon from the "loop" crossing is difficult to determine, but the segments shown represent an estimate of the probable positions. If it is assumed that the velocity of progress here is $\sqrt{gh}$, then the extreme steepness of the canyon walls effectively introduces discontinuities, so that a continuous wave front extending across the canyon cannot be maintained.

In the area between La Jolla Point and the Canyon the diagram is more complex than at the head of Scripps Canyon, but the resulting forms are somewhat similar. The surf-beat advance BCDD'E here is northeastward over the head of La Jolla Canyon and toward the beach. The front E approaches the shore at the south end of the sea wall almost coincident with the arrival of the energy front "e" which has progressed directly over the canyon. The front F is generated by the direct incoming energy front, and develops into G as a "loop" or cross. The arrows aid in following the diagram. The construction is not carried further for this section, but it can be seen that as the two sections of G cross over, one approaches shore to the east, while the other proceeds toward the Point. Between the two canyons the wave FGHH', which progresses over the comparatively uniformly changing bottom, refracts from G to H, and finally, owing to greater velocities at the extremities, appears to almost reach a focus in the region H'.

The wave front to the north of Scripps Canyon is shown by steps C through H. Here the contours are quite regular, and the wave appears to be directed toward the open sea. Due to the peculiar topographic features, however, the wave undergoes great divergence. Of the step-by-step advance I to N, only the wave front position N at the bottom of the page has been reproduced. The section N represents only a small portion of the initial wave front. Actually, the upper portion of H proceeds seaward almost due west, while the inner part of the front N returns to the shore beginning at a point along the beach below and off the figure. The energy of the wave is thus spread between these two limits. The return of the surf beat to the shore should have comparatively little effect upon water height because of the marked divergence in this instance, but is important in that it demonstrates a mechanism of interference. A similar return of the surf beat, but under conditions of convergence, could have a profound effect upon height variation.

This graphical study demonstrates that for the incident waves studied some of the long-period waves return to the sea, while some after development and refraction are directed back toward shore. In the areas of phase-wave convergence the energy front and the outgoing waves are very complex. The energy is high in these regions, but in view of the high dissipation over the canyon heads, there is no definite indication of a pattern of interference which might produce waves of excessive magnitude sweeping out a canyon.

The surf-beat front assumed to be generated by an energy front at a straight beach with uniform slope is shown in Figure 2. The rays obtained indicate that this wave does not proceed indefinitely toward deep water, but reaches a maximum depth or distance from shore. It may be anticipated that these rays return shoreward, but the graphical construction does not provide a means of continuing the paths beyond maximum depth. However, this situation permits of analytical treatment, given below, which does give rise to the important case of "total reflection" from deep water as a result of the wave refraction.

Since the analytical development for this special case treats both the incoming group and the ray path of the long-period wave generated at the shore line, without recourse to graphical means, it provides a check upon the graphical method of construction. Although there is some deviation from one construction path to the next, they are, on the whole, quite similar in form to the ray which has been obtained analytically. The average maximum depth of 27 fm obtained graphically compares favorably with the value of 27.7 obtained from analytical computation. This result is much better than could be expected in the case of a complex diagram such as Figure 4.

Analytical determination of the path of a wave generated by the energy front at a straight beach of uniform slope--The analytical description of the path of a wave group as it approaches a straight beach of uniform slope has been given above. Assuming that this group or energy front generates a long-period wave as it traverses the beach, let us now consider the analytical description of the path of this newly formed wave as it progresses seaward. This path can be expressed in terms of the characteristics of the incoming phase waves and the slope of the beach.

The analytical treatment of this problem by application of Fermat's principle of least travel time such as applied by ARTHUR [1946] is carried out by minimizing the integral

$$I = \int_{\text{path}} (1/c)\, ds = \int_{\text{path}} \left\{ [1 + (y')^2]^{1/2} / (gay)^{1/2} \right\} dx \quad \ldots\ldots\ldots\ldots\ (3)$$

where ds is an element of the path, c the velocity of the disturbance, y' equals dy/dx, and ay is the depth h, a being the slope. The coordinate x is parallel to the shore and y positive outward as shown in Figure 5. A solution of this equation [JOOS, 1934, p. 71] is

$$(ay)\,[1 + (y')^2]\,K = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4)$$

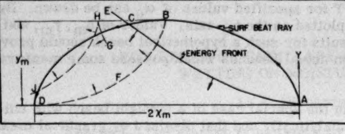

Fig. 5--Path of surf-beat wave generated along a straight beach of uniform slope as determined analytically

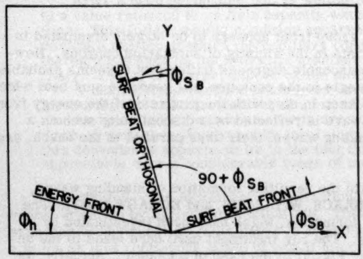

Fig. 6--Diagram of energy front and surf-beat front at the line of generation

where K is a constant which can be determined either graphically or analytically. From the graphical construction of the ray

$$K = 1/\left\{ay_1\ [1 + (y'_1)^2]\right\} \ldots\ldots\ldots (5)$$

where y'_1 is the slope of the ray evaluated at some point (x_1, y_1). Analytically, K can be determined as a function of the constants related to the incoming waves as shown below.

An examination of (5) reveals that the maximum distance from shore which a ray reaches is

$$y_m = 1/aK \text{ and the maximum depth } h_m = 1/K\ldots (6)$$

The solution of (4) in terms of x and y is

$$\pm x = -\sqrt{y/Ka - y^2} + 1/Ka \sin^{-1}\sqrt{Kay}\ldots (7)$$

where x = 0, y = 0 is some point at the shore line. When $y = y_m$

$$x = \pm(\pi/2)\, y_m,\ \pm(3\pi/2) y_m, \text{ etc.}\quad\ldots (8)$$

as a result of the multiple-valued function.

From the solution of the integral (3) the time t_m required for the disturbance to travel from $y = 0$ to $y = y_m$ is found to be

$$t_m = (2 \sin^{-1} 1)/(a\sqrt{gK})\ \ldots\ldots\ldots (9)$$

a multiple-valued function associated with the corresponding x_m values. Therefore,

$$t_m = \pi/(a\sqrt{gK}) = \pi\sqrt{y_m/ag}\ \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The solution indicates a ray path such as ABCD, Figure 5, with x_m and y_m as shown. The time t_m is the time required for the disturbance to traverse the arc AB or BCD, etc. ECA is the incoming generating energy front. Let the point A be the origin x = 0, y = 0, and suppose the energy front to reach A at time t = 0. It will arrive at D when $t = t_m$, the surf-beat ray will reach B at the same time, and the outgoing wave front at that instant will be represented by BFD. The velocity $\sqrt{gh}$ at B is equal to the rate of progress of the generating front along the shore AD. A progressive wave advances parallel to the shore with this velocity.

The constant K has been evaluated in terms of incoming phase wave and group characteristics. In Figure 6 let the depth be sufficiently small, h/L_o of 0.006 or less, so that the incoming group velocity C_G is practically equal to $\sqrt{gh}$, the velocity of the surf beat. The angle of the surf-beat wave ϕ_{SB} can then be set equal to the angle ϕ_h for the energy front. The value of the constant K becomes

$$K = 1/(ay_1)\ [1 + (y'_1)^2] = 1/h\ (1 + \cot^2 \phi_h)\ldots\ldots\ldots\ldots\ldots (11)$$

where $\phi = \phi_h$ is to be evaluated at some depth h determined by the relation $h/L_o = 0.001$. (The selection of depth h corresponding to this value of h/L_o is a choice of convenience. A consideration of the velocity relations and of the depth at which breakers are formed seems to indicate that a reasonable value for h/L_o should be between 0.001 and 0.006.) By use of (2) the computation, upon introduction of simplifying approximations, yields

$$K = (50.0\,\pi \sin^2 \alpha_o)/(g\,T^2)\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where T is the incoming wave period. Therefore, from (10)

$$t_m = \pi/a\sqrt{gK} = 0.251\ T/a \sin \alpha_o\ \ldots\ldots\ldots\ldots\ldots (13)$$

If in use of (13) we consider a given beach, the slope a is constant, and for any given deep-water direction sin α_0 is also constant, while $t_m = AT$ indicating a linear relation between t_m and T. A family of straight lines relating t_m and T for specified values of α_0 can be drawn. By use of (10) the values of h_m, x_m, y_m, are readily plotted on the t_m axis. Values of h_m, y_m, and t_m are readily obtained from such a plot. The results for such a hypothetical beach should prove useful as a guide in estimating similar relations on actual beaches which possess some measure of regularity.

Discussion and conclusions--The agreement in the special case of a straight beach with uniform slope, between the energy front as obtained analytically and that obtained by graphical means, provides support of the graphical construction in cases where the analytical approach is difficult or impossible. Except in the situations where the phase-wave orthogonals approach very close to each other, no difficulty is encountered in the graphical method. The diagram for waves approaching a straight uniform beach from an angle indicates the formation of an echelon pattern of short-crested waves from long-crested swell.

The energy of the surf beat generated by the group front appears to be largely dissipated in regions of complex bottom topography such as exists in the vicinity of submarine canyons. However, along a straight beach which possesses a reasonable degree of uniformity it seems probable that the energy front due to waves initially at an angle to the contours will generate surf beat which gives rise to a progressive wave motion that will keep in step with the progress of the energy front, the velocity being $\sqrt{gh_m}$. If such a progressive wave is reflected by a discontinuity such as a shallow bar or breakwater, the conditions for standing waves, their axes parallel to the beach, are met.

The surf-beat ray paths produce a caustic with the resulting formation of standing waves, their axes normal to the beach, as discussed by ISAACS, WILLIAMS, and ECKART [1951]. The mathematical relationships of this standing-wave formation, which have been investigated by Eckart, will be reported by him in the near future. The ray treatment used here leads to the untenable requirement of infinite wave height (see H, Fig. 5) in the case of a caustic. Actually, as in the case of total reflection of light [JOOS, 1934, p. 343] it will be expected that some disturbance exists beyond h_m, the depth given by the ray treatment for total reflection.

The ray path and travel time obtained in the case of a hypothetical straight beach of uniform slope suggest the possibility of similar conditions in the case of an open bay where multiple reflections from the two sides may take place. If the period and direction of the incoming waves are such that the "group period" is equal to this travel time or an appropriate sub-multiple thereof, this resonant condition may well be the explanation, not only of excessive surging but also of the variations in the height of the surges.

Acknowledgment--This work represents results of research carried out for the Beach Erosion Board, Corps of Engineers, Department of the Army, Contract W-49-055-eng-3.

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(Communicated manuscript received January 15, 1952;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

INFLUENCE OF CAPILLARY CONDUCTIVITY AND DEPTH OF WETTING ON MOISTURE RETENTION IN SOIL

L. A. Richards and D. C. Moore

Abstract--The storage of moisture in soil in the field is discussed and explained in terms of the dynamic moisture-transmitting properties of the soil. Published data on capillary conductivity (unsaturated permeability) are summarized and additional new data for six soils are presented, along with a new pressure-type apparatus for measuring capillary conductivity.

After a deep permeable soil is wetted in the field, the moisture content soon reduces to a value referred to as field capacity which thereafter changes relatively slowly with time. This field-capacity condition apparently corresponds to the moisture content and moisture tension at which the capillary conductivity of the soil becomes small. Changes thereafter depend on the hydraulic gradient and the low residual values for the conductivity. These, of course, depend on the moisture distribution in the whole profile.

A laboratory procedure sometimes used for estimating field capacity consists in measuring the moisture content of a mass of soil that has been wetted and allowed to stand a specified time in contact with dry soil. The dependence of this method on the depth of wetting is illustrated by laboratory tests on three soils and the explanation for this dependence appears to lie in the fact that the capillary conductivity of soils remains appreciable over a considerable range of moisture content below field capacity.

Introduction--The movement and retention of water in unsaturated soil has been given considerable attention by agricultural scientists. These processes also have direct relation to several branches of engineering. This paper is concerned with water relations in soil where there exists a more or less continuous phase of gas at atmospheric pressure, in addition to soil particles and the water. The flow of water in unsaturated soils, that is, film flow, should be distinguished from the gaseous diffusion of water vapor, because the latter is controlled by a different physical mechanism. There is evidence that under isothermal conditions moisture-transfer rates by vapor diffusion are negligible compared with unsaturated flow in the moisture-content range that will permit the growth of plants. This may not be true, of course, in the presence of temperature gradients, in which case vapor transfer may be in a different direction from the film flow. The driving force for unsaturated flow is assumed to be the same as for saturated flow. The Darcy relation, $v = k\,i$, is particularly convenient for practical purposes and will be used in the present paper. The vector sum of the gravity force and the pressure-gradient force acting on water in soil are represented by the hydraulic gradient i. The definition of hydraulic gradient and its relation to the gradient of energy potentials often used for expressing the driving force have recently been reviewed by L. A. RICHARDS [1940] and HUBBERT [1950].

In the above equation the flow velocity v is the volume of water which in unit time crosses unit area in the soil at right angles to the flow. The proportionality factor k is sometimes referred to as the Darcy coefficient of permeability. If, as is indicated in the JACOB [1946] report, it is preferable in considering porous mediums to reserve the term permeability for reference to a specific physical property related only to the pore structure and independent of the nature of the fluid, then we may refer to the Darcy k by some such term as hydraulic conductivity. When the soil pores are not completely filled with water, k depends on the moisture content of the soil and may be referred to as unsaturated permeability or preferably capillary conductivity. In practical units when the driving force is expressed in terms of the hydraulic gradient, capillary conductivity is the calculated flow velocity at unit hydraulic gradient.

Proportionality between flow velocity and driving force at a given moisture content in unsaturated soil has long been assumed and has recently been verified by CHILDS and GEORGE [1950] for a number of granular porous mediums.

Figure 1 illustrates how porous ceramic cups and vacuum gages are used for measuring the equivalent negative pressure and hydraulic head of water in unsaturated soil. The porous cups designated by the letters a, b, c, and d are connected through vertical tubes to the mercury manometers above the soil surface. The units are vacuum-tight and are completely filled with water. Units of this type have been used by soil scientists throughout the world for measuring the condition of soil moisture [L. A. RICHARDS, 1949] and are usually called tensiometers. Equivalent water

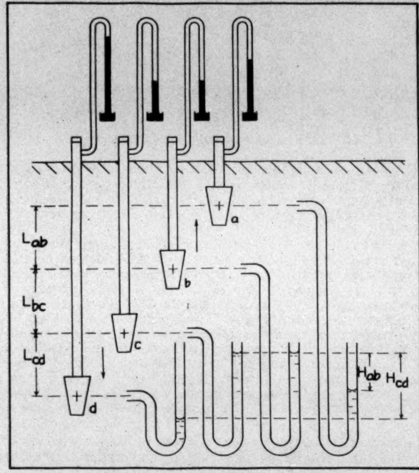

Fig. 1--Tensiometers for measuring hydraulic head in the field; mercury manometers are shown above the soil surface and equivalent water manometers at the right

manometers are sketched in the figure at the lower right to illustrate and clarify the definition of terms. The manometer pattern illustrates conditions that could exist in a soil a few days after a rain. Assume for this example that there is no horizontal variation in the soil or soil moisture. The pressure in the water in the porous cups is less than atmospheric pressure and may be read from the mercury manometers above ground. This pressure is referred to as pressure deficiency or soil-moisture tension and can be expressed in terms of a water column of length equal to the vertical distance from the porous cup down to the surface of the water in the equivalent water manometer at the right. Soil-moisture tension is defined as the negative pressure that must be applied to the water in the porous cup in order to establish hydraulic equilibrium through the porous cup wall with the water in the soil. It is not implied that the water in the soil has the same hydrostatic pressure as the water in the porous cup, but surface-force actions arising at solid-liquid-gas interface boundaries in the soil combine to produce a certain attraction for the soil moisture which is measured by the negative pressure that must be applied to the water in the cup in order to attain a zero transfer rate between the cup water and the soil water. Similarly, the hydraulic head of the water in the soil is defined as the elevation at which water stands in a riser or piezometer connected to the point in question in the soil. If the point lies below a water table, a vertical piezometer tube may be used. If the soil is not saturated, a porous cup must be used to make connection to the water in the soil. In Figure 1, the hydraulic head of the water at the various cups is equal to the elevation of the water surface in the appropriate water manometer. This elevation may be expressed with respect to any desired datum. If the datum is taken at the soil surface, then in unsaturated soil the hydraulic head values are all negative. If the datum is taken at the bottom of the figure, then the hydraulic head of the water at all of the cups is positive. In either case, it is apparent that the hydraulic head of the water at cup b is higher than the hydraulic head of the water at cup a. The difference in hydraulic head between these two cups is H_{ab}. The average hydraulic gradient in the vertical soil interval ab is equal to the difference in hydraulic head divided by the vertical distance between cups, that is, $i_{ab} = H_{ab}/L_{ab}$. According to our hypothesis, moisture flows in the direction of the greatest rate of decrease of the hydraulic head or in the direction of the hydraulic gradient. Consequently, moisture movement in the soil interval ab is upward as indicated by the arrow.

In the example it is noted that the hydraulic head at cup b is the same as at cup c. Thus, the average hydraulic gradient in this interval is zero and the net soil-moisture movement in this interval is zero. The soil-moisture tension in this interval increases in the upward direction and the upward force arising from the tension gradient is just equal to the downward force of gravity. This is the condition that is satisfied when soil water, saturated or unsaturated, is at rest under gravity.

The hydraulic head at cup c is shown in the figure to be greater than at cup d, so $i_{cd} = H_{cd}/L_{cd}$ is downward and moisture movement is downward in this soil interval. Observations have been published on both the direction and the magnitude of the hydraulic gradient in unsaturated soils in the field [RICHARDS and HUBERTY, 1941] and in the laboratory [L. A. RICHARDS, 1950].

The relation between pressure deficiency or soil-moisture tension and the moisture content of the soil has been extensively studied and is complicated considerably by hysteresis effects. For some purposes, as indicated by CAMPBELL and RICHARDS [1950], moisture-release curves (desorption curves) over the plant-growth moisture range can be represented by the hyperbolic equation

$$P_w = aT^{-b}$$

where a and b are constants depending on the soil, P_w represents moisture percentage, dry weight bases, and T is the soil-moisture tension.

Capillary-conductivity data and methods of measurents--Figure 2 summarizes and presents capillary-conductivity measurements made by a number of different workers. Data for a variety of soils are shown plotted to a linear scale.

The hydraulic conductivity of soils that may be regarded as relatively normal with respect to their moisture properties may vary from approximately 0.2 to 20 cm/hr. It is seen from Figure 2 that as the soil-moisture tension increases up to about 25 cm of water, the capillary conductivity drops rapidly. The capillary conductivity for all of the soils represented in the figure has been reduced to a relatively low value before the tension reaches 150 cm of water.

Previously, capillary-conductivity data have mostly been represented on semi-logarithmic graphs and the significance of the rapid decrease of the unsaturated permeability to low values at tensions of the order of 150 cm of water or less has been neglected or overlooked. The shape of this capillary-conductivity curve is related directly to the moisture-storing and transmitting

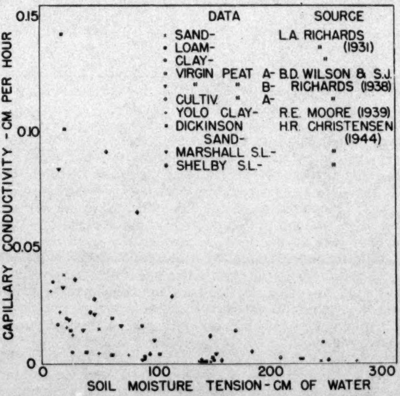

Fig. 2--Presentation of unsaturated permeability (capillary conductivity) on a linear scale to show the low values obtained for most soils at tensions above 150 cm of water properties of soils in the field. The ordinary definition for the field capacity, which is applied for deep, well-drained soils, may be paraphrased as follows: Field capacity is the moisture content of soil two or three days after a heavy rain or irrigation when downward drainage has reduced the moisture content of the soil and the thickness of the moisture films to such an extent that the capillary conductivity is reduced to a low-residual value and further downward drainage of water is negligible. This view is further supported by the fact that the tension range from 30 to 150 cm of water over which the capillary-conductivity data shown in Figure 2 drop to a low-residual value is also the range over which tensiometers are observed to read in various soils at the moisture condition referred to as field capacity.

Comparatively few capillary-conductivity data are available because measuring methods have not been convenient to use. CHILDS [1945] has proposed a novel procedure that was used to very good purpose by CHILDS and GEORGE [1950]. The apparatus shown in Figure 3 is also a new development. It is identical in principle with that previously used by L. A. RICHARDS [1931] but substitutes pressure for vacuum control.

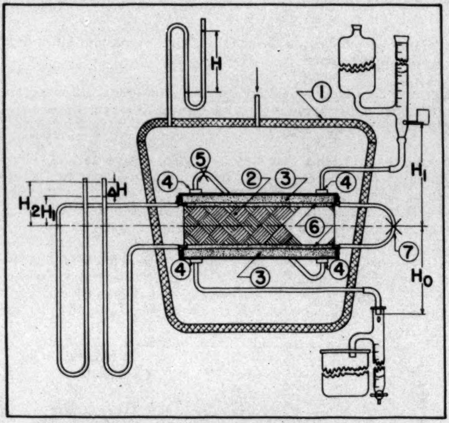

Fig. 3--Pressure type of apparatus for measuring capillary conductivity

An air-tight pressure chamber, 1, is provided with connection to a supply of air at constant pressure. A bubbler type of control unit is convenient for this purpose [L. A. RICHARDS, 1949]. A manometer is provided for reading the pressure of the air in terms of length of water column, H. A column of soil, 2, approximately three to six cm high and 25 cm in diameter, is mounted between two porous plates, 3. These plates on the side opposite to the soil contact are backed with a bronze screen and sheet rubber which is tied on around the periphery. The soil column is supported in a metal cylinder. Rubber tube connections, 4, through the sheet rubber at opposite sides of the plate are made so that when the pinch clamp, 5, is removed a continuous stream of water can be flushed through to remove the air between the rubber sheet and the porous plate. Small porous tubes, 6, are installed at each end of the soil column and connected to manometers at the left. A connecting loop of tubing is provided at the right-hand side with a pinch clamp, 7, which is normally closed. When this pinch clamp is opened, water can be flushed through the manometer and the porous tubes to remove air.

The center line of the soil column can be taken as the reference datum for measuring hydraulic head. Water is supplied from a constant-pressure source through the upper plate to the soil column. As water is withdrawn from the upper buret, air is allowed to bubble into the base of the buret through the small side tube so that atmospheric pressure is maintained at this point. The buret allows accurate measurement of water input over short time intervals and the large side tube makes it possible to take care of longer runs without frequent refills. The head for the input water is thus H_1. Water which has passed through the soil column and the attached porous plate drips into

Table 1--Capillary-conductivity data for six soils, showing the conductivity k in relation to the air pressure H in the pressure chamber

Soil type	Datum no.[a]	Date	H	k	Soil type	Datum no.[a]	Date	H	k
			cm of water	cm per hour				cm of water	cm per hour
Superstition sand		1950	0	6.59	Millville silt loam		1948	0	1.69
	13	Jan 11	10	1.30		3	Dec 9	10	0.13
	15	Jan 13	20	0.65		14	Dec 16	20	0.12
	17	Jan 20	40	0.37		17	Dec 17	40	0.11
	21	Feb 15	60	0.024		19	Dec 20	80	0.059
	22	Mar 13	200	0.00012		23	Dec 23	200	0.0059
Coachella loamy fine sand		1948	0	2.40	Chino silty clay loam		1949	0	0.723
	35	July 30	10	0.82		4	Oct 10	10	0.060
	42	Aug 3	25	0.24		5	Oct 12	20	0.038
	55	Aug 5	50	0.15		7	Oct 17	40	0.013
	72	Aug 12	85	0.063		10	Oct 20	80	0.0064
	74	Aug 16	100	0.019		12	Oct 25	120	0.0030
	77	Aug 17	200	0.00058		14	Nov 4	160	0.0014
						15	Nov 28	200	0.00115
Pachappa fine sandy loam		1949	0	1.48		19	Dec 6	120	0.00116
	1	Aug 2	10	0.51		23	Dec 19	20	0.0049
	3	Aug 4	40	0.43					
	7	Aug 10	80	0.17	Silty clay from Preston, Idaho		1949	0	0.43
	9	Aug 17	120	0.021		1	Mar 7	5	0.043
	11	Aug 22	200	0.0024		4	Mar 9	10	0.036
	12	Aug 29	78	0.014		7	Mar 10	20	0.029
	15	Sep 6	40	0.075		9	Mar 11	40	0.012
	16	Sep 12	9	0.47		11	Mar 15	60	0.0052
						15	Mar 17	80	0.0025
						18	Mar 23	100	0.0012
						20	Mar 28	200	0.00029

[a] Serial number of conductivity calculation after attainment of steady state; see text.

the lower buret and the hydraulic head of the outflow is equal to H_0. If H_1 and H_0 are made equal to zero, then H_2 and H_1 will also approach zero. The equivalent soil-moisture tension of the water in the soil at the center of the column will then be controlled by and becomes equal to the air pressure H maintained in the pressure chamber. The soil-moisture tension and the corresponding moisture content of the soil can be varied at will, depending upon the setting for the air pressure. If, now, an overall driving head such as shown equal to $H_1 - H_0$ is applied, then water will move in through the upper porous plate, through the soil column, out through the lower porous plate and into the receiving reservoir. To measure the head loss in the soil, small porous cups are installed at each end of the soil column. The net hydraulic head used up in causing the water to move through the column is $\Delta H = H_2 - H_1$.

When the inflow rate becomes steady and equal to the outflow rate, it is possible to make a conductivity calculation as follows: The transmission velocity is equal to the volume of water transmitted per unit of time, divided by the cross-sectional area of the soil column. The hydraulic gradient is equal to ΔH divided by ΔL, which is the distance between the tensiometer cups at each end of the column. Then, from the Darcy equation, it follows that the capillary conductivity is equal to the ratio v/i. There is, of course, a difference in soil-moisture tension between the top and bottom of the soil column equal to $\Delta H - \Delta L$, but the average soil-moisture tension value at the center of the soil column in cm of water is

$$H - (H_1 + H_2)/2$$

Table 1 gives samples of capillary-conductivity data obtained in the pressure chamber. The data are not presented graphically because to compare absolute values at high and low tensions would require a logarithmic scale and this tends to obscure the physical interpretation. For these measurements the air-dried soils after passage through a two-mm sieve were packed in the conductivity apparatus. The soil column was about 25 cm in diameter and usually about four or five cm high. The wetting process was accomplished by allowing the column to absorb water through the end plates with the tension at the mid-point of the column, set equal to ten cm of water. Initially, as found by L. A. RICHARDS [1931], successive conductivity curves gave successive decreases in conductivity. This apparently was due to structural changes accompanying repeated moisture fluctuations and in the work reported herein was minimized by cycling the column several times between moisture tensions of ten and 250 cm of water. After this cycling, the normal procedure was to wet the soil column through the porous plates by supplying water at a constant tension of five to ten cm of water. The successive conductivity determinations were then made at successively higher tensions.

The measurements were made at a temperature of 20°C. Riverside city water was used in the tests and the outflow from the columns was reused to avoid structural changes in the soil that occur with continuous leaching. Initially the water had an electrical conductivity of 380 micromhos per cm at 25°C with a soluble sodium percentage of about 44. The hydraulic conductivity of the various soils when saturated with water was measured in conventional permeameters, and these values are given at the top of each column of data in the table with the soil-moisture tension indicated as zero.

The names for the soil types are those used by the Division of Soil Survey of the U. S. Department of Agriculture. The particle-size analyses given in Table 2 were supplied by the Riverside Laboratory of the Division of Soil Survey. The silt and clay were determined by the standard pipet method.

Table 2--Particle-size analyses made by the Division of Soil Survey of the U. S. Department of Agriculture

Soil type	Number[a]	Sand (2.0-0.05 mm)	Silt (0.05-0.002 mm)	Clay (<0.002 mm)
		pct	pct	pct
Superstition sand	2642	89.6	4.8	5.6
Coachella loamy fine sand	2761	85.1	10.5	4.4
Pachappa fine sandy loam	2640	59.4	32.9	7.7
Millville silt loam	66	24.9	60.3	14.8
Chino silty clay loam	377	5.8	58.1	36.1
Clay from Preston, Idaho	3047	1.5	52.9	45.6

[a] Soil accession numbers of the Salinity Laboratory.

Capillary-conductivity values were calculated during the transition period between changes in air pressure. Serial numbers assigned to these calculations, along with the date for the steady-state values, are indicated in Table 1. The head values labeled H_1 and H_2 in Figure 3 were not recorded each time, so the pressure of the air in the chamber, H, is given in the table instead of soil-moisture tension. The values for H_1 and H_2 are needed to calculate the mean tension in the soil column, but this will not differ appreciably from H because H_i and H_o were kept equal and small.

An inspection indicates that the data are in harmony with the previous measurements shown in Figure 2. The most significant fact is the comparatively rapid reduction to low conductivity values as the tension is increased. Two of the soils are the same as those previously studied by L. A. RICHARDS [1931]. Converting units to those used here, the previous pairs of values for tension and conductivity for the clay from Preston were respectively: 9.7 - 0.035; 92 - 0.0021; and 156 - 0.0010. These are seen to be in almost exact agreement with the new data, though obtained with different apparatus nearly 20 years later. The Millville silt loam is from the same area as the "loam" soil previously used. The previous data for this soil were distinctive because of the low rate of change of conductivity with tension over part of the tension range and the same tendency is noticeable in the new data.

A comparison of the new data for the six soils indicates that the sandy soils have higher conductivity at low tension and lower conductivity at high tension (0.2 atmosphere) than the medium- or fine-textured soils.

Evidence for a hysteresis effect is apparent in the data for Pachappa and Chino soil, where data for a decreasing segment of a tension cycle are shown. It is seen that conductivity is much lower at corresponding tension values when the tension is decreasing than when the tension is increasing. This hysteresis in the relation of tension to both the moisture content and the capillary conductivity is a very real effect in soils and is oppositely directed in drying and wetting processes as has been shown by S. J. RICHARDS [1938].

Moisture-content changes in relation to depth of wetting--If the view is correct that field capacity corresponds simply to that moisture content at which unsaturated permeability becomes negligible, then it should be possible by some such procedure as that proposed by WILCOX [1939] to estimate the field capacity of soils from a laboratory determination. To test this idea, experiments were conducted by the writers using a technique similar to that proposed by the PUNJAB IRRIGATION RESEARCH INSTITUTE [1943] for estimating the colloid content of soils. By this method a small predetermined volume of water is added to air-dried soil. The moisture content of the wetted soil is then determined after a certain length of time.

From the capillary-conductivity data shown in Figure 2, one would expect that the moisture content would decrease rapidly at first and then change much more slowly when the capillary conductivity of the wetted portion attained a low residual value. Results from tests of this kind are shown in Figure 4.

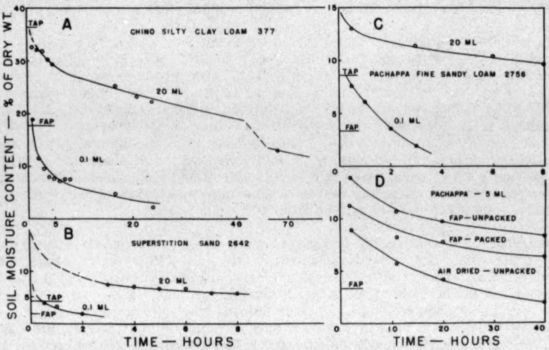

Fig. 4--Rate of change of the moisture content of wet soil which is in contact with dry soil; the curves show the effect of the amount of water applied to the soil; TAP and FAP represent respectively the 1/3-atmosphere percentage and the 15-atmosphere percentage for the soils

At the start, a basket made of metal screen with three-mm openings was placed in a shallow vessel and covered with a layer of soil. To obtain an easily reproducible test condition the soil was air-dried and passed through a two-mm round-hole sieve. It was then mixed and delivered without packing to the wetting vessel through a tremie to avoid particle-size separation. The tremie used was a glass funnel with a stem about eight cm long and 1.5 cm inside diameter. A known volume of water was then applied in 0.1-ml drops from a buret to separate spots on the soil. After a predetermined drainage time the wet soil was removed from the dry soil by raising the basket. The average moisture content of the wetted soil was then calculated from the buret delivery and the decrease in weight of the container of dry soil.

For short drainage times this method gives reproducible results, but, as indicated by the curves in Figure 4, the moisture content of the wetted soil at any given time depends markedly on the volume of water used in wetting the soil.

With long-drainage times the wire-basket method of separation works decreasingly well because with drier soil the adhesion of the moisture film decreases and the separation of wet from dry soil becomes less distinct. The shape of the mass of wetted soil was roughly hemispherical, and the data shown in Figure 4 were obtained by oven drying approximately 1/3 of the volume of the wetted soil nearest the point of application of the water. Tests were made with water volumes of one and five ml also, but these curves lie intermediate between the curves for 0.1 and 20 ml which are shown in the figure. The 15-atmosphere percentage (FAP) approximates the lower limit of soil water that is available for plant growth, and the 1/3-atmosphere percentage (TAP) approximates the moisture equivalent. These moisture-retention values are marked on the moisture scales for curves A, B, and C in Figure 4.

It is clear that the depth of wetting, or the volume of water applied, greatly influence the rate of change of the moisture content in the wetted portion of the soil. For a water volume of 0.1 ml, the moisture content of the wetted portion of the soil may drop below the 15-atmosphere percentage within an hour or two, whereas for a 20-ml volume of applied water the moisture content will remain at a much higher value for a much longer time.

Tests were conducted on cores with undisturbed structure, but difficulties connected with this type of sample make the method impractical for the routine estimation of field capacity.

The unpacked soil condition actually used in these tests was highly artificial and detailed capillary-conductivity data for the soil condition are not available, but the observed effects appear to be readily explainable in terms of conductivity. The point of primary significance in this explanation lies not in the rapid decrease of conductivity to a low residual value at tensions in the 30- to 150-cm range but in the fact that the residual conductivity remains appreciable.

The data in Table 1 illustrate the fact that at all tensions at which the capillary conductivity was measured the conductivity remains measurably greater than zero. It appeared to L. A. RICHARDS [1936] and to RICHARDS and WILSON [1936] that, when the moisture content decreased sufficiently in the one-atmosphere tension range, the capillary conductivity became zero. WILSON and S. J. RICHARDS [1938], however, working with the same apparatus, later identified certain moisture losses from the system and showed that the previous conclusion was not justified. The fact that the capillary conductivity does not reduce to zero appears to be conclusively demonstrated by the work of RICHARDS and WEAVER [1944] and L. A. RICHARDS [1949] since they found that, after attainment of hydraulic equilibrium between a sample of soil and a pressure membrane, each pressure increment produced an immediate water-outflow increment, even at pressures corresponding to soil-moisture tensions in the range from 20 to 100 atmospheres.

During the process of wetting dry soil, the transfer of water across the wetting front will progress at a finite rate. The hydraulic gradient at this front will be high because of the rapid change of soil-moisture tension with distance, whereas within the wetted mass the conductivity is relatively higher and the moisture tension and hydraulic gradient will be lower. With a given rate of water flow near the wetting front, it is obvious that the average rate of change of the moisture content in the wetted zone will vary inversely with the volume of relatively mobile water in the wetted zone. Herein appears to reside the explanation for the difference between the two curves on each of the graphs A, B, and C in Figure 4.

At D in the figure are shown similar curves but for a five-ml volume of applied water, and with the state of packing and the initial moisture content of the soil as additional variables. The top curve shows the change in moisture content with time for unpacked sieved soil initially at FAP, while the second curve shows the more rapid drainage occurring when the soil is packed approximately to field density. The third curve shows that unpacked, air-dried soil gives even more rapid drainage than the packed moist soil. This relatively more rapid movement into dry soil is in line with field observations and is to be expected in view of the larger hydraulic gradient existing at the wetting front.

Field applications--The foregoing data on capillary conductivity and the laboratory tests relating moisture retention, depth of wetting, and drainage time appear to have direct application to field conditions. COLMAN [1944] made a study of the wetting process in a soil profile that initially was at or below the wilting percentage to a depth of more than 60 inches. He made detailed moisture determinations one to four days after rainfalls and found that the moisture content of the surface soil increased with wetting depth until the wetting front had penetrated about 30 inches below the surface. Colman referred to this moisture content which he found in the surface soil, for the sampling interval he used, as field capacity.

A day or two following the application of water to a permeable but dry profile, there is a depth interval in which the moisture content grades from the former dry value to a maximum value for any given depth which is somewhat independent of the depth of wetting. Colman defined the wetting zone as the depth interval in which the moisture content is a function of the wet depth. He further summarized his observations as follows: "When the soil is wet below the 30-inch depth, first the surface layers and then successively deeper layers reach maximum moistures unaffected by further downward penetration of moisture. The wetting zone thus moves downward in the soil as the depth of moisture penetration increases."

The wetting and moisture-retaining process thus described can be explained directly in terms of capillary conductivity. In the wetting zone, the conductivity is low but the hydraulic gradient is high so that the flow velocity is definitely appreciable and prevents the maintenance of moisture at the field capacity if the wetting zone extends to the soil surface. However, as the wetting front proceeds downward through the soil, there comes a time when the moisture content at the wet side of the wetting zone is high enough to be on the rapidly rising part of the capillary-conductivity curve and moisture contents higher than this cannot persist in the overlying soil. It appears to be this physical property that limits or determines the soil moisture content observed in the field above the wetting zone.

Colman also found that the wetting zone occupied a shallower depth interval as the wetting front progressed more deeply into the soil. This circumstance was observed in several profiles. Among the profiles studied was a soil that had been homogenized several years earlier by thorough mixing and in which the wetting process following irrigations was studied by moisture samplings. The explanation for this decrease of the thickness of the wetting zone with depth of wetting would appear to lie in the fact that water is more readily supplied to the wetting zone when the overlying soil layer which is at field capacity is thicker and therefore contains a larger supply of water at the high moisture content where the capillary conductivity is high.

It is generally recognized that field capacity represents a moisture content in field soil that occurs and changes slowly with time because of the dynamic moisture-transmitting properties of the soil and that it does not in any sense represent an equilibrium condition. If the factors that control the rate of transfer of water through the wetting zone tend to make this rate of transfer somewhat constant, then the thicker the zone of wetted soil is, the more slowly will its moisture content change with time. This, of course, is observed in the field.

There is strong indication that field capacity corresponds to that tension and moisture content at which conductivity becomes negligible, but this is yet to be definitely verified for specific cases. Measuring techniques are now approaching the stage where moisture retention and conductivity can be expressed as functions of tension in soils having undisturbed structure. This will make possible a theoretical solution for the soil-wetting process in the field and extend the application of basic principles to the soil-moisture system.

One complicating factor in the problem lies in the fact that there is apparently at least a tenfold variation between soils in the magnitude of the hydraulic gradient during infiltration in the field. It will be possible to bring soils of substantially undisturbed structure to the laboratory for measurement and study, but it may not be convenient to reproduce the boundary conditions that control the hydraulic gradient in the field. This seems to be the main difficulty now being experienced in developing a simple, general, laboratory method for estimating field capacity. Even so, the theory and soil measurements now available are sufficiently complete to be of considerable help in understanding and qualitatively evaluating field-moisture problems.

Acknowledgment--This paper is a contribution from the U. S. Salinity Laboratory, Bureau of Plant Industry, Soils, and Agricultural Engineering, Agricultural Research Administration, U. S. Department of Agriculture, Riverside, California, in cooperation with the 17 Western States and the Territory of Hawaii.

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(Communicated manuscript received July 30, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

FROST PENETRATION INTO SOILS AS INFLUENCED BY DEPTH OF SNOW, VEGETATIVE COVER, AND AIR TEMPERATURES

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Abstract--Frost penetration and removal in the Miami silt loam and the Almena silt loam were measured by resistance readings made on plaster of Paris blocks located at intervals in the soil profile to a 36-inch depth. As the soil water freezes, the unit resistance rises sharply.

Dense vegetative and snow cover insulate the soil and reduce the rate of soil freezing and thawing. Winter wheat gave very little protection to the soil. Graphs are included which show frost penetration and removal on hay, winter wheat, and plowed land on the Miami silt loam. Eighteen inches of snow insulated the Almena soil sufficiently to prevent frost penetration to a depth of 12 inches when the weekly average daily minimum temperatures were as low as -13°F. Eighteen inches of snow did not prevent frost penetration when the average minimum temperature was -21°F. However, 24 inches of snow cover prevented frost penetration at -21°F. Graphs are included showing the frost movement into and out of the Almena soil under varying depths of snow cover. The soil thaws from both the surface and from the lower frost line.

In recent years a number of instruments and methods have been developed to measure soil-moisture conditions. The electrical-resistance method has received much attention and is one of the most promising. In this method, blocks constructed of various materials are placed in contact with the soil. Because of the pore space and absorptive properties of the material in the blocks, an equilibrium between the moisture in soil and block is readily established and the electrical resistance varies accordingly. High moisture content will give low readings and, as the soil moisture decreases, the resistance readings increase. The resistance is measured with a Wheatstone bridge.

BOUYOUCOS and MICK [1948] have reported the use of blocks to determine freezing in the soil. Freezing is indicated by a sharp increase in resistance. "Initially, the freezing of soil water produces a curve similar to a drying curve. As long as unfrozen water remains in the unit, however, a fairly high conductivity is maintained. When all water has been frozen, so that none exists as liquid, the unit resistance rises abruptly. Freezing does not destroy the absorbent, nor does it appear to have any bad effects on the subsequent functioning of a fabric absorbent."

COLMAN and HENDRIX [1949] conducted laboratory studies with fabric blocks and determined the temperature and freezing points of soil with various moisture contents. They noted that in "a completely saturated soil (the resistance) rises from 120 to 150 ohms as the temperature drops from 60° to 28°F. At 28° F the soil moisture freezes and (the resistance) rises abruptly to 250,000 ohms. Dropping the temperature to 10°F brings the resistance up to 1.3 megohms, showing that the increase in resistance per degree change in temperature is much greater in ice than in liquid water." They found a soil at field capacity to freeze at 24.5°F and "upon freezing, its resistance rises from 1250 to 50,000 ohms." This rise is considerably less than that of the saturated soil, but it is still sufficient to distinguish between the frozen and liquid state of the soil water. A third soil with moisture content near the wilting point, and at the freezing temperature but still unfrozen, had a resistance of about 200,000 ohms. Upon freezing at the same temperature, its resistance was 600,000 ohms. They conclude that "reasonably reliable detection of freezing and thawing can be achieved by this method in soils that contain more than wilting-point moisture." However, the detection of freezing and thawing below this moisture level becomes subject to question.

Plan and purpose of field study--The laboratory work of these men has prompted a field study to determine the rate and depth of frost penetration using plaster of Paris blocks. Previously, measurements of frost penetration were obtained at La Crosse using a King soil tube and hammer [ATKINSON and BAY, 1940]. The tube was driven into the soil profile a few inches at a time. The cores were removed from the tube and examined for ice crystals. The depth to which these crystals occurred was recorded as frost depth. The tube would also "give" as it passed from the frozen to the unfrozen soil. This could be sensed more readily under higher moisture conditions than when the soil was dry. The method was very laborious and time-consuming. It also put the equipment to the extreme test, often resulting in broken tubes and hammers. In spite of the difficulties involved in the technique, the accuracy of the method was high. Because of the important part frost action plays in our crop land during the winter and spring periods, it seemed desirable to have information with regard to its rate and depth of penetration and removal. The objectives of this experiment were: (1) to determine the usefulness of the plaster of Paris block as a unit to measure frost movement in the soil, and (2) to determine the effects of air temperature, snow cover, and exposure on frost penetration.

Measurements of frost penetration were started in the field in November, 1949, on the Miami silt loam near Madison, Wisconsin. The Miami soil is grouped with the thick, silty, surface soils having a moderately permeable upper subsoil and lower subsoil covering till material below 36 to 42 inches. The land slope of the area on which the blocks are located is nine per cent.

During the 1949-1950 winter period, hay, winter wheat, and two fall plowed plots, one with an eastern exposure and one with a northwestern exposure, were under study. Plaster of Paris blocks were located at 6-, 12-, 18-, 24-, 30-, and 36-inch levels under each of the above cover conditions. Resistance readings were recorded twice a week during the period of frost penetration and as often as necessary to follow the frost removal from the soil profile. The winter wheat and plowed plots were equipped with runoff collecting equipment. This made it possible to determine runoff and soil loss during the winter and spring thaw.

Procedure and results--Soil samples were taken from depths in the profile representative of the gypsum block locations. Curves of the relation of per cent moisture to resistance were determined for each depth in the Miami silt-loam profile represented by a block. The procedure followed was similar to the method of KELLEY [1940]. Saturated blocks were imbedded in approximately 200 grams of soil contained in small wire baskets lined with cheese cloth. The assembled unit of soil, block, and basket was saturated by placing it in a pan of water for a period of 24 hours. Resistance readings and weights were recorded between equilibrating in a humidity chamber and drying at room temperatures. Then calculating the per cent moisture, corrections were made for water content of the block at the corresponding resistance reading. This correction was made on the basis of a curve developed by C. B. Tanner of the Soil Physics Department, University of Wisconsin. Figure 1 presents the moisture curves for the six different depths. Each curve represents an average of triplicate samples. The curves are generally uniform for all depths except the one representing the plow depth. This difference is probably due to organic matter content, structure, and texture.

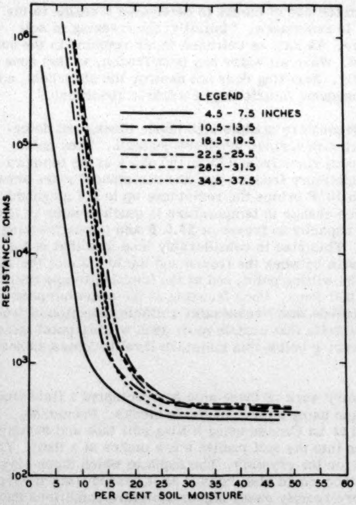

Fig. 1--Calibration curves of gypsum blocks in Miami silt loam

Frost measurements at Madison--Moisture conditions in autumn were near field capacity to a three-foot depth in the hay and plowed plots having eastern exposure. The winter wheat and plowed plots with northwest exposure were at a much lower moisture content, with resistance readings between 5,000 and 10,000 ohms. The question arises, at what resistance can one say that soil freezes? Colman and Hendrix have found that soil at field capacity having a resistance of 1250 ohms will rise to 50,000 ohms upon freezing. It will start thawing at 30,000 ohms at 32°F. These moisture conditions apply to the plots having an eastern exposure. Under the lower moisture conditions of the plots having the northwestern exposure, resistance readings of 100,000 ohms would indicate frozen soil.

Applying these data to our field study and examining the graph of each treatment we determined the depth and rate of frost penetration. Wide fluctuations in resistance occurred at 6-, 12-, and 18-inch depths. They were more pronounced on plowed land or land having little or no vegetative cover, such as winter wheat. Vegetative and snow cover tended to insulate the soil and reduce the wide range of fluctuation in readings.

During the winter of 1949-1950, there was very little precipitation in the form of snow until February 13, 1950. Prior to this date, sub-freezing temperatures caused frost penetration to depths of 2 1/2 ft under hay and plowed plots with eastern exposure, and to three feet under winter wheat and plowed plots with northwestern exposure. Snow between February 13 and 15 provided an eight- to nine-inch covering on the plowed plots, about six inches on the hay plots, and approximately three inches on the winter wheat. Drifting and blowing accounted for the varying depths of snow. This covering of snow prevented further penetration of frost under all treatments except the winter wheat. With a check against further penetration, the warmth of the Earth began thawing the soil at the lower frost line. Frost thawed from the 30-inch level on the two eastern exposure plots and a gradual decrease in resistance was noted on the plowed plot with northwestern exposure. In contrast, on the winter wheat plot with only a few inches of snow, there was further penetration of frost at the same air temperatures.

The resistance readings provide a complete record of frost leaving the soil profile. An examination of the data reveals that thawing takes place at the lower frost line upward as well as from the surface downward. The soil profile was free of frost when the lower line met the upper line near the 18-inch level.

An interpretation of the resistance data in terms of actual inches of frost is presented in Figure 2. The general pattern of frost penetration and its removal from the soil is similar to that found in earlier work using the King soil tube. A notable observation is that the winter wheat plot and fall plowed plot facing the northwest were frozen to a greater depth than the plots facing the east. This may be attributed to the fact that the eastern exposure received more direct sunlight and was not subjected to the prevailing northwest or northerly winds. The various U-shaped areas on Figure 2 represent unfrozen soil. None appears on the hay plot as the vegetative cover insulated the soil sufficiently to prevent thawing until the latter part of March. At this time thawing started from the surface as well as from the lower frost line.

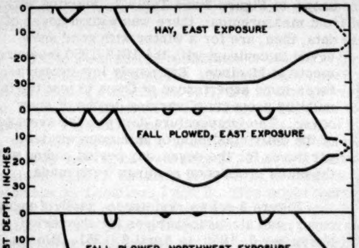

Fig. 2--Frost penetration under various covers and exposures, Madison, Wisconsin, 1949-1950

Soil losses and runoff at Madison--The winter wheat and plowed plots facing the northwest were equipped with tanks for measuring runoff and soil losses during the frozen and thawing periods. These data are shown in Table 1.

The amount of runoff and soil loss occurring this time of the year will depend upon the amount of snow cover, the type of vegetative cover, the amount of surface soil exposed and unfrozen, and the form of precipitation. The soil surface of the winter wheat plot was comparatively smooth with some vegetative cover provided by the autumn growth of wheat. The plowed area was rough and cloddy, having greater surface storage capacity than the winter wheat. Runoff and soil losses from winter wheat were higher during January and February, due mostly to the smooth surface condition with insufficient vegetative cover to protect the soil. Soil losses and runoff were higher from the plowed plot during the March thaws. Two to three times more snow covered the ground of the plowed plot than the winter wheat plot. As this larger amount of snow began to melt and as the soil surface thawed, the plowed plot's surface storage capacity was exceeded and high runoff and loss of soil resulted. When rain and thawing occur at the same time, runoff and soil losses are greater than from thawing alone. Note particularly the rain and thaw of March 25-26. Exceedingly high soil losses have been measured at La Crosse from a gentle rain falling on soil thawed to a depth of four inches.

Table 1--Runoff and soil losses, winter period, 1949-1950, Madison, Wisconsin

Date	Precipitation[a]	Winter wheat			Fall plowed		
		Snow depth	Runoff	Soil loss	Snow depth	Runoff	Soil loss
1950	inch	inch	inch	tons/acre	inch	inch	tons/acre
Jan 13	0.65	0	0.26	0.08	0	0	0
Jan 15	0.06	0	0.01	0	0	0	0
Jan 22-25	1.30	0	0.88	0.94	0	0.02	0.02
Jan 28	Thaw	0	0.02	0	0	0	0
Feb 6	Thaw	1	0.05	0	1	0.05	0
Feb 28	Thaw	3	0.20	0	8	0.03	0
Mar 4-5-6	Thaw	2	0.50	0.22	6	1.03	0.42
Mar 6-7	Thaw and 0.44	4	0.35	0.25	4	0.67	0.54
Mar 23-25	Thaw	2	0.69	0.21	2	0.68	0.28
Mar 25-26	Thaw and 0.66	0	0.63	0.94	0	0.73	1.80
Apr 10	0.66	0	0.01	0	0	0	0
Totals	....	..	3.69	2.64	..	3.21	3.06

[a] Precipitation shown in this table occurred in the form of rain, or rain and snow, and resulted in runoff. An additional 2.16 inches of precipitation in the form of snow occurred during this period.

Frost measurements at Owen--Near Owen, Wisconsin, five sets of gypsum blocks were installed on land that was in second-year hay in a corn, grain, hay, hay rotation. The blocks were set in the soil on September 11, 1950. The soil is an Almena silt loam which has a mottled subsoil over compact, tight, gravelly clay loam. The subsoil has poor internal drainage. The land slope is three per cent. Data from this study are shown in Figure 3 and Table 2. Starting with the middle of November throughout the period of frost measurement, there was a good cover of snow, with a maximum depth of 24 inches. These data, then, are for a winter with good snow cover in contrast with the 1949-1950 measurements at Madison. Extremely low temperatures were experienced at Owen to test the insulating property of varying depths of snow cover. The temperature data are the averages of the daily maximum or minimum air temperatures for the seven-day period ending with the dates resistance readings were made.

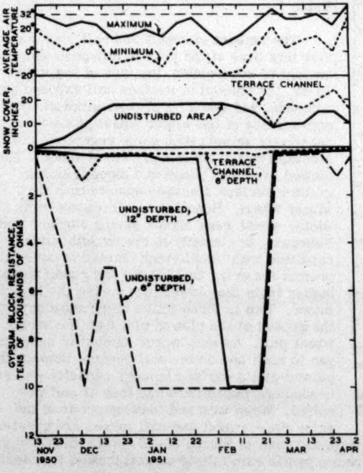

Fig. 3--Effect of snow cover and air temperature on frost penetration, Owen, Wisconsin, 1950-1951

Figure 3 shows resistance, depth of snow cover, and air temperatures for the period November 6, 1950, to April 2, 1951. Blocks were located at 6, 12, and 24 inches below the surface. However, none of the blocks at 24-inch depth showed sufficient change in resistance to indicate frost penetration. Drifting of snow resulted in a greater depth of snow cover at the set of blocks located in a terrace channel than at the location in the undisturbed area. Snow-depth measurements were made at the block locations after February 14. Those snow depths shown before that date are representative of cover on the undisturbed field area. However, one can safely assume that there was a greater depth of snow in the channel after November 23 as on that date there was a strong wind which drifted the terrace channel full as the snow fell. Under this cover the resistance measured at six inches was greater than 3000 ohms only on December 4 when it was 14,000 ohms. On this date, the six-inch block in the undisturbed area showed a resistance of 100,000 ohms. Under the deeper snow cover, the resistance had dropped to less

Table 2--Resistance at six-inch depth with varying depths of snow mantle, Owen, Wisconsin

Date	Air temperature[a]		Location I		Location II		Location III		Location IV		Location V	
	Average maximum	Average minimum	Snow depth	Resist-tance	Snow depth	Resist-tance	Snow depth	Resist-tance	Snow depth	Resist-tance	Snow depth	Resist-tance
1951	°F	°F	inch	ohm	inch	ohm	inch	ohm	inch	ohm	inch	ohm
2/5	3	-21	. .	100000	. .	100000	. .	100000	. .	100000	. .	2100
2/14	15	-16	18	100000	16	100000	15	100000	16	100000	24	2200
2/19	24	1	12	100000	14	100000	14	100000	15	100000	20	3100
2/27	36	21	6	14000	8	1800	8	800	6	2400	8	1000
3/5	28	3	8	3100	10	1300	8	900	13	1200	18	1000
3/12	28	9	6	15000	10	1200	10	1000	13	1200	16	800
3/19	32	13	16	13000	15	1300	15	900	18	1100	20	900
3/26	23	0	18	18000	15	12000	15	950	18	1500	24	800
4/2	39	29	10	6500	10	3000	10	1000	10	1400	16	900
Average			12	33700	12	27600	12	25700	14	26100	18	1300

[a] Air temperature data are averages of daily maximum and minimum for seven-day period ending on date shown.

than 3000 ohms by the time of the next reading whereas under lesser snow cover the resistance dropped only to 50,000 ohms. Under the deep cover no frost penetration was indicated at the 12-inch depth. However, with the more shallow cover on the undisturbed area, frost had penetrated to the 12-inch depth by the time of measurement on February 5. Eighteen inches of snow provided sufficient insulation to prevent frost penetration at 12 inches at temperatures with weekly average daily minimums of -11° and -13°F but did not prevent frost penetration to this depth when the average minimum was -21°F. The soil thawed at the 12-inch level with average maxima of 36 and minima of 21°F.

Table 2 shows the resistance of blocks at five different locations with all blocks located at the six-inch level. In general, the least snow cover was measured at Location I, with increasing depths to Location V. Locations I, II, and III had an average snow cover of 12 inches. Location I showed an average resistance of 34,000 ohms whereas the other two (II and III) had an average of about 26,000. This difference apparently was caused by partial freezing at Location I on March 12 with a six-inch cover, while Locations II and III did not freeze with ten inches of snow. Also, as the soil was thawing on February 27, Location I thawed out more slowly so that the resistance remained higher. Location IV, with an average of 14 inches of snow, indicated about the same resistance as Locations I and II. This might indicate that the differences observed between I, and II and III, are not significant or are influenced by factors other than depth of snow cover. At Location V there was no indication of frost penetration to the six-inch level with an average depth of snow cover of 18 inches and a minimum of eight inches. The eight-inch depth occurred when temperatures were quite mild. A cover of 24 inches of accumulated snow protected the soil against frost penetration to the six-inch depth even with an average minimum of -16°F and maximum of 15°F for the seven-day period ending on February 14.

Conclusions--The following conclusions may be drawn.

(1) The data indicate that gypsum blocks can be used to measure the movement of frost into and out of the soil.

(2) Winters with little snow allowed frost to penetrate as much as three feet into the soil under usual farming practices.

(3) Twenty-four inches of snow provided enough protection to prevent frost penetration at -21°F. Eighteen inches of snow insulated the soil sufficiently to prevent frost penetration to 12 inches at temperatures with weekly average minimums of -11° and -13°F, but did not prevent frost penetration to this depth when the average minimum was -21°F.

(4) Atmospheric temperatures, snow depth, and exposure are important factors affecting frost penetration.

(5) Thawing takes place at the lower frost line as well as from the surface downward.

(6) Soil losses and runoff are high when rainfall and thawing of snow occur at the same time on partially unfrozen soil.

Acknowledgments--The work on which this paper is based was done by the Upper Mississippi Valley Soil Conservation Experiment Station, La Crosse, Wisconsin, conducted cooperatively by the Soil Conservation Service and the Wisconsin Agricultural Experiment Station.

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(Communicated manuscript received August 23, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

SOME FACTORS INVOLVED IN GROUND-WATER REPLENISHMENT

E. S. Bliss and C. E. Johnson

Abstract--Laboratory and field studies of water spreading on fine-textured soils are discussed. Effects on water-intake rates of treatments with cotton-gin trash, grasses, detergents, and other substances are interpreted in terms of fundamental processes. The effects of soil and water properties and changes in these properties as a result of various treatments are given consideration. Microbial activity and other causes of soil sealing are discussed. The results of microbial activity and a management program that ultimately leads to an increase in water-intake rates above normal are presented.

Introduction--This paper is in the nature of a preliminary report of some work done on problems of ground-water replenishment. Certain phases are discussed and explanations attempted. As treated in this paper, ground-water replenishment refers to the artificial conveyance of surface water to an underground reservoir for the primary purpose of increasing the underground supply. Broadly speaking, there are two practical methods of recharging an underground basin artificially. The first and most commonly used method is to spread the water over porous land connected by permeable formations to the basin to be charged. The second method is to convey the water underground through pits or shafts. In isolated cases the latter method has been successful [MUCKEL, 1945], but it has the serious limitations of high cost, possibility of pollution of the reservoir through direct addition of contaminated water, and tendency toward rapid sealing of the shaft, particularly if the water contains even small amounts of silt, organic matter, or other impurities.

Water spreading for ground-water replenishment in this country dates back to 1889 when the City of Denver undertook a small water-spreading project to maintain its domestic supply. Since that date considerable spreading has been done in Southern California, and some elsewhere, but in nearly all cases on coarse alluvial sands, gravels, and boulders. The operations have been carried on for short periods to capture excess flood flow. The problems have been mainly building and maintaining rather crude systems of dikes, revetments, and ditches [MITCHELSON, 1949].

For the past several years, increasingly critical underground-water shortages have been developing in the San Joaquin Valley and many other places where sufficient surface soils composed of coarse gravels and sands are not available for spreading operations. To replenish the San Joaquin basins it is planned to use large amounts of water from the Central Valley Project as well as from other sources, bringing it by canal and spreading over underground basins where shortages exist. Soils in these areas generally range from sandy loams to clay loams and many have hardpan or other impervious layers in the profile. The length of time spreading continues will depend on the amount of excess water available. It may vary from none in drouth years to a continuous period of eight months or even longer in wet years.

Problem--Early attempts at spreading water on several of the valley soils, reported in 1943 by the late H. L. Haehl, Consulting Engineer for the Kern County Land Co., Bakersfield, Calif.,

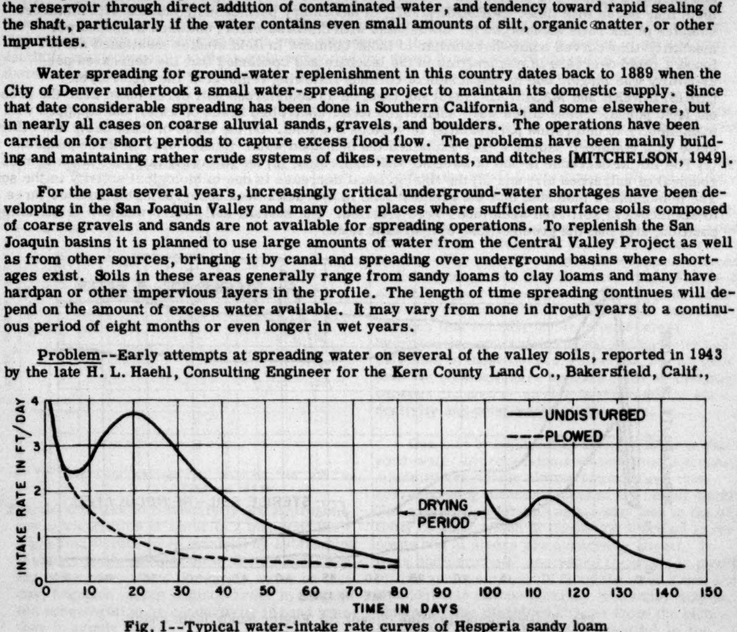

Fig. 1--Typical water-intake rate curves of Hesperia sandy loam under undisturbed conditions and after plowing disclosed serious problems. Initial maximum or peak rates of water intake were frequently too low to accomplish much replenishment. Those areas which attained satisfactory initial rates failed to maintain them. In two to eight weeks rates would drop so low that operation became impractical. Typical water-intake rate curves obtained in field ponds, in these and later experiments, are shown in Figure 1. Following a brief drying, the curve for undisturbed soil repeats itself but at a lower rate.

In 1943 several organizations began a concerted study of the problems involved in groundwater replenishment in these areas. Records of some 60 ponds operated at various times since 1936 and ranging from 0.005 to over 50 acres in size were assembled. The Riverside Regional Salinity Laboratory made a two-year laboratory study that greatly extended information on infiltration rates of soil under prolonged submergence. Several irrigation and water-storage districts contributed land, water, and money to the program. Recognizing the need for additional studies, the cooperators requested that the Division of Irrigation of the Soil Conservation Service establish a special laboratory to deal with replenishment problems involving soil microbiology, soil physics, and hydrology. Present cooperators in this project are: Kern County Land Co., North Kern Water Storage District, U. S. Bureau of Reclamation, California State Division of Water Resources, California State Experiment Station, and Division of Irrigation and Water Conservation of the Soil Conservation Service. Related field studies are being made on two groups of small ponds, either 0.005 acre or 0.01 acre in size. One group of ponds is located on Exeter sandy loam, a soil having intermittent hardpan-like layers at 18 to 24 inches. The other group is on Hesperia sandy loam, which has no highly impervious layers but is markedly stratified.

Previous work--Although considerable literature exists on the subject of infiltration or permeability rates of soil for short time periods, comparatively little study of the subject has been made under conditions of long-continued submergence. While it is known that various physical and chemical properties of soil and water affect permeability, changes in magnitude of these effects may occur with time. BODMAN [1937], in laboratory permeability studies with disturbed samples of six soils submerged for six months with distilled water, obtained S-shaped permeability-time curves somewhat similar to those obtained in field studies mentioned above. He found a rapid decrease in electrolytes in the leachate and concluded that the decreased permeability was largely due to dispersion of colloids following removal of the electrolytes. CHRISTIANSEN and MAGISTAD [1945] ran tests on 43 undisturbed soil cores taken near the present field ponds. These cores were submerged under controlled heads of water and equipped with manometers to determine permeability of various sections. On the basis of these tests and others made with disturbed samples, they explained the S-shaped water-intake curve in three phases: (1) The initial decrease is due to swelling and dispersion; (2) the following increase accompanies removal of entrapped air; and (3) the final gradual decrease is due to biological activity in the soil. The manometers indicated that the greatest loss in hydraulic head occurred in the surface three to six inches of soil even when tubes were inverted. The surface soil, which was now in the lower position, still had the greatest loss of hydraulic head.

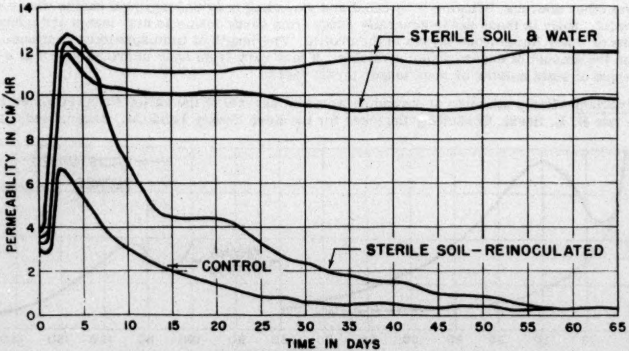

Fig. 2--Permeability-time curves for Exeter sandy loam under prolonged submergence [ALLISON, 1947]

ALLISON [1947] substantiated the microbial hypothesis by running laboratory permeability tests on disturbed samples using sterile soil and water. Permeability curves obtained by Allison are shown in Figure 2. He concluded that there was ". . . no evidence of soil aggregate breakdown due to purely physical causes. The reduced permeability appears to be due entirely to microbial sealing. The soil pores probably become clogged with the products of growth: cells, slimes, or polysaccharides. If any of the observed reduction in permeability was due in part to disintegration of soil aggregates, the dispersion is believed to be due to biological causes, that is, the attack of microorganisms on the organic materials which bind soil into aggregates."

Maintenance of peak permeability of laboratory soils by sterilizing procedures has been further corroborated by T. M. McCalla (private communication). Where physical factors do not interfere, the development of practical sterilizing procedures for field use apparently might increase the quantity of water taken in during a spreading run. With soils having naturally high peak intake rates such procedures could aid greatly in making spreading practical, but maintenance of the low natural peak intake rates common to many valley soils would be of little practical benefit.

Experimental procedure and discussion--The process of microbial sealing of submerged soil seems well established. The effect of such factors as dispersion or swelling of soil colloids, presence of impervious layers, heavy soil texture, or unfavorable water quality in limiting water movements is well known. The first three are examples of inherent soil characteristics which determine its natural permeability. Water quality, however, may be considered apart, since it affects, to a greater or lesser extent, any soil upon which it is used.

Early pond studies by cooperators of this project showed that use of "high sodium" or "soft" waters rapidly depressed intake on soils with naturally high rates to very low levels [MUCKEL, 1950]. Complete chemical analysis of water used in presently operated field ponds indicates they fall into what is commonly accepted as the "Excellent to good" irrigation class [WILCOX, 1948]. However, it was noted that pond water pH and conductivity varied with time and from pond to pond. Similar but much smaller variations were observed in the supply waters. A series of pH and conductivity measurements on the supply water and the water from some 20 operating ponds was therefore started, for a time on a daily basis, and later, for a period of a year, on a weekly basis.

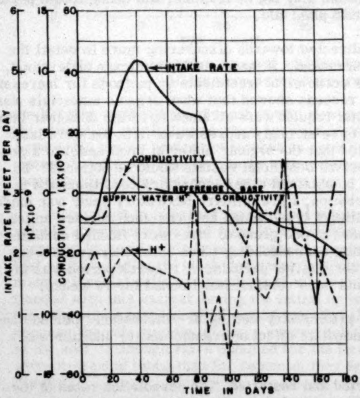

Fig. 3--Changes in conductivity and hydrogen-ion concentration of water in a test pond during a run; values were obtained by subtracting measurements of pond water from corresponding measurements of the supply water on that day; negative values indicate lower hydrogen-ion concentration or conductivity in pond water than in supply water

All pH and conductivity measurements were plotted against time on graphs which also showed pond-water intake rates. No correlation appeared between any of the curves. To reduce the effect of the small fluctuations in pH and conductivity of supply water, measurements of respective values for supply waters were then subtracted from corresponding measurements of the pond water and the differences plotted on intake-rate graphs. The logarithmic pH values were reduced to H+ concentration before subtracting. Figure 3 shows curves obtained from a typical pond when data are plotted in this way. To a fair degree, changes that actually occur in pond-water hydrogen-ion concentration and conductivity are reflected in these curves. However, for all data thus far obtained there appears to be no relation between changes occurring in either pH or conductivity and pond-water intake rates.

One fact brought out by these studies is that pond-water hydrogen-ion concentration and conductivity are nearly always slightly less than corresponding measurements of the supply waters. The explanation for this apparently lies in the activity of algae which at times are abundant in the ponds and at others are apparently absent. In their photosynthetic processes these green plants utilize various minerals from the pond water and considerable carbon dioxide. The latter may be obtained from dissolved CO_2 or from the bicarbonate ion which is relatively plentiful in the supply water. Removal of CO_2 from the bicarbonate ion results in formation of relatively insoluble carbonates. This is borne out by analysis of a pronounced grayish crust common to all ponds shut down after a long run. This crust of dried algae together with other insoluble material consists of about 25 pct carbonates, calculated as calcium carbonate. The removal of various ions from solution by the activity of the algae causes a decrease in pond-water conductivity compared with the supply. The increased pH (or decreased hydrogen-ion concentration) probably results from the relative increase of sodium as carbonate, due in part to elimination from solution of the calcium carbonate and possibly in part to much heavier use of calcium than sodium by the plant. Sodium carbonate would be hydrolyzed to sodium hydroxide with a resulting increase in pH.

One of the principal objectives of field pond studies initiated by cooperators of this project has been the discovery of practical treatments that might increase and maintain high peak rates of water intake under long submergence. MUCKEL [1950] in discussing these studies divided the treatments tried into five broad classes: (1) mechanical, (2) chemical, (3) operational procedures, (4) vegetative, and (5) addition of organic matter. Within each class, individual treatments are found that will increase and others decrease the natural intake rates for the soils considered. Types four and five have been most effective thus far in increasing rates. In particular, the addition of a six-inch layer of cotton-gin trash has, in some cases, increased water-intake rates by as much as 300 to 400 pct over rates before treatment. Growth of a very dense stand of Bermuda grass or Paragrass has been almost as effective. While considerable attention is being directed to these two treatments, both have limitations. Gin trash, although a waste product in the San Joaquin Valley, is costly to haul and distribute and, of course, is not readily available in many areas where water spreading is needed. Minimum quantities necessary are not yet known but it appears that quite heavy applications are required. A specific management program is required to get beneficial results; without it, intake rates may actually be greatly depressed. Also, the possibility of introducing noxious weed seeds to an area from the trash must be considered.

Vegetative growth is a highly desirable treatment if plants offering some additional economic return can be used. Spreading is contemplated on valuable land, and economic benefits from spreading must compete with those of high-priced crops. Some type of pasture appears most logical here. To be adapted for spreading grounds, plants must not only benefit water-intake rates but should also be able to withstand long periods of at least partial submergence and the long dry periods between spreading runs. At times, irrigation may not be feasible, and these drouth periods may continue for years with little rainfall to sustain plant life.

In the laboratory, studies are being largely directed towards discovering more in detail the mechanism by which water-intake rates can be increased. If these mechanisms are understood, it may be possible to find other more effective or economical treatments or methods for increasing and maintaining intake rates. Study of test-pond records showed that when organic materials were added to the soil it was some time before beneficial results were obtained in terms of higher intake rates. Under certain conditions, rates appeared to be strongly depressed at first but were later improved following a drying period. This indicated that the organic material must undergo a preliminary decomposition or "incubation" period before beneficial results would be obtained. To test whether a similar sequence of results would be obtained under laboratory conditions and to test two methods and several time lengths of incubation, a preliminary laboratory study was made using disturbed soil in small percolation tubes with gin trash added as a one-inch surface mulch (approximately 15 tons per acre). Several members of one group of tubes were flooded continuously for periods of two, four, and eight weeks while corresponding tubes of another group were intermittently flooded (irrigated every third day) for similar periods. A 16-week period was included with this group. Unmulched "control" tubes were continuously flooded for 16 weeks.

Percolation rates of flooded tubes during the preliminary wetting or "incubation" period are shown in Figure 4a. As in the field ponds, the immediate effect of organic matter addition was a rapid decrease in rate of water intake.

Following "incubation" all tubes were air-dried and restarted. The percolation rates of the tubes after incubation by continuous flooding and those incubated by frequent irrigation are shown in Figure 4bc. In both methods the rates of the tubes receiving the longest period of incubation were the highest. However, only the eight-week period of incubation by continuous flooding produced appreciably higher rates and a greater volume of percolate than the control. Continuous flooding in the percolation tubes apparently was a more efficient method of incubation than intermittent flooding (irrigation) where the gin-trash mulch dried out between wettings. Had the trash been incorporated in the soil, it would have remained moist and more decomposition obtained. In field ponds, greater increases have sometimes (though not always) been achieved, probably due to

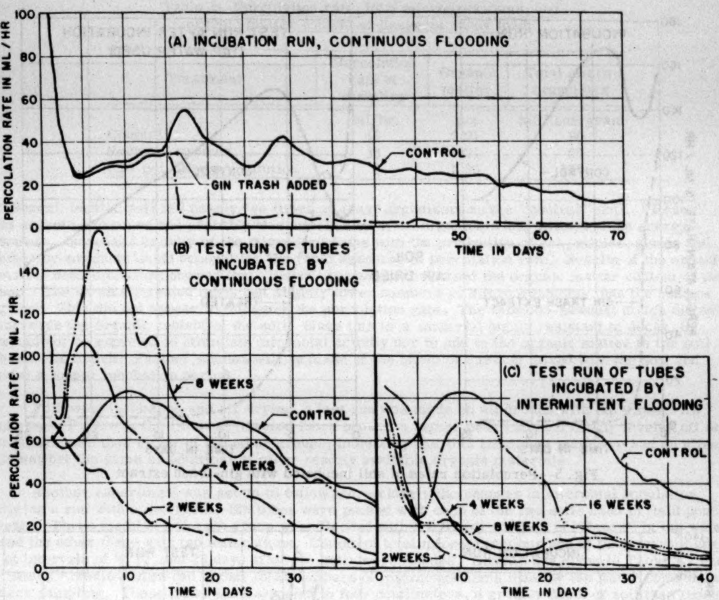

Fig. 4--Percolation rates of soil incubated by continuous or intermittent flooding; 15 tons per acre of gin trash added as a surface mulch

the heavier applications of trash used. However, it has been noticed in these ponds that rates on the second re-run following short incubations are often higher than obtained in the first re-run. Apparently longer periods of incubation result in more beneficial decomposition of the organic material available. Further tests with different rates of gin-trash application and with the trash incorporated in the soil mass are under way.

In an effort to separate the physical effect of the presence of the gin trash from the effect of soluble substances leached from it, a test was made using a water extract of the trash. This was prepared by soaking gin trash in water for several days and then filtering off the liquid. Microbial activity in the gin trash and water mixture was high, and the filtrate undoubtedly contained many microbial cells and products of metabolism. Figure 5 shows the percolation rates of tubes flooded with this extract during the initial or incubation period. Rates obtained with tap water on the subsequent test run after drying are also shown. The effect of the gin-trash extract was to depress the rate rapidly, giving a curve similar to that obtained when gin trash is added directly to the soil. Although rates obtained for the first few days of the test run were higher for the extract-treated tubes than for controls, they dropped rapidly and were low for the remainder of the run.

This preliminary test showed that the gin trash need not be present in or on the soil to produce the sealing effect noted when fresh organic material is added to soil. While beneficial effects were not obtained in this test, it is believed that an insufficient incubation period was used, and further studies are planned. Treatment of water prior to spreading rather than treatment of the spreading area might prove to be an economical and practical way of increasing water intake.

Similar percolation tests have been made in the laboratory on soil treated with a redwood sawdust mulch at ten tons per acre and, on another group of tubes, using "Renex," a non-ionic

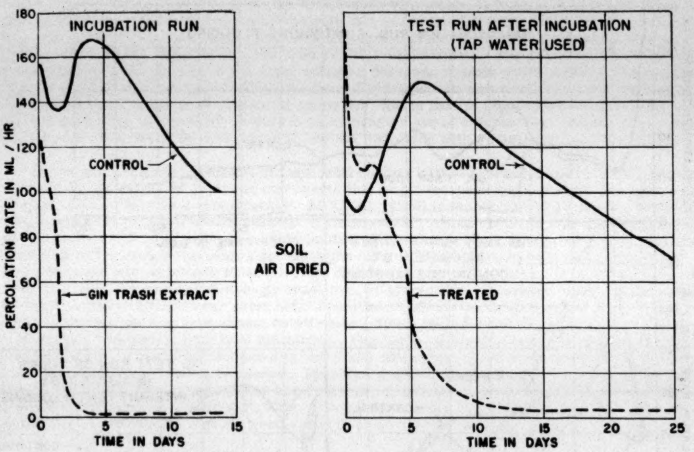

Fig. 5--Percolation rates of soil incubated with gin-trash extract

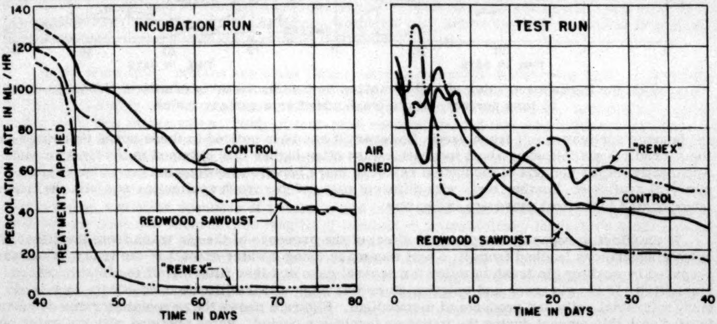

Fig. 6--Percolation rates of soil incubated with 0.01 pct "Renex" solution or ten tons per acre of redwood sawdust mulch, air dried between runs

detergent at 0.01 pct concentration in the water. The "Renex" detergent used was supplied by courtesy of Atlas Powder Co. Figure 6 shows the percolation rates obtained during the "incubation" run and during the test run. The two treated and one untreated control group of six tubes each had undergone a 43-day period of flooding with tap water at the time the treatments were imposed without interrupting the run. The addition of redwood sawdust as a mulch apparently had no effect on the percolation rate since the rate continued to decrease in the same way as the control group of tubes having no treatment. Within seven days after "Renex" was added, however, the percolation rate had declined to about one-tenth of that before treatment. After the "incubation" run, the surface centimeter of soil from two tubes of each group was sampled for counts of total numbers of microorganisms and for organic matter content. Results are shown in Table 1. The

Table 1--Percolation rate, total microorganisms, and organic matter content of Hesperia sandy loam

Treatment	Percolation rate at sampling	Organic matter	Total microorganisms
	ml/hr	pct	millions/gram
Control	46	0.91	90
Redwood sawdust	38	0.91	67
0.01 pct "Renex" solution	3	1.30	175

"Renex" treated soil had nearly two times as many organisms as the "control" soil. "Renex" is an organic compound and probably was attacked by microorganisms and utilized as an energy source. Increased activity of the microorganisms with the production of gas, slimes, gums, and other by-products could account for the rapid decrease in percolation rate. Results of the organic matter determinations showed that "Renex" treatment increased the organic matter content of the soil. The sawdust-treated tubes had slightly lower numbers of microorganisms than the control group. This did not appear to influence the percolation rate. The redwood-sawdust mulch did not increase the organic content of the soil. Since this is a material highly resistant to decay, it would not be expected to stimulate microbial activity nor to add to the organic matter in the soil below the mulch. Further studies will be made of the effect of sawdust mixed with the soil and over a longer incubation period.

Following incubation and air drying, a test run was made on all groups with tap water. As shown in Figure 6, the "Renex" treated tubes showed a higher peak rate and finally leveled off at a higher rate than either of the other groups, indicating that this treatment may have had an effect somewhat the same as additions of other readily available organic materials.

Another experiment was set up to follow more closely the changes in microbial population during a run with a detergent. Six tubes were packed with each of the two soils used in field pond tests. Three members of each group were flooded with a 0.01 pct solution of "Renex" in tap water and the other three with tap water alone. Counts of total microorganisms and of fungi were made at intervals of 8, 12, and 23 days after the soils were flooded. Results are given in Table 2. The "Renex" treated tubes had higher total numbers of microorganisms than the tap water controls at each sampling. These tubes were sampled to four centimeters, a greater depth of soil than those shown in Table 1. Since the highest concentration of the aerobic organisms enumerated are found in the surface, the greater depth of sampling tends to decrease the difference in total numbers between "Renex" treated and control soils. The effect of surface sampling probably contributed to the generally higher counts noted in Table 1.

The fungi population in the "Renex" treated tubes decreased markedly with time. Most fungi are strict aerobes, that is, they require oxygen for growth. In the "Renex" treated soils and, to some extent, in the untreated soil it appeared that anaerobic conditions had developed after the percolation rate became very low. This anaerobic condition was indicated by the odor of hydrogen sulfide and the blackening of the soil in the tube. Very little oxygen was carried into the soil by the slowly percolating water. It is possible that the lack of oxygen limited the activity of the fungi.

Table 2--Percolation rate, total microorganisms, and fungi Hesperia sandy loam and Exeter sandy loam

Soil	Treatment	Percolation rate				Fungi			Total microorganisms		
		Time in days				Time in days			Time in days		
		1	8	12	23	8	12	23	8	12	23
		ml/hr				thousands/gram			millions/gram		
Hesperia sandy loam	Tap water	52	40	29	25	69	70	82	63	58	57
	0.01 pct "Renex" solution	49	8	3	1	56	63	13	76	70	70
Exeter sandy loam	Tap water	38	34	27	17	57	77	57	67	44	41
	0.01 pct "Renex" solution	36	8	5	3	49	61	5	79	48	82

The results so far obtained with "Renex" do not preclude the possibility that the rate changes obtained during "incubation" and "test" runs are not, at least in part, due to chemical or physical effects caused by the detergent. Additional experiments to test this possibility are planned.

The role played by the organic matter added to the ponds and perçolation tubes in increasing water intake is complex. While it is well known in a general way that organic matter added to soil tends to improve subsequent infiltration characteristics, the mechanism of the process is somewhat obscure. McCALLA [1945] has shown that a certain amount of microbial decomposition of such added organic matter is apparently essential before increased soil aggregation and improved short-time infiltration rates result. MARTIN [1945] isolated one fungus and one bacillus organism that caused aggregation of sterile soil particles when incubated with these organisms and then dried. The active aggregating material consisting in part of microbial cells and in part substances synthesized by the cells was attacked to a limited extent by fungi and was readily destroyed by certain other microorganisms.

Chemical analysis of cotton-gin trash samples, the material which has thus far been of greatest benefit to intake rates, indicates this product is as rich in several elements essential to microbial growth (including nitrogen and potassium) as ordinary farm manure. Considerable amounts are readily water soluble and form a rich supply of energy material for microorganisms when added to the soil.

The mechanism by which the added organic matter improves pond-water intake rates is visualized as occurring in three phases:

(a) Incubation period--Following the application of organic material, water is supplied to the soil. With a rich supply of energy material available, microbial activity increases rapidly. Large numbers of microbial cells, and large quantities of slimes, gums, gas, and other by-products are formed. Under these conditions the pores of the soil are rapidly sealed and the water-intake rate is strongly depressed. Length of time needed for incubation under field and laboratory conditions will depend on such factors as the amount and kind of energy material and its availability to the microorganisms present, temperature, moisture, and possibly others. Under field conditions 30 days seem to be a minimum.

(b) Drying period--Following incubation the soil must be dried. Without this drying, no increase in intake rate is obtained. During this time the microbial cells and solid by-products contract, tending to pull the soil particles together into aggregates and forming a more or less water stable coating around these larger particles. Larger pores result from the formation of these aggregates.

(c) Running period--Upon restarting the pond or percolation tube the enlarged and more stable pores in the soil permit more rapid water movement, and, following removal of entrapped air, greatly increased water-intake rates occur. As suggested by Martin and Allison, the substance of the coatings may be attacked by other microbes and the structure destroyed. Also, in the following runs, the more resistant organic residue will be attacked and eventually the pores will again be clogged with products of microbial activity. Thus microbial activity again accounts for the decreasing intake rates which follow the maximum.

Physical measurements--As already pointed out, various surface treatments appear to have markedly increased pond-water intake rates. Such increases must be related to certain physical changes at or near the soil surface. To determine the nature of some of these changes, a number of physical measurements are being made.

Table 3 lists volume weights typical of those obtained on the various ponds. Both field soils, Exeter sandy loam and Hesperia sandy loam, have high volume weights in the undisturbed state. As would be expected, volume weights decreased when organic material was added and also under a heavy growth of vegetation. A fact of particular interest brought out by this table is that the effect of these treatments definitely extends below six inches in only one case, that of Pond 15 with a heavy growth of Bermuda grass. Pond 4A, with an even heavier growth of grass, had almost no roots below the three-inch level. This probably reflects the differences in management of the two ponds.

A major consequence of decreased volume weight is increased total porosity. However, it has been pointed out by many workers that it is the amount of larger, so-called "non-capillary" pores in soil that determine the rate of water movement under saturated conditions. To find what changes have occurred in these larger pores, desorption tests at low tensions by the LEAMER and SHAW [1941] technique have been made on cylinders of undisturbed soil taken from various ponds. Figure 7 shows typical desorption curves obtained on Exeter soil. With gin-trash treat-

Table 3--Volume weights and increase in maximum monthly intake rate of two soils after different treatments

Soil	Location	Depth	Average volume weight[a]	Treatment	Increase in maximum monthly intake rate after treatment[b]
		inch			pct
Exeter sandy loam	Outside ponds	0-3	1.69	None	. . .
		6-9	1.69		
	Pond 2	0-3	1.41	May. 1946, 10 tons/acre alfalfa spaded	250
		6-9	1.64		
	Pond 1A	0-3	1.21	May, 1946, 10 tons/acre lime spaded	No definite increase
		6-9	1.68	June, 1948, 6-inch layer gin trash spaded and submerged 60 days	300
	Pond 2A	0-3	1.24	May, 1946, 10 tons/acre alfalfa spaded and irrigated	Not measured separately
		6-9	1.69	May, 1948, 6-inch layer gin trash surface mulch submerged 60 days	250
	Pond 4A	0-3	1.17	May, 1945, Bermuda grass sodded	
		6-9	1.62		600
Hesperia sandy loam	Outside ponds	0-3	1.59	None	. . .
		6-9	1.65		
	Pond 15	0-3	1.18	Apr., 1946, Bermuda grass sodded	
		6-9	1.46		250
	Pond 17	0-3	1.17	Dec., 1945, 6-inch layer gin trash on surface	Not measured separately
		3-6	1.22	May, 1946, spaded	600

[a] Measurements made after most recent treatment.

[b] Increase calculated from intake rate of each pond in untreated condition.

ment there is not only a great increase in total porosity but also a large increase in "non-capillary" porosity.

The recent incorporation of organic material in Pond 1A served to decrease volume weight slightly more than leaving it on the surface, but it would appear that decrease in volume weight alone does not assure maximum increase in the larger sized pores. Whether the greater volume of non-capillary pores in Pond 2A is due to the alfalfa spaded under in 1946 or to the gin-trash treatment is not now known. Pond 2, however, had alfalfa applied at the same rate at the same time with no additional treatment since. It is evident that some effect of the alfalfa still remained over three years later. For approximately 10 1/2 months of that time, Pond 2 was submerged and a 850-ft depth of water passed through the surface. Apparently the effect of these treatments does not extend below six inches, for the six- to nine-inch curves are closely grouped and very similar to data for untreated soil.

Water-intake-rate curves for these three ponds on runs following sampling are shown in Figure 8. Comparisons of these curves indicate that factors in addition to pore-size distribution of surface material are operating to control intake. Differences in the soil profile, examined to a depth of six feet, probably are responsible for the differences in intake. The surface of all three ponds is a sandy loam. A rather compact sandy clay-loam layer was found at 24 to 30 inches under Pond 2A. A similar but less compact and lighter textured layer occurs at a four-foot depth under Pond 1A. Pond 2 has a light clay-loam horizon from 10 to 24 inches, and is a friable sandy loam below.

The surface of Pond 2 had been undisturbed for over three years following alfalfa treatment and only a very small amount of alfalfa residue remained. Structure of the soil was well estab-

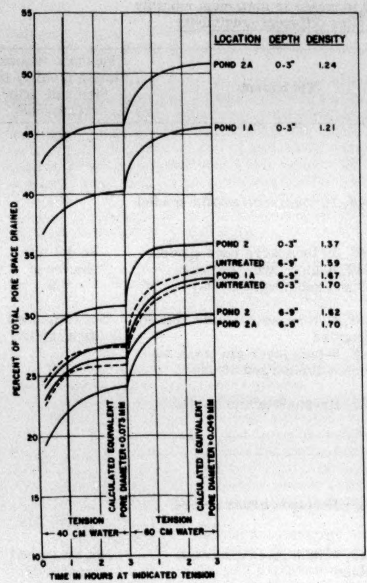

Fig. 7--Desorption curves and apparent density of soil samples taken from test ponds on Exeter soil

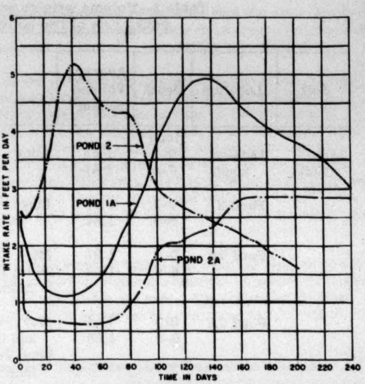

Fig. 8--Water-intake rates of test ponds on Exeter soil

lished after repeated flooding and drying. Thus there was little initial breakdown or dispersion, and the first decrease in the S-curve is almost absent. Apparently little air was entrapped and water could move through the soil fast enough to dissolve it rather rapidly. Prior to the alfalfa surface treatment, peak rates were substantially lower. However, peak rates obtained on runs previous to this, but since the alfalfa treatment, have been slightly higher. It seems probable, therefore, that the present peak is determined by a surface limitation rather than a limitation within the profile. The small amount of energy material added would not be expected to cause a particularly resistant or stable structure. Thus, although a fairly large improvement over the natural intake rate was attained, the rate could not be maintained over a protracted period.

Pond 1A attained approximately the same peak rate as Pond 2, and it is of interest to compare the shape of the curves. Removal of entrapped air from Pond 1A was more difficult because of the lower initial intake rate, caused in part by dispersion and swelling of surface material. Air removal took so long that the peak rate was probably considerably lowered by microbial activity that had occurred during this time. Having attained a peak, however, the rate of decrease was slower in Pond 1A than in Pond 2. For example, Pond 1A maintained a rate of three feet per day or over for 150 days while Pond 2 maintained this rate for only 85 days. The difference is probably due to the larger amount of energy material originally applied to Pond 1A which produced a more stable structure.

Pond 2A possibly had more air entrapped by the relatively impervious layer at 24 to 30 inches than either Pond 1A or 2. The initial rate was so low that little air was dissolved and for 70 days the rate was minimum. Following the gradual removal of air, the pond finally reached a rather low maximum which it maintained for the remaining 80 days of the run. This low and very flat peak points to the possibility that the control section was below the surface. Microbial activity at a depth of 24 inches and over is comparatively low; hence the rate in this section should remain constant for a long period if other physical changes do not occur. If this hypothesis is correct, future runs should not go over three feet per day no matter what treatment is applied to the surface as long as the 24- to 30-inch layer is unaffected. Eventually products of microbial activity will fill surface soil pores to a point where infiltration rate will be less than potential percolation rate of the subsoil and the curve will drop.

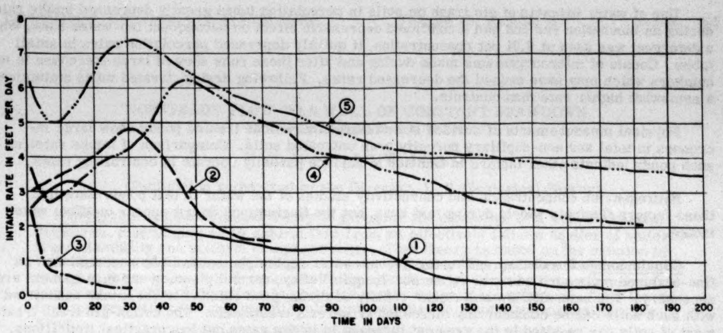

Fig. 9--Water-intake rate curves of a test pond on Hesperia soil; see text for designation of curves

If surface soil pore space and time length of incubation are the major factors influencing sealing, then, since the two factors are similar in the two ponds, the curve for Pond 2A should start dropping at approximately the time the curve for Pond 1A would intersect it. If other factors such as method of application of residue or amount of water passing are important, the curves would not necessarily parallel each other. Unfortunately, the run of these ponds had to be discontinued before this point could be tested.

To illustrate the step-by-step development of improved water-intake rates following the application of organic residues in a field pond, Figure 9 shows intake-rate curves on a pond treated with several such residues. Curve 1 represents initial rate of intake before addition of organic residues. Curve 2 shows intake rate following application of 100 pounds of manure and later a one-inch layer of alfalfa spaded in and incubated by intermittent irrigation and rain for several months, then dried. After a second re-run which had a lower and shorter peak, a six-inch layer of cotton-gin trash was spaded in and submerged for 30 days. Curve 3 shows intake rate during this incubation run. Curve 4 shows the first test run following gin-trash incubation and drying. The second test run, begun after drying the pond for almost a year, is shown by Curve 5. Note that the additional incubation of the organic residue during the period of Curve 4 and the redrying apparently resulted in further structure improvement, and a considerable increase in amount of water absorbed in a six-month period.

Summary--Replenishment of underground water supplies must be done in many areas by water spreading on relatively fine-textured soils. A more or less rapid decline in water-intake rate on such soils poses serious problems. This decline appears to be associated with microbial activity as well as other factors that affect infiltration. In laboratory percolation studies, high rates can be maintained as long as the system is kept sterile.

Experiments on field ponds show that relatively high intake rates can be obtained and maintained for longer than normal time periods by growth of dense stands of Bermuda grass or by proper management of organic materials incorporated in the surface soil. Only limited investigations have been made of reasons for improved water intake under grass. Improved intake rates through use of organic residues apparently are due to microbial activity. This activity seems to result ultimately in improved aggregation, stability of structure, and larger pores. Management or organic residues includes incubation and drying before benefits are achieved. Laboratory and field studies show water-intake rates are severely depressed during incubation.

Laboratory studies using cottin-gin trash mulch in percolation tubes showed eight weeks continuous flooding was more effective as an incubation than four weeks which in turn was better than two weeks. Only the eight-week incubation increased rates over untreated control tubes. Incubation by intermittent irrigation for up to 16 weeks did not improve intake rates beyond that of control tubes.

Use of water extracts of gin trash on soils in percolation tubes greatly depressed intake rates during an incubation run and had a continued depressive effect on subsequent tap-water runs. When a detergent was used at 0.01 pct concentration, it quickly depressed percolation rates in small tubes. Counts of microorganisms made during and after these runs showed large increases in total numbers which may have caused the decreased rates. Following drying, treated tubes maintained a somewhat higher rate than controls.

Physical measurements of surface soils in organic residue treated ponds show large increases in total and non-capillary porosity over untreated soils. Comparison of intake rates on such ponds indicate other factors in addition to surface porosity operate in controlling rates.

Hydrogen-ion concentration and conductivity studies of the water in test ponds show that these factors fluctuate widely during test runs, but the fluctuations do not appear to affect water intake.

Conclusions--If water spreading for ground-water replenishment is to be accomplished on fine-textured soils such as occur in the San Joaquin Valley, careful planning and management are necessary. The naturally low and rather rapidly declining peak rates of water intake associated with such soils can be considerably increased by several treatments. The cotton-gin trash treatment of soils has resulted in the greatest increase in intake rates but has practical limitations. Further study is needed of its effects as well as of management needed to produce similar effects with other organic residues. Grasses have increased intake rates almost as much as gin trash and may provide additional economic return where they are adapted. The possibility of increasing intake rates by adding substances directly to water also requires further study.

Studies here reported and to be made in the future are concerned with fundamental processes involved in water intake under spreading conditions. It is felt that only by better understanding of such processes can practical answers to field problems be obtained. Such an understanding would permit use of information over a wide range of soils and climatic conditions.

Acknowledgments--Although an attempt is made to credit in formal references the chief sources of information and ideas contributed by others to the preparation of this report, doubtless the acknowledgments for this aid are incomplete. The continuous efforts, support, and suggestions of A. T. Mitchelson in the development of the laboratory and the work have been stimulating and encouraging. Grateful acknowledgment is made to Leonard Schiff for reviewing this report and giving helpful suggestions; to the North Kern Water Storage District and to the Kern County Land Co. for their excellent cooperation in support of the program, for the provision of experimental ponds, and for supplying water measurements.

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(Manuscript received September 19, 1951; presented at the Twenty-Sixth Meeting, Fresno, California, February 9, 1951; open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

NONSTEADY FLOW TO A WELL OF CONSTANT DRAWDOWN IN AN EXTENSIVE AQUIFER

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(Publication authorized by the Director, U. S. Geological Survey)

Abstract--A mathematical theory is given for the discharge of a well of constant drawdown, discharging as by natural flow from an effectively infinite aquifer of uniform transmissibility and uniform compressibility. This theory is based on the solution by L. P. Smith of the analogous problem in heat conduction. The mathematical function involved in the solution, which cannot be integrated directly, is evaluated by numerical integration. A table of its values is given for a wide range of its argument. This function is compared with other, asymptotic solutions, and simple, useful approximations are given.

Two graphical methods are outlined for determining the coefficients of storage and transmissibility from variations in the rate of discharge of wells flowing at constant drawdown. Data from the Grand Junction, Colo., artesian basin are treated by these methods. In the Grand Junction artesian basin there are about 25 flowing wells ranging in depth from 600 to 1600 ft, most of which obtain water from the Entrada sandstone. A few of the wells obtain water from a sandstone in the overlying Morrison formation and a few tap the underlying Wingate sandstone.

The procedure of the tests is outlined, and the "ink-well" mercury gage used to measure the artesian pressures is described. Recovery tests were run on the same wells after the discharge tests. Values of transmissibility obtained from the recovery tests check those obtained by means of the discharge tests.

Introduction

When a well that penetrates an extensive confined aquifer is pumped at a constant rate, the drawdown increases roughly as the logarithm of the time. The exact solution of this problem, for an effectively infinite and perfectly elastic aquifer of uniform thickness, was obtained by THEIS [1935]. It expresses the drawdown in terms of the exponential integral. For small values of its argument (or large values of time) this function may be approximated by a constant plus another constant times the logarithm of the time. This solution has been applied with notable success to the determination of the coefficients of transmissibility and storage of confined beds and to the prediction of drawdowns at distances and times beyond those immediately observed. It is the purpose of this paper to set up the solution for the related problem of a well of constant drawdown discharging, as by natural flow, from an effectively infinite bed of similar characteristics. It is shown that under these conditions the discharge declines roughly as the inverse of the sum of a constant and the logarithm of the time. Application of the solution is illustrated by the treatment of data on flowing wells in the Grand Junction area, Colo.

The theory was developed by Jacob and the field tests were conducted by Lohman. The interpretation of the field data is the work of both.

Theory

Heat-conduction analogue--The analogous problem of nonsteady heat flow in an infinite solid bounded internally by a cylinder has been solved by SMITH [1937]. He analyzed the case in which the uniform initial temperature is constant and the temperature on the internal boundary has stepwise variation, to determine the flow of heat across that boundary and its variation. For application here to the analogous discharge of a single well at constant drawdown, only the first step of variation will be considered. In other words, the problem will be to determine the variation of discharge of a well tapping an infinite aquifer of uniform thickness, uniform permeability, and uniform compressibility. Initially the head is assumed uniform throughout the aquifer. At zero time the head at the well face drops instantaneously to some lower constant value.

Expressed mathematically the initial condition is

$$h = h_0 \text{ for } t = 0 \text{ and for all values of } r \qquad (1)$$

where h_o is the uniform initial value of head, t the time, and r the distance from the well.

The boundary conditions are

$$h \rightarrow h_o \text{ as } r \rightarrow \infty, \text{for } t > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and

$$h = h_o - s_w \text{ for } r = r_w \text{ and for } t > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where s_w is the drawdown of the well and r_w is the radius of the well.

The differential equation governing the nonsteady flow is

$$\partial^2 h/\partial r^2 + (1/r)\, \partial h/\partial r = (S/T)\, \partial h/\partial t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

S being the storage coefficient of the aquifer, and T its transmissibility; that is, the product of the transmission constant of the sand, K, and its thickness, b [see JACOB, 1950, p. 366].

The solution of (4) that satisfies conditions (1), (2), and (3) is

$$Q = 2\pi T\, (h_o - h)\, G\,(\alpha) = 2\pi T\, s_w \cdot G\,(\alpha) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where Q is the discharge of the well and where

$$\alpha = Tt/S\, r_w^{\,2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The values of the function G (α) are given by the following integral obtained by SMITH [1937]

$$G\,(\alpha) = (4\,\alpha/\pi)\int_0^{\infty} x e^{-\alpha x^2} \left\{(\pi/2) + \tan^{-1}\,[Y_o\,(x)/J_o\,(x)]\right\} \ldots\ldots\ldots\ldots (7)$$

in which J_o (x) and Y_o (x) are Bessel functions of zero order of the first and second kinds, respectively.

Evaluation of the function by numerical integration--The integral in (7) is not tractable by any of the ordinary means of integration. Accordingly, it was found necessary to resort to numerical methods, replacing the integral by a summation, as follows

$$G\,(\alpha) = (4\alpha/\pi) \sum_{\nu} x_{\nu}\; T\,(\alpha,\, x_{\nu})\, \Delta x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where

$$T\,(\alpha,\, x_{\nu}) = e^{-\alpha x^2} \left\{(\pi/2) + \tan^{-1}\,[Y_o\,(x)/J_o\,(x)]\right\}$$

Using mean values of x and T in each interval

$$G\,(\alpha) = (\alpha/\pi) \sum_{\nu=0}^{\infty} [x_{\nu} + x_{\nu+1}] \cdot [\,T(\alpha,\, x_{\nu}) + T\,(\alpha,\, x_{\nu+1})]\, \Delta x$$

Summations were carried out for α equal to different integral powers of 10. For even powers of 10 it was found convenient to put $\Delta x = 10^{-(n+2)/2}$ where $10^n = \alpha$; then $\alpha\, \overline{\Delta x}^2 = 10^{-2}$. Also, x_{ν} was divided into ν intervals, each Δx wide, giving $x_{\nu} = \nu \Delta x$. Thus

$$G\,(10^n) = (1/100\,\pi) \sum_{\nu=0}^{\infty} (2\nu + 1) \left\{T\,(10^n,\, \nu 10^{-1-n/2}) + T\,[10^n,\, (\nu + 1)\, 10^{-1-n/2}]\right\}$$

To improve the approximation further, the first interval was divided into 10 small intervals, giving

$$\begin{aligned} G\,(10^n) = (1/100\,\pi) \Big[(1/100) \sum_{\mu=0}^{9} (2\mu + 1) &\left\{T\,(10^n,\, \mu 10^{-2-n/2}) + T\,[10^n,\, (\mu + 1)\, 10^{-2-n/2}]\right\} \\ + \sum_{\nu=1}^{\infty} (2\nu + 1) &\left\{T\,(10^n,\, \nu 10^{-1-n/2}) + T\,[10^n,\, (\nu + 1)\, 10^{-1-n/2}]\right\}\Big] \ldots\ldots\ldots\ldots (9) \end{aligned}$$

with $T(\alpha, 0)$ taken equal to $T(\alpha, 10^{-2-n/2})$.

For odd powers of 10 it was found convenient to put $\Delta X = 10^{-(3+n)/2}$ and $T(\alpha,0) = T(\alpha, 10^{-5/2-n/2})$ getting finally

$$G(10^n) = (1/1000\pi)\Big[(1/100)\sum_{\mu=0}^{9}(2\mu+1)\Big\{T(10^n, \mu 10^{-5/2-n/2}) + T[10^n, (\mu+1)10^{-5/2-n/2}]\Big\}$$

$$+\sum_{\nu=1}^{\infty}(2\nu+1)\Big\{T(10^n, \nu 10^{-3/2-n/2}) + T[10^n, (\nu+1)10^{-3/2-n/2}]\Big\}\Big] \ldots\ldots\ldots \quad (10)$$

Computed values of G are given in Table 1. The even values of n (= log α) for which summations were performed range from -4 to +18; the odd values are -1 and +1. They are plotted in Figure 1.

Table 1--Values of G (α) for values of α between 10^{-4} and 10^{12}

	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10	10^2	10^3
1	56.9	18.34	6.13	2.249	0.985	0.534	0.346	0.251
2	40.4	13.11	4.47	1.716	0.803	0.461	0.311	0.232
3	33.1	10.79	3.74	1.477	0.719	0.427	0.294	0.222
4	28.7	9.41	3.30	1.333	0.667	0.405	0.283	0.215
5	25.7	8.47	3.00	1.234	0.630	0.389	0.274	0.210
6	23.5	7.77	2.78	1.160	0.602	0.377	0.268	0.206
7	21.8	7.23	2.60	1.103	0.580	0.367	0.263	0.203
8	20.4	6.79	2.46	1.057	0.562	0.359	0.258	0.200
9	19.3	6.43	2.35	1.018	0.547	0.352	0.254	0.198
10	18.3	6.13	2.25	0.985	0.534	0.346	0.251	0.196

	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}	10^{11}
1	0.1964	0.1608	0.1360	0.1177	0.1037	0.0927	0.0838	0.0764
2	0.1841	0.1524	0.1299	0.1131	0.1002	0.0899	0.0814	0.0744
3	0.1777	0.1479	0.1266	0.1106	0.0982	0.0883	0.0801	0.0733
4	0.1733	0.1449	0.1244	0.1089	0.0968	0.0872	0.0792	0.0726
5	0.1701	0.1426	0.1227	0.1076	0.0958	0.0864	0.0785	0.0720
6	0.1675	0.1408	0.1213	0.1066	0.0950	0.0857	0.0779	0.0716
7	0.1654	0.1393	0.1202	0.1057	0.0943	0.0851	0.0774	0.0712
8	0.1636	0.1380	0.1192	0.1049	0.0937	0.0846	0.0770	0.0709
9	0.1621	0.1369	0.1184	0.1043	0.0932	0.0842	0.0767	0.0706
10	0.1608	0.1360	0.1177	0.1037	0.0927	0.0838	0.0764	0.0704

Comparison with other, asymptotic solutions--It will be noted that for small values of α (or small relative values of t) G appears to approach the line plotted for $1/\sqrt{\pi\alpha}$, as it should to give the discharge from an infinite aquifer bounded by a plane when $r_w \rightarrow \infty$. Actually, on the other side $G(\alpha) \rightarrow 1/2 + 1/\sqrt{\pi\alpha}$ as $\alpha \rightarrow 0$. Interpolated values of G in the low range of α in the table were determined by plotting values of the difference $G(\alpha) - 1/\sqrt{\pi\alpha}$ against the several integral values of n (= log α) and graphically interpolating values of the difference for intermediate values of α.

For large values of α (or large relative values of t) the G approaches $2/W\,(1/4\alpha)$, where W stands for the negative exponential integral of the negative argument $(-1/4\alpha)$. This is also as it should be because the drawdown [JACOB, 1950, p. 367] in a well of radius r_w discharging at the rate Q for a period of time t is

$$s_w = (Q/4\pi T)\, W\,(1/4\alpha) = (Q/2\pi T)\, W\,(S\, r_w^{\,2}/4Tt)/2 \ \ldots\ldots\ldots\ldots \quad (11)$$

By comparing (11) with (5) it is seen that 2/W in the latter corresponds to G in the former. Inasmuch as the two approach each other asymptotically, this is another way of saying that the ratios of discharge to drawdown in two isolated wells -- one with constant discharge, the other with constant drawdown -- approach equality at sufficiently large relative values of t. Figure 2 is drawn to illustrate this statement. It is understood, of course, that the two wells tap two aquifers of equal T and equal S, or that they tap the same aquifer and are sufficiently spaced to prevent appreciable interference.

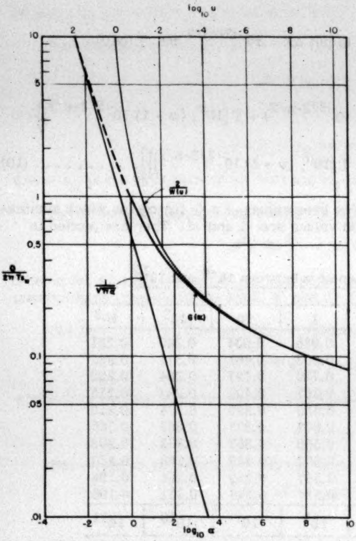

Fig. 1--Discharge of well of radius r_w at steady drawdown s_w from infinite elastic aquifer of transmissibility T and storage coefficient S

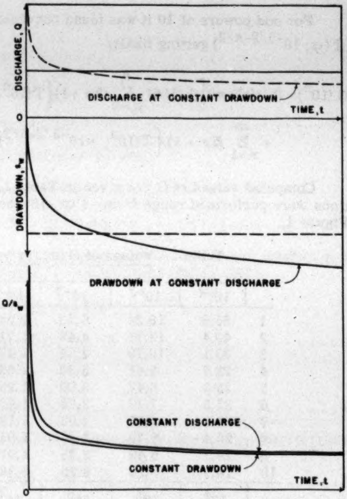

Fig. 2--Ratio of discharge to drawdown at constant drawdown and at constant discharge

Interpolated values of G in the high range of α in the table were similarly determined by plotting values of the logarithm of the difference (2/W) - G against log α for the different integral values of log α, and reading off intermediate values of the difference. Beyond $\alpha \approx 5 \times 10^{11}$ the difference is less than one in the third significant figure, and there is no point in continuing the table, because G can be computed directly for larger values of α as twice the reciprocal of the exponential integral. (For a table of the exponential integral see WENZEL [1942]. In fact, the difference is only one per cent of G at $\alpha \approx 80{,}000$, about two per cent of G at $\alpha \approx 5000$, and about five per cent at $\alpha \approx 200$.

Further approximation of G--For large relative values of t the function W $(S r_w^2/4Tt)$ can be approximated by 2.30 log $(2.25\ Tt/S\ r_w^2)$ [see JACOB, 1950, p. 368]. The difference is only about 0.02 per cent of W at $\alpha = 200$, where the difference between G and 2/W is five per cent of G. Thus, to almost the same approximation, G can be replaced by 2/2.30 log $(2.25\ Tt/S\ r_w^2)$ instead of by 2/W, and the discharge can be determined from

$$Q = 4\pi T s_w / 2.30 \log (2.25\ Tt/S\ r_w^2) \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Equation (12) can be written in the inverted form

$$4\pi T s_w / Q = 2.30 \log (2.25\ Tt/S\ r_w^2) \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Graphical determination of the coefficients S and T--Data on the variation of discharge with time of a well at constant drawdown may be plotted to determine S and T, using (5) and (6). First, on logarithmic paper a graph is drawn of G (α) versus α. On transparent logarithmic paper, to the same scale, values of Q/s_w are plotted against values of t/r_w^2, or values of Q for a single well may be plotted against values of t. Then by placing the experimental data over the theoretical curve the best fit of the data to the curve can quickly be made, and from the coordinates of any point in both coordinate systems the values of T and S can be determined. This same method has been used with the exponential-integral solution for a well of constant discharge [see JACOB, 1940; or WENZEL, 1942].

When the smallest observed values of time are relatively large (that is, large relative to $S r_w^2/T$) a somewhat simpler graphical method may be employed, using the approximate relation (13). Experimental values of s_w/Q for one well, or several non-interfering wells, are plotted against $\log (t/r_w^2)$. According to (13), if the conditions of the theory are met, these points should fall on a straight line. From the slope of that line T may be determined. For

$$\partial (s_w/Q)/\partial \log (t/r_w^2) = 2.30/4\pi T$$

whence

$$T = 2.30/[4\pi\, \partial (s_w/Q)/\partial \log (t/r_w^2)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Then, having determined T, the value of S may be found by substituting the coordinates of some convenient point on the line into (13). For instance, at the intercept on the logarithmic time axis

$$s_w/Q = 0 = \log [(2.25\, T/S)\, (t/r_w^2)_o]$$

whence

$$S = 2.25\, T\, (t/r_w^2)_o \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Field tests

A geologic and hydrologic investigation of the Grand Junction artesian basin in Mesa County, Colo., which was undertaken by Lohman in 1946 as a part of the cooperative ground-water investigation by the Colorado Water Conservation Board and the U. S. Geological Survey, required the accurate determination of the coefficients of transmissibility and storage of several artesian aquifers from tests on individual flowing wells. Methods of testing involving the simultaneous use of more than one well were not practicable because of the high cost of putting down observation wells to the required depths (600-1600 ft) and because the great demand for the water for domestic use precluded the possibility of shutting in more than one well at a time. The desired coefficients were readily obtained by application of (5) and (6) or (14) and (15) to the data obtained by flow tests under conditions of constant drawdown and variable discharge. Values of T were obtained also by application of the recovery method [THEIS, 1935]. The test procedure is described briefly and the results are analyzed in order to demonstrate the applicability of the formulas to actual field conditions.

Geologic and hydrologic setting--The Grand Junction artesian basin, as at present developed, comprises an area of about 35 sq mi that lies in and southwest of the City of Grand Junction and includes about 25 flowing artesian wells ranging in depth from less than 600 ft to as much as 1639 ft (depending upon topographic situation and distance from outcrop). The measured artesian heads range from a few feet to as much as 169 ft above the land surface. Owing to the low transmissibility of the aquifers, however, the yields are relatively low, ranging from less than one gallon per minute to 20 or 30 gpm. As more than half the wells are crowded within an area of only about four square miles, natural declines in artesian head and flow have been augmented by mutual interference between wells to the extent that rather alarming declines have taken place in some wells. Despite the low yields more wells are being drilled, as the water is in great demand for hauling to rural and suburban areas that are devoid of domestic ground-water supplies. Fear that some of the wells may stop flowing and have to be pumped led to the need for detailed study in order to obtain factual data for predicting future trends.

The wells are supplied from one or more of three sandstones of Jurassic age that dip three to seven degrees northeast. Most of the wells are only three to four miles northeast from the recharge area; others are from one to about six miles from it. In the recharge area the strata have been dragged sharply upward along a faulted monoclinal fold, and the water-bearing sandstones have been badly crushed and sheared. Recharge is provided in part by exposure of the broken, upturned beds to the scanty rainfall, but it occurs mainly at points where these upturned beds are crossed by ephemeral streams that issue from the colorful, deep canyons of the adjacent Colorado National Monument.

Most of the wells in the artesian basin obtain water from the Entrada sandstone, but a few wells tap a shallower sandstone (Salt Wash Sandstone member) in the lower part of the Morrison formation and a few tap the deeper Wingate sandstone. The Entrada is about 150 ft thick and consists of cemented fine-grained salmon or white sandstone. Its confining bed is the overlying Summerville formation (mostly siltstone), which is succeeded above by the Morrison formation. The Kayenta formation, comprising 20 to 80 ft of sandstone and red shale, separates the Entrada from the underlying Wingate sandstone. Although the Wingate is about 300 ft thick, its buff fine-grained sandstone generally is more firmly cemented and has a lower transmissibility than the Entrada. The Wingate is underlain by about 100 ft of the Triassic Chinle formation (mostly red siltstone), which in turn rests upon Precambrian granite, gneiss, and schist. The sandstone in the lower part of the Morrison is highly lenticular and only 20 to 30 ft thick, and it yields less water than the others. The Morrison, which is about 600 ft thick and is mainly variegated shale, is overlain by the Cretaceous Burro Canyon formation, Dakota sandstone, and Mancos shale. Fairly good drillers' logs of most of the wells permitted generally good identification and correlation of the water-bearing and other formations.

Flow-test apparatus and procedure--Flow and recovery tests on 11 of the wells and repeated measurements of shut-in artesian head for most of the wells were made during the period 1946-51, using an ink-well mercury gage [LOHMAN, 1947]. Tests could not be run on the other wells because of physical difficulties or lack of owners' permission. Figure 3 shows the stainless-steel valves, the ink-well reservoir, the lower part of the glass tube, and the Invar scale calibrated to read the mercury column directly in feet and tenths of feet of water (hundredths of a foot are readily interpolated). Correction for slight depression of the reservoir level is read on the small scale at the right of the reservoir and added to the reading on the main scale. The left-hand valve admits water from the well; the right-hand valve is used to expel air and dirty water from above the mercury in the reservoir and, with the left-hand valve, to control the rise or fall of the mercury column. The gage is built in two sections; the lower section shown in Figure 4 reads up to 66.5 ft, and the range is increased to 134 ft by adding an extension. Heads of more than 134 ft were read by adding a short length of glass tubing to the top section of the gage and measuring the rise above 134 ft.

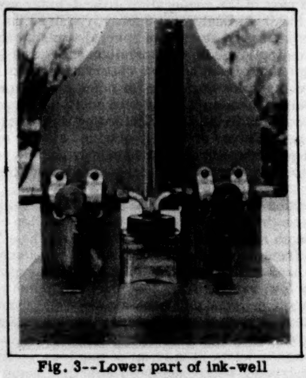

Fig. 3--Lower part of ink-well mercury gage

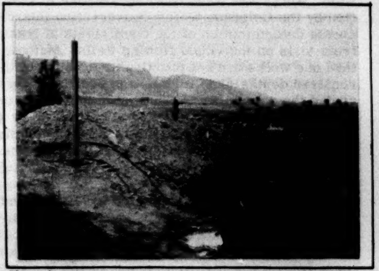

Fig. 4--Typical setup for flow test on Well 20, in Wingate sandstone; Colorado National Monument in background

Each well in turn was "shut in" in the afternoon or evening before it was to be tested, by adding the proper fittings, a 1/2-inch globe valve for connecting the gage and a 1 1/2- or 2-inch gate valve for use during the flow test. Most of the wells were shut in for 12 to 14 hours before testing. Because most of the wells normally are allowed to flow continuously, this period was none too long for aquifers having such low transmissibility, but it was the longest that was practicable.

On the following morning the gage and a swinging discharge pipe were connected (see Fig. 4), and the static head was carefully measured. Then at a recorded time the gate valve was opened rapidly and left open for a period of about three hours. Discharge measurements were made (at recorded times) at gradually increasing intervals ranging from 30 to 60 seconds for the first 15 minutes to 10 or 15 minutes during the third hour, by swinging the discharge pipe into position over a calibrated five-gallon container (carefully leveled) and timing the filling with a stop watch. A calibrated 32.5-gallon barrel was used in measuring some of the larger discharges. Then the gate valve was closed at a recorded time, and measurements of the slowly recovering artesian head were made at recorded times during a period of about three hours. The first measurement of head was made two or three minutes after closing the valve; thereafter measurements were made at gradually increasing intervals ranging from one minute during the first 20 or 30 minutes to 15 minutes during the third hour.

Graphical determination of the coefficients T and S in Geological Survey units--In order to determine values of T and S in units commonly used by the Geological Survey, (5) and (6) may be written

$$T = 229\ Q/G\ (\alpha)\ s_w \qquad (16)$$

in which T is in gpd/ft, Q is in gpm, and s_w is in ft.

$$S = 9.28 \times 10^{-5}\ Tt/\ \alpha\ r_w^{\ 2} \qquad (17)$$

in which T is in gpd/ft, t is in min, and $r_w^{\ 2}$ is in ft^2.

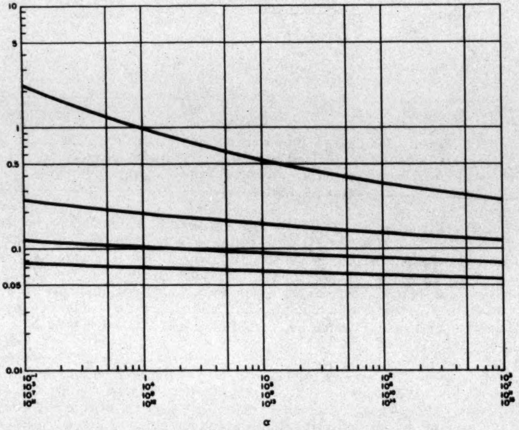

Fig. 5--Type curve obtained by plotting values of α from 10^{-1} to 10^{14} against corresponding values of G (α)

The type curve (Fig. 5) may conveniently be prepared by plotting values of α versus G (α) from Table 1 on three- by five-cycle logarithmic paper (convenient size 11 by 14 1/2 inches). Values of Q versus t from field tests are then plotted on transparent logarithmic paper to the same scale, as shown in Figure 6. By superimposing the experimental curve on the type curve (using a light table, and keeping coordinate axes parallel) a matching point may be found to give specific values of α, G (α), Q, and t which, with measured values of s_w and r_w, may be used in determining first T, from (16), and then S, from (17).

Care must be exercised in measuring the drawdown, s_w, under various test conditions. When the discharge pipe (Fig. 4) does not run full and hence there is no "back pressure," the drawdown is the "shut-in" head as measured above the fluid level in the highest part of the discharging system, generally about the midpoint of the horizontal member of the discharge pipe. When the discharge pipe runs full and hence acts as a siphon, the drawdown is the "shut-in" head as measured above the actual discharge point, provided that there is little or no "back pressure." If there is appreciable "back pressure," drawdown is the "shut-in" head as measured above the actual discharge point, minus the "back-pressure" head. Wherever conditions at the site permit, it is generally desirable to tap the mercury gage into the rising section of the well-discharge pipe at any convenient point below the level corresponding to the actual discharge point. Thus, for any type of flow condition, the drawdown, s_w, is determined by measuring with the mercury gage the "shut-in" head above the gage and the head above the gage during discharge and recording the difference.

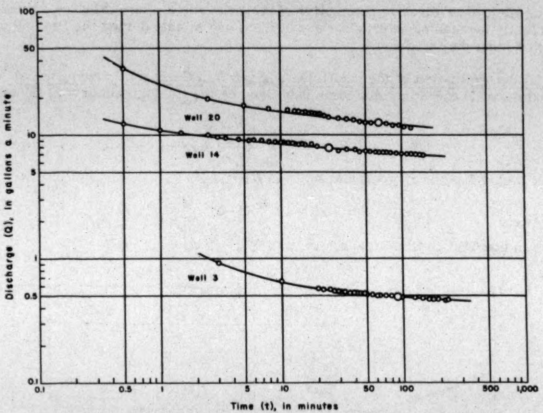

Fig. 6--Typical discharge-time curves for wells in each aquifer; large circles indicate points used in matching with type curve (Fig. 5); Well 20, Wingate sandstone; Well 14, Entrada sandstone; Well 3, sandstone of Morrison formation

The effective well radius, r_w, generally is the most difficult quantity to evaluate accurately. In the Grand Junction area, values of r_w were obtained by careful inspection of the drilling and casing records and, where possible, by questioning the drillers as to the exact size of bit used, the amount of bit wobble suspected, the possibility of and probable amount of caving, and other similar factors. It is suggested that well calipers be used, if possible, particularly when there is any doubt regarding the reported value of r_w. T can be determined from (16) even if r_w cannot be evaluated or measured, but S cannot be determined without a value for r_w.

The simpler straight-line graphical method using (14) and (15) may be employed with considerable saving in time and hence is recommended over the method just described. Using semi-log paper, experimental values of s_w/Q are plotted on the linear scale and corresponding values of t/r_w^2 are plotted on the logarithmic scale. The slope may be determined simply by taking the change in s_w/Q over one log cycle of t/r_w^2 (for which the value of $\log_{10} t/r_w^2$ is unity), so that (14) may be written simply

$$T = 2.30/4\pi \, \Delta \, (s_w/Q) \qquad (18)$$

In order to determine T and S in units commonly used by the Geological Survey, (18) and (15) may be written

$$T = 264/\Delta \, (s_w/Q) \qquad (19)$$

$$S = 2.09 \times 10^{-4} \, Tt/r_w^2 \qquad (20)$$

in which T, s_w, Q, t, and r_w^2 are in the units given for (16) and (17). Typical solutions of (19) and (20) are shown in Figure 7.

The method may be modified by plotting values of 1/Q against $\log_{10} t$ in the same manner, whence (19) may be written

$$T = 264/s_w \, \Delta \, (1/Q) \qquad (21)$$

in which T, s_w, and Q are in the units given for (16). This method eliminates computing values of t/r_w^2 and is particularly useful when r_w is not known. But in this case only T can be determined.

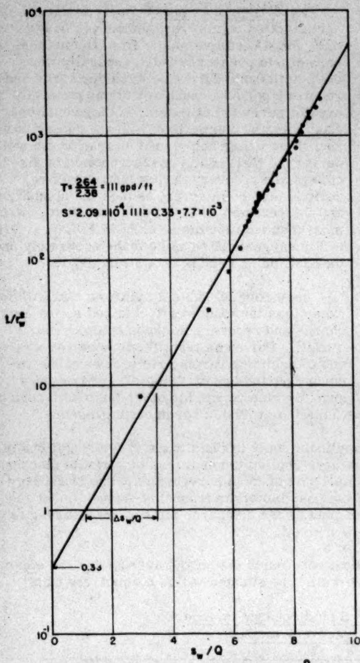

Fig. 7--Curve of s_w/Q versus log t/r_w^2 for Well 20, in the Wingate sandstone

It may be seen from Figure 7 that if the slope of the straight line were appreciably steeper (as it is for some tests), additional log cycles of t/r_w^2 would have to be crossed before the curve would intersect the ordinate $s_w/Q = 0$, so that S could be determined from (20). If the curve should have to be extended unduly in this manner, errors of plotting might result in an erroneous value of S.

For curves having such steep slopes a more accurate method may be employed for determining the value of S, as follows: Equation (13) may be written

$$s_w = (2.30\,Q/4\pi T)\log_{10}(2.25\,Tt/Sr_w^2)\;.\;.\;(22)$$

Solving this equation for S

$$S = 2.25\,Tt/r_w^2 \cdot \log_{10}^{-1}(4\pi Ts_w/2.30\,Q)\,.\;.\;(23)$$

In Geological Survey units, (23) may be written

$$S = 2.09\times10^{-4}Tt/r_w^2 \cdot \log_{10}^{-1}[(s_w/Q)T/264]\,.\;(24)$$

in which T, t, r_w^2, s_w, and Q are in the units given for (16) and (17). The value of T from (19) and values of t/r_w^2 and s_w/Q from a specific point on the straight line (such as in Fig. 7) are substituted in (24) to obtain the value of S.

The denominator in (24) may be simplified to facilitate computation by noting that (19) may be written

$$\Delta(s_w/Q) = 264/T\;.\;.\;.\;.\;.\;.\;.\;.\;(25)$$

in which s_w, Q, and T are in the units given for (16). If T is determined from (19), and hence the value of $\Delta(s_w/Q)$ is known, (24) and (25) may be combined to give

$$S = 2.09\times10^{-4}\;Tt/r_w^2\;\log_{10}^{-1}\;[(s_w/Q)/\Delta(s_w/Q)]\,.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;(26)$$

in which T, t, r_w^2, s_w, and Q are in the units given for (16) and (17). Equation (23), of course, can be similarly simplified.

Checking value of T by recovery method--The measurements of artesian head made during the recovery period were used to obtain values of T by the recovery method [THEIS, 1935] for comparison with values obtained by the methods described above. Using semilog paper, values of the residual drawdown, s, were plotted on the linear scale against corresponding values of t/t' on the logarithmic scale (t = time since discharge began; t' = time since discharge stopped). The slope of the resulting straight line is obtained simply by taking the change in residual drawdown, Δs, for one log cycle of t/t' (for which the value of $\log_{10} t/t'$ is unity), so that the recovery formula in Geological Survey units may be written

$$T = 264\,Q/\Delta s\;.\;(27)$$

in which T is in gpd/ft, Q is in gpm, and Δs is in ft.

The discharge, Q, is the average discharge during the preceding period of flow. This method is strictly applicable only under conditions of constant discharge and variable drawdown, but it gives sufficiently accurate values of T under the constant-drawdown, variable-discharge conditions of the flow tests described above. A typical solution of (27) is shown in Figure 8. (On Fig. 8, the value 7.48 for Q is not to be confused with the number of gallons in a cubic foot.)

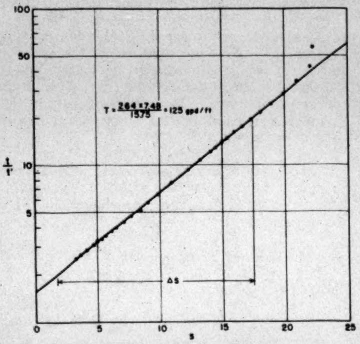

Fig. 8--Recovery curve for Well 14, in the Entrada sandstone

Evaluation of results--Small deviations of some plotted points from the curves in Figures 6, 7, and 8 resulted mainly from fluctuations in barometric pressure, which generally were quite noticeable during both the discharge and recovery periods. Such deviations generally are not serious, but if desired they could be eliminated by comparison with the record of a recording barograph placed at or near the well during the test. Early measurements of discharge and recovery during the respective periods may be in error, as both the discharge and the recovery change rapidly at first. Later measurements (points at right in Fig. 7, at left in Fig. 8) generally can be made accurately and they fall on or very near a straight line.

Inspection of Table 2 indicates that with few exceptions the values of T obtained by the discharge and recovery methods compare fairly closely. For some tests the flow period was one of declining barometric pressure, the recovery period one of rising barometric pressure, or vice versa; for other tests both periods were accompanied by a steadily rising, steadily declining, or variable barometric pressure.

That the Theis recovery formula is strictly applicable only to tests made under conditions of constant discharge and variable drawdown, but was here applied to conditions of variable discharge and constant drawdown, probably accounts for a small part of the discrepancies. Use of average discharges is believed to have eliminated most of the possible error from this cause. Under the conditions of the test, however, the values of T obtained by the discharge method are believed to be more nearly accurate than those by the recovery method.

Note that the value of T for Well 21 is only about one-fourth that of the average for the other five wells that tap the Entrada sandstone. The five wells are situated within a relatively small

Table 2--Values of T and S by discharge method, and values of T by recovery method

Formation	Well no.	T (gpd/ft)		S
		Discharge method (Equation 19)	Recovery method (Equation 27)	(Equation 20 or 26)
Entrada sandstone	1	120	110	. . . . .[a]
	2	240	230	5×10^{-6}
	8	160	150	9×10^{-4}
	11	90	80	1.4×10^{-5}
	14	120	125	8×10^{-5}
	21	33[b]	. . .[c]	7×10^{-5}
	Average	150	140	5×10^{-5}[d]
Entrada sandstone and sandstone in Morrison formation	17	490	320	6×10^{-6}
Entrada and Wingate sandstones	15	300	290	1.6×10^{-4}
Wingate sandstone	20	110	110	8×10^{-3}
Sandstone in Morrison formation	3	9.2	47	3×10^{-5}
	7	36	35	4×10^{-4}
	Average		41	1.1×10^{-4}[d]

[a] 0.084, obviously spurious. [b] Excluded from average. [c] Not determined. [d] Logarithmic average or harmonic mean.

area, whereas Well 21 is about five miles northwest from the group. The lower transmissibility for Well 21, therefore, seems to reflect a natural variation in the character of the aquifer. The seeming discrepancies for Wells 17 and 15 are believed to result in part from the fact that each well flows from two different aquifers. In each well the deeper aquifer contains water under higher head than the shallower aquifer, so that some water flows from the deeper aquifer into the shallower aquifer during the "shut-in" period. During the flow period water is discharged from both aquifers, but loss of head during the recovery period slows the apparent recovery and hence gives a lower value of T.

The lens of sandstone in the Morrison formation is continuous from the outcrop to and beyond Wells 3 and 7, which tap it, but at the outcrop it lenses into siltstone and claystone within relatively short distances along the strike. The exact configuration and dimensions of the lens in the vicinity of the wells is not known. If the flow tests on Wells 3 and 7 could have been continued longer (which was not possible as the owners required the use of the wells), it is possible that the effects of the lateral boundaries of the lens would have been detected by a change in slope of the otherwise straight line obtained by plotting s_w/Q versus $\log_{10} t/r_w^2$. Changes in slope were observed in the curves for both wells and four changes in slope were observed in the recovery curve for Well 3, but these changes occurred too early in the flow period to indicate boundary effects. Rather, they may have indicated possible variations of the storage coefficient, S, with time (because of plastic deformation in the confining beds of siltstone and claystone), or they may reflect leaks in the well seals or casing, or both. The average value of T (9.2 gpd/ft) for Well 3 obtained by the discharge method does not compare at all closely with the value (47 gpd/ft) obtained from the average of four slopes by the recovery method.

The values of S are believed to be accurate except insofar as the values of r_w may be in error. Accurate records of the lengths and diameters of casings are available for most of the wells, but the diameters of the effective part of the wells had to be assumed for the older wells and were supplied from memory by the drillers of the newer wells. Moreover, even if the values reported are correct, any caving of the walls of the uncased effective parts of the wells would make the true values of S larger than reported. Since r_w appears as r_w^2 in the denominator of (17) and (20), it will be seen that values of r_w that are too small result in the values of S being too large, and vice versa.

Acknowledgments--The authors are indebted to Rodney Hart of the U. S. Geological Survey for assistance in computing some of the values in Table 1, and to M. A. Warren, also of the U. S. Geological Survey, for making preliminary determinations of T and S by (16) and (17). Thanks are expressed also to T. G. McLaughlin of the U. S. Geological Survey, to W. R. Smith and C. C. Williams, formerly of the U. S. Geological Survey, and Mahmood Hussain for assistance at various times in running the flow tests; and to W. J. Powell of the U. S. Geological Survey and W. H. Lohman for assisting in the geologic mapping and in running flow tests.

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(Communicated manuscript received November 29, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

MOST FREQUENT PARTICLE PATHS ON THE UNIT SPHERE

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Abstract--The former model of plane continuous random motions is transferred to the surface of a sphere. The formulas for the most frequent paths among these orbits depend on incomplete elliptic integrals of the first and the third kind. Particular paths which hit the poles can be expressed by elliptic functions. They may be evaluated easily, using existing tables.

Before any attempt can be made to define spherical most frequent particle paths in a natural manner, the basic facts about the plane curves of this type should be recapitulated. A particle is moving in a plane at random. After each unit distance it is allowed to change its direction. The angle of deflection is a random variable which is normally distributed. It may be known that the way of the particle from A to B has the length s. The most probable of all ways which are consistent with these conditions is called a most frequent particle path.

It has been proved [von SCHELLING, 1951] that this definition yields the intrinsic equations[a] of all most frequent particle paths in the plane. If κ denotes the curvature, the results are

$$\kappa = 0 \text{ (straight lines)} \ldots\ldots\ldots\ldots \quad (1)$$

$$\kappa = \text{constant} \neq 0 \text{ (circles)} \ldots\ldots\ldots\ldots \quad (2)$$

$$\kappa = 2mk \cos \operatorname{am}(k, ms); \quad 0 < k \leq 1; \quad m > 0 \ldots\ldots\ldots\ldots \quad (3)$$

$$\kappa = (2m/k) \operatorname{delta am}(k, ms/k); \quad 0 < k \leq 1; \quad m > 0 \ldots\ldots\ldots\ldots \quad (4)$$

The solutions (3) and (4) may be condensed into only one equation

$$\kappa = (2m/k) \operatorname{delta am}(k, ms/k); \quad k^2 > 0; \quad m > 0 \ldots\ldots\ldots\ldots \quad (5)$$

The intrinsic equations are local relations between the curvature κ and the arc length s. They are independent of the surface on which the particles are moving. Equations (1) through (5) remain true for any surface if the geodesic curvature κ_g is substituted for the ordinary curvature κ. Of course, the integration will become a serious problem in general. But it is possible for the unit sphere.

If λ and β denote longitude and latitude of a point on the sphere and if s is the arc length as before, the radius vector $\vec{r}(s)$ of a spherical curve may be written

$$\vec{r}(s) = \left\{\cos \beta(s) \cos \lambda(s); \quad \cos \beta(s) \sin \lambda(s); \quad \sin \beta(s)\right\} \ldots\ldots\ldots \quad (6)$$

It follows

$$\cos^2 \beta \, (d\lambda/ds)^2 + (d\beta/ds)^2 = 1 \ldots\ldots\ldots\ldots \quad (7)$$

The geodesic curvature κ_g becomes

$$\kappa_g = \sin \beta \, (d\lambda/ds) \, [1 + (d\beta/ds)^2] + \cos \beta \, [(d^2\beta/ds^2) \, d\lambda/ds - (d^2\lambda/ds^2) \, d\beta/ds] \ldots \quad (8)$$

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[a] Jacobian elliptic functions are written in this paper as follows:

am (k, ϕ)	amplitude (k, ϕ)
sin am (k, ϕ)	sine of amplitude (k, ϕ)
cos am (k, ϕ)	cosine of amplitude (k. ϕ)
delta am (k, ϕ)	delta amplitude (k, ϕ)

where k is the modul.

It is obvious that the case (1), that is, $\kappa_g = 0$, yields the great circles, and that case (2), $\kappa_g =$ constant $\neq 0$, defines the small circles on the sphere. Therefore, circles are degenerated spherical most frequent particle paths.

Case (5), the general case, is equivalent to the following relation

$$\kappa_g = \epsilon^2 \sin \beta \quad \ldots\ldots \quad (9)$$

where ϵ^2 is a constant. Of course, this equation is not evident. But it is a complete analogue to a formula for plane most frequent particle paths. Starting from (9) as a guess, the integration is relatively easy. It is not possible to describe the procedure here step by step. The final results must depend on two parameters which may be denoted by C and c. To abbreviate we keep two other constants in the formulas, ϵ^2, contained in (9), and the modulus k of the elliptic integrals. Both constants are functions of C and c, namely

$$\epsilon^2 = 1/[(C^2 - c^2 + 1)^{1/2} - C] \quad \ldots\ldots \quad (10)$$

$$k^2 = 2c/(C + c) > 0 \quad \ldots\ldots \quad (11)$$

With these notations the general most frequent particle paths on the unit sphere are described by the equations

$$\left.\begin{aligned} &\sin \beta = (2/\epsilon^2)\,(\epsilon \sqrt{c}/k)\ \text{delta am}\ (k, \epsilon \sqrt{c}\, s/k) = \kappa_g/\epsilon^2 \\ &\lambda = \epsilon^2 s/2 - k\,(\epsilon/2\sqrt{c})^3\,[2\,(C^2 - c^2) + 1 - 2C\,(C^2 - c^2 + 1)^{1/2}] \\ &\int_{\psi=0}^{\text{am}(\epsilon\sqrt{c}\, s/k)} \left\{1/[\sin^2 \psi - 1/k^2 + (\epsilon/2\sqrt{c})^2]\,(1 - k^2 \sin^2 \psi)^{1/2}\right\} d\psi \end{aligned}\right\} \quad \ldots\ldots \quad (12)$$

Tables for the incomplete elliptic integral of the third kind do not exist. The general orbits (12) can be discussed only indirectly. That may be done by using the formulas (9), (12.1), and

$$d\lambda/ds = (\epsilon^2/2)\left(1 + [2\,(C^2 - c^2) + 1 - 2C\,(C^2 - c^2 + 1)^{1/2}]/\cos^2\beta\right) \quad \ldots\ldots \quad (13)$$

In general, the paths oscillate either in two separated symmetrical zones on the northern and southern hemispheres or they fluctuate in a central belt symmetrically to the equator.

There are special orbits which do not involve an elliptic integral of the third kind. That is the case for

$$k = 1 \quad \text{or} \quad C = c \quad \ldots\ldots \quad (14)$$

It follows

$$\sin \beta = 2\sqrt{C\,(1 - C)}/\cosh\,[s\sqrt{C/(1 - C)}] \quad \ldots\ldots \quad (15)$$

With increasing arc length s the curve converges asymptotically to the equator. The longitude λ can be expressed by elementary functions, although in a highly complicated manner. A simple example for this family of orbits is the case C = 1/2.

$$\vec{r}\,(s) = [-\,(\tanh s)\,(\cos s);\quad -\,(\tanh s)\,(\sin s);\quad 1/\cosh s] \quad \ldots\ldots \quad (16)$$

for which $\lambda = s$, $\cos \beta = \tanh s$, $\sin \beta = 1/\cosh s$. It is a good exercise to calculate the geodesic curvature from (8). The result is $\kappa_g = 2/\cosh s = \epsilon^2 \sin \beta$, as it should be.

The second partial case occurs if the factor of the integral in (12) vanishes. Then

$$C = (1/2c) - c \quad \ldots\ldots \quad (17)$$

If we separate again $k < 1$ and $k > 1$, we get the following two classes of paths

$$\vec{r}\,(s) = \left\{-\,k \sin \text{am}\,(s/k) \cos\,(s/k);\quad -\,k \sin \text{am}\,(s/k) \sin\,(s/k);\quad \text{delta am}\,(s/k)\right\} \quad \ldots \quad (18)$$

with $k = 2c$; $\epsilon^2 = 2/k = 1/c$; $\kappa_g = 2/k$ delta am (s/k); $\lambda = s/k$, and

$$\vec{r}\,(s) = \left\{- \sin \mathrm{am}\,(s) \cos (ks);\quad - \sin \mathrm{am}\,(s) \sin (ks);\quad \cos \mathrm{am}\,(s)\right\} \qquad (19)$$

with $k = 1/2c$; $\epsilon^2 = 2k = 1/c$; $\kappa_g = 2k \cos \mathrm{am}\,(s)$; $\lambda = ks$. The area between the curve, the equator, and two limiting meridians λ_1 and λ_2 becomes for both solutions (18) and (19)

$$\text{Area}\,\Big|_{\lambda_1}^{\lambda_2} = \sin^{-1}\left[(1/2c) \cos \beta\right]\Big|_{\beta_1}^{\beta_2} \qquad (20)$$

where β_1 and β_2 denote the latitude at the intersections between the curve and the two meridians.

Using the notation $F\,(k, \phi)$ for the incomplete elliptic integral of the first kind, (17) and (18) may be written respectively

$$\lambda = F\,(k, \phi); \quad \cos \beta = k \sin \phi \qquad (21)$$

$$\lambda = kF\,[k, (\pi/2) - \beta\,] \qquad (22)$$

The curves can be drawn easily, using any table of the elliptic integral of the first kind. Class (21) starts at a pole and returns to the same pole; class (22) connects the poles with each other. The zones which were mentioned above, stretch to the poles for these special cases.

For $k = 1$, (18) and (19) become identical and agree with the former particular example (16). The formulas (21) and (22) degenerate to

$$\lambda = \int_{\psi=0}^{\beta} d\psi/\sin \psi = \log_e \tan \beta/2 \qquad (23)$$

Opinions or conclusions contained in this paper are those of the author. They are not to be construed as necessarily reflecting the views or endorsement of the Navy Department.

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(Communicated manuscript received May 2, 1951, and, as revised, March 17, 1952; open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

SV AND SH

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Abstract--The motion of the ground at the Earth's surface produced by continuous sinusoidal transverse waves (angle of incidence i) is discussed theoretically. If incident SV-waves (vibrating in the plane of propagation) are totally reflected (i greater than the critical angle i* of about 35°, epicentral distance Δ in average shallow shocks roughly between 30 and 3000 km), the reflected SV-waves should show a phase displacement with reference to the incident waves (Fig. 1), and the ground particles should move in ellipses. This elliptic ground motion is not a surface wave in the proper sense since it does not propagate energy along the surface and does not have a velocity of its own; it corresponds to the linearly polarized motion for $i \leq i^*$.

Horizontal (u) and vertical (w) ground motions are plotted in Figure 2, which also gives the apparent angle of incidence $\bar{i}$ on the assumption that u and w occur simultaneously. Characteristic examples of calculated ground motion are reproduced in Figure 3. Equations are given to find the amplitudes and polarization of the incident SV-waves and the amplitudes of the SH-components from the recorded S-waves. Advantages of using records of Benioff strain seismographs to identify SV- and SH-waves are discussed. Characteristic S-phases recorded by a variety of instruments at Pasadena are reproduced and calculations for SV, SH, $\bar{i}$, and ϵ are tabulated. No evidence is found for a suspected delay of SV relative to SH.

In longitudinal waves P the direction of motion of a particle is determined uniquely by the direction of the ray. However, transverse waves S may vibrate in any direction in a plane perpendicular to the ray, depending mainly on the direction of motion at the source. To a first approximation, it can be assumed that the motion of the ground particles in S-waves is linearly polarized. WIECHERT [1907, p. 454] seems to have been the first to point out that for amplitude calculations of reflected and refracted waves and for finding of ground motions the displacement due to a traveling S-wave has to be resolved into two components which JEFFREYS [1926] indicated by SH and SV. SH vibrates purely horizontally and perpendicular to the plane of propagation. The vibrations in SV are in the plane of propagation and perpendicular to the ray. For calculations involving SV and SH the two components must be treated separately and the results must be combined. This procedure does not imply that actually two different waves are propagated. Only under very extreme conditions, such as anisotropy in much greater amount than can be assumed inside the Earth [STONELEY, 1951], is an appreciable difference to be expected between the arrival times of two S-waves vibrating in different directions perpendicular to a given ray. But even then, an S-wave with vibrations in a given direction need not split into an SH-wave and a separate SV-wave.

General equations for the partition of the energy of a wave incident at a discontinuity between two different homogeneous media into the reflected and refracted waves were first developed by KNOTT [1899]. He pointed out that the total incident energy is always proportioned among the (one or two) reflected and (zero to two) refracted waves, and that where one or more of his calculated energy factors are imaginary, for example, of the form p + iq, the expression $\sqrt{p^2 + q^2}$ determines the energy.

WIECHERT [1907] discussed physical processes at the Earth's surface in case of an incident P-, SV-, and SH-wave. He included equations for the apparent angle of incidence $\bar{i}$ as a function of the true angle of incidence i; i is the angle between the direction of the ray and the vertical, whereas $\bar{i}$ is calculated from the components u (horizontal in the plane of propagation) and w (vertical) of the ground displacement (for SV, $\tan \bar{i} = w/u$). Wiechert also developed equations for the amplitudes of waves reflected and refracted at the surface and for the horizontal and vertical components of the ground motion.

For SV, Wiechert considered the two cases where the reflected longitudinal wave is real or imaginary. If A is the ratio of the transverse (v) to the longitudinal velocity (V), the critical value of i = i* which separates the two cases is given by

$$\sin i^* = A = v/V \qquad (1)$$

The apparent angle of incidence ($\bar{i}$) of SV is given [WIECHERT, 1907, Eq. (142) and (163)] by

$$\tan \bar{i} = (2 \sin i/\cos 2i) \sqrt{A^2 - \sin^2 i} \qquad (2a)$$

$$\tan \bar{i} = (\tan 2i/\cos i) \sqrt{\sin^2 i - A^2} \qquad (2b)$$

depending on whether (2a) $i \leq i^*$ or (2b) $i > i^*$. In case (2b) for sinusoidal incident waves, the surface particles should move in ellipses with the axes perpendicular and parallel to the surface [WIECHERT, 1907, p. 459]. Later JEFFREYS [1926, p. 329] stated "the surface particles oscillate backwards and forwards in straight sloping lines" when the square root in (2a) is real, "whereas when it is imaginary they describe ellipses with their axes horizontal and vertical." In the latter case the reflected SV-waves show a phase difference δ with respect to the incident SV-waves. WIECHERT [1907, Eq. (159)] gave (3), below, for δ, again on the assumption of continuous sinusoidal waves (which is made throughout the following discussion)

$$\tan \delta = 2m/(1 - m^2) \qquad (3)$$

where

$$m = \cos^2 2i/(4 \sin^2 i \cos i \sqrt{\sin^2 i - A^2}) \qquad (4)$$

Values of δ have been calculated and plotted in Figure 1 assuming Poisson's ratio $\sigma = 1/4$. If σ increases, the initial portion of the curve shifts to smaller angles $i = i^*$ (see Table 1). The minimum ($\delta = -180°$) remains at $i = 45°$ regardless of Poisson's ratio.

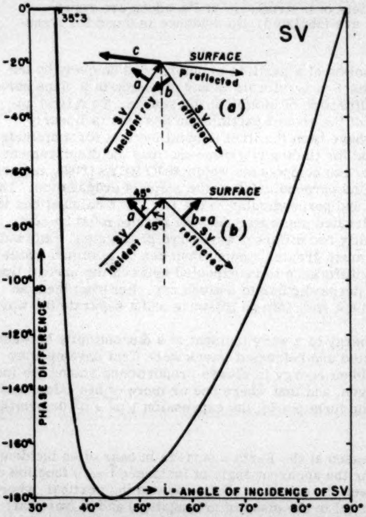

Fig. 1--Phase difference between reflected and incident SV-waves as a function of the angle of incidence i for total reflection, if Poisson's ratio $\sigma = 1/4$; insert: sketches of the reflected waves, if an SV-wave is incident (a) with $i = 35°\pm$, (b) with $i = 45°$; a, b, and c indicate the amplitudes of the incident SV, the reflected SV, and the reflected longitudinal wave respectively; in (b), c = 0

If $i > i^*$, the phase angle δ results in elliptic ground motion. Such elliptic motion of surface particles (and similar motion of particles in surfaces between media of different properties in the interior of the Earth) in the case where the angle of incidence of the reflected or refracted wave is complex has often been called "surface waves." However, this expression is misleading. The fact that this elliptic surface motion decreases rapidly with depth is not sufficient to call it a "surface wave," since this decrease is similar to that in instances where the movement of the surface particles is polarized in a straight line (for example, in SV for $i < i^*$). The two types of surface motion differ only in the fact that one is polarized and the other is not. On the other hand, this elliptic surface motion contrasts in essential points with actual surface waves. It does not propagate any energy along the surface since all the arriving energy goes into the reflected wave (or additional reflected or refracted waves in the case of internal discontinuities). WIECHERT [1907, Eq. (149)] found that the apparent velocity $\bar{V}$ of this elliptic motion along the Earth's surface is given by

$$\bar{V} = v_o/\sin i_o \qquad (5)$$

where v_o is the velocity of the transverse waves and i_o the angle of incidence of the SV-ray at the surface. Wiechert did not point out that $\bar{V}$ is equal to the velocity $\bar{v}$ with which the incident S-waves appear to travel along the surface. Naturally, this varies with the distance Δ, since i_o is a function of Δ. Thus, this elliptic surface motion does not propagate itself. It disappears (except

Table 1--Selected values of u^* as a function of A or σ

A = v/V	σ	u*	i*
			°
0.707	0.000	∞	45.0
0.633	0.167	9.17	39.2
0.588	0.237	5.20	36.0
0.577	0.250	4.90	35.3
0.568	0.262	4.63	34.6
0.500	0.333	3.46	30.0
0.000	0.500	2.00	0.0

for diffraction effects) if the incident S-waves enter a "shadow zone" at the Earth's surface.

Numerical calculations of the components u and w of the ground displacements in SV have been discussed by GUTENBERG [1944, p. 99]. Figure 2 shows calculated values of u and w as a function of the angle of incidence i, if Poisson's ratio $\sigma = 1/4$. For $i < i^*$, u is the sum of three terms arising from the incident SV, the reflected SV, and the refracted P respectively

$$u = \cos i + [(L^2 - 1)/(L^2 + 1)] \cos i$$
$$+ [2AL^2/(L^2 + 1)] \tan 2i \sin \lambda \text{. . . . .} (6)$$

where

$$L^2 = \cot 2i \cos 2i/A^2 \sin 2\lambda \quad \text{and} \quad \sin \lambda = (\sin i)/A \text{ . . . .} (7)$$

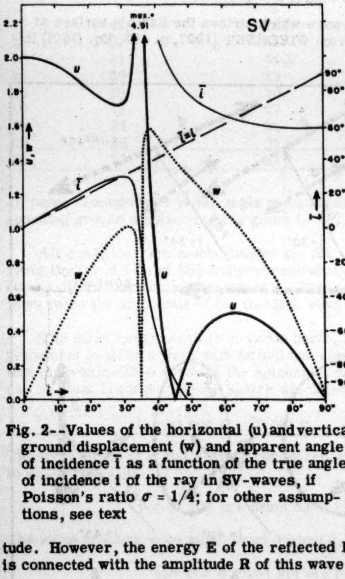

Fig. 2--Values of the horizontal (u) and vertical ground displacement (w) and apparent angle of incidence $\bar{i}$ as a function of the true angle of incidence i of the ray in SV-waves, if Poisson's ratio $\sigma = 1/4$; for other assumptions, see text

λ is the angle of incidence of the reflected longitudinal wave and A the velocity ratio v/V as before. The values of u are expressed in terms of the amplitude of the incident SV-wave. Equation (6) follows from GUTENBERG [1944, Eq. (22), (3), and (7b)], considering that the vibrations in SV are perpendicular to the ray. For i = 0, L is infinite, the first and second terms in (6) are cos i = 1, the third term is zero, and u = 2. The vibrations in the incident and in the reflected SV-waves are equal and in the same direction. With increasing i the third term becomes dominant. When i approaches the critical angle i*, the value of L again approaches infinity and u approaches the value

$$u^* = 2 \cos i^* + 2A \tan 2i^* = \sqrt{1 - A^2}/[(1/2) - A^2] \text{.} (8)$$

Selected values of u* as a function of A (or Poisson's ratio σ) are given in Table 1.

From (8) it follows that the relatively large ground displacements for i = i* are due mainly to the term 2A tan 2i* which results from the amplitudes of the reflected longitudinal wave (P); This is indicated in Figure 1a for i slightly less than i*. Since, under average conditions, i* is near 35° (last column of Table 1), the horizontal ground motion from the reflected P is three to four times the amplitude of the incident wave, while the ground motion from each of the two SV-waves is about 0.8 times the incident amplitude. However, the energy E of the reflected P-wave approaches zero as i approaches i* since E is connected with the amplitude R of this wave through the relationship

$$E = (R^2 \sin 2\lambda)/\sin 2i \text{. . . . .} (9)$$

which follows, for example, from WIECHERT [1907, Eq. (139)] or from GUTENBERG [1944, Eq. (3)]; if i approaches i*, the amplitude R approaches the rather large value 2A tan 2i*, but 2λ approaches 180°, and the energy E approaches zero.

The conditions near the critical angle of incidence were apparently first discussed by JEFFREYS [1926, p. 329], who indicated that most of the relatively large horizontal ground motion u is then "contributed by the irrotational part of the disturbance." If i > i*, values of u can be calculated, for example, from GUTENBERG [1944, Eq. (33-35)]. For i > i* the effect of the real longitudinal wave which was just discussed for i < i* is replaced by a "compressional movement near the surface, not in the nature of a wave (unless we like to regard it as a wave propagated in an imaginary direction) but capable of affecting the movement of the surface'' [JEFFREYS, 1926, pp. 328-329]. It is represented by equations for elliptic surface motion of a compressional type [WIECHERT, 1907, Eq. (148)]; it is zero for $i = 45°$.

Unfortunately, whenever amplitudes change rapidly with distance, one must always suspect that terms of higher order become important, and a theoretical treatment of the type indicated by CAGNIARD [1939] should be attempted. This may also aid in finding at least approximately the ground motion immediately following the first arrival of SV-waves and may give some information concerning the difference between the actual ground motion and that calculated on the assumption of a steady state (continuous sinusoidal motion).

If i increases beyond the critical angle and approaches 45°, u decreases toward zero regardless of the value of Poisson's ratio σ; for $i = 45°$ the horizontal components of the reflected and incident SV-waves are always equal but in opposite directions due to the phase difference of 180° (see Fig. 1 and 1b). Figure 2 exhibits the horizontal component u as well as the vertical component w of the ground motion in SV as a function of i. The following values of w do not depend on σ: $i = 0°$, $w = 0$; $i = i^*$, $w = 0$ (see Fig. 1a); $i = 45°$, $w = \sqrt{2}$ (see Fig. 1b); $i = 90°$, $w = 0$. If i approaches 90°, w/u approaches $2\sqrt{1 - A^2}$, which is 1.63 if $\sigma = 1/4$.

Figure 3 shows the ground motion d for an SV-wave which strikes the Earth's surface at selected angles i but with the same amplitude a. From WIECHERT [1907, p. 457, Eq. (160)] it

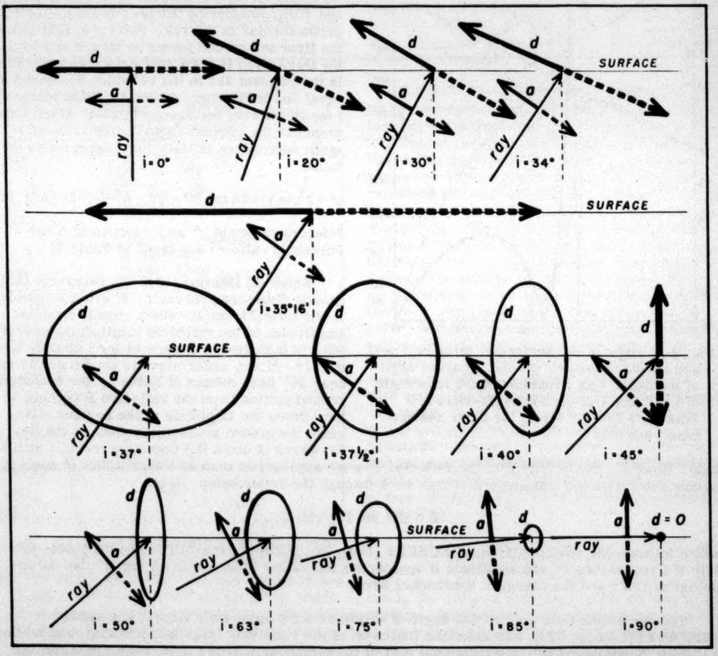

Fig. 3--Examples of the theoretical ground displacement d if continuous sinusoidal SV-waves with a given amplitude a are incident at the Earth's surface under the indicated angles i and if $\sigma = 1/4$

follows that the sense of movement along the ellipse reverses at i = 45° from counterclockwise to clockwise. The surprisingly great variations in d which are indicated by Figure 3 are to be considered not only in the study of teleseisms but also in the investigation of destructiveness in nearby earthquakes where S frequently causes the largest ground motion. The angle of incidence i increases from i = 0 at the epicenter to a maximum which is reached at the distance corresponding to the point of inflection of the travel-time curve. Thus, the critical angle $i = i^*$ can be expected twice. It is reached first at a distance of the order of the focal depth. Details depend on the change of velocity of S with depth which is not yet well known for the crustal layers in any region [GUTENBERG, 1951]. From this critical distance to the second critical distance of roughly 3000 km (to be discussed later) the ground motion produced by SV should show complications connected with the total reflection of SV. Relatively large horizontal ground motion is to be expected near the two critical distances.

Table 2--Calculated angles of incidence for transverse waves

Δ	i	
	v_o = 3.67 km/sec	v_o = 4.0 km/sec
°	°	°
20	41±	46±
25	33	37
30	31	34.5
40	29	33
50	27	30
60	25	27
70	23	25
80	20	22.5
85	19	21

Equations (1) and (2) permit the calculation of the apparent angle of incidence $\bar{i}$. For $i > i^*$, $\bar{i}$ has been calculated on the assumption that the vertical and horizontal motions are in phase. For $\sigma = 0.273$, values of $\bar{i}$ as a function of i have been plotted by Gutenberg [GEIGER and GUTENBERG, 1912, Fig. 3, p. 636]. Values of i for $\sigma = 0.25$ are reproduced in Figure 2. As long as $i < 30°$, i and $\bar{i}$ differ by less than four degrees. Consequently, it can be assumed that in this range (corresponding to epicentral distances of a few kilometers and again greater than roughly 45°; Table 2) i and $\bar{i}$ are equal in SV within the limits of error of observation. For $i > 30° \pm$, $\bar{i}$ decreases with increasing i (Fig. 2); $\bar{i} = 0$, if $i = i^*$; $\bar{i} = \pm 90°$, if $i = 45°$; $\bar{i} = \tan^{-1} (2 \sqrt{1 - A^2})$ which is near 60°, if i = 90°. Note again that in all these discussions i is the angle of incidence of the SV - ray while the direction of the corresponding ground displacement is given by 90° - i.

All conditions are much simpler for SH. The ground motion is purely horizontal (which prevents finding of i from SH) and perpendicular to the plane of propagation which contains SV and especially its horizontal component u. Theoretically, the surface ground amplitude of SH is always twice the amplitude of the incident wave.

The three components (N towards north, E towards east, Z upward) of the total ground displacement in S for a wave with an incident component a of SV and b of SH and arriving at the station in an azimuth α towards the epicenter, positive from north (α = 0) towards east (α = 90°) can be found from an obvious sketch which leads to

$$N = au \cos \alpha - 2b \sin \alpha \qquad E = au \sin \alpha + 2b \cos \alpha \qquad Z = aw \quad \text{.........} (10)$$

The first terms in N and E are a consequence of SV, the second terms of SH; Z is completely due to SV. From (10)

$$a = (N \cos \alpha + E \sin \alpha)/u = Z/w \qquad b = (E \cos \alpha - N \sin \alpha)/2 \quad \text{.........} (11)$$

The value of u has to be taken from Figure 2. The azimuth β of the incident SV-wave is given by

$$\tan \beta = (a \cos i \sin \alpha + b \cos \alpha)/(a \cos i \cos \alpha - b \sin \alpha) \quad \text{..........} (12)$$

If

$$\gamma = \beta - \alpha \quad \text{..................................} (13)$$

the angle ε of polarization, that is, the angle between the plane of vibration of S and the plane of vibration of P, can be found, following GALITZIN [1911; see also GUTENBERG, 1932, p. 50] from

$$\tan \epsilon = \tan \alpha \cos i \quad \text{..........................} (14)$$

If the observed vertical ground amplitude Z was not used in the calculation of a (Eq. 11), it can be used now to calculate w = Z/a. The apparent angle of incidence can be found from

$$\tan \bar{i} = Z/(N \cos \alpha + E \sin \alpha) \quad \text{. . . . .} \quad (15)$$

Since i does not differ appreciably from $\bar{i}$ for $i < 30° \pm$, (15) should give also a good approximation for the angle of incidence i of the S-waves in this range (epicentral distances roughly over 45°).

The main difficulty in practical calculations depending on i is caused by the fact that i has to be found from the equation $\sin i = \bar{v}/v_0$ and that there is uncertainty as to what value should be used for v_0 (velocity of S near the surface); the value of $\bar{v}$ (apparent velocity of S) as a function of the distance Δ is fairly well known. For Southern California, v_0 = 3.67 km/sec seems to be the best average value of v_0 in the upper ten km [GUTENBERG, 1951]. However, in many records of distant earthquakes the S-waves have periods of 12 sec or more corresponding to wavelengths of the order of 50 km or more. The velocity at a depth of 50 km is about 4 1/2 km/sec, so that an average velocity of about 4 km/sec seems to be more reasonable for relatively long S-waves. A theory of waves arriving through a layer of a thickness comparable to the wavelength is required for a good approximation. For this reason, i was calculated on several assumptions for v_0. Results for shallow shocks are given in Table 2. For very deep shocks (h = 600 km), i is about 2° smaller at $\Delta = 30°$, and the difference decreases to about 1/2° for $\Delta = 85°$.

By comparison of Table 2 with Table 1 it follows that, for distances greater than 30°, i is probably smaller than the critical angle. Comparison of Figure 2 and Table 2 shows that $u = 1.8 \pm 0.1$ if $80° > \Delta > 30°$. For distances over 80° there is usually too much uncertainty as to which phase is S; the displacement produced by S is frequently superposed by displacements from ScS, SKS, SKKS, etc. On the other hand, effects of the complications arising from the imaginary longitudinal wave at distances less than 30° as well as the effect of the low-velocity layer below a depth of 80 km ("20° - discontinuity") cast doubt on results concerning SV and SH obtained from a study of S-waves in seismograms if $\Delta < 30°$. This may be in part the reason for the suspected delay in SV which has been reported in situations such as these by NEUMANN [1930] and by BYERLY [1938]. Neumann constructed very interesting curves for the horizontal ground motion. While some of his figures show a dominant SV [see his Fig. 11] or SH [his Fig. 12], others are very complicated [for example, his Fig. 7, 9, 10] and are within the critical range for SV. They agree with the theoretical expectation. No clear conclusion can be drawn from these figures as to whether or not SV arrives later than SH.

BYERLY [1938] in one of his fundamental studies to find the direction of faulting, pointed out that sometimes a "curtsey," prevailingly in the SH-component, precedes the actual S-waves. However, most of the seismograms used by him are within or near the range of distances where complicated SV-records are to be expected. Finally, LEHMANN [1935, p. 128] observed two S-phases near $\Delta = 80°$, the first without and the second with a vertical component. However, she does not state that she considered them to correspond to SV and SH respectively. Actually, the direction of the ground motion in S which is indicated by her European seismograms of an Aleutian earthquake seems to be prevailingly NW - SE while the direction of the epicenter is in general towards NNW; this suggests that in these instances the first motion of S corresponds to prevailing SV-phases with minor components of SH. Lehmann also pointed out that the late beginning of S in the vertical records may be due in part to insufficient magnification of the vertical instruments. In addition, the vertical component of S is usually much smaller than the horizontal, since the SH part of S has no vertical component, and since near $\Delta = 80°$ the vertical component of SV should be less than 1/2 of the horizontal. On the other hand, the best vertical records at her disposal had been written by Galitzin instruments with a period of about 12 sec, while the corresponding horizontal instruments had periods of about 24 sec. For the prevailing periods in the S-phases (12 to over 20 sec) this would give a magnification of the verticals of only about 1/3 of that for the horizontals. Thus, in the Galitzin seismograms used by her, trace amplitudes in the vertical component of S of less than 1/10 of those in the horizontal components are to be expected and a still smaller ratio in records of mechanically recording seismographs.

To study the actual behavior of recorded S-waves, Pasadena records of about 60 shocks with epicentral distances between 25° and 80° were selected. No records prior to 1944 were used, since until then the available instruments were insufficient to calculate reliable values of the ground motions N, E, and Z in the three components of S and to discover effects of inadvertent changes in instrumental constants. The records of the two long-period Wood-Anderson torsion seismographs (one of which, south-north, was installed in February, 1946) rarely indicate such changes in constants. The long-period Benioff NS-component (which was the first model of this type) had most of the time a smaller magnification than the EW-component and was replaced in 1951. Calculations based on seismograms recorded during several months in 1947 as well as the size of the recorded microseisms indicate for this period a magnification of the vertical component appreciably higher than usual. Consequently, amplitudes of Z for this period were not used. In general, calculated values for the azimuth β can be assumed to be more accurate than those for the angle of incidence i.

Very valuable information was derived from the records of the Benioff strain seismographs. Unfortunately, the records are frequently disturbed by the presence of persons and by work in the laboratory; the EW-component is shorter and less sensitive than the NS-component, and it was in operation only part of the time. Since the time when it became evident that the value of the two components is even greater than originally assumed, both are recording continuously. Similar to difference in their recording of Rayleigh- and Love-waves [BENIOFF and GUTENBERG, in press], two strain seismographs perpendicular to each other record an SV-wave in the same direction (supposing that compressions record in the same direction for both), while the corresponding records of an SH-wave show a phase difference of 180°. This results from the response of the strain seismograph [BENIOFF, 1935] which for SV is proportional to $(1/2)\sin 2i\cos^2\alpha$, and for SH to $(1/2)\sin i\sin 2\alpha$ (i = angle of incidence of the ray, α = azimuth of the epicenter relative to the direction of the strain rod). For SV the sign depends only on whether the motion results in a compression or in a dilatation of the ground, but a difference in the azimuth α does not change the sign of the variable response factor, while for SH the response for two perpendicular but otherwise identical components is theoretically always equal in amplitude but opposite in sign. In addition, compressions in SV correspond to a vertical component downward, rarefactions to an upward motion. Since this is opposite to the relationship in P, it can be used in identification of certain waves; for example, SP and PS should both show strain records in phase, but combined with vertical motion of opposite direction.

To facilitate the interpretation of the S-records, Table 3 may be used. For each quadrant in azimuth towards the source, there are two alternatives with opposite directions for SV as well as for SH; all four combinations between the directions of SH and SV are possible.

Table 3--Directions in SH- and SV-waves (C = compression, R = rarefaction, dilatation)

Direction of source	Direction in SH				Direction in SV				
	Pendulum		Strain		Pendulum			Strain	
	NS	EW	NS	EW	NS	EW	Z	NS	EW
NE	S	E	C	R	N	E	Up	R	R
	N	W	R	C	S	W	Down	C	C
SE	N	E	C	R	S	E	Up	R	R
	S	W	R	C	N	W	Down	C	C
SW	N	W	C	R	S	W	Up	R	R
	S	E	R	C	N	E	Down	C	C
NW	S	W	C	R	N	W	Up	R	R
	N	E	R	C	S	E	Down	C	C

Seismograms of shocks with epicentral distances less than 25° and over 80° were not used for the reasons given in the theoretical discussion. Seismograms of shocks at epicentral distances between 25° and 40° frequently show such complications that the beginning of the S-phase is in doubt. This is especially true for shocks of magnitude of 7 1/2 or more. Most seismograms of shocks of magnitude 7 3/4 had to be rejected on account of complications in the phases and of the fact that S emerged from too large a background. On the other hand, seismograms of shocks of magnitude less than 7 usually were too small for accurate measurements of amplitude ratios and for reliable calculations of β, ϵ, w, and $\bar{i}$. In instances of strongly prevailing SH, the vertical amplitude was usually too small to furnish data for w and $\bar{i}$.

Table 4 contains essential data for each shock which was investigated. In some instances the "eS" of shocks referred to in footnote c may be too small to be recognized. In a few instances, data for two successive waves are listed.

The latitude and longitude of the epicenter are given to the nearest degree. These data, as well as those in the next two colums (focal depth h and magnitude of the shocks), were taken from GUTENBERG and RICHTER [1949] for the years 1944-1946 and from tabulations in the Station Bulletin at the end of each year for 1947-1950. Epicentral distances Δ were taken from the cor-

Table 4--Elements for S in shocks recorded at Pasadena (for symbols, see text)

Date	Lat	Long	h	Magnitude	Δ	α	β	Component			Amplitude		ε	ī	w
								N	E	Z	a	b			
1944	°	°	km		°	°	°	micron	micron	micron	micron	micron	°	°	
Feb 29	14S	70W	200	7	67	130	53	+15	+20	+ 5	+ 2 3/4	- 12	-75	28	1.0
Jun 28	15N	92W	n	7	30	125	126	+22	-30	-12?	-22	- 1/2	+ 1	37	0.8
Jul 27	54N	165W	70	7.1	38	316	239	-10	- 6 1/2	+ 1?	+ 1 1/2	- 5 3/4	-75	22?	0.7±
Sep 23	54N	160E	40	7.4	58	318	201	+16	+ 7	+ 5	+ 4 1/4	+ 8	+61	35	1.2
Oct 2	42N	142E	75	7.0	74	312	356	(- 1/4)	+ 2	- 1 1/4	- 1	+ 1/2	+42	38±	1.3±
Oct 23	1N	80W	n	6.9	48	127	197	+10	+ 3	- 2±	- 2	- 5	+68	29±	1.0±
Dec 12[c]	51N	179E	50	7.0	47	312	276	- 1/2	+ 7 1/2	- 5	- 3 1/2	+ 2 1/4	-34	41	1.5
1945															
Apr 15[c]	57N	164E	70	7.0	56	320	346	-19	+ 4	-10	- 9	- 6	+24	34	1.1
1946															
Jan 11	44N	129E	580	7.2	80	320	278	+ 3/4	- 5 1/2	+ 1 1/2	+ 2 1/4	- 2	-40	21±	0.7±
Jan 12	59N	147W	50	7.2	32	333	34	+12	+ 8	+ 5	+ 4	+ 5 1/2	+52	35	1.2±
Feb 4	53N	176W	160	6 3/4	45	315	300	+ 1 1/4	- 2	+ 2	+ 1 1/2	- 1/4	-14	41±	1 1/4±
May 21[c]	14N	60W	n	7	55	98	35	+ 1 1/2	+ 1	+ 1 1/2	+ 1/2	- 3/4	-61	d	d
Aug 2	26S	70W	50	7 1/2	75	139	88	(+ 1/2)	+ 6	+ 3	+ 2	- 2 1/2	-49	39	1 1/2
Aug 21	24S	177W	100	7	80	230	286	- 1 1/4	+ 3	- 1 1/4	- 1	- 1 1/2	+54	34	1 1/4
Aug 28	26S	63W	580	7.2	80	132	207	- 4	- 2	+ 1	+ 3/4	+ 2 1/4	+74	35	1 1/4
Sep 30	13S	76W	70	7	61	135	60	+ 1	+ 3/4	(1/4)	0	+ 1/2	-72	d	d
Nov 1	51N	174W	40	7.0	43	312	268	(+ 1/2)	+16	-12	- 6 1/2	+ 6	-42	47±	1.8±
Nov 10[c]	8S	77W	n	7 1/4	57	132	171	+ 5	(- 1/2)	- 2 1/2	- 2 1/4	- 1 1/2	+37	34	1.1
Nov 12	20S	173W	n	7 1/2	75	230	170	+10	+ 1	- 6	- 7	+ 3 1/2	-59	39	1 1/2
1947															
Jan 26	12N	86W	170	7.2	36	120	163	+40	-12	-11?	-17	- 14	+40	20±	0.7±
Jan 29	26S	63W	580	7 1/4	79	133	200	- 8	- 3	(+ 2)	+ 1 3/4	+ 4	+65	34±	1.1±
Apr 14[c]	45N	149E	n	7.1	69	310	239	+ 4	+ 7	?	- 1 3/4	+ 4	-70	?	?
Apr 24	8N	39W	50	7	76	90	68	- 1 1/4	- 3 1/4	- ?	- 1 3/4	+ 3/4	-20	?	?
Aug 7	20N	75W	50	7	41	100	176	- 2	(- 1/2)	- ?	(- 1/4)	+ 1	+74	?	?
Oct 16[c]	64N	148W	n	7	35	337	312	-16	+18	-10	-13	+ 5 1/4	-24	24	0.8
Nov 1	11S	75W	n	7.3	61	130	215	-15	-10	0	+ 1	+ 9	+85	d	d
1948															
Jan 4	21S	179W	600	7.1	79	235	207	-14	- 7	+ 3 1/2	+ 8	- 4	-25	14	0.4
Jan 6[c]	17N	98W	80	7	25	130	50	-50	-60	- 1?	- 8	+ 39	-79	d	d
Jan 22	22S	177W	140	7.0	79	233	141	- 6	+ 4	(+ 1)	+ 1/4	- 3 1/2	+88	d	d
Jan 27	20S	178W	630	7.2	79	234	163	-30	+ 7	+ 6	+ 5	- 14	-65	27	1.2
Apr 21	19N	69W	40	7.3	45	97	8	-40	- 6	?	(- 1/2)	+ 20	±90	d	d
May 11	17S	70W	70	7.1	68	132	132	+10	-11	- 6	- 8 1/2	0	0	22	0.7
May 14[c]	54N	161W	n	7.5	36	318	334	+40	-20?	+30?	+25?	+ 6?	+15	35?	0.8?
Jun 29	16S	173W	60	7.0	72	234	315	+ 3 1/2	- 5	(+ 1/2)	+ 1/2	+ 3	+80	d	d
Aug 25	23S	65W	50	7.1	77	132	160	+ 4 1/2	- 1 1/2	- 1 3/4	- 2 1/2	- 1 1/4	+26	23	0.7
Sep 8	21S	174W	n	7.8	77	230	184	+160	+10	-40	-60	+ 59	-44	21	0.7
Sep 10	43N	147E	40	7.1	70	310	282	0	-10	+ 3	+ 4 1/4	- 3 1/4	-26	21	0.7
Nov 19	10N	83W	80	7.0	39	120	53	+ 2	+ 4	+ 1	+ 1 1/2	- 3	-75	20±	0.7±
1949															
Jan 24	22S	176W	110	7.1	78	230	313	+ 8	- 8	+ 3/4	+ 1/2	+ 5 1/2	+82	35±	1.2±
Feb 2[c]	53N	173W	220	7	42	315	381	+ 7	+ 3	+ 1 1/4	+ 1 3/4	+ 3 1/2	+64	31±	1.0±
Apr 25	20S	69W	110	7.5	71	132	70	+20	+50	+ 2?	+13	- 25	-59	6?	0.3?
Aug 6	18S	174W	70	7.6	75	233	167	-15	+ 4	+ 4?	+ 3 1/2	- 7 1/4	-63	34?	1.1?
Sep 27[c]	60N	149W	50	7	33	331	288	- 7	+18	- 2?	- 9	+ 7	-40	8?	0.2?
Nov 3	48N	154E	160	6.9	64	314	230	+ 2	+ 2 1/2	(1/2)	- 1/4	+ 1 1/2	-82	d	d
Nov 27	18S	173W	60	7.2	74	231	180	+ 7	0	- 3	- 2 1/2	+ 2 3/4	-48	35±	1.2±
Dec 21	19S	64W	620	6.9	73	129	158	- 7	+ 2	(+ 1)	+ 3 1/2	+ 2	+27	10±	0.3±
Dec 22	16N	93W	100	6 1/2?	28	123	56	+ 4	+ 6	+ 1 1/4	+ 1 1/2	- 3 1/4	-63	27±	0.9±
1950															
Jan 12	18S	178W	550	7.0	78	231	230	- 4 1/2	- 4	+ 3 1/2	+ 3 1/2	- 1/2	- 1	31	1.0
Feb 28[c]	46N	144E	340	7.8	72	313	25	+200	+90	+50	+40	+110	+70	35	1.2
Jun 7	4S	77W	110	7.0	54	125	163	- 6	+ 2	+ 3	+ 3	+ 2	+36	31±	1.0±
Jul 9	8S	71W	650	7	61	126	118	-13	+24	+ 9	+15	- 1 3/4	- 7	18	0.6
Jul 9[c]	...	...	..	..	..	...	132	-20	+20	+13	+14 3/4	+ 2 1/4	+ 5	25	0.8
Aug 14	27S	63W	630	7 1/4	80	132	107	+ 3 1/4	-12	- 1 3/4	- 6 1/2	+ 2 3/4	-22	9±	0.3±
Aug 14[c]	...	...	..	..	..	...	147	+15?	- 9	- 2 1/2	- 9 1/4	- 2 1/2	+14	9±	0.3±
Oct 5[c]	11N	85W	n	7.7	38	120	98	+ 9	-40	-15?	-24	+ 6	-20	20±	0.6±
Oct 23	14N	92W	n	7.3	31	123	133	+50±	-60	-50±	-30	- 5	+ 8	45±	1.6±
Oct 23[c]	...	...	..	..	..	...	129	+140	-180?	-140	-130	- 11	- 5	31±	1.1±
Dec 1	14N	47W	60	7	66	89	47	+15	+15	+ 3 1/2	+ 8 1/2	- 7 1/2	-38	33	0.4
Dec 9	23N	67W	100	8.0	74	134	66	+30	+70	+12	+16	- 35	-66	22	0.8
Dec 10	14S	76W	80	6 3/4	62	135	222	- 6	- 5	(1/2)	+ 1/4	+ 4	+86	d	d
Dec 14	19S	176W	200	7.7	76	235	173	-200	+30	+28	+50	- 90	-59	17	0.6
Dec 14	17N	97W	n	7.3	25	130	50	+70?	+90?	?	+13?	- 54?	-75?	?	?
1951 (tentative)															
Jan 6[c]	7N	81W	100	7 1/2	43	122	106	- 6	-20	- 9	- 8	+ 8 1/2	-15	30±	1.1±
Feb 13	15S	175W	250	7	72	236	231	- 4	- 5	+ 5	+ 3 3/4	- 1/4	- 5	38	1.3
Feb 13	56N	155W	n	7	34	324	309	-12	+14	- 5	-11	+ 2 1/2	-13	15	0.5
Apr 14	24S	66W	200	7	75	134	64	+ 2 1/2	+ 5	+ 2	+ 1	- 2 1/2	-68	45±	1.8±
Apr 14	61N	136E	n	6 3/4	68	330	240	+ 2	+ 4	0	0	+ 2 1/4	±90	d	d

[c] Measured amplitudes do not refer to the first half wave of S which is too small to be used for the present purpose.

[d] Values too small for reliable calculation.

responding calculation sheets for each shock and checked on a map in stereographic projection with the center at 34°N, 118°W (near Pasadena). This map was also used to measure with an accuracy of about ±1° the azimuth α towards the epicenter (starting with 0° towards N, and increasing towards E). The azimuth β of the transverse waves was calculated from (12). It was taken within ±90° of α.

The components N, E, and Z of the ground motion were calculated to the nearest tenth of one micron but are given in Table 4 only to the nearest quarter or integer. The sign "+" indicates motion towards N, E, and Z respectively. The problem of the accuracy of the instrument constants, which enters here, has been discussed above. In addition, values of a few microns are relatively less accurate than greater amplitudes. Calculations of the ground amplitudes were made on the inadequate assumption of continuous sinusoidal waves, but this affects the results for all components of a given instrument type in a similar way for a given wave form and only ratios of amplitudes enter in the essential results.

Amplitudes a and b of the SV- and SH-components of the arriving S-waves were calculated from (11). A plus sign for a indicates that its horizontal component is towards the epicenter and its vertical component is upward; b is positive, if the direction of the ground motion in SH is to the right of the direction towards the source (azimuth $\alpha + 90°$). The values and signs of a and b correspond to the same wave, which, as has been mentioned, is not necessarily the first motion in S.

The angle of polarization ϵ was calculated from (13) and (14). Usually it agrees with $\beta - \alpha$ within 3°. It is positive if the plane which is determined by the tangent to the ray and the direction

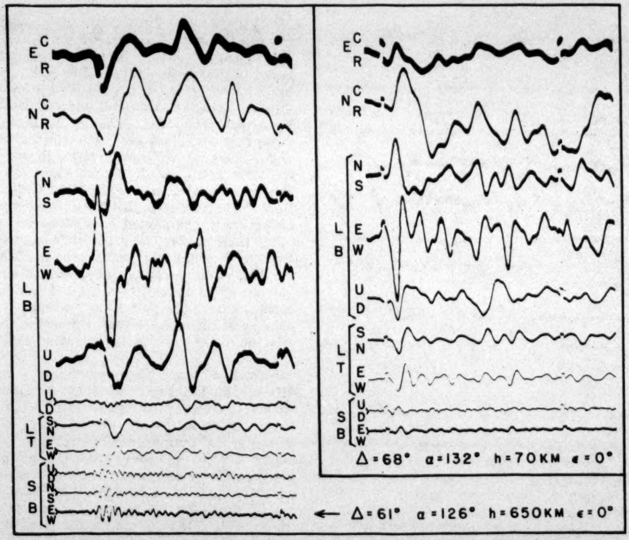

Fig. 4--Pasadena seismograms of shocks in South America, magnitude 7±, showing prevailing SV-waves; length of each slightly over one minute as indicated by two minute marks; top to bottom: EW-strain followed by NS-strain (C = compression, R = rarefaction); LB = long-period Benioff seismograph (T_g = 90± sec, last of LB on left records: T_g = 14 sec); LT = long-period Wood-Anderson torsion seismograph (T = six seconds); SB = short-period Benioff seismographs (T_g = 1/4± sec); time marks indicate one minute; Δ = epicentral distance, α = azimuth of epicenter at station, h = focal depth, ϵ = angle of polarization; left: July 9, 1950, at 04h; right: May 11, 1948, at 08h; for details, see Table 4

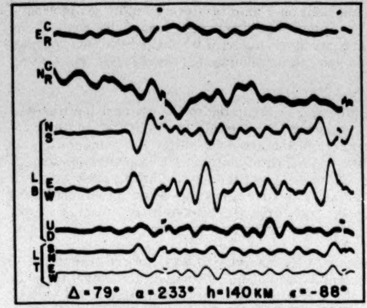

Fig. 5--Typical SH-record (January 22, 1948, at 13h, source in Tonga sector); arrangement and symbols as in Figure 4

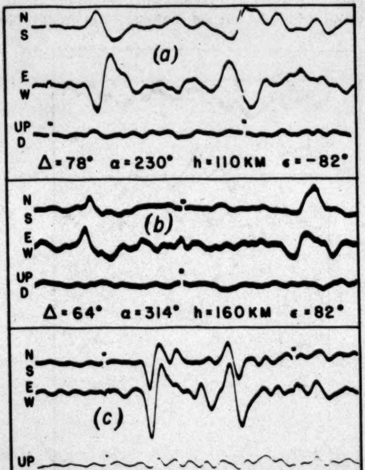

Fig. 6--Strongly prevailing SH-waves recorded by long-period Benioff instruments (T_g = 90± sec); (a) January 24, 1949, at 09h, epicenter in Tonga sector; (b) November 3, 1949, at 01h, Kurile Islands; (c) December 10, 1950, at 02h, South America; symbols as in Figure 4

of the vibration in S intersects the surface in a direction with an azimuth greater than α, that is, to the right of the plane of propagation. It was always taken between -90° and +90°. The last two columns give the apparent angle of incidence $\bar{i}$ of the ray and the ratio w of the vertical ground amplitude Z to that of the incident amplitude a. They were calculated from (15) and the last equation of (10) respectively. All resulting values of i and w are positive since, in all shocks of Table 4, a and Z were found to have the same sign. This may indicate that all of these shocks had epicentral distances greater than the critical distance Δ^* of SV, for which $i = i^*$. However, it is not impossible that the change in sign of $\bar{i}$ and w which is indicated for distances smaller than Δ^* ($i > i^*$) by the theory on the assumption of a steady state (continuous sinusoidal waves) does not occur in the first SV-wave or in SV-impulses. The usually complicated records of S from sources at distances less than about 30° may be an independent indication that the critical epicentral distance Δ^* is between 25° and 30°.

Figures 4 to 7 illustrate the preceding discussion. Figure 4 shows two examples of almost pure SV-waves. In both the two strain records (top E-W strain, below N-S strain with higher magnification than E-W) are in phase. The following records (LB and LT) were used in the determination of the SV and SH components (note that the LT south-north component always records southward motion towards the top, while the other north-south instruments record it towards the bottom). While the short-period Benioff instruments (SB) have higher magnification for short-period waves than any of the other instruments, they aid little in investigations of S-waves, for which even the Benioff vertical seismograph connected with a galvanometer of about 14 sec period (sixth record from bottom in left-hand part of Fig. 4) is usually of little use. This illustrates the discussion of the results described by Lehmann.

Figures 5 and 6 give examples of dominant SH-waves. In these records the vertical component is correspondingly very small, but there is no indication that the weak SV begins later than SH. All four shocks are from intermediate depth, and the relatively large phases following S are probably sSH-waves with polarization apparently opposite to that in SH, at least in Figure 6. Since SH theoretically does not change its phase if it is reflected at the Earth's surface, it seems to be more likely that sS and S started from different sides of the fault plane (sS upward, i = 25°±). This assumption is not unreasonable, since the fault planes involved seem to form angles greater than 25° with the vertical [BENIOFF, 1949]. If the direction of the fault trace forms an angle less than 90° with the direction towards the station, the minimum angle required to result in a start of sS and S from opposite sides of the fault plane is even smaller than 25°. The displacement of the two strain records in Figure 5 (top) is in opposite direction, as it should be theoretically in SH. No case with clear

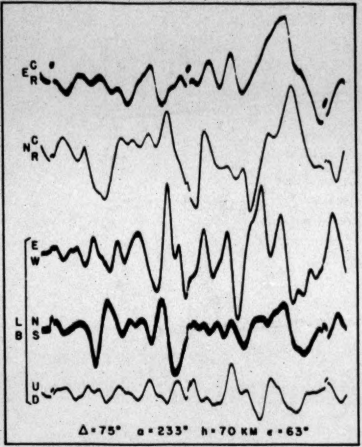

Fig. 7--Example of SV-wave superposed on larger SH-wave; August 6, 1949, at 00h, epicenter in Tonga sector; arrangement and symbols as in Figure 4

S-waves was found indicating a discrepancy with the theory in this respect. In the three records of Figure 6; S is rather short-period, and consequently the magnification of the pendulum seismographs for S was large relative to that for the strain seismographs which furnished only very small records.

Figure 7 gives an example of records of a shock in which SH prevails slightly over SV. The directions shown in Table 5 should be expected under the circumstances of Figure 7 ($\alpha = 233°$). The actual recorded directions and amplitudes agree well with the theoretical expectation. The second large phase in Figure 7 (especially in EW) is probably a combination of ScS and sS, while the large movement at the right end of the records has prevailing P- and SV-components (the strain records are in phase) and probably consists mainly of SP and PS.

Values of $\bar{i}$ and w calculated from the seismograms (Table 4) are plotted in Figures 8a and 8b respectively, together with theoretical curves based on Table 2 and Figure 2. It was assumed that Poisson's ratio is 1/4, but various values for v_0 were used. Both figures indicate that $v_0 = 3\ 3/4 \pm$ km/sec gives the best agreement; this value agrees with the expectation from the theory. However, no significant difference between results based on S-waves with different periods could be established beyond the limits of error.

Table 5--Directions to be expected from shock of Figure 7

Wave		Pendulums		Strains	
SH	South	East	. . .	Comp. E	Dil. N
SV	South	West	Up	Dil. E	Dil. N
SH + SV	South large	East small	Up small	Comp. E small	Dil. N large

Observed values of the polarization angle ϵ, given in Table 4, were plotted as a function of distance as well as azimuth. There was no clear correlation in either case. Finally, the direction of the plane of polarization at the various epicenters was plotted on a map. It was assumed that this direction is at the epicenter to the right of the direction towards Pasadena if it was found to be at Pasadena to the left of the direction towards the epicenter. In most instances, especially in the Tonga sector, these directions were within ±45° of the line perpendicular to the fault direction at the source. Directions along the fault zone (two sectors of 90° each) were very rare. In a relatively large fraction of the shocks under investigation, the plane of the fault zone passes nearly through Pasadena (for example, for epicenters in parts of South America, Central America, the Aleutians, and northern Japan). Combining results from P- and S-waves can be expected to give more detailed information on fault movements. However, such problems were considered to be outside the scope of the present paper.

Acknowledgments--The research reported in this document has been made possible through the support and sponsorship extended by the Geophysical Research Division of the Air Force Cambridge Research Center under contract AF 19(122)436. It is published for technical information only and does not represent recommendations or conclusions of the sponsoring agency. The figures were drafted and arranged by John M. Nordquist.

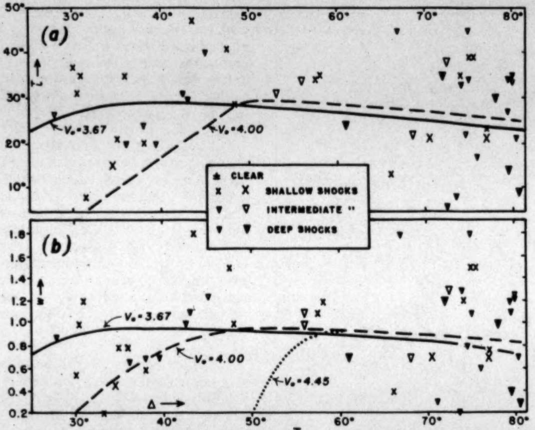

Fig. 8--(a) Apparent angle of incidence $\bar{i}$ of S, calculated from recorded S-waves (Table 4) as a function of the epicentral distance Δ and theoretical curves, based on the travel-time curve of S for different values of v_0 (velocity of S near the surface), if Poisson's ratio $\sigma = 1/4$; (b) corresponding values of w (ratio of vertical ground motion to incident amplitude in SV-waves)

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(Communicated manuscript received December 26, 1951; open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

PHYSICAL ASPECTS OF THE CONVECTION-CURRENT HYPOTHESIS OF OROGENESIS

A. E. Scheidegger

Abstract--Theories of orogenesis fall into two principal groups according to whether they depend upon contraction or convection currents. Some physical aspects of the convection-current hypothesis are first considered and it is then pointed out that it is essentially thermal data which support the convection-current hypothesis, whereas the occurrence of earthquakes points towards a fracture theory. It is shown that, in order to maintain the observed heat flow through the surface of the Earth, the velocity of the convection currents need only be of the order of a few millimeters per year and hence may be much smaller than assumed heretofore. The stresses and the viscosity of the material in which such slow convection currents occur are estimated and it is shown that they are of the same order of magnitude as that which is usually assumed in the fracture theories. Thus the occurrence of fracture within the currents is advanced as a possible explanation for the different orogenetic phenomena. It is shown, with examples, that this is possible if the fracture phenomena are of short duration as compared with those connected with the currents.

One of the most puzzling problems in geophysics has been to suggest a mechanism that would give rise to the Earth's magnetic field. The most satisfactory explanation so far made of the origin of the field itself and of the cause of several of its properties is that it arises from convection currents in the liquid iron core of the Earth [BULLARD, 1950; ELSASSER, 1950; THOMPSON, 1950]. To maintain those currents a certain heat flow is necessary. Bullard has stated that the requisite heat flow could not be maintained very well merely by conduction through the outer mantle.

If this is so, the required heat flow amplifies our inadequate data from measurements of heat flow at the surface. It also has lead to the assumption that another set of convection currents exist in the mantle above the core to transport this heat and hence to the conclusion that these currents might be the cause of orogenesis and a factor in the shaping of continents.

PEKERIS [1935] has worked out a complete possible scheme of convection currents in the outer mantle of the Earth (in three dimensions), and the only fault of his result from a geological point of view is that the actual arrangement of continents and oceans on the Earth bears not the least resemblance to his patterns.

GRIGGS [1939] and VENING MEINESZ [1948] have gone even farther and explained in great detail the nature of a continental margin including island arcs by the assumption of convection currents. However, their investigations deal only with the cross sections of continents and island arcs. The actual pattern of these arcs in plan and the manner in which they join each other, intersect, or split apart is in contradiction to the most elementary principles of hydrodynamics, namely, that vortex lines do not cross each other and must be closed [SCHEIDEGGER and WILSON, 1950].

It seems that the thermal data point toward the probable existence of convection currents, but that the explanations with which they have been introduced are not adequate to explain the observed facts about mountains and continents. Let us therefore investigate these hypothetical convection currents from a physical standpoint and see what they might do and what they could not do.

Since heat must be conveyed from the bottom to the top, the material must flow in that direction. Furthermore, the axis of such a convection current must be closed. Therefore, the patterns that are possible are easily seen to be of the general type of those shown in Figure 1, where the axial rings lie approximately parallel with the surface of the Earth.

The effect on the surface in Figure 1, type a, would be that material would accumulate over the current (see Fig. 2). In type b, a hole would result (see Fig. 3). If the radius R of the convection current becomes large, then ring-shaped structures would result.

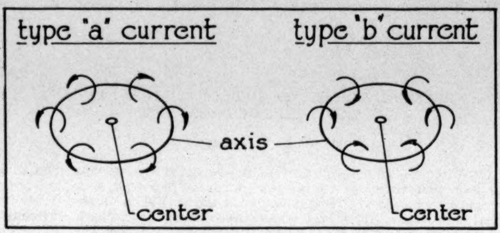

Fig. 1--Possible types of convection currents

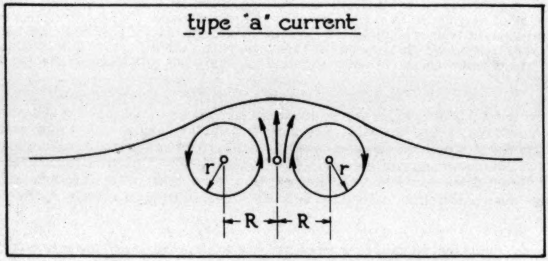

Fig. 2--Material accumulated on top of an outward, type a convection current

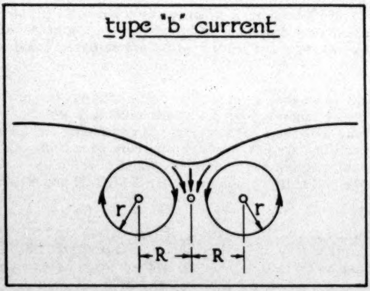

Fig. 3--Hole caused by an inward, type b convection current

We can calculate the rotating speed (t = time of revolution) for such a convection current which would be necessary to give the observed heat flow. For we note that the heat H transported by such a revolving torus is

$$H = 2\pi^2 r^2 Rn\,\Delta T/t$$

The volume of the torus is $2\pi^2 r^2 R$, the temperature difference between the two levels is ΔT, and the specific heat per unit volume is n. Thus the heat flow h per unit area is

$$h = H/\pi(R+r)^2 = 2\pi[r^2 R/(R+r)^2]n\,\Delta T/t$$

Hence

$$t = [2\pi r^2 R/(R+r)^2] \times n\,\Delta T/h$$

We must now make reasonable assumptions for the quantities occurring in the last equation. It will be noted that for an approximation only powers of ten will matter. Thus we may assume that r is of the order of a hundred km, R of a thousand km, n of the order of unity (1 cal/deg cm^3), ΔT of the order of a thousand degrees, and h (Bullard's value for the maintenance of the Earth's magnetic field) is 6×10^{-7} cal/cm^2 sec.

Thus we obtain

$$t \cong [2\pi\,(10^7)^2\,10^8/(10^8)^2] \times 10^3/6 \times 10^{-7} \cong 10^{16}\ \text{sec} \sim 3 \times 10^8\ \text{years}$$

We see that the time of revolution will be of the order of several times 10^8 years.

The speed of the material where it goes fastest is

$$2\pi r/t \cong 2 \times 10^{-1}\ \text{cm/year} = 2\ \text{mm/year}$$

The speed of the material is thus of the order of a few millimeters per year.

The stresses that occur due to viscous drag can also be estimated. Denoting the shear stress by τ, the coefficient of viscosity by μ, one has [LAMB, 1932, p. 588]

$$\tau = \mu r\, d\omega/dr$$

ω being the angular velocity of the torus: $\omega = A/r^2$. A is a constant of integration which has to be determined, so that for $r = R$ we have $\omega = 2\pi/t$. Hence we find

$$d\omega/dr = -2A/r^3 = 4\pi R^2/tR^3 = 4\pi/tR$$

and

$$\tau = 4\,\pi\mu/t$$

This equation connects the stresses with the coefficient of viscosity. If the material is to behave as though it was rigid in certain instances, then the limiting shear stress τ_0 (beyond which rupture will occur) must be rather low (of the order of 100 kg/cm^2); but the viscosity rather high (at least 10^{22} poise). Our last equation indicates that those requirements are indeed compatible. Choosing

$$\tau_0 \cong 100 \times 10^6 = 10^8\ \text{dynes/cm}^2$$

yields

$$\mu < \tau_0 \times t/4\pi = (1/4\pi)\,10^8 \times 10^{16} \sim 10^{23}\ \text{poise}$$

with the value found above for t.

There is, therefore, no difficulty in assuming slow convection currents, moving at a rate of about two mm a year. Such convection currents are quite sufficient to remove enough heat from the core to maintain the Earth's magnetic field. The breaking strength and "viscosity" required would be about those of limestone. This is not quite the same movement as that assumed by Pekeris. His convection currents move at a somewhat higher speed, namely, about one cm/year. He therefore had to assume a smaller viscosity.

In order to see how these convection currents might affect the geology of the Earth, we might consider that the case of the outward current (type a) might be the cause of a continent (see Fig. 4). Alternatively, the opposite case might represent an ocean (see Fig. 5).

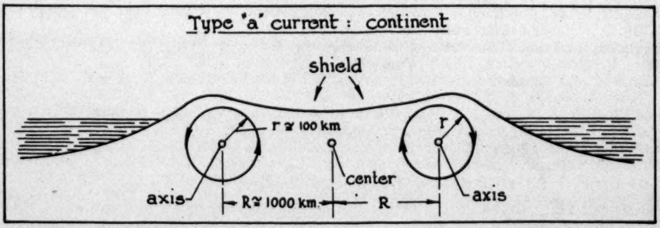

Fig. 4--Cross section of a continent as caused by a type-a convection current

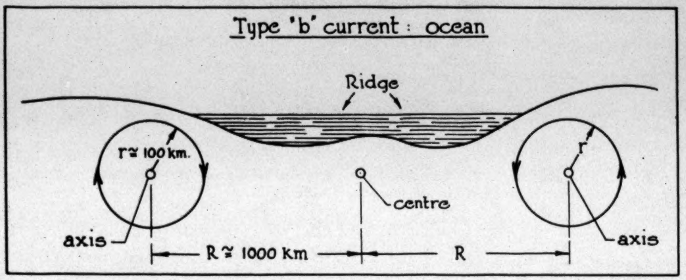

Fig. 5--Cross section of an ocean as caused by a type-a convection current

In these instances, we see that convection currents might provide an explanation of the existence of continents and oceans. They would also provide an explanation of why all continents and ocean shields are of the form of blocks, and not of the form of a single large belt around the globe. Motions that could be connected with these convection currents would be continental uplift and marginal flexure by the drag at the edges of a continent. These motions would be of the same order as the speed of the convection currents, that is, a few millimeters per year.

No explanation, however, can be obtained from this theory for deep-focus earthquakes, for the arcuate structure of mountain belts, and for the rather rapid events of orogenesis. On the contrary, it is true that the very presence of deep-focus earthquakes presents a serious difficulty to the assumption of convection currents. JEFFREYS [1945] states that the essential feature of any sort of viscosity is that, for any initial distribution of stress, the medium tends to move so as to reduce the stress differences. Consequently, if the requisite stress differences to produce earthquakes and mountains cannot be produced in a material with finite strength, they cannot be produced in a viscous one. If there is a viscous yield, earthquakes and mountains occur in spite of it and not because of it.

This is a point of view which we should like to emphasize. The mechanism that produces island arcs, mountain belts, and earthquakes must be a different one from the convection currents considered here. We have pointed out in an earlier publication [SCHEIDEGGER and WILSON, 1950] that sliding fracture might be able to produce a pattern of island arcs. The two phenomena, convection currents and sliding fracture, could exist side by side if the characteristic time for the convection is long compared with the characteristic time for the fractures; in other words, if the movements around the island arcs are measured at least in centimeters per year, as compared with the millimeters per year characteristic for the convection currents proposed.

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(Communicated manuscript received October 15, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

GEOTHERMAL INVESTIGATIONS ON THE ARCTIC SLOPE OF ALASKA

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(Publication authorized by the Director, U. S. Geological Survey)

Abstract--Preliminary results of geothermal investigations now in progress near Barrow, Alaska, indicate that, below the level to which seasonal changes penetrate, ground temperatures do not fall below about -10°C. The depth of the 0°C isogeotherm ranges from 670 to 1300 ft, increasing with the distance from the nearest large body of water. Thermal disturbances caused by drilling in and through the permafrost persist for many months, so that thermal readings taken in freshly drilled holes appear to have little value. Some correlation between terrain, electrical resistivity of the ground at shallow depths, and ground temperatures is indicated.

Introduction--An unparalleled opportunity for the study of ground temperatures in an area of permanently-frozen ground extending to considerable depths has been offered by the drilling program now under way in Navy Petroleum Reserve No. 4 on the Arctic Slope of Alaska. Field work was started in April, 1949, and has been carried on since then by the U. S. Geological Survey, under the joint sponsorship of the Survey, the Office of Naval Research, the Bureau of Yards and Docks, and, during the past year, the Snow, Ice, and Permafrost Research Establishment of the U. S. Corps of Engineers. This paper summarizes some of the work already accomplished.

Method of measurement--Ground temperatures are measured by means of individually calibrated thermistors, which are attached to multiconductor cables at regular intervals. These cables are then inserted in drilled holes and allowed to freeze permanently into place. As most of the holes are uncased, in only a few places is it necessary to consider the possible disturbing effects of metallic casing.

Thermistors are small temperature-sensitive elements with negative thermal coefficients so that their resistance decreases as the temperature rises. They are of different sizes and sensitivities, and have a great advantage over most temperature-sensitive elements in that their inherent sensitivity is of a very high order. At temperatures near -10°C the thermistor used in this work changes in resistance approximately 300 ohms/°C, so that variations of as little as 0.01°C are readily measurable. Resistances are read by means of an ordinary Wheatstone bridge through circuits that automatically provide the necessary corrections for resistance of the lead wires. The absolute accuracy of the temperature measurements recorded by means of thermistors depends largely upon how accurately the thermistors are calibrated. It is believed that in most cases an absolute accuracy of about 0.03°C has been attained, although in certain individual cases errors of as much as 0.1°C are almost certainly present. Temperature variations can be recorded within 0.01°C of their true values, even with considerable errors in thermistor calibration.

Ground temperatures--Near Barrow the minimum ground temperatures below the level to which seasonal changes penetrate range from about -7.3°C close to the shore of the Arctic Ocean to about -9.9°C a few miles inland. These minimum values are found about 100 ft below the surface but, as the vertical temperature gradient at this depth is extremely slight, it is usually impossible to assign an exact depth to the point of minimum temperature. At all shallower depths, ground temperatures fluctuate with the seasons, with a phase lag of 180° at 40 or 50 ft. At the greater depths the normal steady increase of temperature downward is found. In one well drilled on the shore of the Arctic Ocean (South Barrow No. 1) and now in approximate thermal equilibrium, a reciprocal gradient of 78.4 ft/°C was observed between 98.5 and 498.5 ft below the surface; in another which is about a mile from the nearest large body of water (Simpson No. 13) the reciprocal gradient was 84.3 ft/°C between 120 and 632 ft; and in a third about seven miles inland (South Barrow No. 3) the reciprocal gradient was 133.8 ft/°C between 155 and 595 ft. This last well is not yet quite in thermal equilibrium, but is so close to it that the computed gradient cannot be appreciably in error. Figure 1 shows the thermal logs of these three holes.

Note that the curves for South Barrow No. 1 and for South Barrow No. 3 are very smooth, but do not extend below 500 and 600 ft respectively. The two South Barrow wells are cased; the Simpson is uncased. Each well has a different gradient, with that for the uncased well intermediate be-

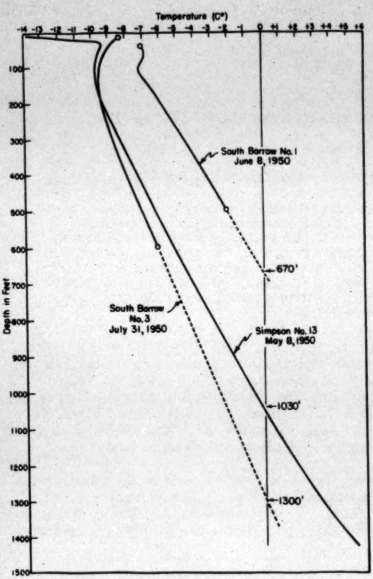

Fig. 1--Three typical thermal logs from the Barrow area: South Barrow test well No. 1, South Barrow test well No. 3, and Simpson core test No. 13

tween the other two, so that it does not seem probable that the characteristics of these logs are governed by the presence or absence of casing. The gradients do not seem to correlate with any marked change in stratigraphy as evidenced by the driller's log, nor is there more than a vague suggestion of correlation with the electric logs. It seems most probable that the changes in gradient are caused by a change in thermal conductivity in the earth materials, although there is no proof whatever that this is so.

Seasonal variations in temperature--Seasonal variations are as much as 30°C at the surface of the ground, but they diminish rapidly with depth and become almost imperceptible at about 70 ft, where the seasonal change is less than 0.2°C. The seasonal waves of rising and falling temperatures observed in one 65-ft hole close to Barrow Village are shown in a simplified form in Figure 2. Figure 3 gives the actual maximum and minimum temperatures recorded in this hole at depths from two to 65 ft below the surface over a period of slightly less than a year. The maximum temperatures fall along an extremely smooth curve, but the curve drawn through the minimum temperatures contains noticeable irregularities below about 30 ft which unfortunately do not show clearly on the scale of the figure. These irregularities probably result from the fact that the hole had not reached complete thermal equilibrium when the 1949 minima were recorded for these depths, and the 1950 minima had not penetrated to these depths during the period covered by this plot. It is anticipated that these irregularities will disappear when several years' observations have

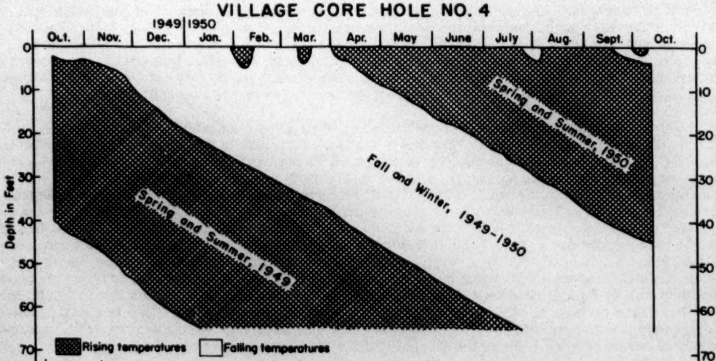
VILLAGE CORE HOLE NO. 4

Fig. 2--Simplified thermal regime, Barrow Village core hole No. 4, showing dates at which temperatures were rising or falling at various depths

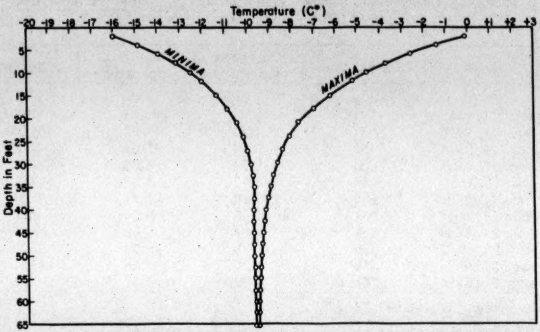

Fig. 3--Amplitude of seasonal changes at Barrow Village core hole No. 4 as shown by the maxima and minima observed at various depths between October 28, 1949, and October 6, 1950

been accumulated. As expected, both the maximum and the minimum curves seem to be exponential in character, and of the form $Y = ae^{-bx} + C$, where Y is the maximum (or minimum) temperature at depth x, a, b, and C are constants, and e is the base of the Naperian logarithms. Although the data are insufficient to permit exact determinations, these curves both seem to be asymptotic to about -9.7°C. This may therefore be the approximate average annual temperature of the soil at the surface.

Thickness of the frozen layer--The thickness of the frozen layer varies from place to place and, at least in the neighborhood of Point Barrow, seems to be largely controlled by the distance from a large body of water. The greatest depth at which sub-freezing temperatures have been measured in this area is 1030 ± 5 ft, at Simpson No. 13, approximately a mile west of Smith Bay. Unfortunately no other equilibrium readings at depths penetrating the complete thickness of the frozen zone are as yet available. Extrapolation of data from shallower depths, admittedly not an entirely safe procedure, suggests that at South Barrow No. 1, on the shore of the Arctic Ocean near Barrow, the 0°C isogeotherm lies at about 670 ft and that some seven miles inland at South Barrow No. 3 it may be as much as 1300 ft below the surface. The measured depth of 1030 ft at Cape Simpson and the estimated depth of 1300 ft at South Barrow No. 3 constitute, as far as the writer is aware, record permafrost depths for the North American Continent. Russian scientists are understood to have found a depth of about 2000 ft for the permafrost at Nordvik, Siberia [I. V. Poire, as quoted by R. F. Black in "Permafrost," Smithsonian Inst. An. Rep. for 1950, p. 276].

Thermal disturbance caused by drilling--In any form of drilling operations the ground temperatures immediately adjacent to the drill hole are greatly disturbed and, on the Arctic Slope, require from many weeks to one or even two years or more to regain thermal equilibrium. In drilling through permafrost this disturbance is always in the direction of higher temperatures except below the base of the frost if the well is deep enough to penetrate the full thickness. "Cooling curves" have been plotted for several drilled wells and in all so far analyzed they approximate rather closely a hyperbola of the general form $Y = x/(ax + b) + C$. The constants a, b, and C vary from one well to the next and from one depth to another in the same well. It is realized that a more complicated expression is called for by theory, but the simple hyperbola gives such a close fit to the observed data that it is thought wisest to wait until more data are at hand before attempting a more complicated formula. One example of such a cooling curve covering a period of 337 days for a depth of 595 ft at the South Barrow No. 3 well is given in Figure 4. The circles represent observed values, and the curve was computed from the formula $Y = (7 - x)/(0.31x + 8.06) - 3.34$, where Y is the temperature at 595 ft in degrees C, and x is the elapsed time in days from the arbitrarily chosen date, September 22, 1949. Obviously this date must be chosen after the fluids in the well have completely frozen, or complications due to the latent heat of fusion would be introduced. This formula gives such a close approximation to the observed temperatures (see Table 1) that it should be possible to estimate with fair accuracy the final equilibrium temperature at this depth. When this is done, the formula just quoted indicates that the equilibrium temperature in

Table 1--Comparison between observed and computed temperatures at a depth of 595 ft, South Barrow test well No. 3

Date	Elapsed time[a]	Observed temperature	Computed temperature	Difference
	days	°C	°C	°C
Sep. 29, 1949	7	-3.34	-3.34	0.00
Oct. 29, 1949	37	-4.83	-4.88	-0.05
Nov. 14, 1949	53	-5.16	-5.22	-0.06
Dec. 18, 1949	88	-5.54	-5.63	-0.09
Jan. 18, 1950	119	-5.76	-5.83	-0.07
June 7, 1950	259	-6.18	-6.19	-0.01
June 15, 1950	267	-6.20	-6.20	0.00
June 29, 1950	281	-6.22	-6.22	0.00
July 13, 1950	295	-6.23	-6.23	0.00
Aug. 3, 1950	316	-6.26	-6.25	+0.01
Aug. 31, 1950	344	-6.29	-6.28	+0.01

[a] Since September 22, 1949.

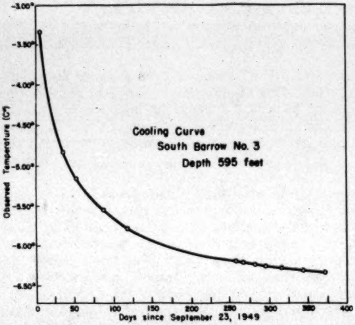

Fig. 4--Cooling curve for South Barrow test well No. 3

this particular well at 595 ft will be about -6.57°C, and therefore that on August 31, 1950, it was within about 0.29°C (6.57 - 6.28) of its final temperature at this particular depth. These observations on cooling rates also underline the fact that temperature measurements made shortly after drilling ceases may differ from undisturbed ground temperatures by as much as several degrees.

Possible correlation between terrain and electrical and thermal properties of the permafrost--Although no extensive study of ground resistivity has yet been made in this area, the very limited data at hand suggest that the portions of the tundra which stand higher and are better drained than the average have, at depths down to 100 ft or slightly more, electrical resistivities that are lower and ground temperatures that are higher than in neighboring areas. The temperature variations, which in one place amount to as much as 1.5°C at a depth of 95 ft, are probably caused by the different heat diffusivities of the wet and the dry tundra areas. The variations in resistivity are probably largely controlled by the number of mobile ions present, and this in turn is largely a matter of ground temperatures and the percentage of ground ice present except where differences in the salinity of soil moisture introduce further complications. Extremely saline formations are definitely present at 100-120 ft in wells approximately a half mile away. In one 3150-ft traverse taken across exceptionally wet and exceptionally dry strips of adjacent tundra, and where the electrode separation was adjusted so as to give an effective depth of penetration of 50 ft, the apparent resistivities ranged from less than 2000 ohm-cm in the driest to about 320,000 ohm-cm in the wettest areas crossed by the traverse.

These findings may be typical of the tundra at large, or may be the result of local conditions not effective elsewhere especially since this is an area of old beach ridges and former lagoons; much more work will be needed before this point can be settled. As the maximum distance to saline water from any of these electrical stations was only 5/8 mile, it may well be that infiltration of saline water has altered the electrical and even the thermal properties of the area. Three selected points along the traverse were core-drilled but, because a rotary drilling rig was used, the cores were recovered in a thawed and flushed condition so that neither determinations of salinity nor estimations of ice content were possible.

Further evidence that surface conditions affect thermal conditions to a considerable depth is found in a line of 100-ft holes spaced 1320 ft apart, and crossing both extremely wet and moderately dry areas. These were seismic holes, and were drilled and shot on June 10-11, 1949. Thermal cables were installed on June 11, immediately after the completion of the last hole in the series. The only thermistors available at the time unfortunately had a lower order of sensitivity, and had been less carefully calibrated than those used in the rest of the investigations. It is believed that, although the actual temperature determinations may be in error, the changes in temperature with time have been recorded with fair accuracy. In two dry holes, bottom-hole temperatures fell about 4.4° and 4.2°C between July 2 and September 10, 1949, while in two wet holes the corresponding decreases were only about 1.1° and 0.9°C, indicating a considerable difference in thermal behavior of the two types of ground. These figures can be considered only as suggestive, not only because of the poor quality of thermistors used to obtain the temperatures, but because it is not certain that the same amount of explosive was used in each of the holes, and hence that the same amount of heat of combustion was added to each. Nor is anything known of the relative amounts of heat introduced into these various holes by the drilling operations.

If further work substantiates these suggested correlations between terrain, ground temperatures, and electrical characteristics, it may be possible to develop a technique so that the amount of ground ice present at shallow depths can be estimated from surface resistivity measurements without resorting to drilling.

Other investigations--Other experiments now under way include studies of the heat diffusivity of frozen ground, and of the effects on observed temperatures of various types of well casing and of such different hole-filling materials as air, water, drilling mud, and diesel fuel oil. Some work is also being done on the effects of large bodies of water on the subjacent ground temperatures. The effects of buildings and other artificial constructions in this area on the permafrost are also being investigated. In this connection it has been noted that, over a period of about three years, a heated quonset hut 40 × 100 ft, with a five- to six-inch monolithic concrete slab floor poured directly upon a beach gravel foundation, is capable of raising the ground temperature beneath it a matter of one or two degrees at a depth of 50 ft beneath its floor.

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(Communicated manuscript received December 10, 1951;
open for formal discussion until January 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

DISCUSSION OF

"A GRAVITY RECONNAISSANCE OF THE ISLAND OF OAHU"

by GEORGE PRIOR WOOLLARD

[Trans., v. 32, pp. 358-368, 1951]

Chester K. Wentworth (Hawaiian Volcano Observatory, Hawaii National Park, Hawaii, T. H.)--The addition of 26 stations to the gravity network for the Island of Oahu as reported in this paper represents a notable advance in geophysical knowledge. It is gratifying to have precise data for 33 well-distributed stations on this volcanic island of 604 sq mi and it should be of interest to all students of Earth science to query how the measured values of gravity fit into various concepts and mathematical models in isostatic reasoning. I have no facilities for critical examination of the various methods proposed for explaining the local and regional variations in the value of gravity and must assume that the technical procedures have been properly carried out. Neither do I "ride any specific horse" among the several philosophies of isostatic compensation or the lack of it.

My concern in this note is that Woollard's selection of geologic data for Oahu and the sweep of necessarily generalized assumptions give, in parts of his paper, a quite incorrect impression and may lead to serious error. It must be recognized that geophysical explorations are often conducted where few surface data are available and where there is but meager information from drill holes. But this does not justify the use of general data for particular types of rocks where specific information is available in recent publications or from geologists who can be consulted on the ground. At any rate certain of the points made and reasoning presented appear to warrant further discussion from the standpoint of actual geologic conditions on Oahu.

First, there appears throughout Woollard's paper confusion in the use of the terms vesicular and vesicularity. This is illustrated by the fourth paragraph on page 362, where he says "if the vesicular structure of the Hawaiian lavas is considered this is not unreasonable since such lavas have been known to occur with density values as low as 1.0 gm/cc." In the last part of this passage he is undoubtedly speaking of a low, measured density caused by numerous gas bubbles in the rock as solidified. But, as will appear below, the deficiency in mass of islands of the Hawaiian group, so well shown by Woollard's measurements, is only in small part due to the gas-bubble vesicularity that can be readily and directly measured. On the other hand, the present writer knows of no valid measurements of total voids, or of deficiency of mass, other than the kind made by Woollard himself and other geophysicists as reported in the paper in question. If the term "vesicular structure" is used in the narrower sense, the numerical data offered constitute an unimportant and insufficient part of the argument because of the limited amount and inadequate sampling in the available data if applied to the whole formation and because it is the volume of varied and numerous openings not included in this narrow sense which is mainly responsible for the total voids shown by deficiencies of gravity and by the great permeability found by hydrologists. On the other hand, if the term "vesicular structure" is used in the broader sense of total voids, the data quoted above are not applicable and can be misleading. No valid measurements of the total voids are known.

It is well known that the rock structure of Oahu, as of other islands of Hawaii, has a density much less than the grain density of the olivine basalts and other rock types. The deficiency of mass, it is true, is partly in the vesicles left by gas bubbles contained in the lavas as they cooled. In some lavas the vesicularity may be 50 pct or more, but, despite the impressively cellular character of such rock-in-hand specimens, these vesicles are not the chief cause of the high porosity and permeability so important in ground-water problems, nor of the density deficiency which we are here considering. The fact is that large volumes of extrusive volcanic rocks, particularly of basaltic composition like those of Hawaii, miles in horizontal extent and hundreds of feet in thickness, have extraordinary numbers and forms of openings quite distinct from vesicles and vesicularity in the usual sense. The denser parts of lava flows, whether pahoehoe or aa, are nearly always riven by contraction joints due to cooling, which divide them into units with lateral dimensions as small as, or smaller than, the thickness. The lava rarely halts with sufficient excess heat to mold tightly against or on the pre-existing surface, itself usually extremely rugged. As a result, there are many irregular openings between successive flows and along the margins of any given flow. The very rough and topographically rugged surface of aa flows is composed of aa clinker, and the basal parts of aa flows are usually formed of the same. This material as it lies probably has 20 to 40 pct large voids of extremely irregular form. Pahoehoe flows are not only composed of moderately to highly vesicular glass and finely crystalline basalt but also have lava tubes of one to 20 or more feet in diameter and scores or hundreds of feet in length, as well as blisters and partly hollow tumuli up to many feet in diameter. The composite result of these and various other openings in both the aa and pahoehoe basaltic lava flows is that the overall deficiency of mass is a large fraction of the whole and is rather difficult to estimate, even for those intimately acquainted with the field conditions over an entire area.

It should be emphasized that the various structures and openings mentioned above are most characteristic of basalt and occur less generally or not at all in andesites or trachytes. The result is that the mass deficiency due to such openings is much less with the latter rocks and because of this it is the considered opinion of the writer and other geologists experienced in Hawaii that, in the ground, in any large volume of andesite or trachyte, despite their lesser grain density, there is as much, if not more, mass as in the same volume of average basalt. This would discredit Woollard's use of the sequence from earlier basalts to later andesites as indicating sequence from greater to lesser mass density (p. 366). Moreover, this lithologic sequence is true only for the Waianae dome; no such differentiation is known in the Koolau dome.

In his discussion of the possible effects of size of vent in explaining the positive anomaly of larger basaltic vents as against the lack of such in the later "acidic" lava vents (p. 367, first paragraph), Woollard appears to have confused the later, more acidic lava flows of the Waianae dome, with the later, smaller volcanic craters secondary to the Koolau dome, which yielded rocks in the main of ultrabasic composition. There is no such series of small diameter, later stage vents in the Waianae range, and there are no later stage, acidic lava formations on the Koolau dome where the small diameter vents occur.

For the larger vent areas of the Koolau and Waianae calderas, with thick flows and dike complexes, Woollard's supposition of a higher density than the ordinary flank flows seems entirely in accord with the observed facts. However, as for the reasoning that the smaller vents show no significant positive anomaly due to the presence of similar intruded material of greater compactness, it must be pointed out that each of these later vents was the site of only one, or in some instances, two brief eruptions, mostly explosive. It is quite as likely that such eruptions, by the disruption of the surface rock in the course of the episode, and by the lack of repeated opportunity for intrusive filling, left a central structure lighter than the normal rock, rather than heavier. This difference in kind of vent structure is in my opinion a more probable cause of failure to find a positive anomaly.

In item (9) of his abstract, Woollard surmised that the earlier lavas were not "vesicular." Again it is not clear if he is using the term vesicular in the limited sense, or in the sense of total porosity or voids. If in the former, we should state that there is no direct evidence that these lavas are other than surface, basaltic flows and probably quite as vesicular as those we see in the process of making on Hawaii today. If these flows are assumed to have been formed in water deep enough to cause them to be generally more dense than surface flows, the case is somewhat weakened by the circularity of reasoning because it is the subsidence of several thousand feet that is suggested as reconciling the anomalies found. Taking another possible cause of greater mass, it is possible that there may be some decrease in total voids in the very deep lavas due to the imposed load or to intrusion or mineralization outside the vent zones, though there is very little evidence to this effect in the deeper rocks exposed on Oahu.

It is not implied that the general course of the speculation elaborated by Woollard in presenting his measurements would be seriously modified by a more cautious use of local geologic data. On the contrary, the offering of various possible explanations would be more generally effective if the several, relatively unimportant, supposedly factual agreements were omitted, or buried in a more extended summary. It is to be hoped that means will be found for more extensive gravity measurements on other islands of the group, particularly the large island of Hawaii, with its five component volcanoes and more complex history.

George Prior Woollard (University of Wisconsin, Madison, Wisconsin, Author's closure)--Wentworth takes the writer to task on several counts in his analysis of the gravity observations on the Island of Oahu. Specifically, the criticism can be broken down into five parts: (a) failure to use actual density values for the rocks on Oahu, (b) failure to consider in detail the differences in the surface geology of the island, (c) failure to use the correct terminology in referring to the highly permeable rocks forming the island mass, (d) failure to consult with local geologists on the investigation, and (e) speculative reasoning with little supporting factual data.

At the outset it must be emphasized that the detailed consideration of the local geology that Wentworth misses in the paper would contribute little to the study of the regional reconnaissance gravity survey under discussion. This is evident from a consideration of the size of the gravity anomalies present and the changes in the geology that occur within a single anomaly area. Some of his objections, however, are well taken and we shall endeavor to explain the basis for the action in question.

The apparent neglect of the local lithology in determining density values to be used in the analyses of the gravity data by the writer resulted from the fact that it was not feasible to gather suites of rocks for density determinations at the time, and an examination of the literature for density determinations on the rocks of Oahu yielded such poor results, both in terms of number of determinations, spread in values, as well as coverage of rock types reported present, that these data were judged inadequate for purposes of the analysis. They were therefore discarded in favor of statistically determined mean density values for the same rock types listed in Washington. This procedure, while not ideal, at least permits a first-order approximate evaluation of the densities of the rocks present to be made and is believed to be preferable to using the limited number of local data or omitting this part of the analysis entirely.

Wentworth is correct in his criticism that a consultation with Hawaiian geologists might have been helpful in resolving apparent disagreements between the gravity data and the geology. However, there was no opportunity at the time of the measurements to arrange for any cooperative action with local geologists, since the measurements were the result of an unplanned program that materialized during a two-day enforced layover at Hickam Air Force Base. Subsequent failure to contact local geologists resulted from a study of the literature on the geology of the Hawaiian Islands which appeared to be reasonably complete. This was especially true for the Island of Oahu, the area of the survey.

That I used the term vesicular in a very broad sense, not confined to openings resulting from gas bubbles alone, is unfortunate. This apparently led to unintentional confusion. The point being made was that the low density indicated for the island mass as a whole was due to high porosity rather than any marked difference in mineral composition of the rocks forming the island and the surrounding rock of the Pacific Ocean floor. That the openings resulted from clinkering at flow contacts, lava tunnels, etc., as well as the gas vesicles present, was thought to be obvious.

The failure of the writer to not differentiate between the local geology of the Wainaie and Koolau caldera areas was predicated by the lack of sufficient gravitational data needed for making any detailed correlation with local geology rather than ignorance of the change in geology present. The fact that the regional gravity-anomaly pattern shows no difference in the fundamental mass distribution present in the two eruptive areas despite the difference in the local geology in the two areas was the factor that determined to what extent changes in local geology should be considered. Both areas have maximum residual anomalies of about 110 mgal centered over the caldera areas and any difference in anomaly pattern is probably related more to the distribution of the gravity data than to the geology. With more abundant gravity data in the two areas, it might be possible to show a correlation with the observable geologic variations, but the present data did not warrant making such a detailed geologic analysis. That a correlation existed between the two caldera and the gravity values was in itself surprising.

Finally, it was apparently with well-justified trepidation that the writer ventured to extend his analysis so far as to try to account for the fact that although the two early eruptive centers had very large gravity effects associated with them, the later sites of volcanic activity appear to have no appreciable change in the gravity field associated with them. In this connection, Wentworth points out that the late stage volcanics in the Honolulu series were basaltic, and that the lack of a change in density the writer felt might be present as a result of compositional change at depth is not present. Wentworth's theory concerning the lack of a change in gravity field at these sites of late volcanic activity as a result of the explosive nature of their emplacement was also considered by the writer. However, this theory was discarded since it was felt that the lack of a density differential with the flow material which would require a porosity of 25 pct in the basaltic throat material probably would not be maintained over the entire length of the feeder pipe through the flow material.

We are thus confronted by a difference in personal opinions which cannot be resolved on the basis of the observable factual data. Any geologic interpretation of gravity data must by the very nature of the force field involved be speculative since the number of variables involved permit an infinite number of solutions. The writer in this case examined those solutions which appeared to be geologically reasonable and selected the one which he thought was the most reasonable. He naturally cited what evidence there was that might substantiate his choice. Wentworth objects to this evidence as being irrelevant and misleading since the surface geology is not in agreement. It undoubtedly would have been much more convincing if the local surface flow material had indicated the change in composition postulated at depth. However, the fact that the surface geology does not agree with a reasonable subsurface geologic condition hypothecated on geophysical data and other geologic observations in the area is not an a priori reason for condemning that hypothesis. If this were true, geophysics would have little or no value in geology.

DISCUSSION OF

"TIDE-PRODUCING FORCES AND ARTESIAN PRESSURES"
by WILLIAM O. GEORGE and FREDERICK E. ROMBERG

[Trans., v. 32, pp. 369-371, 1951]

Tom Culbertson (Consulting Geologist, P. O. Box 896, Fort Stockton, Texas)--I was an employee of the Texas State Board of Water Engineers in charge of the Fort Stockton field office at the time that the data for this paper were gathered. Since I have done both extensive and detailed ground-water work in the Fort Stockton area, including the setting and maintaining of the water-stage recorder at the Gonzales well, I believe I can add some pertinent hydrologic remarks about this paper.

The authors state that the principal water-bearing formation is a limestone. This opinion is not shared by myself and most likely the majority of other geologists in this area. ADKINS [1927] favored the basal Cretaceous sands as the principal source of water to Comanche Springs. I believe that the most extensive and the principal water-bearing formation is a sand and sandstone. The crevices and channels reported in wells and exposed at the springs are only a localized condition resulting from structural weakness and solution caused by a high water surface. The piezometric surface in sand and crevice wells is essentially identical; this suggests that there exists but one principal aquifer in this area. W. N. White, former District Geologist in Texas for the U. S. Geological Survey, in a personal communication to me in 1948, reported Comanche Springs to be the most reliable springs in Texas. This reliable flow strongly supports the concept of a sand aquifer, whose catchment area, or source, is of vast and varied extent, and a great distance from its outlet.

The specific capacity of a well in this area generally indicates whether the water is being transmitted directly to the well by open conduits, such as crevices or channels, or by the less permeable sand and sandstones. The wells having crevices or channels have prolific yields of anywhere from 1000 to 4500 gal/min with only a few feet of drawdown; the sand and sandstone wells yield usually less than 400 gal/min with drawdowns of as much as 100 ft. The Gonzales well, on test, made less than 400 gal/min with the pump set at about 150 ft. Furthermore, the driller of this well reports that if he obtained the water in the crevice or cavity from 45 to 49 ft, he would not have drilled down to the sand.

The authors report that no measurable effects on the observed well were caused from the withdrawals made by the city pumps. I believe that no attempt was made to correlate city pumpage with the variation of the water level in the Gonzales well because the city did not have meters on all their wells at that time. I believe that withdrawals for the city affected and will continue to affect the level of the water in the Gonzales well. The source of water is from a single artesian aquifer. I believe it would have been interesting to compare the withdrawals by the city, which have a definite diurnal rhythm, with the other curves shown in this paper.

Some interference effect from the pumpage of an irrigation well about two miles southwest of the Gonzales well had been identified on previous hydrographs of the Gonzales well. Intermittent pumping of the irrigation well had been occurring subsequent to obtaining the gravity-meter observations. Although the effects may be considered inconsequential, I believe some mention of this should have been made in this paper.

Since the Fort Stockton area is typified by erratic storms and spotty precipitation, I believe it significant to point out that the barograph was not located at the locality of the well but in a hotel room about two miles north.

Finally, the relationships shown and the suggestion made that the fluctuations in atmospheric pressure may be less important than the Earth tide in causing the diurnal fluctuations of the water level are considered premature if not altogether unwarranted. Until such time as the artificial forces due to withdrawals from wells are considered and a more careful atmospheric pressure record is obtained, it will be my impression that the atmospheric pressure changes are more significant in accounting for the diurnal water-level variations than the Earth tides suggested. I would like to point out that PARKER and STRINGFIELD [1950, p. 446] suggest caution in the interpretation of fluctuations similar to those shown on the hydrograph of the observation well, as follows: "Semidiurnal variations in atmospheric pressure with highs near midnight and noon, and lows near 06h and 18h, cause water-level fluctuations that, in places, may be mistaken for effects of oceanic tides." And I might add, Earth tides.

William O. George (U. S. Geological Survey, Austin, Texas) and Frederick E. Romberg (Geophysical Service, Inc., Houston, Texas; Authors' closure)--The authors are indebted to Culbertson for calling attention to water-level records obtained by means of a water-stage recorder subsequent to the observations of Earth tides made November 30 - December 1, 1949. From the observations made at that time, it appeared that "withdrawals (by city wells) had no measurable effect on the water level in the observed well." Although there is yet no evidence that the city wells did affect the observed wells, the statement should be modified to indicate that some interference could have and may have occurred. Notes recorded on the chart by Culbertson for June 1, 1950, indicate strong interference caused by an irrigation well about 1.7 miles from the observed well but in the opposite direction from the city wells.

The automatic control of city pumping is designed to keep the tank nearly full. In summer when consumption is high, pumping is nearly continuous and in winter the pumps shut off for short intervals during the day and for longer intervals during the night.

The hydrographs, recorded intermittently over a period of about a year, show the same type of diurnal fluctuations and fortnightly change in amplitude presented by ROBINSON [1939] in his scholarly analysis of similar data obtained from wells in Iowa and New Mexico. The daily retardation at Fort Stockton was not uniform; on some days there appeared to be none. However, the accumulated retardation had a daily average of nearly 50 minutes.

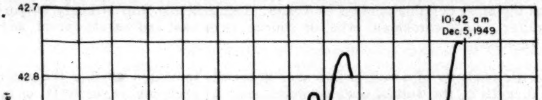

Fig. 2--Fluctuations of water levels in Well F-131, Pecos County, Texas, November - December, 1949

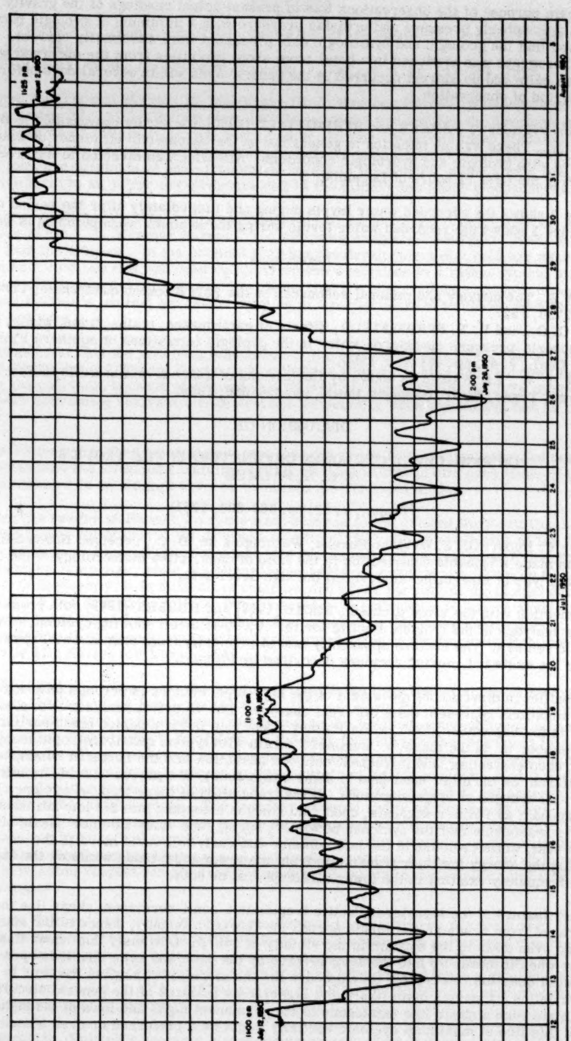

Fig. 3--Fluctuations of water levels in Well F-131, Pecos County, Texas, August, 1950

The main purpose of the observations was to present actual readings of the gravity meter in relation to atmospheric pressure and artesian pressure with a minimum of geologic background. It is believed that the geologic and hydrologic data presented were essentially correct. The recording barograph was stationed in a hotel room about two miles from the observed well. The weather was calm and no storms occurred in the general area within several days before and after the period of observation.

The observations of PARKER and STRINGFIELD [1950] are hardly comparable to those at Fort Stockton. There was no intention to generalize on the barometric efficiency of water wells which varies with geologic and hydrologic conditions. Attention was directed to the divergence of curves during the first 12 hours of December 2.

Figure 2 shows the recorded water levels during and immediately after the period of observations. Figure 3 shows the recorded water levels during the summer when pumping is greatly increased.

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DISCUSSION OF

"ON HIGH-LEVEL CYCLONES ORIGINATING IN THE TROPICS"
by C. E. PALMER

[Trans., v. 32, pp. 683-696, 1951]

Lester F. Hubert (U. S. Weather Bureau, Washington 25, D. C., received November 21, 1951)--Palmer has made a valuable contribution to the field of descriptive meteorology of the tropics by applying this type of streamline analysis to the high-level data.

In connection with the same problem, RIEHL [1948] and HUBERT [1949] both presented streamline analyses of the tropical Pacific for the 40,000-ft level, showing cyclonic as well as anticyclonic centers. The centers apparently moved primarily from east to west, however, in contrast to the eastward-moving cyclones described by Palmer.

The conflict in direction of movement of the high-level centers is perhaps only apparent because the situations represent quite different seasons. The 40,000-ft analyses with east-west movement is for September, 1945, while Palmer's analysis is for a period much earlier in the year. September is, in the mean, a "summer" month with typical quasibarotropic tropical atmosphere, deep easterlies, and infrequent cold-air advection into the northern hemisphere tropics. March, on the other hand (and in some cases June), is a period of cold-air intrusion behind upper-air troughs of large amplitude. If the translation of these tropical cyclones toward the east depends, as Palmer suggests, upon cold troughs extending into the low latitudes, then it seems quite reasonable that the cyclones of winter, spring, and early summer should often move toward the east, while cyclones in the late summer and early fall could move to the west, somehow deriving the energy for their translation from the lower level trade winds or the upper level easterlies frequently existing in the lower stratosphere, or both.

Another feature of the high-level Pacific tropics is a semi-permanent shear line that can often be traced from Southern California across the Hawaiian Islands, thence to the Marshall Islands, and even down to the equator in the vicinity of 160°E. Obviously this shear line is apparent only when it moves up into the area covered by the data, and even then it is open to a certain amount of question. Accepting its existence for the moment, this opens the door to an interesting speculation. It seems quite likely that large-scale features of the general circulation might maintain such a shear line (analogous to the permanent highs and lows of the higher latitudes), and this line of maximum cyclonic vorticity might be a preferred or even a necessary condition for the tropical cyclogenesis described by Palmer.

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Jerome Namias and William M. McMurray (U. S. Weather Bureau, Washington 25, D. C.; received December 27, 1951)--Palmer very lucidly proposes a model for the development of high-level cyclones which differs from that described by Palmén. Whereas those discussed by Palmén are cut-off pools emanating from the temperate-latitude upper-level westerlies, Palmer suggests that cold lows can originate at near-tropopause levels in the tropics, enjoy a long life, and ultimately give birth to an active low-pressure cell in the middle and lower tropospheres.

He chose to illustrate this intriguing hypothesis by re-analyzing a particular series of 500-mb charts done at the U. S. Weather Bureau. The alternate analyses he suggests seem to us less probable than the original. In the original analysis the development of a cold low near Hawaii was identified as a cold low that had been cut off from the westerlies in a rather typical manner. It is the purpose of this discussion to present evidence that the observed data support this conclusion rather than Palmer's, so that the reader may be in a better position to choose between the two series of analyses.

We emphasize that the following discussion of the relative merits of the two analyses is not prompted by the thought that the Northern Hemisphere Historical Map Series cannot be improved upon. On the contrary, because of limited data as well as human fallibility, it is likely that alternative, more convincing analyses can be devised--especially when the analyst has no time restruction.

Since Palmer proposes a completely new analysis of the data over an extended period, we have re-examined the situation carefully. Our contention is that in this case some of the data have been strained in an attempt to force them into his proposed model.

Unless the reader of Palmer's article had recourse to the printed Northern Hemisphere Historical Map Series for March, 1949, it would be difficult to make an evaluation of the two sets of analyses. This is so because the Historical Map Series were of necessity reproduced in approximately postage-stamp size, while the alternate re-analyzed solution was blown up to a full third of a page. Consequently, it is difficult to compare and assay the role of the plotted data. In our re-examination and comparison, it has been necessary to transcribe Palmer's analysis onto the large printed maps. Because of the difficulties of presenting the entire relevant data, the reader, in order to follow the arguments presented below, must refer to the Northern Hemisphere Historical Daily Map Series for March, 1949.

The first and most important disagreement is with the insertion of a closed cyclonic center at 500 mb in the vicinity of Kwajalein (8° 43′ N, 167° 44′ E) on Palmer's re-analysis for March 15, 1949. The existence of this center, as opposed to the original construction of a broad trough extending southwestward toward Kwajalein, is the very foundation of his argument that this vortex moved eastward until, some five or six days later, it appeared at Hawaii. He postulates five hypotheses as to what could have happened to this center and finally concludes that it must have moved to Hawaii. Objections to this conclusion are: Firstly, the data at hand do not necessarily favor a closed center rather than a trough. The fact that a center shows up five or six days later in the vicinity of Hawaii by no means proves the earlier existence of this center. Secondly, the continuity of motion of this hypothetical center is difficult to follow. For example, on his chart for March 18 (his Fig. 13) a closed low has been entered just to the southwest of a north wind of force 5. Those who still have some respect for the gradient wind relationship in the tropics can hardly condone this discrepancy.

Because Palmer's re-analyzed charts as displayed in his figures are for a very limited area, one cannot determine from them how the analysis would conform to data away from the periphery. On maps of a larger area the following question arises: On March 15 (Palmer's Fig. 10), the construction of a strong high-pressure cell north of Wake Island would force a gradient of unlikely strength between a point at 30°N, 155°E, and one at 39°N, 154°E, where a ship reports a height of the 500-mb surface of 17,350 ft.

On March 16, Palmer's analysis places a deep trough along 155°E longitude. This trough is inserted to the east of a principal surface trough and the strong cold front which appear on corresponding sea-level maps (note the time of the sea-level analyses). Differential analysis, by taking into consideration the additional information incorporated in the surface map, would have revealed this inconsistency. Also on his map for March 16, the contours are incorrectly drawn or labeled, since the Honolulu report of 18,990 ft does not fit with the interpolated value of less than 18,900 ft read from the contours.

In the original analysis a polar trough was drawn on the map for March 18 south and east of Honolulu. After some westward motion of the lower portion of this trough to 165°W on March 19, it becomes detached from the parent trough. The subsequent evolution into a closed cut-off low follows as fast westerlies spread across to its north. The reader may choose between this solution and that proposed by Palmer.

In conclusion, we believe that Palmer's re-analysis in this case offers no improvement on the original. We suggest that it would not be difficult to find other cases--particularly in the data-rich area of the Caribbean and southeastern United States during fall--that would illustrate very tenacious upper vortices giving birth subsequently to lower tropospheric cyclones. However, there is still a good deal of question as to whether those vortices originate in the tropics. Moreover, it is hoped that further studies of these important phenomena will throw light on the perplexing problems of how and especially when one of the high-level cyclones will give birth to a cyclone in the lower troposphere.

C. E. Palmer (Institute of Geophysics, University of California, Los Angeles 24, California, Author's closure)--Hubert's comments and suggestions, particularly his reconciliation of Riehl's, his own, and my descriptions of the movement of circulations at upper levels are well founded. I appreciate his drawing attention to his and Riehl's results in this discussion.

The points raised by Namias and McMurray may be divided into two classes: those that refer to technical aspects of synoptic analysis and those that are based upon implicit hypotheses concerning tropical meteorology held by these writers. I shall deal with the former class first. My analysis for March 16 has incorrectly labeled contours, as pointed out, and for this, attributable to a drafting transcription error, I apologize; the qualitative features of the analysis are not changed by correction of the error. The second technical point is that concerning continuity on March 18. If the writers will refer again to the original paper, they will see that, wherever possible, we have plotted observations of which we had a record in Los Angeles and supplemented them from other sources only when such data were missing; therefore, the wind at 20,000 ft was plotted for Johnston Island in preference to that at 500 mb and the fact that this wind is off-level is displayed on the map. However, the United States Weather Bureau maps show that the wind at 500 mb was east of north, possibly even north-east--this was the wind used in my analysis. As for the point about unlikely strength of the wind at 30°N, 155°E, I have no information on this and, in view of the very great strength of the gradients sometimes reported from the Japanese area, it would seem unwise for anyone to offer too firm an opinion at present.

So much for the technical points. The others, referring to positions of troughs aloft with respect to those at the surface, are based upon assumptions that relations usually observed in temperate latitudes will hold in the tropics. This is not necessarily true. It is not justifiable to assume mechanically that the air aloft to the west of troughs in the tropics is always cold. Then the use of differential analyses based on assumed lapse-rates can seldom add reliability to tropical analysis.

The purpose of that part of the paper which is criticised was to apply the hypothesis previously developed to the only maps in existence which are at the same time competently analyzed and easily available--those of the Historical Series. The features emphasized are the ones that are common to those maps and the analyses in the paper. The important point is that both the analysis in the Historical Series and that of my paper show strong cyclonic vorticity at 500 mb over the Marshall Islands on March 15, 1949, followed by a migration of this local maximum toward the Hawaiian Islands. Whether a low-contour center existed in the 500-mb surface, or whether a cyclonic wind center existed at that surface, is quite immaterial and cannot be decided on the observations available. By analogy with the situation of June 8-11, 1946, a cyclonic center in the wind field is possible or even likely.

The upper-air data over the Marshall Islands during atomic test periods are very plentiful for the regions of importance to the study of the subtropical high-level cyclones. It is possible that a study of these will be equally as rewarding as one in the Western Hemisphere where there are almost no high tropospheric data in regions between 0° and 15°N.

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

DISCUSSION OF

"FREE FALL OF DROPS THROUGH AIR"
by WALTER HITSCHFELD

[Trans., v. 32, pp. 697-700, 1951]

Robert W. Moorman (State University of Iowa, Iowa City, Iowa)--Hitschfeld has presented methods of solving the basic equation of accelerated motion which are noteworthy for their simplicity and utility. The excellent agreement with experimental data, evidenced by his Figure 2, shows conclusively the applicability of his assumptions within this limited range of Reynolds numbers. He does not, however, indicate a lower limit of drop size to which his technique should be employed. Previous investigations with solid spheres would cast considerable doubt on the advisability of the unrestricted use of the steady-state resistance in the analysis of accelerated free fall. The following résumé of the principal publications pertaining to the general case of the unsteady-state phenomenon should serve to illustrate the need for such a lower limit.

A general solution for the very slow motion of solid spheres was obtained analytically by BASSET [1910] and by VILLAT [1943]. They linearized the Navier-Stokes equations to disregard convective acceleration terms. Their results, though quite complicated in form, have been interpreted by S. F. Shen in an unpublished work performed at the Iowa Institute of Hydraulic Research. The velocity-time diagram obtained from this analysis is rather startling in form. The indicated rate of velocity increase is considerably less than that obtained from the simplified relationship based upon the assumption of the steady-state resistance force. The velocity approaches the terminal value of Stokes but with a distinct oscillatory pattern, the acceleration changing markedly in magnitude and even reversing in sign. This oscillatory variation appears to be more or less pronounced depending upon the ratio of the density of the solid to that of the fluid. Apparently the drag force in the general case of unsteady motion, at least within the range of very low Reynolds numbers, need not be at all analogous to that of the steady-state condition.

Velocity-time curves of a very similar character were obtained by SCHMIDT [1920] and by LUNNON [1928]. They performed experiments with pilot balloons ascending in the atmosphere and solid spheres settling in water at Reynolds numbers, the range of which extended far above that of the author's experiments. SCHILLER [1932] in discussing these results sought to explain the occurrence of rapidly changing acceleration by purely inertial effects, that is, the "virtual mass" of the fluid. He assumed, as did the author, that the drag force was a function of the instantaneous Reynolds number alone. To correlate Schiller's suggested equation with the experimental data would require that the "virtual mass" vary not only in magnitude but also in algebraic sign, a result that seems to the writer to have no physical significance.

The extension of the development of Basset and Villat, qualitatively speaking, to the case of motion beyond the laminar range would appear to be basically unsound. However, CARSTENS [1950], in his experiments dealing with a sphere oscillating in simple harmonic motion, found as a peculiarity of this type of accelerated motion that the laminar flow pattern persisted up to instantaneous Reynolds numbers exceeding several thousand. He concluded that the establishment of a stable separation which is characteristic of flow beyond the Stokes range is a function of time as well as the Reynolds number. It seems reasonable to assume that this condition will prevail in some measure in the case of free fall.

None of the foregoing should be construed to detract from Hitschfeld's excellent development. In presenting some of his interpretations of available material concerning the phenomenon of accelerated motion, the writer has attempted to establish the need for imposing limitations on the use of the methods suggested. Admittedly no quantitative value for the minimum drop size has been derived. The insufficiency of experimental information, which precludes the determining of that value, is a handicap to many branches of physical science. With this in mind, the writer is currently engaged in a study of certain aspects of the phenomenon.

In the author's paper, Eq. (9) is suggested as the criterion for fixing the drop size at which fluid deformation becomes serious and as such will mark the upper limit of applicability of his technique. A re-examination of the paper by BOND and NEWTON [1928], from which that equation was obtained, will show conclusively that the critical radius had nothing to do with the deformation of a fluid sphere. This relation was the result of a study of the motion of a drop through a fluid of much greater viscosity. As such drops were found to move sometimes as fluid spheres and sometimes as though effectively solid, the authors suggested that Eq. (9) defines the size which marks the limit between these two types of motion. Their results were inconclusive as to the validity of

this premise. Other investigators [see McNOWN, 1951] have explained this change from "fluid" to "solid" behavior as the result of contamination of the surface of the drop rather than a surface tension phenomenon.

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Walter Hitschfeld (Dept. of Physics, McGill University, Montreal, Quebec, Canada, Author's closure)--Moorman has made a very valuable addition to my paper by drawing attention to limitations of my theory, which I had referred to only in passing. His discussion of the complicated dependence of the drag force on time and various time derivatives of the displacement is particularly interesting. I hope that we can look forward to a more detailed clarification of this matter on his part and on that of his laboratory.

The question of the lower size limits of the drops, while a real problem in the general treatment of moving particles, is of little immediate significance in geophysics. My original treatment is valid for even the smallest cloud droplets encountered in the atmosphere.

Moorman's criticism of the applicability of the work of Bond and Newton is entirely justified.

DISCUSSION OF

"POROSITY FACTOR FOR CASE OF LAMINAR FLOW THROUGH GRANULAR MEDIA"

by J. B. FRANZINI

[Trans., v. 32, pp. 443-446, 1951]

Pier Lúigi Romita (Politecnico di Milano, Milano, Italy)--Franzini's paper is a useful contribution to the study of flow through granular media, and the author is perfectly right in taking into consideration the influence of porosity. As a matter of fact, porosity, which somehow accounts for the arrangement and compaction, and in part also for the shape of the elements of the medium, not only is the most easily measurable geometric characteristic of the medium, but also has the greatest influence on the resistance to flow.

Porosity in connection with the mean diameter of the granules brings forth, even if in a statistical way, the real cross-section size of the pores of the medium, the larger the assemblage, namely, the more casual, and hence isotropic, the arrangement of the granules, the greater the approximation. The size of the pores in turn determines the real velocity of flow, so that all the elements which, for a given fluid and for a given type of surface of the granules, make up the resistance to flow, ultimately depend on porosity.

The author finds that the results of his experiments show the most satisfactory agreement with the FAIR-HATCH [1933] expression of the porosity factor, which, he concludes, represents best the effect of porosity. There is no doubt that the functional relationship between diameter of granules, porosity, and size of pores, on which the Fair-Hatch formula rests, is the most rational among those presented so far.

The writer questions, however, the soundness of the basic assumption necessary to arrive at the Fair-Hatch formula, that the ratio of bulk velocity of flow to real pore velocity be equal to the porosity ratio a, or, in other words, that the "surface porosity" be equal to the "volume porosity." This assumption is made in a great number of studies on the subject, but the writer has never been able to find a correct derivation or a convincing application of it. The only attempt at a derivation

of which the writer is aware is presented in a study by FILLUNGER [1913], who practically follows the reasoning of DELESSE [1848], the first one who made this assumption, known as Delesse's law. Fillunger takes into consideration a parallelopiped with sides x, y, z and porosity ratio a. The volume of voids is $V_p = a\,xyz$. Differentiating with respect to z, the volume of voids contained in an elementary layer dz is found: $V'_p = a\,xy\,dz$. Division by dz provides then the value of the area of voids in a plane normal to z and successive division by xy the value of the surface porosity ratio, which is equal to the volume porosity ratio a. It seems to the writer that this derivation is correct only if the assumption is made that the porosity ratio a does not depend on z, in the sense that the z dimension of the porous parallelopiped considered does not contribute to its value. This comes to assuming a priori the proposition that should be proved. At any rate such an assumption is correct in general only when the pores are distributed according to some particular direction or pattern (for example, when the pores are all parallel to the z direction and have a constant aggregate cross-section), that is, when the isotropy of the distribution of pores in natural materials is discarded. Should, on the other hand, the xyz parallelopiped be looked upon as a schematical representation of the aggregate solid volume of the medium, Fillunger's derivation would hold only if the void volume were accordingly represented with a hollow coaxial parallelopiped, having two sides x′ and y′ different from x and y, and the third one equal to z. But since, as already mentioned, the voids distribution in a sufficiently large granular assemblage must be regarded as uniform along all directions, such a representation is not correct. The voids volume should rather be sketched as a hollow coaxial parallelopiped with sides $\alpha x, \alpha y, \alpha z$, α being less than one. If this is done, the value $a = \alpha x \cdot \alpha y \cdot \alpha z / xyz = \alpha^3$ is found for the volume porosity ratio, while, by derivation of the expression $\alpha x \cdot \alpha y \cdot \alpha z$ with respect to αz, which is the variable that substitutes z when the void volume is considered, the surface porosity becomes $a_s = \alpha^2 = a^{2/3}$.

On the other hand, even if simple patterns of pore distribution, both two- and three-dimensional, but geometrically isotropic, are considered in order to determine the difference existing between the volume porosity ratio a, the surface porosity ratio a_s, and what one might call, with an obvious meaning, the line porosity ratio a_ℓ, conclusions mostly different from Fillunger's are drawn. It can be seen, for example, that in the pattern presented in Figure 5a (the voids are represented by the hatched areas) the line porosity ratio is equal to ℓ/L, the surface porosity ratio to $(\ell/L)^2$, and the volume porosity ratio of the corresponding isotropic three-dimensional assemblage to $(\ell/L)^3$. Hence the ratio is respectively $a_\ell = 1/2$, $a_s = 1/4$, $a = 1/8$. In the pattern presented in Figure 5b the porosity ratios are $a_\ell = a_s = a = \ell/L = 1/2$, and in the assemblage of Figure 5c $a_\ell = \ell/L = 1/2$; $a_s = 3\,(\ell/L)^2 = 3/4$, $a = 7\,(\ell/L)^3 = 7/8$. Of the three now considered isotropic assemblages the one that approximates best the most probable pattern of the voids of a natural medium seems to be the first one, where the solid elements provide each other the most complete mutual support and, when not geometrically regular, can give rise to the common ovoid interconnected pores.

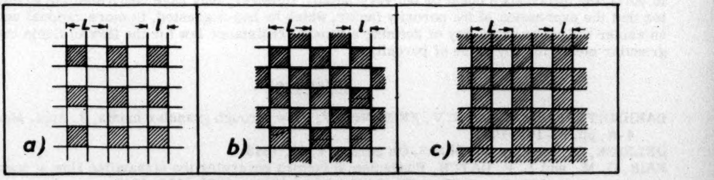

Fig. 5--Elementary patterns of pore distribution; hatched areas represent voids

As it is seen, the main cause of incorrect evaluation concerning the relation between volume porosity and surface porosity lies in the difficulty of "spatially seeing" the porous assemblage, and of lending due attention to the distribution of voids and solids along the third dimension which escapes the common two-dimensional representations; in a word, taking into account the real geometric isotropy of the assemblage. This difficulty can be overcome with following qualitative reasoning. Consider a cavity of a given permeable system and suppose that the porosity undergoes a certain variation; both the cross-section and the length of the cavity will be consequently varied. Hence the variation of the volume porosity depends on the variation of all three dimensions of the cavity, that is, on the cube of the characteristic linear dimension of the cavity, while the variation of the surface porosity depends on the square of this dimension. If the two porosities are equal before the variation, they are no longer so after the variation, and vice versa.

As a conclusion, the writer thinks that the real nature of things would be better represented by setting the surface porosity ratio equal to $a^{2/3}$, which also makes the ratio of bulk velocity of flow to real pore velocity equal to $a^{2/3}$. The same conclusion was reached by BAKHMETEFF and FEODOROFF [1937].

When this assumption is made, retaining the expression of the average size of the pores given by Fair-Hatch and others, the porosity factor considered by the author instead of

$$F(a) = (1 - a)^2/a^3$$

becomes

$$F(a) = (1 - a)^2/a^{8/3}$$

as the writer [ROMITA, 1951] has shown in a recent study on the flow through granular media. The law of variation expressed by this relation is clearly very similar to that represented by the Fair-Hatch expression of the porosity factor. As a consequence, while the two expressions lead to rather differing absolute values of the porosity factor, still, for a limited range of porosities, the relative values, referred to a particular value chosen as conventional unity within that range, differ very little. This is the reason why, as Figure 6 shows, both expressions agree almost equally well with Franzini's results.

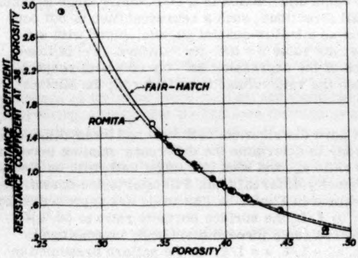

Fig. 6--Relative resistance coefficients versus porosity according to the Fair-Hatch and Romita formulas

The writer has also tried another comparison of the two expressions of the porosity factor based on experimental results obtained by GIVAN [1934] and by MAVIS and WILSEY [1936] on homogeneous assemblages with varying porosities. From these data the values of the resistance coefficient and of the Reynolds number have been computed, applying both porosity factor formulas, and plotted in a graph. The formula that better takes into account the influence of porosity should cause the experimental points to fall more closely on a single curve in such a system of coordinates. Owing to the limited range of porosity of these data (0.35 to 0.41), the differences could be but very small. However, they have confirmed the writer's opinion that the expression of the porosity factor, which he has suggested, is more rational and offers an easier and more exact way of defining a general resistance law for the flow of fluids through granular media for any value of porosity.

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Joseph B. Franzini (Civil Engineering Department, Stanford University, Stanford, California, Author's closure)--Romita has mentioned a point which perhaps warrants further investigation, namely, is the real pore velocity equal to the bulk velocity divided by the porosity or is it equal to the bulk velocity divided by the porosity to the two-thirds power?

Romita states that failure to give due attention to the distribution of voids and solids along the third dimension leads to errors in determining the relationship between volume porosity and surface porosity. This may be the case, but since porosity is merely a proportionality factor having no dimensions, should it be used solely as a volumetric parameter? The reasoning used by Fair and Hatch seems perfectly logical.

$$\text{Porosity} = \text{void volume}/\text{total volume} = (\text{void area} \times \text{length})/(\text{total area} \times \text{length}) = \text{void area}/\text{total area}$$

Taking this relationship and applying the necessary condition of continuity to the flow system leads one to the conclusion that the real pore velocity is equal to the bulk velocity divided by the porosity. Apparently Romita is not the only one who doubts the validity of the above statement. This is an argument which is difficult to resolve and mav never be settled to the satisfaction of all.

Transactions, American Geophysical Union Volume 33, Number 4 August 1952

REVIEWS, ABSTRACTS, AND BIBLIOGRAPHY

REVIEWS AND ABSTRACTS

PERRET, FRANK ALVORD, Volcanological Observations, Carnegie Institution of Washington Publication 549, 162 pp., 1950.

This is the fourth and unfortunately the last of this splendid series of studies of volcanoes, for Frank Perret died in 1942 before the paper had received final review. Earlier papers had been on The Vesuvius Eruption of 1906, The Eruption of Pelee, 1929-1932, and The Volcano-seismic Crisis at Montserrat, 1933-1937. The paper summarizes 40 years of patient and systematic observations of volcanoes, a life's work for which all students of volcanism will forever honor Frank Perret.

In Chapter 1 there is a discussion of the different types of volcanic structures and a presentation of a notable range of these. In Chapter 2 there is a detailed consideration of the "volcanic liquid" and the mechanism of flow. Chapter 3 takes up The Gaseous Elements in which he discusses the relation of gases to eruption, their composition, temperature, and modes of collection. Chapter 4 takes up Electric and Atmospheric Phenomena, and Volcanic Vegetation. Chapter 5 on Volcanic Ejectamenta considers the various types of pyroclastic materials, and the mode of their formation. Chapter 6 on Gravitational and Volcano-seismic Phenomena deals with the mechanisms of transportation of volcanic materials. Chapter 7 on Experimentation--Field and Laboratory, discusses the collection of lava and gases, the measurement of temperatures and equipment. Chapter 8 concludes with a discussion of Diagnosis and Prediction.

In all this discussion Perret develops his conclusions by citing direct observations from the many volcanic regions he has studied. The illustrations, nearly all Perret's own photographs, are a very valuable addition to the paper. The Carnegie Institution and Leason H. Adams, who helped in the editing, are to be congratulated for their valuable contribution to the science of volcanology.

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NOTES AND PERSONALIA

NOTES

Fulbright Awards for 1953-1954--Announcement has been made of the availability of Fulbright Awards for 1953-1954 for university lecturing and post-doctoral research in Europe, the Near East, Japan, Pakistan, the Union of South Africa, and the colonial dependencies of the United Kingdom. Among the awards to be made are: (1) for a specialist in seismology who, in addition to teaching courses bearing on exploration by seismic methods, will also be expected to teach a course in general geophysical exploration methods (France); and (2) a specialist in petroleum exploration research by electrical and magnetic methods involving six months of laboratory work and two or three months of field work (Italy). The closing date for making applications for these awards is October 15, 1952. Further information and application forms may be secured from the Conference Board of Associated Research Councils, 2101 Constitution Avenue, Washington 25, D.C.

Research projects of the Arctic Institute of North America--The Arctic Institute has made awards for carrying on 14 research projects, among which are the continuation of Project Snow Cornice, which is a study of the Seward-Malaspina glacier system and the Juneau Ice Field on the border of Alaska and Canada, and the study of Saskatchewan Glacier in Canada. The projects are supported financially by the Office of Naval Research.

Department of Geophysics established at the University of Alaska--A new Department of Geophysics for undergraduate and graduate instruction in geophysics has been established at the University of Alaska. A limited number of part-time assistantships are being offered. The facilities of the Institute of Geophysics will be available for research. The research program now emphasizes physics of the atmosphere, especially the aurora, and problems of the Arctic. For detailed information, write to the Department of Geophysics, University of Alaska, College, Alaska.

Kalinga Prize of UNESCO for science writing--A call has been made for candidates for the £1000 Kalinga Prize for science writing for the year 1953. Nominations should be sent to The Director, Department of Natural Sciences, UNESCO, 19, Avenue Kleber, Paris 16^{e}, France, before October 1, 1952. Nominations should include full name, title, and address of the nominee, together with four copies of the work upon which the nomination is based, which might be a book or books, a series of papers, or lectures or radio talks. Further information may be secured by writing the address given above. It has recently been announced that the winner of this prize for 1952 is Louis de Broglie.

Scholarships of the National Research Council of Canada--The National Research Council of Canada has awarded scholarships for 1952-1953 aggregating $227,900, including 18 for study abroad. In addition, five post-doctoral fellowships have been granted.

Third Alaska Science Conference--The Third Alaska Science Conference will be held September 22-28, 1952, at Mt. McKinley Park Hotel. C. T. Elvey, Director of the Geophysical Institute of the University of Alaska, is Chairman of the Geophysics Section which is planning to have sessions on the physics of the aurora, radio-wave propagation, meteorology, and other topics. The meeting will also include the annual meeting of the Alaska Division of the American Association for the Advancement of Science. Details may be secured by writing to C. T. Elvey at College, Alaska.

Third Annual Field Conference of New Mexico Geological Society--This conference will be held October 3-5, 1952, in the Rio Grande Trough in central New Mexico. The trip will start from Santa Fe where registration will take place on October 2. Interested persons should write to the headquarters of the Society, P. O. Box 27, Campus Station, Socorro, N. M.

Japanese deep-sea investigations--Deep-sea investigations, using a bathysphere, are to be undertaken soon in Japanese waters. Funds to support the work are coming from various sources, including a substantial contribution from the newspaper *Yomiuri Shimbun*. U. Nakaya is chairman of the committee on the administration and use of the bathysphere, and Koji Hidaka is a member of the committee. It is anticipated that N. Inouye will make the first descent.

Meeting of European geophysicists--The European Association of Exploration Geophysicists held its first annual meeting in London, May 22-23, 1952. This Association was established at The Hague in December, 1951, and now has a membership of more than 400. At the meeting in London, 87 members and visitors were in attendance, and nine technical papers were presented. It is planned that the next meeting will be held in Hannover, Germany, in November, 1952.

Seismological research station in Pakistan--The first seismological research station in Pakistan is now being constructed with the aid of UNESCO technical assistance, under the direction of the Geophysical Division of the Pakistan Meteorological Service in Quetta, Pakistan.

PERSONALIA

E. E. Conant, formerly West Texas Supervisor for the United Geophysical Company with headquarters in Midland, Texas, has taken up duties as Assistant to the Manager of the Geophysical Prospecting Company of Canada with headquarters in Calgary.

Pierre Danel, Head of the Laboratoire Dauphinois Hydraulique in Grenoble, has been awarded one of the first honorary degrees of Doctor of Technology by the Hebrew Institute of Technology.

Mortimer Datz is now engaged in research work at the Geofysiske Institutt in Bergen, Norway.

Carroll E. Dobbin, Geologist, U. S. Geological Survey, was awarded an honorary degree of Doctor of Engineering by the Colorado School of Mines.

Dean K. Fuhriman, until recently Associate Professor of Civil Engineering at Colorado Agricultural and Mechanical College, has recently taken his doctorate from the University of Wisconsin, and has accepted a position as Associate Professor of Irrigation and Drainage at Utah State Agricultural College.

C. E. Jacob, Professor of Geophysics at the University of Utah, is now in Formosa under the sponsorship of the Mutual Security Agency as a consultant to the Chinese Nationalist Government on ground-water problems.

Ade E. Jaskar of Brush, Colorado, has taken up duties as Soils Engineer with Knappen, Tippetts, Abbett, and McCarthy, Consulting Engineers, and is stationed in Tripoli, Libya.

Esper S. Larsen, Jr., President of the Section of Volcanology, AGU, has been elected a Foreign Member of the Geological Society of London in recognition of his contributions to geological science.

Walter C. Lowdermilk has been awarded one of the first honorary degrees of Doctor of Technology by the Hebrew Institute of Technology.

Raymond C. Moore, Professor of Geology at the University of Kansas, and Director of the Kansas State Geological Survey, is Visiting Professor in The Netherlands during the current year, teaching at Rijks Universiteit in Utrecht. He is also to lecture at the Sorbonne in Paris, Louvain and Liége Universities in Belgium, and universities in Sweden and Norway. He was recently elected Membre Correspondant of the Société Géologique of Belgium and a Foreign Member of the Geological Society of London.

Francis W. Murray, Captain, U. S. Air Force, has taken up duties in the Directorate of Scientific Services, Air Weather Service, with headquarters in Washington, D. C.

Ukichiro Nakaya of Tokyo, Japan, specialist in the study of snow crystals, has arrived in the United States for two years of study and research with the Snow, Ice, and Permafrost Research Establishments with headquarters at Wilmette, Illinois.

Francis J. Pettijohn, formerly Professor of Geology at the University of Chicago, has been appointed Professor of Geology at Johns Hopkins University.

Donald A. Rice, Chief, Section of Gravity and Astronomy, U. S. Coast and Geodetic Survey, will attend the General Assembly of the International Astronomical Union at Rome in September, 1952.

Lorenzo A. Richards, Soil Physicist of the U. S. Salinity Laboratory, has been awarded one of the first honorary degrees of Doctor of Technology by the Hebrew Institute of Technology.

William M Scaife, Chief, Division of Coastal Surveys, U. S. Coast and Geodetic Survey, retired on April 30, 1952. Earle O. Heaton has been named as his successor.

E. R. Shepard, formerly in charge of foundation explorations by geophysical methods for the Corps of Engineers, has taken charge of the newly organized Geophysical Exploration Division of the Georator Corporation, with headquarters in Arlington, Va.

Newbern Smith, Chief of the Central Radio Propagation Laboratory of the National Bureau of Standards, has been given the Harry Diamond Memorial Award for 1952 for his fundamental work in radio-wave propagation.

Joshua L. Soske of the Geophysical Engineering Corporation has been appointed to the Henry Salvatori Associate Professorship in Geophysics at Stanford University. The Salvatori Chair was established by a gift from Henri Salvatori, President of the Western Geophysical Company, who also donated equipment for the new Geophysical Laboratory at Stanford.

Andrew Thomson, Controller of the Meteorological Division, Department of Transport of Canada, has been awarded the 1952 Gold Medal of the Professional Institute of Public Service of Canada for his outstanding contribution to national and world well-being.

Arthur C. Trowbridge, Professor of Geology and Head of the Department at the University of Iowa, is retiring. He has been with the Department since 1911, and has served as its Head since 1934. His successor as Head of the Department is Arthur K. Miller.

Aaron C. Waters, formerly Professor of Geology at Stanford University, has been appointed Professor of Geology at Johns Hopkins University.

George D. Whitmore of the U. S. Geological Survey has been elected President of the American Society of Photogrammetry.

Abel Wolman, Professor of Sanitary Engineering at Johns Hopkins University, has been appointed consultant to the U. S. Air Force on sanitary engineering work.

Joseph W. Gross, Consulting Engineer with headquarters in Sacramento, California, died April 26, 1952. He was born in Mitchell, South Dakota, September 13, 1884, and received his baccalaureate degree from the University of California in 1907. His professional career was devoted chiefly to hydraulic engineering problems. Until 1920, his diverse experience in various organizations gave him the broad background that made his work as consulting engineer successful. He opened his own office in 1920 and continued actively in this work until his death. He had been a member of the American Geophysical Union since 1947.

Andrew Cowper Lawson, Professor Emeritus of Geology of the University of California, died June 16, 1952, in Berkeley, California. He was born in 1861 in Anstruther, Scotland, and took his baccalaureate degree from the University of Toronto in 1883. He took his master's degree two years later from the same university, and the doctorate from Johns Hopkins University in 1888. He was awarded several honorary doctorates. He began his professional career with the Geological Survey of Canada, entered upon duties at the University of California in 1890, becoming Dean of the College of Mining before his retirement in 1928. He was recognized as an authority on earthquakes and in other matters bearing on geology, mining, and geophysics. He had been a member of the American Geophysical Union since 1947.

Oscar Rosenzweig, Hydraulic Engineer for the Corps of Engineers in the Philadelphia District, died March 4, 1952, at the age of 53. He was born in Russia and received his baccalaureate degree from the University of Pennsylvania. He entered the service of the Corps of Engineers in 1919 and devoted his entire career to this work. He had become an authority on many problems of hydrology and flood-control work, and on salinity problems in tidal estuaries. He had been a member of the American Geophysical Union since 1944.

MINUTES OF EXECUTIVE COMMITTEE MEETING, AMERICAN GEOPHYSICAL UNION

Meeting of May 7, 1952

The meeting was called to order at 19h 30m in the Board Room of the National Research Council - National Academy of Sciences Building, 2101 Constitution Avenue, Washington, D. C., by President Walter H. Bucher.

Present--Twenty-six persons were present, namely: L. R. Alldredge, K. Hilding Beij, Walter H. Bucher, Wilbur S. Burbank, Perry Byerly, Frank L. Culley, H. N. Eaton, Jno. A. Fleming, R. Clifton Gibbs, Harry H. Hess, M. King Hubbert, Woodrow C. Jacobs, J. W. Joyce, W. D. Lambert, Walter B. Langbein, Esper S. Larsen, Jr., J. Putnam Marble, Leonard M. Murphy, A. C. Redfield, W. W. Rubey, M. B. Smith, Waldo E. Smith, Harlan T. Stetson, F. J. Veihmeyer, James T. Wilson, and Ralph N. Wilson. The following were excused: L. H. Adams, Fred Anderegg, Robert Balk, H. R. Byers, Ernst Cloos, Felix Chayes, Beno Gutenberg, H. W. Hemple, Benjamin Holzman, M. A. Mason, G. F. McEwen, Mary Rabbitt, F. W. Reichelderfer, Irwin Roman, Lorenz G. Straub, W. T. Thom, Jr., Victor Vacquier, and Hurd C. Willett.

Approval of Minutes--The minutes of the meeting of October 15, 1951, published in the Transactions of December, 1951, pages 934-940, were approved.

General Secretary's Report

General Activities

Executive Committee--There has been only one change in the Executive Committee since our last meeting; P. A. Weiss of the University of Chicago has succeeded Ralph E. Cleland as one of the representatives of the NRC on our Committee.

Financial Report--A detailed, audited financial statement for the fiscal year ending June 30, 1951, was published with the minutes of the October meeting.

Our cash balance as of May 1, 1952, was $19,343.76 as compared with $18,499.83 a year ago and $15,370.44 two years ago. In addition the Permanent Fund totals $7,175.20 as of May 1, 1952, as compared with $3,384.56 on May 1, 1951.

Membership--Our membership as of October 1, 1951, was given as 4,718. As of May 1, 1952, this is 4,775, a gain of 57.

Membership, October 12, 1951	4,718	
New members, October 12, 1951 - April 30, 1952	215	
Reactivated	1	4,934
Resignations	125	
Deceased	6	
Inactive	10	
Dropped	18	159
Membership, April 30, 1952		4,775

Those reported as deceased since our last meeting are as follows: Laurie Reid Burgess, August 23, 1951; Daniel Lyman Hazard, September 21, 1951; Charles I. Singer, December 10, 1951; Theodore Zuschlag, January 14, 1952; T. Wayland Vaughan, January 16, 1952; and Paul Felix Nemenyi, March 1, 1952. W. F. Sprengnether, Jr., representative of the corporation membership of his company, was drowned August 30, 1951.

Members who had not paid dues 1950 to date were reported in accordance with By-Law 13, with the recommendation that they be dropped but that one more attempt be made to bring them into good standing.

Corporation Membership--Our corporation members now total 11. Five new ones have been added since our meeting in October: Crop-Hail Insurance Actuarial Association, D-Y-M Corporation (elected at this meeting), The Geotechnical Corporation, Magnolia Petroleum Company, and Tidelands Exploration Company. One corporation member has been lost in the untimely death of W. F. Sprengnether, Jr.

Meetings and Regionalization--The Twenty-Seventh Regional Meeting was held in Pasadena, California, February 8-9, 1952, with the Sections of Meteorology and Hydrology joining. At this Regional Meeting by-laws were adopted for the regional organization (approved by the Executive Committee at this meeting). This Region will hold its usual meeting in February of 1953, and it is hoped that an increasing number of Sections will join in this activity. Steps have been taken to organize the Hawaiian Chapter of this Region.

The Thirty-Third Annual Meeting had a registered attendance of 924, with a total attendance well exceeding 1,000. The Sections of Meteorology and of Terrestrial Magnetism and Electricity joined with the American Meteorological Society in their portion of the meeting.

The Twenty-Eighth Regional Meeting is planned for Chicago in September, 1952, in connection with the Centennial of Engineering. Because Horace R. Byers, Chairman of the General Committee, was elected President of the American Meteorological Society, he desired to resign from the chairmanship. Wallis S. Hamilton of Northwestern University has been designated as his successor. Plans are progressing well and we believe that there will be a good attendance.

The AAAS with whom we are now officially affiliated desires that we co-sponsor meetings bearing on geophysics to be held at their Annual Meeting in St. Louis just after Christmas, 1952. The AAAS meeting in December, 1953, will be in Boston, in 1954 in San Francisco, in 1955 in Chicago, and in 1956 in New York City.

Lorenz G. Straub has polled the States in the Middle West, west of the Mississippi River, to determine whether or not there is sentiment for a regional organization there as far as the hydrologists are concerned. He has had a very favorable response and recommends that steps be taken to organize a Regional Committee. Tentative plans are being laid for a Regional Meeting to be held in Minneapolis in September, 1953, in connection with the International Association of Hydraulic Research of which Straub is the President.

Publications--We are currently publishing our Transactions in an edition of 5,400. The April issue was mailed out on April 28. A total of 4,835 were sent out at that time. About 200 additional copies have been sent by International Exchange.

UNESCO made a payment through the International Union of Geodesy and Geophysics in the amount of $500 to help defray the costs of the publication of the Colloquium on Plastic Flow and Deformation within the Earth.

Student subscriptions are about on the same level as last year, the total being about 25. A number of those who were student subscribers last year are now members. The rate for the current calendar year is $4.00, the same as for 1951.

The Index for the Transactions of the first 25 years, 1920-1944, was finished in the late fall and publication completed about December 10. An edition of 1,500 was ordered. A thousand of these were sent to purchasers and the remainder is in stock for sale.

Administration--Arrangements for quarters for the Union have been continued by the courtesy of the Carnegie Institution of Washington through next January.

Elections--Next year, 1953, is an election year for AGU and nominating committees should be reactivated. A notice was sent out to the Section officers pointing out that the time table for elections was originally set up in 1946 and the details were published in the December, 1946, Transactions, pages 910 to 914. Some slight changes were made in connection with Seismology and Hydrology in 1949; these were published in the August Transactions of 1949, page 627. All nominating committees are urged to get busy in order that the time table may be maintained. This involves the idea that our Executive Committee meeting this fall must be held about November 10 as Executive Committee action upon the report of the Union's Committee on Nominations will be required.

News Items--It is of interest to note that five of the 30 new members elected to the National Academy of Sciences are members of the American Geophysical Union. These are Horace R.

Byers, David T. Griggs, Harry H. Hess, Chaim L. Pekeris, and Lyman Spitzer, Jr.

Frank Press of Columbia University and Gordon G. Darkenwald of Hunter College were designated as AGU representatives to the Third National Conference of the U. S. National Commission for UNESCO held in New York City, January 27-31, 1952. A brief report from Press indicated that he felt that this meeting was well worth while.

Joseph C. Boyce was designated as representative of the American Geophysical Union at the inauguration of the new Chancellor of the University of Chicago on October 18, 1951.

Ernst Antevs of Globe, Arizona, was designated to represent the Union at the inauguration of a new President of the University of Arizona, November 16, 1951.

E. Wm. Heinrich of the University of Michigan acted as our representative at the inauguration of the new President of the University of Michigan, November 27, 1951.

The Pennsylvania Water and Power Company arranged an interesting session on hydrology, held in Harrisburg, April 22-23, 1952, which was attended by various members of the American Geophysical Union including Walter B. Langbein, Vice President of our Section of Hydrology.

Activities as Committee on Geophysics of the National Research Council

Bibliography of Hydrology--The International Association of Hydrology is sponsoring an International Bibliography, encouraging the various National Committees to undertake this work for their countries and, inasmuch as this is of considerable value in this country, various federal agencies concerned with water resources have underwritten the cost of the work of compiling the Bibliography of Hydrology for the United States and Canada for the years 1941-1950, inclusive. The compilation has been completed by Ray K. Linsley. The editing and preparation of the master copy are in progress and it is anticipated that the master copy will be turned over to the Bureau of Reclamation by June 30, 1952. This will be printed by the Government Printing Office.

Preparation of "Employment Outlook for Earth Scientists"--We have continued to work with the Bureau of Labor Statistics in the preparation of their publication "Employment Outlook for Earth Scientists" and we are anticipating that this will be published at an early date (published in late May, 1952).

Education in Geophysics--The Office of Education in its survey of the facilities of colleges and universities has in its current records data and information concerning the current status on education in geophysics. This will be extracted, thoroughly checked, and presumably ready for publication in the late fall of 1952, and will replace the material published in the December, 1945, and August, 1946, Transactions.

American Committee for the Third Polar or Geophysical Year--At the meeting of the International Union of Geodesy and Geophysics in Brussels in 1951, the plan was offered to arrange a Third International Polar Year in 1957-58. The International Union voted its support of this activity, but declined to take the lead in its sponsorship or organization. At the Executive Committee meeting last October this body took similar action. The plan to proceed is being developed and a call has come to the National Research Council to establish an American National Committee for this particular job. The organizations chiefly interested in the Polar Year are apparently the Mixed Commission on the Ionosphere of the International Council of Scientific Unions and the International Scientific Radio Union (URSI).

Meeting of the International Union of Geography--The International Union of Geography is to hold its meeting in Washington, August 8 to 15, 1952. The National Research Council has invited the AGU to designate representatives.

The President's Report on the Brussels Meeting--In the fall of 1951 the President filed his report on the Ninth General Assembly of the International Union of Geodesy and Geophysics held in Brussels last August, which he attended as Chief Delegate from the United States. The report was very well received by the State Department as a constructive contribution to the literature relating to the International Union. It points out many aspects of Union activity which we often tend to overlook, forgetting that we function also as the American National Committee of the International Union. One of the projects which the President desires to emphasize is the need for an international roster of those actively working in various aspects of geophysics as represented by each Section. He feels that a great international service could be rendered by the compilation of such a roster as was done by Joyce in the case of Terrestrial Magnetism and Electricity, and for Oceanography by the Allan Hancock Foundation. Ways and means should be found through the International Associations or Union or otherwise to prepare and keep such rosters up to date.

K. Hilding Beij,
General Secretary

Action on the General Secretary's Report--The report was accepted.

President Bucher took the opportunity to underline the importance of and need for an international roster of the active workers in the various branches of the broad field of geophysics.

International Matters--Honorary President Fleming, always in close touch with international scientific matters, presented a report as shown in Appendix 1.

Nominations to the National Research Council for the American Committee for the Third Polar or Geophysical Year were made as follows: Lloyd V. Berkner, Chas. R. Burrows, J. Wallace Joyce, Elliott B. Roberts, and Newbern Smith. It was agreed that the meteorologists and oceanographers might also offer nominations, these to be in by May 17. (Subsequently these reports have come in. Harry Wexler has been added to the nominees, representing meteorology. Oceanography asks to be kept informed and offers cooperation.)

The nomination by the Section of Seismology of Perry Byerly to be representative to the broadened Association of Seismology and Physics of the Earth's Interior for the balance of the current triennium was approved.

Action on Recommendations of the General Secretary--The pre-war debt of the Maruzen Company of Japan amounting to about $85 was forgiven.

The D-Y-M Corporation was elected to corporation membership.

The recommendations of the General Secretary with respect to members who had not paid dues from 1950 to date were approved.

Reports of Standing Committees

Statutes and By-Laws--Chairman McClain B. Smith recommended approval of the By-Laws adopted by the Pacific Southwest Region. These were approved (see Appendix 2).

Meetings--Chairman Marble presented the report shown in Appendix 3. This was accepted.

Budget--Chairman Eaton presented the budget prepared by his Committee for the fiscal year ending June 30, 1953, as shown in Appendix 4. This, after extensive discussion, was approved, including the recommendations in the report.

Reports of Special Committees

Cosmic-Terrestrial Relationships--Chairman Stetson presented a summary of the Committee's report shown in Appendix 5. This was accepted.

National Research Council Reports

National Research Council--Chairman Rubey discussed Council policies and problems, indicating sources of Council income, its obligations, and its work.

Division of Physical Sciences--Chairman Gibbs presented a brief report relating to the National Science Foundation scholarship program which is administered by the National Research Council, pointing out that there were about 2,800 applicants for 569 pre-doctoral fellowships, and 300 applicants for 55 post-doctoral fellowships. The awards were made in proportion to the number of applicants in the various fields. He also reported briefly on the work of the Committee on Geophysics Advisory to the Office of Naval Research.

Affiliated Organizations

American Geological Institute--W. T. Thom, Jr. (not present) submitted the report shown in Appendix 6. Our other representative on the Board of Directors of the Institute (George P. Woollard) has concurred in this.

American Association for the Advancement of Science--J. Putnam Marble presented the report shown in Appendix 7. Our other representative on the Council, Waldo E. Smith, has concurred in this.

Adjournment--The meeting adjourned at 22h 35m.

K. Hilding Beij, General Secretary
Waldo E. Smith, Executive Secretary

APPENDIX 1--REPORT ON INTERNATIONAL RELATIONS AND RELATED MATTERS, 1952

On July 1, 1951, the National Academy of Sciences (NAS) established a new Section on Geophysics consisting of 17 members, of which 16 had been elected through the efforts of the Temporary Nominating Group on Geophysics during the previous nine years.

At the spring, 1952, meeting of the Academy, Chaim L. Pekeris and Horace R. Byers were elected members under the sponsorship of the Section. David T. Griggs was an intersectional nomination by the Sections of Geology and of Geophysics. Another geophysicist, Harry H. Hess, was elected on nomination of the Section of Geology. Harald U. Sverdrup, now a citizen of Norway and formerly a member of NAS when he was a citizen of the United States, was elected a Foreign Associate of NAS. (Byers, Griggs, and Pekeris later chose Geophysics, thus giving that Section 20 members as of July 1, 1952.)

Details of the General Assembly of the International Union of Geodesy and Geophysics (IUGG) at Brussels in 1951 are given in the report of our President who was officially the head of the delegation from the United States appointed by the Department of State.

The Executive Board and Bureau of the International Council of Scientific Unions (ICSU) held a meeting upon the invitation of NAS in Washington at the Academy-National Research Council Building in October, 1952; nearly all members of the Board attended this very successful meeting, the last day of which was devoted to a public Symposium on Interational Relations in Science with three addresses by members of the Board from abroad and three American speakers including the President of the National Research Council of Canada; each address was discussed and the Symposium was quite successful in presenting an insight into the operation of the Council and its Unions.

The National Research Council has now appointed an Office (formerly called Division) of International Relations with Wallace W. Atwood as Executive Secretary and Roger Adams as Chairman. This Office has already done much to clarify International relations in science. The NAS has established a bimonthly News Report which carries current items of International interest, including calendars of meetings.

General Assemblies of various Unions are set in 1952 as follows: Scientific Radio at Sydney, Australia, in August; Geography at Washington in August; Theoretical and Applied Mechanics at Istanbul in August; Astronomy at Rome in September; and ICSU at Amsterdam in October. The reestablished Union of Mathematics met at Rome in March.

Meetings of Joint Commissions are set in 1952 as follows: Ionosphere at Canberra, Australia, in August; Solar and Terrestrial Relationships at Rome in September; Radiobiology at Stockholm in September; and Spectroscopy at Columbus in September.

Deposits of some funds for ICSU and for IUGG have been maintained in the Riggs National Bank in Washington. During the year, payments for adherence to IUGG have been received from the governments of Colombia (1949-52), Canada (1952), and United States (1951); the present balance is $8,394.01. The deposits for ICSU have been dollar grants-in-aid from UNESCO; the present balance is $29,904.50.

Copies of the Bulletin ICSU have been made by the Office of International Relations of the National Research Council and distributed promptly to interested persons in the United States. Pertinent matters relating to meetings of ICSU, its Joint Commissions, and its Unions, with brief notes on meetings, are reported in the Academy-National Research Council News Report. Interested persons should address Dr. Wallace W. Atwood, Executive Secretary, Office of International Relations, National Research Council, Washington 25, D. C.

The final report on the International Polar Year 1932-33 has been published; it is the volume "Bibliography for the Second International Polar Year 1932-33" just issued from Copenhagen for distribution from the office of the Polar Year Archives (V. Laursen, Meteorological Institute, Charlottenlund, Denmark).

Our Committee, as the Committee on Geophysics of the NRC, should make a recommendation regarding official delegates to the several General Assemblies of Unions and of ICSU. The Office of International Relations has already asked the Committee on International Scientific Unions to make nominations for delegates to the General Assembly of ICSU to be held in Amsterdam on October 1-3, 1952, in accordance with General Secretary Stratton's invitation as follows: "It is hoped that your country will be represented by delegates. Details of local arrangements will be forwarded later. Any resolution that your national adhering organization may wish to submit to the General Assembly should reach me not later than May 31, 1952." It is hoped that the Committee on International Scientific Unions will meet about May 17.

Another matter to be considered is the Third International Polar Year (to be called Geophysical Year) in 1957-58 as proposed by the International Scientific Radio Union. The international Committee on this matter is now in formation, but the work will have to be organized under national committees appointed for the purpose. W. W. Atwood's letter of February 13 requests that this matter be called to our attention and asks our assistance in the formation of a national committee. He is concerned with the designation of some one on such a committee to act as correspondent with the International Committee to be set up by ICSU.

(Walter D. Lambert might be considered for selection as a delegate of the United States to the General Assemblies of ICSU and of International Union of Astronomy this summer, if funds for travel could be provided.)

Jno. A. Fleming

APPENDIX 2--BY-LAWS FOR THE PACIFIC SOUTHWEST REGION, AMERICAN GEOPHYSICAL UNION

Preamble

To foster an effective and strong program of scientific activity and thus to permit the American Geophysical Union to attain more fully its goals of service to its members and to the broad field of Geophysics, the Pacific Southwest Region of the American Geophysical Union is hereby established in accordance with Article 10 of the Statutes and By-Law (9) of the American Geophysical Union.

(1) This shall be known as the Pacific Southwest Regional Branch of the American Geophysical Union, and shall comprise the States of Arizona, California, and Nevada, the Territory of Hawaii, and all other territories, possessions, dependencies, or mandates of the United States in the Pacific Ocean south of the 40th parallel. Its objectives shall be to promote the aims and objectives of the American Geophysical Union within the area covered by this Branch.

(2) Members of this organization shall be members in any grade of the American Geophysical Union who reside within the area covered by this Branch.

(3) To defray the expense of meetings a registration fee, fixed by the Regional Committee, shall be charged.

(4) The Regional Committee shall include at least one and not more than three representatives of each Section electing to take part in the regional activity of this Branch. An individual shall not represent more than one Section. It shall not consist of more than nine members except as provided in paragraph 7. Elections shall be held by mail ballot toward the close of each triennium, beginning with the triennium ending June 30, 1953. The Committee shall draw up its own rules for the elections. Upon its election, the Committee shall be organized by the Chairman of the previous Committee. The new committee shall designate a Regional Chairman and Regional Secretary-

Treasurer, whose terms shall correspond to the terms of the Committee.

(5) The Regional Committee may designate such sub-committees as are appropriate for carrying out the work of the Branch. In addition, each Section active in the Region may have its Regional Committee to facilitate work in that field. One of the representatives of the Section on the Regional Committee shall be Chairman of the Sectional Committee.

(6) Vacancies on the Regional Committee shall be filled by the Chairman of the Regional Committee of the Section concerned or, if there is no such Committee, by the Section President.

(7) Arrangement of the regional meetings (probably not more than one a year) will be one of the important functions of the Regional Committee. These shall be at times and places set by the Committee, bearing in mind that if, in a given year, one Section desires to hold a meeting and the other seven do not, the seven shall not stand in the way of the one. Similarly, should members of a Section not previously active in the Branch desire to partake in a planned meeting, arrangements shall be made to facilitate this. Provisions shall be made to accept representation of the Section on the Regional Committee, which may thus continue with a membership in excess of nine until the end of the triennium. All regional meetings, any expenses of which are met by the Union, are subject to the approval of the Executive Committee of the Union.

(8) Localities within the Region where there is a concentration of members may, on the desire of a number of such members, organize a local committee along the lines of the organization of the Regional Committee, with the aid and support of the Regional Committee. Such local Committees will have a degree of autonomy consistent with high scientific endeavor, but maintaining close contact with the Regional Committee and the Executive Committee of the Union. Recognizing that local meetings will be of the luncheon or evening type which may be arranged on short notice, prior approval is not necessary; however, the Regional and National organizations should be advised promptly of plans.

(9) As soon as possible after June 30 of each year, the Regional Secretary shall submit a brief report of the year's activities to the General Secretary of the Union.

(10) The By-Laws may be amended by a majority of the members in the region voting, subject to the approval of the Executive Committee of the Union. In case of conflict with the Statutes and By-Laws of the Union, the Union takes precedence over the Branch.

APPENDIX 3--REPORT OF THE COMMITTEE ON MEETINGS

John Putnam Marble, Chairman

Woodrow C. Jacobs
Herbert S. Riesbol
Walter E. A. Ruska
Glen E. Stout
Frank J. Veihmeyer

During the year 1951-52 there have been held the Twenty-Seventh Regional Meeting by the Pacific Southwest Region in Pasadena, California, in February, 1952, and the Thirty-Third Annual Meeting in Washington on May 5-6-7, 1952. Plans are well under way for the Twenty-Eighth Regional Meeting in Chicago in September, 1952, to be held in connection with the Centennial of Engineering. Arrangements are being made for the Twenty-Ninth Regional Meeting in California in February, 1953. Preliminary plans are being made for a Thirtieth Regional Meeting at Minneapolis in September, 1953.

We plan to co-sponsor one or more sessions on various topics in geophysics at the American Association for the Advancement of Science meeting in St. Louis on December 27-30, 1952, but presumably without formal meetings of any of our Sections.

The Thirty-Fourth Annual Meeting will be held in Washington next year on the Monday, Tuesday, and Wednesday immediately following the sessions of the National Academy of Sciences. Our dates will presumably be May 4-5-6, 1953.

Further details of all meetings will be published in the Transactions as plans develop. Ample notice will be given, as far in advance as possible.

Once more we extend our heartiest thanks to the Secretaries, the Section officers, and the various local groups who have ensured the success of all the meetings.

APPENDIX 4--REPORT OF THE COMMITTEE ON BUDGET FOR THE FISCAL YEAR ENDING JUNE 30, 1953

Herbert N. Eaton, Chairman

Jno. A. Fleming	H. W. Hemple
Nathan C. Grover	Waldo L. Schmitt

The Committee on Budget of the American Geophysical Union has conferred with the General and Executive Secretaries of the Union regarding the budget for the fiscal year July 1, 1952, to June 30, 1953, and proposes the following budget. A probable small excess of receipts over expenditures is indicated, and, as usual, the actual excess will undoubtedly be somewhat larger than that indicated, since the Committee has attempted to overestimate expenditures and underestimate receipts slightly in order to provide some factor of safety.

Operating Statement

Estimated receipts:			
National Research Council, grant		$ 400	
From Reserve, Directory		750	
Income:			
Dues, 7 corporation members, 4400 individual	$31,500		
Sales, *Transactions*	6,400		
Sales, reprints	2,000		
Miscellaneous	50		
Interest on investments	150	40,100	
Total receipts			$41,250
Estimated expenditures:			
Publication of *Transactions*:			
Printing	$10,000		
Preparation	7,800		
Mailing	800		
Co-editors	1,500	$20,100	
Reprints		2,000	
General office expenses:			
Salaries	$12,900		
Stationery and supplies	1,000		
Postage and communications	1,200		
Equipment	200	15,300	
Expenses, meetings:			
Annual	$ 700		
Regional	200	900	
Expenses, Sections		100	
Expenses, Committees		400	
Travel of officers		50	
Miscellaneous (including Bowie Medal)		50	
Transfer to Reserve, Emergency		250	
Directory:			
From Reserve, Directory	$ 750		
From Income	750	1,500	
Total expenditures			40,650
Excess of receipts over expenditures			$ 600

Transfer of Funds

During past year transferred from Cash Reserve to Permanent Fund	$ 3,815
Transfer from Reserve, Directory, to Operating Receipts	750
Transfer from William Bowie Medal Fund to Cash Reserve	70

Statement of Assets as of June 30, 1952 and 1953 (estimated)

		1952	Changes		1953
General Funds:					
Cash Reserve	$13,575		+$600 from operations + 70 from Bowie Medal	$14,245	
Emergency	750		+ 250 from operations	1,000	
Directory	750	$15,075	- 750 to operations	0	$15,245
Permanent Fund:					
U. S. Bonds (cost)	$ 2,368		None	$ 2,368	
Savings	4,807	7,175	None	4,807	7,175
Robert E. Horton Fund		1,180	None		1,180
Bowie Medal Fund		70	- 70 to Cash Reserve		0
Bills Receivable less Bills Payable		2,500	None		2,500
Totals		$26,000			$26,100

The Committee has been asked to make recommendations to the Executive Committee on several matters. The results of its consideration follow.

Because the Cash Reserve has increased beyond the point that is necessary to maintain a comfortable balance at all times of the year, the Committee has recommended to the General Secretary that he transfer $3,500 from the Cash Reserve to the Permanent Fund. This will result in an additional income of about $100 annually. The transfer has already been made at the suggestion of the Committee on Budget.

The Committee recommends that the fund "Reserve for International Travel" be changed to "Reserve, Emergency," and that the General Secretary be specifically empowered to authorize expenditures from that fund without awaiting approval of the Executive Committee. Owing to the fact that it was not necessary to draw upon this fund for travel to the last international meeting, there is at present a balance of $750 in the fund, and the Committee recommends that $250 be transferred to the fund this year, bringing it up to $1,000. Nothing in this recommendation should be interpreted as curtailing any recognized authority that the General Secretary may now have in regard to the expenditure of funds of the Union.

The William Bowie Medal Fund has been nearly exhausted, and the Committee recommends that the small balance be transferred to the Cash Reserve, it being assumed that the Union will continue to award this Medal and will defray the cost from operating expenses.

A study of the possibility of achieving savings in the cost of printing the Transactions has been made by the General and Executive Secretaries. A recent estimate has shown that appreciable savings could be accomplished by making a change. However, such a change would result in a change of format of the Transactions, more rigid restrictions on the number of pages and on the dates for submitting copy, increased cost of mailing, and the leaving of a printer with whom relations have been entirely satisfactory and cordial. The Committee recommends that no change in printers be made at this time.

The matter of Life Membership in the Union by reason of retirement was called to the attention of the Committee. Our requirement of 15 years' membership in order to qualify is very low, and before long we may be carrying a heavy financial burden on this account. The requirements in other organizations appear to be considerably higher. It is recommended that a Committee be appointed to study this question and to make recommendations at the next meeting of the Executive Committee.

Again the matter of the amount of the annual contribution made by the National Research Council has been brought to the attention of the Committee. It is recommended that the National Research Council be approached by the President and the General Secretary of the Union, in an effort to have the annual contribution of $400 increased.

Several members of the Union have expressed a desire to contribute more than their regular dues to the Union. It is recommended that a line "Contributions" be printed on the bills for dues to call the attention of the members to the fact that gifts of this sort will be welcome.

Although the following matter does not properly fall within the scope of the Committee on Budget, the Committee has been asked for its opinion. It is recommended that a letter be written to the Division of International Relations of the National Research Council calling attention to the fact that at the Brussels meeting of the International Union of Geodesy and Geophysics the hope was expressed that in the future the annual dues of Great Britain, France, and the United States of America should be increased from 800 pounds sterling to 1000 pounds each for Great Britain and France and to 1500 pounds for the United States, and that the American Geophysical Union hopes that the Division of International Relations will recommend that such an increase in the contribution of the United States be made for the next fiscal year.

APPENDIX 5--REPORT OF THE SPECIAL COMMITTEE ON COSMIC-TERRESTRIAL RELATIONSHIPS FOR THE YEAR ENDING JUNE 30, 1952

Harlan T. Stetson, Chairman
E. O. Hulburt, Vice Chairman

Horace R. Byers	Serge A. Korff	W. T. Thom, Jr.
Ross Gunn	Robert R. McMath	George P. Woollard

In addition to the activities of the individual members of the Special Committee on Cosmic-Terrestrial Relationships, the Committee has had the cooperation of other workers in pertinent fields including meteorology, solar physics, cosmic rays, ionospheric research, and radio-wave propagation, and certain geological investigations that may have some bearing on extra-terrestrial phenomena. The field is a very broad one and it has been the object of this Committee to stress the interrelation of such varied investigations as has come to its attention that may focus interest on the interdependence of the problems involved.

E. O. Hulburt, Vice Chairman, reports for the activities from the Naval Research Laboratory as follows:

Upper-atmosphere rocket work--From measurements on the Viking flight of August 7, 1951, it was found that the atmospheric density at 160 and 219 km was 1.2×10^{-6} and 1.0×10^{-7} grams per cubic meter, respectively. The wind at 200 km was 80 meters per second from the southeast [R. J. HAVENS and H. E. LaGOW, Mem. Soc. R. Soc. Liege, Belgium, tome 12, fasc. 1-2, p. 184, 1952].

Cosmic gamma-ray experiments showed that low-energy gamma rays (0.1 to 15 Mev) were present in considerable numbers throughout the atmosphere below 40 km, and in small numbers above. The origin of this radiation in the atmosphere is the Bremstrahlung process of the electrons of the soft component, followed by multiple Compton scattering. Above 40 km, about 25 pct of the observed radiation can be attributed to an albedo effect associated with the curvature of the Earth. It was suggested that the remaining 75 pct is an albedo due to diffusion upward of atmospheric radiation. On an energy basis, the soft gamma radiation above 40 km amounted to only 1/2000 of the incoming charged-particle radiation. In a result unpublished as yet, balloon flights in Minneapolis showed that the charged-particle radiation contains one primary alpha particle for each 5.7 primary protons, approximately the same figure obtained previously in a rocket flight at White Sands, New Mexico [G. J. PERLOW and C. W. KISSINGER, Phys. Rev., v. 84, pp. 572-580, 1951].

Work on the vertical distribution of ozone was completed, and data between 35 and 70 km were found in agreement with a new photochemical calculation based on the densities and solar intensities measured from rockets and including the oxygen recombination reaction [F. S. JOHNSON, J. D. PURCELL, and R. J. TOUSEY, J. Geophys. Res., v. 56, pp. 583-594, 1951]. The direct measurement of solar x-rays and ultraviolet by means of photon counters was reported [H. FRIEDMAN, S. LICHTMAN, and E. T. BYRAM, Phys. Rev., v. 83, pp. 1025-1030, 1951].

Radio astronomy--The program in radio astronomy places particular emphasis on work at shorter wavelengths and higher resolution. Measurements are being made with radiometers using an eight-foot-diameter antenna at 8.5-mm wavelength. The 50-foot parabola is completed and measurements of its quality are nearly complete. Its gain and pattern are as predicted for three-cm wavelength and tests are now in progress at eight mm. Daily readings of the intensity of solar radiation at three-cm wavelength are made from sunrise to sunset. There is a correlation between the variations in intensity and sunspot activity. Solar outbursts have been observed at three-cm wavelength and at a wavelength of 8.5 mm [NRL Aleutian Radio Eclipse Expedition, JOHN P. HAGEN, F. T. HADDOCK, and GROTE REBER, Sky and Telescope, v. 10, no. 5, March 1951; Temperature gradient in the Sun's atmosphere measured at radio frequencies, JOHN P. HAGEN, Astrophys. J., v. 113, no. 3, May 1951].

Twilight sky brightness and polarization measurements--The measurements were made during clear weather at various points over the sky for altitudes of the Sun from five degrees above the horizon to 15° below the horizon at two stations, one in Maryland at an altitude of 100 ft and one on Sacramento Peak, New Mexico, at an altitude of 9500 ft. The brightness of the Sacramento Peak sky was about 2/3 to 1/2 of that of the Maryland sky, due to lesser haze [to be published in J. Optical Soc. Amer.].

Total solar eclipse of February 25, 1952--A group of 14 people journeyed to Khartoum, Sudan, to observe the total solar eclipse of February 25, 1952. The main experiments were rapid spectra of the solar crescents near second and third contacts with two cine-spectrographs, flash spectra with three large grating spectrographs, and solar emissions at eight mm and ten cm with special radio telescopes. In addition, there was an 18-ft corona telescope and two long-focus Mitchell cameras for slow-motion pictures of the eclipse, one in black and white and one in color. Due to good weather and good preparation, all the experiments were carried out successfully and are to be reported in various scientific journals when the data have been analyzed.

Charles F. Brooks, who has long continued investigations of solar radiation at the Blue Hill Observatory, states:

At the Blue Hill Meteorological Observatory of Harvard University detailed observations of solar radiation in its various phases continued through the 19th year: the intensity of the solar beam at normal incidence by the Observatory, and the total radiation on a horizontal surface, diffuse radiation on a horizontal surface, also total on vertical surfaces facing south, north, and west (new this year), by the U. S. Weather Bureau's Solar Radiation Field Testing Unit at Blue Hill, and normal incidence and total on a horizontal surface by the Bureau in Boston. There are some 70 Weather Bureau and cooperating stations in North America recording total solar radiation on a horizontal surface, and at seven of these the intensity of the solar beam at normal incidence is also measured (Blue Hill, Boston; Madison, Wis.; Lincoln, Nebr.; Albuquerque, N. M.; Table Mountain, Calif.; and Climax, Colo.).

A study, Measurements of diffuse solar radiation received on a horizontal surface at Blue Hill Observatory, Milton, Massachusetts, by I. F. Hand and F. A. Wollaston, is in press, and another, on Distribution of solar energy throughout the United States and southern Canada, based on records of 49 stations, will be presented at the Corvallis, Ore., meeting of the American Meteorological Society by Hand, in June. Studies of diffuse solar radiation by S. H. Wollaston are about ready for publication.

The Blue Hill series of aurora observations continues, though under increasing handicap owing to the increasing illumination and smoke of Greater Boston, to the north. The record, which began in 1885, now totals more than 67 years.

Serge A. Korff of New York University, reporting on Solar effects in cosmic-ray neutron studies, informs the Committee:

Two recent experiments have served to throw new light on the problem of solar effects in cosmic-ray neutrons. First, balloon flights have been conducted in northern latitudes, as far north as 69°N geomagnetic. The increase noted at these northern latitudes over that at 55° suggests that the Sun's general magnetic field probably has a somewhat lower value than the minimum previously set at 50 gauss, and is probably as low as five gauss. Further, the neutron counting rates at 88,000 ft showed a slightly larger value during the day than at night, suggesting that perhaps a few neutrons originating in the Sun's upper atmosphere reach the Earth. These results are preliminary, and as they rest upon the results of only a few flights, further experiments are in progress [Diurnal effects on cosmic-ray neutrons at high altitudes, M. J. SWETNICK, H. A. C. NEUBURG, and S. A. KORFF, Bull. Amer. Phys. Soc., v. 27, p. 14, abs. F-6, Jan. 1952].

The important work of the McMath-Hulbert Observatory in solar observations needs no emphasis and the report from this institution for the year has been received as follows:

At the McMath-Hulbert Observatory of the University of Michigan, the program of observation and analysis of the solar spectrum with high dispersion continues. This work includes both the photographic and infra-red regions of the spectrum and deals with spectra of detailed features of the Sun as well as with the fundamental solar spectrum. During the last year more than 800 solar lines between $\lambda 13,000$ and $\lambda 25,242$ have been identified and their wavelengths and intensities measured. During 1951 a special study was made of the intensity variations of the helium line, $\lambda 10830$, from the center to limb of the Sun, and over such solar features as active dark flocculi, faculae, filaments, and flares. An echelle grating has recently been acquired which makes possible the photography of four or five thousand Angstroms of the spectrum with high dispersion in a single exposure. Spectrograms of this type have been made of selected features within prominences at the limb and flares on the disk. A Lyot filter has been installed and is used as a guiding instrument in making spectrograms of the above type.

Continuous, monochromatic, photographic records of rapidly changing solar activity form part of the regular program of the Observatory. Spectroheliographic studies of the disk show the development of sunspot regions, active dark flocculi, and the outbreak of flares. Observations at the solar limb record prominence motions and include simultaneous spectrographic records in the neighborhood of H . Solar flares at or near the limb were studied in detail during the last year.

Charts containing solar, ionospheric, and geomagnetic data for the last four years have been kept up to date. Photometric light curves of more than 50 flares have been derived. These light curves, as well as the 200 Mc/sec solar-noise records secured at Cornell, provide quantitative data to assist in the identification of active solar regions associated with terrestrial disturbances.

Donald H. Menzel reports somewhat at length on the results of progress made at Harvard and its auxiliary observational stations relative to cosmic-terrestrial relationships. The Harvard Observatory maintains two observational stations: one, the High Altitude Observatory in cooperation with the University of Colorado at Climax, Colorado; and second, the station at Sacramento Peak near Alamagordo, New Mexico. This latter station is operated by Harvard in cooperation with the U. S. Air Force. At both of these stations, coronagraphs are in operation for maintaining daily surveys of the corona and solar prominences. At Sacramento Peak, there is in operation a continuous "flare" patrol. The headquarters of the High Altitude Observatory at Boulder, Colorado, acts as a head clearing house for the release of information periodically forwarded to the National Bureau of Standards. Menzel states:

During the past year, Roberts, Evans, and I have been working on the problem of prominence classification. We feel that we have discovered new relationships that enable us to sharpen considerably the meaning of a prominence classification and to attach a physical significance to the various classifications. This new classification will shortly be published.

I have been particularly concerned with the basic theories of solar activity and have become convinced that magnetic forces are fully as important as gravitational ones in the support and formation of solar prominences. In the broadest sense, there are two kinds of prominences: those that go up and those that go down. The former are very much in the minority, at least if we exclude the difficult-to-observe polar spicules. These may prove to be extremely important objects. A large part of the downcoming material may have reached heights through the operation of these spicules. The other major source of outward-moving material is to be found in the surges, which are most conspicuous in the vicinity of sunspots.

A detailed theoretical analysis indicates that magnetic forces can, especially when combined with turbulence, raise quantities of matter to considerable heights. Surges and spicules probably result from the squeezing - pumping action associated with the magnetic lines of force.

The material in the corona is largely supported by magnetic force lines, bent under the action of gravity. The slowly moving coronal material will condense here and there, and gradually erode great canyons or holes in the magnetic field, through which the material will flow and become luminous. These great rivers of luminous gas are the prominences. The details of the theory still need to be worked out, but the broad outline of the cause and evolution of prominences seems to be pretty clearly indicated.

In this connection my colleagues, especially Max Krook and P. L. Bhatnagar, and I, have been studying the problem of the dynamics of ionized media, especially those media associated with magnetic fields. The results will be reported at the meeting at the conference in Australia of URSI in August.

Barbara Bell and I have been studying the peculiar distribution of dark hydrogen flocculi on the solar surface. These clouds seem to follow a fairly regular form and we believe that the distribution fits in well with the general theory of prominences outlined above.

One final point of general interest is that I have expanded somewhat the theory of magnetic storms and especially of auroras, as first described by Chapman and more recently described by D. F. Martyn. As Chapman has shown, an ion cloud from the Sun naturally divides into a stream that flows around the Earth to either side, leaving the Earth itself essentially untouched. Martyn showed that the stream might flow over the lines of force into the auroral zone. What I have shown is that the small condensations that may occur in the stream can break holes in the Earth's magnetic field, which normally protects the Earth from the ion stream from the Sun. As the magnetic force lines collapse, however, material from outside space begins to pour in, being concentrated as if it were flowing through a funnel. The outer portion of the funnel, which looks something like an elongated phonograph horn or vacuum cleaner, then "sucks" the cosmic material in, concentrating it to the lower edge where it flows out near the surface of the Earth to produce the aurora. This theory seems to fit very well with the observations of Meinel, Gartlein, and others, who find broadened and displaced hydrogen lines in the spectrum of the aurora borealis. A further analysis of the physical conditions responsible for these lines indicates that the density in the ion cloud just above the surface of the Earth is of the order of a million per cubic cm, whereas in outer space it is probably of the order of 100 per cubic cm. The theory makes practical use of the series of papers, Physical processes in planetary nebulae, published in the Astrophysical Journal some years ago by myself and collaborators.

G. W. Pickard and H. T. Stetson report further on a study of Tropospheric effects in ionosphere-supported radio transmission [G. W. PICKARD and H. T. STETSON, Proc. Inst. Radio Eng., v. 37, no. 6, June 1949]. Pickard has recently completed an investigation of relations between radio reception and pressure changes at a near midpoint on the transmission path. The transmission data used covers a period of 23 years from 1926 to 1949 utilizing field-intensity recordings of nighttime reception from WBBM, Chicago, as received in the Boston area.

First taking as an epoch the highest barometric pressure at Buffalo for each month of the 23 years, it seems to be pretty well established that the high Boston fields occur on the high Buffalo pressure days. A still more marked correlation was found by using as epochs days showing the greatest pressure rise at Buffalo from the preceding day. Using these interdiurnal pressure rises as epochs, the Boston fields followed the changing Buffalo pressures, being low before the epoch and rising rapidly through the epoch to a maximum two days later. For a mean interdiurnal pressure rise of 16 millibars at Buffalo, the mean field rise at Boston was 29 pct. It has also been found that taking as an epoch the occurrence of thunderstorms at Buffalo active during the recording hours, the Boston fields rose rapidly to a maximum two days following.

Corresponding studies of field intensities with either high Boston pressure or high interdiurnal pressure rise at Boston gave no distinct correlation. It appears, therefore, that the correlation between the Buffalo pressure and the Boston field is real. This is also evidenced by the statistical check of obtaining an amplitude of over five times the probable error.

W. T. Thom, Jr., of Princeton, speaking as a geologist, has raised a question for investigation concerning possible relations between the Earth's dynamic processes responsible for mountain-building and any extra-terrestrial relations and influences which may affect those processes. He calls attention to an investigation by Donald B. Lawrence of the University of Minnesota on Glacier fluctuation for six centuries in southeastern Alaska and its relation to solar activity [Geogr. Rev., v. 40, no. 2, April 1950] as a contribution worthy of note on the basis of lines of evidence cited.

Speaking for the meteorologists, H. R. Byers comments:

In the field of meteorology continued interest in cosmic-terrestrial relationships prevailed, especially stimulated by H. C. Willett's studies of possible relationships of sunspot conditions to large-scale atmospheric phenomena, reported on last year.

In an article entitled The great Pacific anticyclone of winter 1949-50: a case study in the evolution of climatic anomalies [J. Met., v. 8, pp. 251-261, 1951], J. NAMIAS pointed out that atmospheric changes or trends even on a large scale may not require outside influence. Writing of self-developing mechanisms, he concludes that "large-scale atmospheric states . . . carry with them their own seeds for creating new states." The Namias philosophy also is implied in his contributions in the Compendium of meteorology, published late in 1951 by the American Meteorological Society.

The issue of cosmic influences came up again in some articles on "blocking action" by DANIEL F. REX [Tellus, v. 2, pp. 196-211 and 275-301; v. 3, pp. 100-111] and by BREZOWSKY, FLOHN, and HESS [Tellus, v. 3, pp. 191-194]. Blocking is characterized by the breaking up of the normal zonal circulation into a cellular circulation brought about by an apparent blocking of the zonal flow at a particular location, all features remaining quasi-stationary and persisting. A change to persistent, abnormal weather occurs. The question was put by Rex as to what causes the circulation to start misbehaving this way. He mentioned the possibility of solar influences as deduced by Willett. Brezowsky, Flohn, and Hess examined all cases of blocking in Europe from 1881-1950. They conclude: "If one arranges the annual frequencies of blocking highs in accordance with the sunspot maxima or minima, we obtain at best a small peak coinciding with the sunspot minima . . . which cannot be verified statistically. Any influence of solar activity is weak and uncertain, as is also confirmed by Wexler in his investigation of hemispheric pressure charts 1899-1939." The authors find some evidence of a 22-23 year oscillation which they concede may be connected with an effect of the solar-activity period.

As do all branches of meteorology, the field of cosmic relationships of the atmosphere receives ample treatment in the new Compendium of meteorology of the American Meteorological Society. The article by Craig and Willett on Solar-energy variations and anomalous weather changes gives a thorough summary of the problem. A good case is presented for Simpson's theory of the ice age as being a period of increased rather than decreased solar heating. Evidence of direct solar influence on the atmospheric circulation is summarized.

A series of papers on the upper atmosphere is contained in the Compendium of meteorology. Those of special interest from a cosmic sense are, in order of appearance: General aspects of upper atmosphere physics, by S. K. MITRA; Photochemical processes in the upper atmosphere and resultant composition, by SYDNEY CHAPMAN; Ozone in the atmosphere, by F. W. PAUL GÖTZ; Radiative temperature changes in the ozone layer, by RICHARD A. CRAIG; The ionosphere, by S. L. SEATON; Night-sky radiations from the upper atmosphere, by E. O. Hulburt; Aurorae and magnetic storms, by L. HARANG; and Meteors as probes of the upper atmosphere, by FRED L. WHIPPLE. The Compendium also contains an article on The atmospheres of other planets, by S. L. HESS and H. A. PANOFSKY.

In view of certain puzzling apparent variations in transoceanic longitudes from the intercomparison of simultaneous time determinations, the possibility of the existence of a variation in the transmission time of time signals has been reinvestigated. Top experts in the field of

ionospheric transmission of radio waves have communicated to your Committee that there appears to be no existing evidence for a difference in propagation times of waves transmitted from east-to-west as compared to the west-to-east direction. This is a fundamental postulate in the interpretation of any periodic variation of longitudes that may appear significant.

Your Committee is gratified with the increased interest in the field of cosmic-terrestrial relationships as has been evidenced, for example, by a conference on The Sun in the service of Man, held under the auspices of the American Academy of Arts and Sciences in Boston last year, and the increasing interest among meteorologists as well as ionospheric investigators concerning solar effects not only upon the upper atmosphere but in large-scale atmospheric phenomena at low levels.

APPENDIX 6--REPORT OF AMERICAN GEOPHYSICAL UNION'S REPRESENTATIVES ON DIRECTORATE OF AMERICAN GEOLOGICAL INSTITUTE FOR 1951-1952

Directors' meetings were attended at Detroit, Michigan, on November 6-7, 1951, and at Los Angeles on March 23, 1952.

During the year the Institute's attention continued to be centered upon improvement of national scientific manpower policy and upon Earth-science education in all phases. Specific projects discussed at length included the following:

(1) The establishment of a Geologic Manpower Advisory Council.
(2) The preparation of a comprehensive Earth-science glossary.
(3) The establishment of an Earth-science abstract journal.
(4) The establishment of a professional geological journal.
(5) Participation by the Executive Director in discussions looking toward the establishment of a blanket committee (with the American Chemical Society, American Association for the Advancement of Science, and others) to deal with scientific public relations.

The opinion of your representatives with respect to these several projects is as follows (the instructions of the Union's Executive Committee as to future position being hereby requested):

(1) Seems in line with urgent present needs.
(2) Seems highly desirable.
(3) This is rated as desirable.
(4) A matter of dubious necessity.
(5) Definitely undesirable - as being far too much of a blank-check authorization of activity which may be quite uncontrollable.

It has just been learned, with great regret, that David Delo, present Executive Director of the American Geological Institute, is resigning as of July 1, 1952, to accept a college presidency. The selection of a qualified successor naturally and necessarily confronts the Institute with a major hurdle.

W. T. Thom, Jr.

APPENDIX 7--REPORT OF REPRESENTATIVES OF AMERICAN GEOPHYSICAL UNION ON COUNCIL OF AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

As an affiliated society of the American Association for the Advancement of Science, the American Geophysical Union appointed John Putnam Marble and Waldo E. Smith as its representatives on the Council of the American Association for the Advancement of Science. The former was able to attend the Council meeting in Philadelphia last December. A considerable volume of business, most of it routine, was expeditiously transacted at the two sessions. A report of the Council meetings has already appeared in Science. No matters which particularly concern the American Geophysical Union were brought up.

Your representatives feel that affiliation with the American Association for the Advancement of Science has proved to be of use to the Union, and shows promise of being of considerable value.

John Putnam Marble
Waldo E. Smith

SPECIAL ANNOUNCEMENTS

New directory of members, closing date, September 10, 1952--As announced in the April, 1952, issue, it is the plan to issue a new directory of members of AGU as of September 10, 1952. This will probably be a part of the October issue. Members are urged to see to it that their correct addresses are a matter of record in the office of AGU by September 10, 1952.

Meeting on the evaluation of cloud-seeding operations at East Lansing, Michigan, September 4, 1952--The American Geophysical Union is co-sponsoring three sessions of the Summer Meeting of the Institute of Mathematical Statistics on The evaluation of cloud-seeding operations, to be held September 4, 1952, at Michigan State College, East Lansing, Michigan. This is our first announcement of this meeting, but it is hoped that it will reach the hands of interested persons in time to make last-minute arrangements to attend. Arrangements for accommodations should be made through J. Sutherland Frame, Head, Department of Mathematics at Michigan State College, who is Chairman of the Committee on Arrangements. An informal liaison group representing AGU consists of Harry Wexler, Robert C. Bundgaard, and Vincent J. Schaefer. The American Meteorological Society is also acting as co-sponsor.

Regional Meeting in Chicago, Illinois, September 11-12, 1952--Attention is called once more at this last moment to the extensive program that has been developed for the Twenty-Eighth Regional Meeting of the AGU to be held at the Del Prado and Conrad Hilton Hotels in Chicago, in connection with the Centennial of Engineering. Eight sessions bearing on hydrology, meteorology, and seismology, and their applications are being presented in this meeting, which is being held jointly with the American Meteorological Society. The sessions include a symposium on The natural setting and engineering problems of the Great Lakes; and sessions on Evaporation from lakes and reservoirs; Ground water - the Great Lakes region; Evaporation, convection, and precipitation; Applied climatology; Applied forecasting; Air pollution; and Seismology as it applies to the design of structures. For details, write to the Chairman of the Committee, Wallis S. Hamilton, Department of Civil Engineering, Northwestern University, Evanston, Illinois.

Sessions on geophysics at the AAAS meeting in St. Louis, December 27-30, 1952--The American Geophysical Union is co-sponsoring the sessions on geophysics being arranged in connection with the meeting of the American Association for the Advancement of Science to be held in St. Louis, Missouri, December 27-30, 1952. Edward M. Brooks of the Institute of Geophysical Technology, St. Louis University, St. Louis, Mo., is in charge of arrangements of the sessions on geophysics. This will serve as a call for papers (titles and abstracts should be forwarded to Professor Brooks at the very earliest date) germane to a general session in geophysics or to a specific theme. Write to him on these matters, and watch these SPECIAL ANNOUNCEMENTS for further details.

Regional Meeting in California, February, 1953--The Twenty-Ninth Regional Meeting will be held at Stanford University, California, with February 6-7 as the probable dates. Watch these SPECIAL ANNOUNCEMENTS for further information, or write to the Chairman of the Committee for the Pacific Southwest Region, J. W. Johnson, College of Engineering, University of California, Berkeley 4, California.

Thirty-Fourth Annual Meeting, Washington, D. C., May 4-6, 1953--Attention is called to the preliminary announcement in the June issue of the *Transactions* [p. 486] giving the dates for the Thirty-Fourth Annual Meeting (as shown above), issuing the first call for titles and abstracts of papers, and requesting suggestions. Watch the SPECIAL ANNOUNCEMENTS in October for more details.

Regional Meeting in Minneapolis in August-September, 1953--A Regional Meeting will be held in Minneapolis, Minnesota, in late August or early September, 1953, in connection with the meeting of the International Association of Hydraulic Research and of the Division of Hydraulics of the American Society of Civil Engineers. The Regional Committee appointed for this meeting consists of Martin E. Nelson of St. Paul, Minnesota (Chairman); Paul C. Benedict of Lincoln, Nebraska; M. C. Boyer of Iowa City, Iowa; Lorenz G. Straub of Minneapolis, Minnesota; and Roy F. Warner of St. Paul, Minnesota. As plans progress, further word will be carried in SPECIAL ANNOUNCEMENTS. Inquiries should be addressed to the Chairman of the Committee at the U. S. Engineer Office, U. S. Post Office Building, St. Paul 1, Minnesota.

Tenth General Assembly of the International Union of Geodesy and Geophysics--The Tenth General Assembly of the International Union of Geodesy and Geophysics and its seven constituent Associations will be held in Italy in September, 1954. Further information on this Assembly will be given in these SPECIAL ANNOUNCEMENTS from time to time.

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TRANSACTIONS
AMERICAN GEOPHYSICAL UNION

Volume 33 OCTOBER 1952 Number 5 Part I

TRIANGULATION CLOSURE INVESTIGATION

J. E. Lilly and P. Brunavs

Abstract--Latitude and longitude closure equations are developed, taking account of the second powers of differences of latitude and longitude. In a test on a triangulation net, it is found that adjustment by means of equations based on first-order terms only leaves appreciable residual discrepancies, which are reduced to less than 0".001 by inclusion of second-order terms.

Latitude and longitude closure equations are usually based on the following formulas

$$\text{Latitude:}\quad 0 = 7238.24\,(\phi_N - \phi_{N'})'' + \Sigma\left\{(\delta_A v_A - \delta_B v_B)\,(\phi_N - \phi_C)' \pm a_1\, v_C\,(\lambda_N - \lambda_C)'\right\} \ldots\ldots (1)$$

$$\text{Longitude:}\quad 0 = 7238.24\,(\lambda_N - \lambda_{N'})'' + \Sigma\left\{(\delta_A v_A - \delta_B v_B)\,(\lambda_N - \lambda_C)' \pm a_2\, v_C\,(\phi_N - \phi_C)'\right\} \ldots\ldots (2)$$

where

ϕ_N, λ_N	Preliminary computed values of the latitude and longitude of the terminal point
$\phi_{N'}, \lambda_{N'}$	Fixed values of the latitude and longitude of the terminal point
ϕ_C, λ_C	Preliminary computed values of the latitude and longitude of the C-station of any triangle
v_A, v_B, v_C	Adjustment corrections to the A, B, and C angles of the same triangle
δ_A, δ_B	Tabular differences for the A and B angles
a_1, a_2	Constants for any one adjustment, being functions of the terminal latitude

The symbols (") and (') following the quantities in parentheses indicate that these quantities are expressed in seconds and minutes of arc, respectively.

The present investigation indicates that, for greater accuracy, the quantities $(\phi_N - \phi_C)'$ and $(\lambda_N - \lambda_C)'$ should respectively be replaced by

$$\left\{(\phi_N - \phi_C)' + b_1\,(\lambda_N - \lambda_C)'^2\right\} \quad \text{and} \quad \left\{(\lambda_N - \lambda_C)' + b_2\,(\lambda_N - \lambda_C)'\,(\phi_N - \phi_C)'\right\}$$

where b_1, b_2, are functions of the terminal latitude of the net.

Thus the newly developed formulas for latitude and longitude equations would be

$$\begin{aligned}\text{Latitude:}\quad 0 = 7238.24\,(\phi_N - \phi_{N'})'' &+ \Sigma(\delta_A v_A - \delta_B v_B)\left\{(\phi_N - \phi_C)' + b_1\,(\lambda_N - \lambda_C)'^2\right\}\\ &+ \Sigma \pm a_1 v_C\left\{(\lambda_N - \lambda_C)' + b_2\,(\lambda_N - \lambda_C)'\,(\phi_N - \phi_C)'\right\} \ldots (3)\end{aligned}$$

$$\begin{aligned}\text{Longitude:}\quad 0 = 7238.24\,(\lambda_N - \lambda_{N'})'' &+ \Sigma(\delta_A v_A - \delta_B v_B)\left\{(\lambda_N - \lambda_C)' + b_2\,(\lambda_N - \lambda_C)'\,(\phi_N - \phi_C)'\right\}\\ &+ \Sigma \pm a_2 v_C\left\{(\phi_N - \phi_C)' + b_1\,(\lambda_N - \lambda_C)'^2\right\} \ldots\ldots\ldots\ldots (4)\end{aligned}$$

Values of a_1, a_2, b_1, b_2, for latitudes from 26° to 70° are given in Table 1.

For some time it has been noticed that adjustments including latitude and longitude closure equations based on the standard formulas do not give entirely satisfactory closures. Consider, for example, the Edmonton-Kitscoty net. In this net, preliminary geodetic positions were computed to four places of decimals. Closures around individual triangles were good, never exceeding 0".0005 in latitude or longitude, the average being about 0".0002. Latitude and longitude equa-

Table 1--Values of constants a_1, a_2, b_1, and b_2

ϕ	a_1	a_2	b_1	b_2
°				
26	+1.903	-2.330	-0.58×10^{-4}	$+1.41 \times 10^{-4}$
27	+1.886	-2.350	-0.59	+1.47
28	+1.869	-2.372	-0.61	+1.54
29	+1.851	-2.395	-0.62	+1.60
30	+1.833	-2.419	-0.63	+1.67
31	+1.814	-2.444	-0.65	+1.74
32	+1.794	-2.471	-0.66	+1.81
33	+1.774	-2.499	-0.67	+1.88
34	+1.754	-2.528	-0.68	+1.95
35	+1.733	-2.559	-0.69	+2.03
36	+1.711	-2.591	-0.69	+2.10
37	+1.689	-2.625	-0.70	+2.18
38	+1.666	-2.661	-0.71	+2.26
39	+1.643	-2.698	-0.71	+2.35
40	+1.619	-2.738	-0.72	+2.43
41	+1.595	-2.779	-0.72	+2.52
42	+1.571	-2.823	-0.73	+2.61
43	+1.545	-2.868	-0.73	+2.70
44	+1.520	-2.917	-0.73	+2.80
45	+1.494	-2.968	-0.73	+2.90
46	+1.467	-3.021	-0.73	+3.00
47	+1.441	-3.078	-0.73	+3.11
48	+1.413	-3.137	-0.73	+3.22
49	+1.385	-3.200	-0.72	+3.34
50	+1.357	-3.266	-0.72	+3.46
51	+1.329	-3.337	-0.71	+3.58
52	+1.300	-3.411	-0.71	+3.71
53	+1.270	-3.490	-0.70	+3.85
54	+1.241	-3.574	-0.69	+3.99
55	+1.210	-3.663	-0.68	+4.14
56	+1.180	-3.757	-0.68	+4.30
57	+1.149	-3.858	-0.67	+4.47
58	+1.118	-3.966	-0.65	+4.65
59	+1.086	-4.081	-0.64	+4.83
60	+1.055	-4.204	-0.63	+5.03
61	+1.022	-4.336	-0.62	+5.24
62	+0.990	-4.478	-0.60	+5.46
63	+0.957	-4.631	-0.59	+5.70
64	+0.924	-4.797	-0.57	+5.96
65	+0.891	-4.976	-0.56	+6.23
66	+0.857	-5.171	-0.54	+6.53
67	+0.824	-5.383	-0.52	+6.85
68	+0.789	-5.615	-0.51	+7.19
69	+0.755	-5.870	-0.49	+7.57
70	+0.721	-6.151	-0.47×10^{-4}	$+7.99 \times 10^{-4}$

$a_1 = 10^6 \, M \sin 1'' \cdot (N/R) \cos \phi$

$a_2 = -10^6 \, M \sin 1'' \cdot (R/N) \sec \phi$

$b_1 = -30 \sin 1'' \cdot (N/R) \sin \phi \cos \phi$

$b_2 = 60 \sin 1'' \cdot (R/N) \tan \phi$

tions were then set up, using the standard forms. After adjustment, geodetic positions were again computed to four decimals, the closures around individual triangles being about the same as before. The closure on the terminal station, before adjustment, was 0″.1007 in latitude and 0″.4950 in longitude; after adjustment, there still remained a discrepancy of 0″.0052 in latitude and 0″.0028 in longitude. Similar results were obtained in other nets, and the present investigation was undertaken in an effort to eliminate the cause of these residual discrepancies.

For the purpose of closure equations, a simple chain of triangles is selected connecting the initial and terminal points of the net. The effect produced on the computed position of the terminal point by adjusting the angles of each triangle of the chain is calculated and expressed in terms of the corrections to the angles of that triangle. To form the closure-condition equations, the sum of these effects over all the triangles of the chain, thus expressed in terms of the v's, is equated to the required change in the computed latitude and longitude of the terminal point.

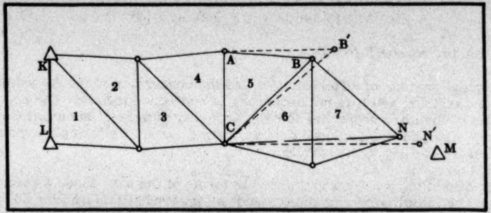

Fig. 1--Triangulation network requiring adjustment so that computed position N will coincide with known position M

Figure 1 represents a chain of seven triangles connecting the initial line KL with the terminal point M. The computed position of the terminal point is N, and the adjustment must so alter the angles of the chain as to make N coincide with M.

The full lines in the sketch represent the position after the application of the adjustment corrections to the angles of the first four triangles. The broken lines represent the effect of the adjustment of the angles of triangle 5, the angles of triangles 6 and 7 remaining unchanged. The adjustment of triangle 5 moves B to B′ and N to N′. The change in azimuth of the line CN is the angle NCN′, which is equal to the angle BCB′, and the length CN becomes CN′, where CN′/CN = CB′/CB. Now putting s and α for the length and azimuth of the line CN, we have for the effect of the adjustment of triangle 5

$$d\alpha = NCN' = BCB' = v_C$$

$$d \log s = \log CN' - \log CN = \log CB' - \log CB = \delta_A v_A - \delta_B v_B$$

Hence if we can express the change in the position of N as a function of $d\alpha$ and d log s, we can immediately replace these quantities by their values in terms of the v's. (If the C-angle were on the opposite side of the chain, as in triangles 1, 3, and 6, we would have $d\alpha = -v_C$.)

Following the application of the adjustment corrections to the angles of triangle 5, corresponding corrections must be applied to the angles of triangle 6, producing a further change in the computed position N. Finally, the application of the adjustment corrections to the angles of triangle 7 must bring N into coincidence with M.

The fundamental formulas adopted for the computation of geodetic positions are those developed by Tobey

$$\Delta\phi = (N/R)\,u - (N/2R)\,v^2 \tan\phi_n - (3/4)\,u^2\,\delta\,\sin 2\phi$$

$$\Delta\lambda = v \sec\phi_n + (1/6)\,v^3 \sec\phi_n \tan^2\phi_n$$

where

N Length of the normal, at latitude ϕ, terminated at the minor axis of the ellipsoid
R Length of the radius of curvature of the meridian section at latitude ϕ
ϕ_n Latitude, on the sphere of radius N, of the terminal point
$u = (s/N) \cos\alpha + (s^3/3N^3) \sin^2\alpha \cos\alpha$
$v = (s/N) \sin\alpha - (s^3/6N^3) \sin\alpha \cos^2\alpha$
$\delta = e^2/(1 - e^2)$

We thus have for the line CN (writing $\Delta\phi$ and $\Delta\lambda$ for $(\phi_N - \phi_C)$ and $(\lambda_N - \lambda_C)$)

$$\Delta\phi = f\ (s, \alpha, \phi)$$
$$\Delta\lambda = F\ (s, \alpha, \phi)$$

where $\phi = \phi_n$ above. Then by differentiation

$$d\Delta\phi = (\partial f/\partial s)\ ds + (\partial f/\partial \alpha)\ d\alpha + (\partial f/\partial \phi)\ d\phi$$
$$d\Delta\lambda = (\partial F/\partial s)\ ds + (\partial F/\partial \alpha)\ d\alpha + (\partial F/\partial \phi)\ d\phi$$

where f is written for $\Delta\phi$ and F for $\Delta\lambda$.

The mechanical process of differentiation and the conversion of the derivatives to usable forms is long and tedious, and it is not necessary to reproduce it here. The process was carried through to terms of the third order (the fourth and higher powers of the quantities s/N, $\Delta\phi$, $\Delta\lambda$, δ, were neglected). Also, ds was replaced by $s\ ds/s = s\ d \log s$. We thus obtained $d\Delta\phi$ and $d\Delta\lambda$ expressed as functions of $d \log s$, $d\alpha$, and $d\phi$.

We already have $d \log s$ and $d\alpha$ expressed in terms of the v's. Since ϕ refers to the terminal point of the line, the initial point remaining fixed, we may substitute $d\Delta\phi$ for $d\phi$, and use the approximate value of $d\Delta\phi$ as obtained from the first two terms above. We thus have the two increments $d\Delta\phi$ and $d\Delta\lambda$ expressed in terms of the v's. Summing these increments over all the triangles of the chain, and equating to the required change in position of the terminal point to effect closure, we have the latitude and longitude condition equations.

Considering only first-order terms in the final result, we obtain the standard formulas, Eq. (1) and (2). Considering second-order terms, we get the formulas given above as Eq. (3) and (4). As a result of checking the formulas on a traverse some 260 miles long, in azimuth 45°, it was decided that the third-order terms may be neglected. The newly developed formulas were then tested on a fictitious net. Two closure adjustments were made: A using the standard formulas (first-order terms only), and B using the new formulas (first- and second-order terms). The results are shown in Table 2.

Table 2--Comparison of terminal positions after adjustments in which latitude and longitude closure equations omitted second-order terms (adjustment A) and included second-order terms (adjustment B)

Item	Latitude	Longitude
	° ′ ″	° ′ ″
Initial point	48 37 13.5780	105 26 25.9160
Terminal point (preliminary)	51 18 02.5039	100 40 11.6513
Terminal point (adjustment A)	51 18 02.9922	100 40 10.9648
Terminal point (adjustment B)	51 18 02.9997	100 40 11.0008
Terminal point (fixed)	51 18 03.0000	100 40 11.0000
Residual error (adjustment A)	0.0078	0.0352
Residual error (adjustment B)	0.0003	0.0008

Thus, if a net of this size, and with this closing error, were adjusted for closure by the standard method, there would be residual discrepancies of 0″008 in latitude and 0″035 in longitude, which would have to be distributed through the net. Using the newly developed formulas, the residual discrepancy is less than 0″001 in both latitude and longitude. The extra work involved in including second-order terms in the condition equations is actually very little -- merely the computation of the quantities $b_1 (\lambda_N - \lambda_C)'^2$ and $b_2 (\lambda_N - \lambda_C)' (\phi_N - \phi_C)'$, and the addition of these quantities as corrections to the quantities $(\phi_N - \phi_C)'$ and $(\lambda_N - \lambda_C)'$ before entering on the latitude and longitude equation sheets.

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(Manuscript received March 4, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 6, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

THE APPLICATION OF RAY-TRACING METHODS TO THE REDUCTION OF SHORAN TO MAP DISTANCES

Arthur S. G. Grant

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Abstract--Ray-tracing methods are used to obtain formulas for the arc length of the shoran radio ray and for the corresponding geodetic map distance, in terms of the angle of inclination of the ray at the ground station, the height above the smooth Earth surface, and the vertical rate of change of the squared modified index of refraction N^2. The method is applicable to cases where N^2 can be expressed as a multi-linear function of the elevation. Examples are worked out comparing the reduced distances with those obtained by the standard method.

Introduction--In shoran geodetic surveying it is usually assumed that the path of the radio ray is an arc of a circle, the radius of which is obtained from measurements of the vertical rate of change of the atmospheric dielectric constant. In most cases, this assumption is correct to a high degree of approximation. However, in cases where the curvature of the ray varies over wide limits, this can lead to erroneous results. Thus an exact method of reduction is of interest in these special cases and also for theoretical purposes.

Development of formulas--Let ds (Fig. 1) be an element of length of a ray, inclined at an angle B to the horizontal, at height z above the surface of the Earth of radius a.

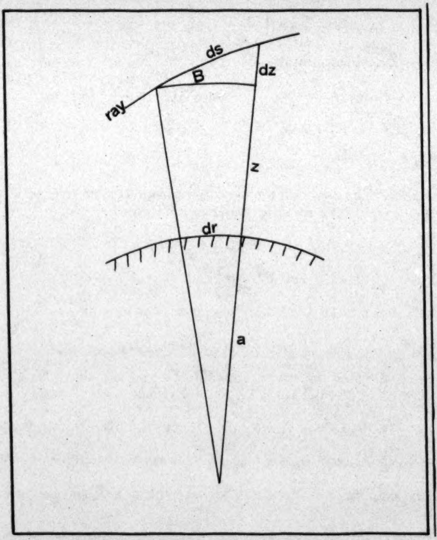

Fig. 1--Sketch of elementary triangle used in development of ray-path formulas

Snell's law in polar coordinates may be written

$$N \cos B = C = N_0 \cos B_0 \ldots\ldots (1)$$

where the symbols have the following meanings: $N = n\,(1 + z/a)$, the modified index of refraction; n, the index of refraction; B, the angle of inclination of the ray; C, a constant; N_0, B_0, the values of N and B at height z_0.

From (1) and the relations apparent in Figure 1, the following differential equations are readily derived

$$dr = \pm C\,(1 + z/a)^{-1}\,[N^2 - C^2]^{-1/2}\,dz \ldots\ldots (2)$$

$$ds = \pm N\,[N^2 - C^2]^{-1/2}dz \ldots\ldots (3)$$

and if it is assumed that $N^2 = N_0^2 + A\,(z - z_0)$, where A is a constant, we have

$$dr = \pm N_0 \cos B_0\,(1 + z/a)^{-1}\,[N_0^2 \sin^2 B_0 + A\,(z - z_0)]^{-1/2}\,dz \ldots\ldots (2')$$

$$ds = \pm\,[N_0^2 + A\,(z - z_0)]^{1/2}\,[N_0^2 \sin^2 B_0 + A\,(z - z_0)]^{-1/2}\,dz \ldots\ldots (3')$$

If $A > 0$ and $r = s = 0$ when $z = z_0$ upon integration, there is obtained

$$r = 2a\,N_0 \cos B_0\,[A\,(a + z_0) - N_0^2 \sin^2 B_0]^{-1/2}\Big\{\tan^{-1}\Big([N_0^2 \sin^2 B_0 + A\,(z - z_0)]^{1/2}\,[A\,(a + z_0) - N_0^2 \sin^2 B_0]^{-1/2}\Big) - \tan^{-1}\Big([N_0 \sin B_0]\,[A\,(a + z_0) - N_0^2 \sin^2 B_0]^{-1/2}\Big)\Big\} \ldots\ldots (4)$$

$$s = (N_0^2/A)\Big\{[\sin^2 B_0 + (A/N_0^2)\,(z - z_0)]^{1/2}\,[1 + (A/N_0^2)\,(z - z_0)]^{1/2} - \sin B_0\Big\} + (N_0^2/A)\cos^2 B_0 \log\Big\{\Big([1 + (A/N_0^2)\,(z - z_0)]^{1/2} + [\sin^2 B_0 + (A/N_0^2)\,(z - z_0)]^{1/2}\Big)(1 + \sin B_0)^{-1}\Big\} \ldots\ldots (5)$$

If $B_0 = 0$, these expressions simplify to

$$r = 2a\,N_0\,[A\,(a + z_0)]^{-1/2} \tan^{-1}\,[(z - z_0)^{1/2}\,(a + z_0)^{-1/2}] \ldots\ldots (6)$$

$$s = A^{-1/2}\,(z - z_0)^{1/2}\,[N_0^2 + A\,(z - z_0)]^{1/2} + (N_0^2/A) \log\Big\{[1 + (A/N_0^2)\,(z - z_0)]^{1/2} + [(A/N_0^2)\,(z - z_0)]^{1/2}\Big\} \ldots\ldots (7)$$

When $A < 0$ the above expression for r becomes imaginary, so that an alternate form must be used. In this case the formula for the map distance r becomes

$$r = (a\,N_0/D) \cos B_0 \log\,[(D - E)\,(D + N_0 \sin B_0)/(D + E)\,(D - N_0 \sin B_0)] \ldots\ldots (8)$$

where $D^2 = N_0^2 \sin^2 B_0 - A\,(a + z_0)$ and $E^2 = N_0^2 \sin^2 B_0 + A\,(z - z_0)$.

The expression (5) for s is still valid for negative A as long as

$$z - z_0 < -\,(N_0^2/A) \sin^2 B_0$$

and this is the elevation at which the ray becomes horizontal and above which it cannot penetrate. Substitution of $z - z_0 = -\,(N_0^2/A) \sin^2 B_0$ in (8) and (5) yields

$$r' = (a\,N_0/D) \cos B_0 \log\,[(D + N_0 \sin B_0)/(D - N_0 \sin B_0)] \ldots\ldots (9)$$

$$s' = (N_0^2/A) \cos^2 B_0 \log\,[\cos B_0/(1 + \sin B_0)] - (N_0^2/A) \sin B_0 \ldots\ldots (10)$$

where r' and s' are the map and arc distances between the source and the point at which the ray becomes horizontal.

The case where $B_0 < 0$ and $A > 0$ is of interest. The map and arc distances to the point at which the ray becomes horizontal are obtained from (4) and (5)

$$r'' = -(2a\,N_0/D')\cos B_0 \tan^{-1}[(N_0/D')\sin B_0] \tag{11}$$

$$s'' = (N_0^2/A)\cos^2 B_0 \log[\cos B_0/(1+\sin B_0)] - (N_0^2/A)\sin B_0 \tag{12}$$

where $D' = D\sqrt{-1}$.

The special case when $A = 0$ leads to a simple solution of (2′) and (3′)

$$r = a \cot B_0 \log[(a+z)/(a+z_0)] \tag{13}$$

$$s = (z - z_0)/\sin B_0 \tag{14}$$

When B_0 is eliminated from (13) and (14) and r expanded, we obtain

$$r \doteq s - (z+z_0)\,s/2a - (z-z_0)^2/2s + \dots \tag{15}$$

This expansion is in agreement with the formula used in the Geodetic Survey of Canada [ROSS, 1949] as far as the third terms.

An expression for the radius of curvature of the ray is of interest. Referring to Figure 1, it is apparent that

$$1/R = (\cos B)/(a+z) - dB/ds \tag{16}$$

If (1) is written $d\,(N\cos B)/ds = 0$, and $A/2N$ substituted for dN/dz in (16), we have

$$1/R = (\cos B)/(a+z) - (A/2\,N^2)\cos B \tag{17}$$

or in terms of A, z, z_0, N_0, and B_0

$$R = 2\,(a+z)\,[N_0^2 + A\,(z-z_0)]^{3/2}\Big/\Big\{N_0 \cos B_0\,[2\,N_0^2 + 2\,A\,(z-z_0) - A\,(a+z)]\Big\} \tag{18}$$

Some of the formulas above will be expressed in a form more readily adaptable to calculation. Equations (6) and (7) become upon substitution of the small angle approximations and expansion into series form

$$r \doteq 2\,N_0\sqrt{(z-z_0)/A}\,[1 - (z+2\,z_0)/3a] \tag{19}$$

$$s \doteq 2\,N_0\sqrt{(z-z_0)/A}\,[1 + A\,(z-z_0)/6\,N_0^2] \tag{20}$$

Equations (9) and (11) reduce to the approximate form

$$r \doteq -\Big\{(N_0^2 \sin 2B_0)/[A\,(1+z_0/a)]\Big\}\Big\{1 + (2\,N_0^2\sin^2 B_0)/[3\,A\,(a+z_0)]\Big\} \tag{21}$$

and (10) and (12) reduce to

$$s \doteq -(2\,N_0^2/A)(\sin B_0)\,[1 - (1/3)\sin^2 B_0] \tag{22}$$

The above formulas for s refer to the actual physical length of the ray path. The shoran electronic equipment measures this length in terms of time, under the assumption of a constant velocity. Thus the relation between the shoran measured distance S and the actual length s is given by

$$dS = (n/n_s)\,ds \tag{23}$$

where n_s is the assumed value of the refractive index under standard conditions. From the relations in the early part of the paper

$$dS = N\,ds/[n_s\,(1+z/a)] \tag{24}$$

$$dS = N^2\,(N^2 - C^2)^{-1/2}\,dz/[n_s\,(1+z/a)] \tag{25}$$

and

$$dS = [N_0^2 + A\,(z-z_0)]\,dz\Big/\Big\{n_s\,(1+z/a)\,[N_0^2\sin^2 B_0 + A\,(z-z_0)]^{1/2}\Big\} \tag{26}$$

If it is assumed that $S = 0$ when $z = z_0$ and $N_0^2\sin^2 B_0 + A\,(z-z_0) > 0$, there is obtained upon integration

$$S = (2a/n_s) [N_0^2 - A (a + z_0)] [A (a + z_0) - N_0^2 \sin^2 B_0]^{-1/2} \left\{ \tan^{-1} (E/D') - \tan^{-1} [(N_0/D') \sin B_0] \right\} + (2a/n_s) \left\{ [N_0^2 \sin^2 B_0 + A (z - z_0)]^{1/2} - N_0 \sin B_0 \right\} \quad \ldots . (27)$$

When $(N_0^2 \sin^2 B_0)/(z - z_0) < A < 0$, the following form must be used

$$S = [a/(n_s D)] [N_0^2 - A (a + z_0)] \log \left\{ [(D - E) (D + N_0 \sin B_0)]/[(D + E) (D - N_0 \sin B_0)] \right\} + (2a/n_s) [E - N_0 \sin B_0] \quad \ldots\ldots (28)$$

The special case when A = 0 leads to

$$S = [(a N_0)/(n_s \sin B_0)] \log [(a + z)/(a + z_0)] \quad \ldots\ldots (29)$$

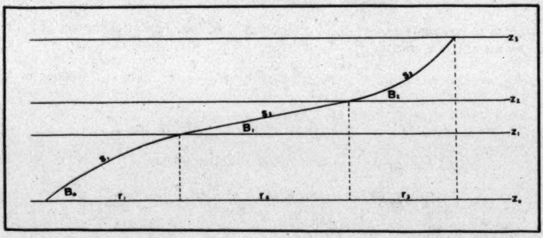

Fig. 2--Assumed three-section ray path used in computed example

The following example is given as a check on the formulas (see Fig. 2 and Table 1). Let

$a = 3967.822$ mi $\qquad \sin B_0 = 0.005 \qquad z_0 = 0$

$N_0^2 = 1.000566 \qquad A_1 = -2 \times 10^{-4}$/mi $\qquad z_1 = 0.125$ mi

$N_1^2 = 1.000541 \qquad A_2 = 0 \qquad z_2 = 0.13$ mi

$N_2^2 = 1.000541 \qquad A_3 = 4 \times 10^{-4}$/mi $\qquad z_3 = 1.13$ mi

Table 1--Comparison of two methods of reduction

Ray number	Arc length s	Map length M	Map length r	Radius of curvature
	mi	mi	mi	mi
1	48.8379	48.8366	48.8368	2840
2	42.0445	42.0431	42.0432	3970
3	99.4406	99.4224	99.4222	19180
1-2	90.8824	90.8790	90.8800	2870
2-3	141.4851	141.4663	141.4654	11600
1-2-3	190.3230	190.3075	190.3022	18785

Substitution of the appropriate constants in (8) and (5) yields the map and arc distances of the first portion, in (13) and (14) those of the second portion, and in (4) and (5) those of the third. The combined distances are found by addition. The arc distances were also reduced to map distances using the formula employed in the Geodetic Survey of Canada [ROSS, 1949].

$$M = S - (1.2601)\, 10^{-4}\, S\, (H + K) - (H - K)^2/(2S) + (0.26466)\, 10^{-8}\, S^3 - S^3/(24\, R^2) \quad \ldots . (30)$$

where the symbols have the following meanings: M, geodetic map distance; S, shoran arc distance; H, airborne-station elevation; K, ground-station elevation; R, radius curvature of ray; Earth radius assumed 3967.822 mi.

The conventional method of obtaining the mean ray curvature is from the relationship

$$1/R = (1/n)\ (\cos B)\ (dn/dz) \doteq (n_2 - n_1)/(z_2 - z_1) \qquad (31)$$

It is seen that the agreement between the reduced map distances is less than 0.0003 mi for the individual arcs, but that differences up to 0.0053 mi occur in the combined cases. The reason for this is that in general the value of the radius curvature obtained from (31) is not representative of the true mean radius curvature unless it is constant over the entire path.

Conclusion--Formulas are given which will permit exact reduction of shoran to map distances. Although the labor involved in computation is considerable, the method should be of value in checking the accuracy of simplified methods.

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(Manuscript received March 7, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 6, 1951; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

LOW-FREQUENCY ATMOSPHERIC RADIO NOISE IN FLORIDA

A. W. Sullivan, H. M. VanValkenburg, and J. M. Barney

Abstract--On the basis of data obtained at frequencies of 15, 52, 189, and 530 kc at Gainesville, Florida, a study was made of diurnal and seasonal variations in average noise level, and of average frequency spectra. Both diurnal and seasonal variations in noise intensity were found to depend on frequency and on propagation conditions. The seasonal variations were found to conform fairly closely to sine functions at 15 and 52 kc, and somewhat less closely at the higher frequencies.

The diurnal variation of the frequency spectrum of noise also indicated to some degree the effect of propagation conditions on noise level. The possibility is pointed out that the average noise spectrum in the frequency range 10-500 kc at the source of the noise is constant. The noise level was found to be almost directly proportional to wavelength during periods when propagation conditions had no appreciable effect.

During sudden ionospheric disturbances the noise levels rose, indicating the incidence of ionospheric conditions similar to those experienced at night.

Introduction--A program of research in the field of atmospheric noise measurement and description has been conducted by the University of Florida since early in 1947. Among the principal objectives of this research were those of devising instruments and techniques for the determination of the statistical properties of noise, and of conducting a long-term study of the time variations in the average value of the noise envelope in the frequency range 10-500 kc. This paper discusses the conditions under which the study of the time variations were made and presents some of the results obtained.

Apparatus--The site chosen, approximately eight miles from the city of Gainesville, Florida, is well suited for this work. Practically no man-made noise is present, and atmospheric noise levels are quite high from March to September.

This study made use of vertical antennas with a height of 23 m (70 ft), the effective height being about 7.0 m. Although these antennas exhibit directional characteristics in a vertical plane and respond only to vertically polarized noise, they have the advantage of giving good coverage in a horizontal plane so that noise is received from any azimuth.

Any good radio receiver can serve as an amplifier for radio noise. In these studies Signal Corps receivers types BC 969 and BC 779 were used. They were modified to include average-reading detectors having a 60-second time constant.

Proper calibration of the noise meter is one of the major problems encountered in noise measurement. Ideally, one might expect that a calibration signal similar to the noise would be required in order to insure accurate relative measures of noise with different noise meters. Experimental measurements were made to determine the usefulness of a sine wave as a calibration signal. It was found that a sine wave was satisfactory for average-reading noise meters but not for quasi-peak-reading instruments. For this reason only average values of noise level are reported in this paper.

Reduction of data--Data obtained with the system described above are expressed in decibels above one microvolt per meter at one kilocycle of bandwidth, assuming that the output response of the noise meter is proportional to the square root of the effective bandwidth of the noise meter.

A median value of noise intensity is extracted from the noise-meter record for each hour of the day. The hourly values are averaged in different ways to determine diurnal and seasonal variations. Diurnal variations are obtained by averaging all readings for any given hour for an entire month, quarter, year, or other period. Seasonal variations are found by computing daily and monthly averages of noise intensity throughout the year. These resulting averages can be plotted against time or frequency.

Diurnal variation of average noise level--The diurnal changes in noise level are those associated with the rotation of the Earth. During the hours of darkness, noise levels are generally higher than during the daytime. This is partly due to changes in propagation conditions, but even at frequencies where such effects are subordinate, noise levels are actually higher during the late afternoon.

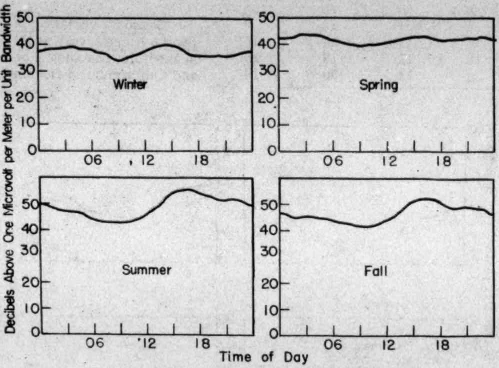

Fig. 1--Mean diurnal variation of the average value of atmospheric noise on 15 kc

At 15 kc, Figure 1, during the winter months a noise level was observed with lowest values about 35 db and highest about 40 db. A peak was reached at about 14h 00m, and a minimum occurred shortly after 09h 00m. (All times stated in this paper refer to 75th Meridian or Eastern Standard Time.)

Even less variation is exhibited during the spring months, but the overall noise level has increased to approximately 43 db.

A maximum noise level occurs during the summer months, and the effect of afternoon thunderstorms becomes more pronounced. A minimum noise level of 43 db occurs between sunrise and noon, while a maximum noise level of 55 db occurs at about 17h 00m. The autumn noise levels vary diurnally in about the same manner as those in the summer.

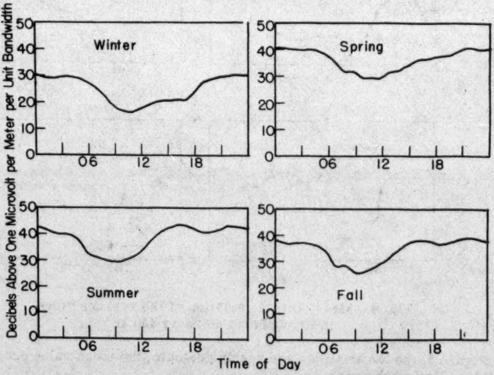

Fig. 2--Mean diurnal variation of the average value of atmospheric noise on 52 kc

Table 1--Magnitude of diurnal variation

Frequency	Winter	Spring	Summer	Autumn
kc	db	db	db	db
15	6	4	12	9
52	14	12	14	14
189	18	12	19	15
530	22	22	20	19

Other frequencies exhibit time variations similar to those at 15 kc but with higher magnitudes, as shown in Figures 2 to 4 and in Table 1.

Seasonal variation of average noise level--Noise levels at this locality vary seasonally, reaching a peak in June or July and a minimum in December or January. As shown in Figure 5, these variations are nearly periodic, the mean value occurring about March 21 and September 21 of each year.

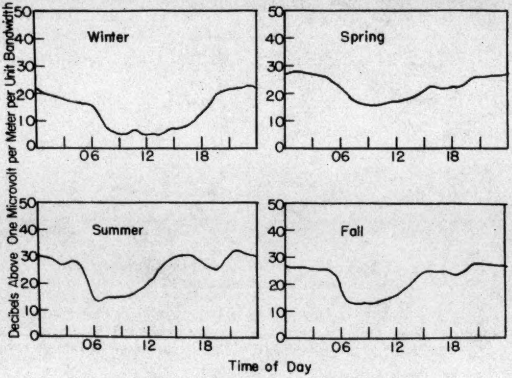

Fig. 3--Mean diurnal variation of the average value of atmospheric noise on 189 kc

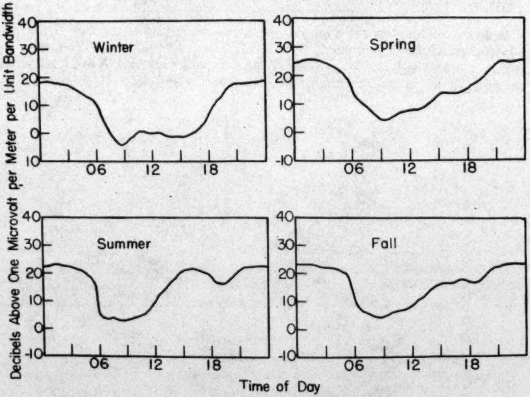

Fig. 4--Mean diurnal variation of the average value of atmospheric noise on 530 kc

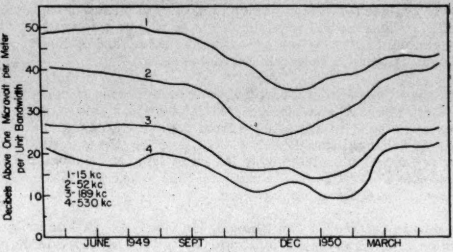

Fig. 5--Monthly means of atmospheric noise levels

As a first approximation to a mathematical description of this variation, a sine wave plus a constant term has been fitted to one year of noise data on each frequency. The resulting equations are expressions for the daily average noise level as observed in this study, and are as follows:

$$N_{15\ kc} = 43.7 + 7.0 \sin t_1 \quad \text{(1)}$$

$$N_{52\ kc} = 32.9 + 6.6 \sin t_1 \quad \text{(2)}$$

$$N_{189\ kc} = 20.6 + 8.5 \sin t_1 \quad \text{(3)}$$

$$N_{530\ kc} = 14.8 + 5.4 \sin t_1 \quad \text{(4)}$$

where $t_1 = 2\pi(t - t_o)/365.25$, t = interval from beginning of year to date required (day number), t_o = interval from beginning of year to last March 21 prior to date required, and N_{kc} = noise levels in decibels above one microvolt per meter at one kc of bandwidth.

In testing the closeness of fit of each of these equations to the actual measured values of noise, the results shown in Figure 6 were obtained.

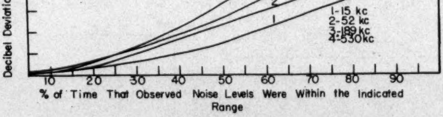

Fig. 6--Equations expressing daily mean noise levels as a function of time

The sine-wave seasonal variation is followed most closely on the lower frequencies. The larger departures at higher frequencies are attributed to the effect of propagation conditions on noise received above 100 kc.

Variation of average noise level with frequency--The spectrum of average noise level below 500 kc has been found to exhibit diurnal and seasonal variations. Short-term variations also appear, but in order to obtain an estimate of the spectrum most likely to occur at any time, seasonal averages have been obtained for various times of day. The mean atmospheric noise spectrum at any point on the Earth's surface at any particular time will be determined by the combined spectra of the individual lightning crashes which constitute the noise, and by the frequency-selective properties of the propagating paths of these noise crashes.

In an effort to show the effect of the propagating path, curves of noise spectra are presented for 04h 00m, 24h 00m, 12h 00m, and 16h 00m. At 24h 00m, a stable nighttime condition is generally reached, in which noise is being received from distant sources so that propagation conditions play a major role. At 04h 00m, shortly before local sunrise, the nighttime condition is being disturbed since some areas which contributed to the noise level at 24h 00m are being illuminated by the Sun, and ionospheric absorption has increased. This effect will lower noise levels at frequencies where sky-wave propagation is taking place.

Daytime spectra are obtained at 12h 00m and 16h 00m. At 12h 00m, noise is generally being received from distant sources since the local activity is low. Here again the effects of conditions along the ground wave paths are encountered.

A good estimate of the spectra of atmospheric noise when it is not subjected to propagation conditions is obtained at 16h 00m. At this time local activity has reached a maximum, and measured spectra most nearly approximate those of the combined noise crashes at their source.

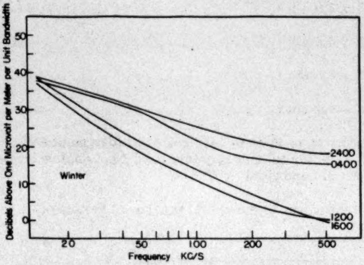

Fig. 7--Mean diurnal variation of the frequency spectrum of average noise 10-500 kc in winter

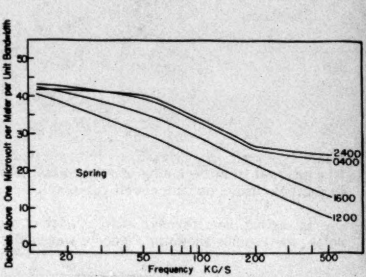

Fig. 8--Mean diurnal variation of the frequency spectrum of average noise 10-500 kc in spring

Measured average noise spectra for each season are presented in Figures 7 to 10. During the winter months, Figure 7, the diurnal variation is small at the lower frequencies, 10-50 kc, while considerable change is noted at the higher frequencies. This indicates that propagation conditions are exerting a major influence on the shape of the received noise spectrum.

During the spring months, Figure 8, propagation conditions tend to have less effect, and the 16h 00m spectrum has risen above the 12h 00m spectrum as a result of increased afternoon

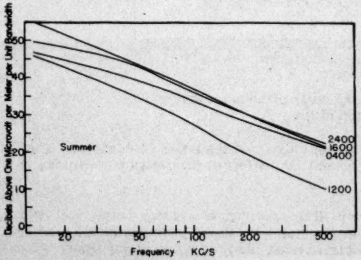

Fig. 9--Mean diurnal variation of the frequency spectrum of average noise 10-500 kc in summer

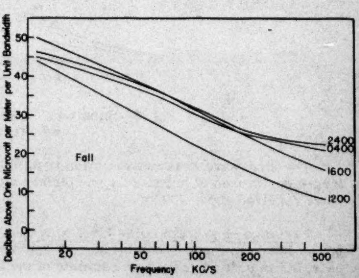

Fig. 10--Mean diurnal variation of the frequency spectrum of average noise 10-500 kc in autumn thunderstorm activity. The nighttime spectra exhibit a bend which was not present during the winter.

The summer spectra, Figure 9, are especially interesting, particularly at 16h 00m. At this time, noise levels are very high and are caused by nearby sources. Under these conditions, the effect of propagation conditions should be negligible, and the spectrum received should be essentially the average spectrum of the noise source. Within the range of these measurements, it was found that at 16h 00m noise levels were directly proportional to the wavelength, when the average spectrum was taken over the entire summer period. All spectra computed during the summer are generally similar in shape, and propagation conditions seem to have little effect on the measured spectra. Autumn spectra, Figure 10, are similar to those obtained during the summer, with propagation conditions slightly more in evidence.

The effect of sudden ionosphere disturbances on low-frequency noise and 100-kc signal--A study has been made of the effect of SID's (sudden ionosphere disturbances) on low-frequency noise. One year's noise records were checked against reports of SID's observed in the United States, published by the Central Radio Propagation Laboratory (CRPL), Washington, D. C. The results were not, in general, very satisfactory since there appeared to be little effect noted in most cases. Normally, it would be expected that SID's would affect noise levels, but in this study the receiving antennas were very close to the noise sources, so that propagation effects were slight. However, there were instances in which the noise levels were seen to increase suddenly during the same periods of time that SID's were reported by CRPL.

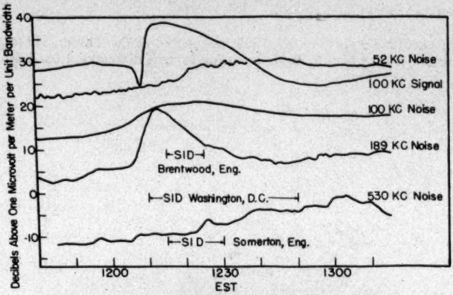

Fig. 11--Sudden ionosphere disturbance of September 19, 1950

A particularly striking example, Figure 11, occurred on September 19, 1950, between 12h 00m and 13h 00m. Little effect was noted at 15, 52, and 530 kc, but at 100 and 189 kc a considerable change in both noise and signal intensity occurred. Both noise and signal on 100 kc were measured with the same meter, with signal "on" for ten minutes and "off" for five minutes. Although some interpolation was necessary on the 100-kc noise record, it is apparent that the noise level increased. The 100-kc signals were received from Allaire, New Jersey.

The 189-kc noise record is continuous, however, and a 13-db change in intensity was observed. It is possible that this change was due to the reception of a c-w signal which was not normally received, but which now reached the receiver as a result of the use of higher power during abnormal conditions. Nevertheless, the results of these observations tend to indicate that an SID generally causes an increase in noise level at these frequencies.

These results in general support the theory that D-region ionization becomes abnormally high during the course of an SID, so that the D region becomes a good conductor. This, in effect, causes a condition similar to that normally encountered at night, and noise sky waves are received from over the entire area affected by the SID. The change in noise intensity noted at 189 kc during the SID discussed here is very nearly the same as is normally observed in the diurnal variation of noise on this frequency.

One interesting aspect of this particular case is that noise levels at 15, 52, and 530 kc were only slightly affected, while at 100 and 189 kc a considerable increase in noise level was observed. This tends to indicate either that the geographical area affected by the SID did not include all sources contributing to local noise levels or that the D region was effective primarily for frequencies in the approximate range of 100 to 200 kc. Since the SID was observed in both the United States and England, it seems reasonable that most noise sources contributing to the local noise level would be affected by the SID. If this assumption be true, then the possibility exists that a good conducting D region was presented only to frequencies lying in the range 100 to 200 kc.

Conclusions--To summarize these results, perhaps the most important information obtained is that, in general, noise levels are almost inversely proportional to frequency in the range 10-500 kc. A potentially useful consequence is the possibility that the average spectrum of the noise source itself is thus distributed (intensity directly proportional to wavelength), and that any deviation from this relation could be attributed to the frequency-selective properties of the propagating medium. If this is true, atmospheric noise might be used as a tool in studying propagation conditions.

It is intended that these results will also serve as an estimate of the time and frequency variations in average noise level below 500 kc in this region.

Acknowledgments--The research work described in this paper was supported by the U. S. Air Force under AMC Contract No. W28-099-ac-152.

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(Manuscript received February 25, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 5, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

GAMMA RADIATION FROM URANIUM X_2

V. F. Hess, W. F. Burns, and W. D. Parkinson

Abstract--The amount of ionization due to hard gamma rays from UX_2 (Pa^{234}) in equilibrium with a known amount of uranium was found by subtracting the calculated radiation due to RaC from the measured total hard gamma radiation. Corrections for self-absorption, geometry, and differential absorption in the lead filter are applied. Ionization is measured in terms of radium standards. The result obtained is that of the hard gamma radiation from the products of uranium, 5.2 pct (±0.4 pct) is due to UX_2, the rest being from RaC. This figure is about twice as high as the previously accepted one. Nevertheless, it shows that the contribution of the gamma radiation of UX_2 to the total radiation from all radioactive products contained in ordinary rocks is insignificant and does not play any important role in the total ionization of the lower atmosphere by terrestrial radiations.

Introduction--The amount of ionization due to the various radiations from uranium and its products are fairly well known today. One exception is the gamma radiation from UX_2 (Pa^{234}). This is difficult to measure because it is largely masked by the equally hard gamma radiation from RaC and ThC, the latter being a frequent impurity. The only previous determination of this figure is one by SODDY and RUSSELL [1909] who separated UX_2 chemically from a large quantity of uranium.

We were recently supplied with about 40 gm of thorium-free uranium ore from Joachimstahl, the uranium content of which was known to be 45.7 pct, with considerable accuracy. J. P. Marble, in a private communication, writes as follows about the pitchblende:

"The whole specimen was broken up in June, 1948, into smallish fragments on a clean rock-cutter, and except for four small chips, reserved for some other studies, was powdered on a clean mulling plate at the U. S. Geological Survey until the entire sample passed through a standard U. S. 80-mesh screen. The entire powder was then separated into light and heavy fractions by settling in fresh bromoform, which had recently been washed with distilled water several times and dried over calcium chloride. From 15 to 35 settlings were made on individual portions of the sample, weighing from 100 to 250 gm each. The separations were carried out on a given sample until, under the binocular microscope, no further diminution of light-colored material could be seen in the "heavy" portion. From time to time the powder was checked under the petrographic microscope. I estimate that there is about ten per cent of silica in the remaining material, and about 15 pct of calcite. It proved impossible to make a cleaner separation than this, as the silica and calcite were so very intimately mixed with the pitchblende. Finer grinding would probably not have helped. There was, to judge by the microscopic examinations, a fraction of a per cent of secondary uranium minerals present.

"The final heavy fractions were washed thoroughly with washed acetone, and dried on a steam bath until no odor of organic material could be noted. The individual portions were then shaken vigorously at various times for a total period of at least one hour in stoppered glass bottles. The material sent you was taken from this main sample.

"As regards freedom from thorium: In connection with the analysis for uranium content, which I furnished you earlier, I made three separations of the rare-earth group as insoluble oxalates in a solution free from NH_4^+ salts in very weakly acid solution, after having separated U + Fe + rare earths from Ca, Mg, etc., by ammonia precipitation several times repeated, and having earlier removed Pb, Cu, Bi, etc., by H_2S precipitation in dilute acid solution. This rare-earth oxalate precipitate would have contained any thorium present. On three analyses, made with 1-, 2-, 3-gram samples, respectively, I was unable to detect any thorium by the peroxynitrate test. My experience has been that this method will determine readily 0.1 mg ThO_2 in a one-gram sample of the original material.

"Furthermore, I supplied Elizabeth Rona, then at Argonne Laboratory, later at Oak Ridge Institute of Nuclear Studies, with some of the original powdered material for Io/Th ratio studies. She wrote me under date of September 18, 1950 '. . . the ratio of ionium-thorium found by me in this mineral was 25 to 1, determined by the extremely sensitive method of analysis.' "

Using the null method of ionization measurement described by HESS [1946], we were able to measure the excess of total ionization over that due to RaC alone, although this excess amounts to only about five per cent of the total.

Procedure--The ionization chamber used had a volume of 44 liters and was shielded by ten cm of iron on top, bottom, and three sides. In every case the amount of ionization was expressed in terms of radium equivalent by comparison with a known radium standard. Four different radium standards were calibrated by the senior author, using a set of sealed radium tubes provided by the U. S. Radium Corporation, New York, ranging from 2267 to one μgm of radium. The "background" radiation, with no source present, was determined before and after each set of readings.

The ratio of the gamma radiation of UX_2 to that of RaC from the uranium ore should be as high as possible for these measurements. Now the former radiation is softer than the latter, and hence the minimum of filtering material should be used consistent with the filtering out of all softer radiations. The amount of filter used was 1.7 cm of lead, which excludes all gamma radiation from RaB. This introduces an error due to the difference in absorption coefficient of the UX_2 and RaC radiation. Taking these coefficients in lead to be 0.72 and 0.50 cm^{-1} respectively, the correction factor to be applied to the radium equivalent of the UX_2 radiation is

$$e^{(0.72-0.5)1.7} = 1.45$$

Corrections--The radium standards are so small that they can be considered as point sources. The uranium ore, on the other hand, occupies a considerable volume. It was contained in a sealed, cylindrical glass tube of 1.1-cm diameter and extended over a total height of 10 cm. This introduces two further corrections to be applied to the measured effect from the ore. The first is due to the absorption by the ore itself. The tube containing the ore was held with its axis parallel to, and 35 cm away from, the axis of the cylindrical ionization chamber.

The absorption coefficient μ in the ore itself was determined experimentally by imbedding a much stronger radium source (2267 μgm) in the uranium ore and measuring the decrease due to absorption. The value $\mu = 0.1238$ cm^{-1} was obtained.

The effective volume V of an element of the ore is related to the actual volume V_o by

$$V = V_o \, e^{-\mu s}$$

where s is the absorbing path of material for this element of volume. This can easily be obtained by graphical integration. Taking into account the fact that not all rays travel perpendicular to the axes of the test tube and ionization chamber, we find the correction factor for self-absorption to be 0.938. Therefore, observed results must be divided by this or multiplied by 1.066.

The second correction is due to the fact that all the uranium ore cannot be put at the same point as the radium standard. Let the uranium ore extend from height h_1 to h_2 and let the position used for the radium standard be h_o. Also let

U	Total ionization from ore
F (h) dv	Ionization from volume element dv of ore at h
R (h)	Ionization from radium standard when at h
v	Total volume of ore
v'	Volume of ore giving the same ionization as radium standard (assuming the ore concentrated at the position of the radium)
U'	Total ionization from ore if it were all at h_o

Then, by definition of v'

$$R (h) = F (h) v'$$

so that

$$U = \int F (h) \, dv = (1/v') \int R (h) \, dv$$
$$= v/v' \, (h_2 - h_1) \int_{h_1}^{h_2} R (h) \, dh$$

since the cross section is uniform and

$$U' = F(h) . v = (v/v') R(h_o)$$
$$= U(h_2 - h_1) R(h_o) / \int_{h_1}^{h_2} R(h) \, dh$$

Now R (h) can be measured by placing the radium standard at various heights between h_1 and h_2 so that $[1/R(h_o)] \int R(h) \, dh$ can be determined. This gives the geometrical correction factor $U'/U = 0.988$.

Results--The following is a typical set of readings, showing the method of procedure. The mean of six background measurements (three before and three after) is 1.89 I (ion pairs/cc sec). After correction for background the ionization due to a 67.2 µgm radium standard is 85.9 I and that due to the uranium ore is 6.63 I, each being the mean of ten readings. The latter, corrected for self-absorption and the geometrical factor, is 6.97 I. Hence the radium equivalent of the uranium ore is

$$67.2 \times 6.97/85.9 = 5.45 \ \mu gm$$

Now, using 3.4×10^{-7} for the radium-to-uranium ratio, the amount of radium in the uranium ore can be calculated to be 5.25 µgm, so that the radium equivalent of the UX_2 is 5.45 - 5.25 = 0.20 µgm. This must now be corrected for the differential absorption by being multiplied by 1.45. Hence, the result of this set is 0.20 × 1.45 = 0.29 µgm.

Four different radium standards were used. These were all intercompared so that all results could be referred to one "correct" standard. Table 1 shows the results obtained when each of the standards in turn is assumed correct.

Table 1--Results obtained from four different radium standards

Standard assumed correct	Number of sets	Radium equivalent	
		of ore	of UX (corrected for differential absorption)
		µgm	µgm
No. 1 (67.2 µgm)	15	5.45	0.29
No. 2 (10 µgm)	17	5.48	0.33
No. 3 (5 µgm)	15	5.38	0.19
No. 4 (4.2 µgm)	6	5.54	0.42
	Weighted mean		0.29

Now the total amount of uranium in the tube was 16.03 gm. Thus 0.29 µgm is the mass of radium which would give the same amount of ionization due to hard gamma rays as does the UX_2 in equilibrium with 16.03 gm of uranium. This may also be expressed by saying that of the ionization due to hard gamma rays emitted by the products of uranium, 0.29 × 100/(5.25 + 0.29) = 5.2 pct is due to UX_2. The figure given by SODDY and RUSSELL [1909] was almost exactly half of this. Nevertheless, it shows that the contribution of the gamma radiation of UX_2 to the total radiation from all radioactive products contained in ordinary rocks is insignificant and does not play any important role in the total ionization of the lower atmosphere by terrestrial radiations.

The probable error in our figure due to observational errors, as derived from the sets of ten readings, is about 0.02 µgm or seven per cent. This does not take into account inaccuracies in the radium standards or in the analysis of the uranium ore. It is pleasing to note, however, that the mean of standards 2, 3, and 4 is the same as the value for standard 1, for we consider the last to be the most accurate.

Acknowledgment--This study was supported in part by contract AF 19 (122)-409 of the U. S. Air Force.

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(Manuscript received April 8, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 5, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

A COMPARISON OF THE POWER LAW AND A GENERALIZED LOGARITHMIC FORMULA IN MICROMETEOROLOGY

Herman Lake

Abstract--It is shown that the variation of wind speed with height as given by Deacon's form for the power law is similar to that expressed by Lettau's generalized logarithmic law. The exponent in the power formulation may be evaluated by an extension of the theory of turbulence in the atmospheric surface layer.

Introduction--In the early part of this century, the vertical gradient of the velocity of homogeneous fluid flow in the boundary layers of pipes, ducts, channels, and flat plates was expressed as a function of the distance from the surface by a power law. It was soon discovered that the exponent was a function of the surface roughness or Reynolds number. PRANDTL [1932] pointed out that the power law will yield the logarithmic law when the exponent equals unity.

Micrometeorologists found empirically that a logarithmic height representation of the wind velocity in the very lowest layers of the atmosphere was valid for adiabatic conditions. For conditions other than adiabatic, the logarithmic law no longer held true. Early attempts to extend the logarithmic law to non-adiabatic stratification did not prove very successful, as cumbersome and complicated mathematics had to be introduced into the picture. Therefore, researchers represented the variation of wind in the vertical as a power function of height having a fixed exponent (that is, a homologous velocity profile) which representation worked fairly satisfactorily when the exponent was determined by the thermal stratification of the air.

The power law is an aid to the mathematical tractability of atmospheric diffusion. However, its principal disadvantages lie in the height and time variation of the exponent. Therefore, SVERDRUP [1936] regarded the power law as merely a suitable interpolation formula. It is intended to express the effects of the surface roughness and the thermal stratification on the profile configuration by utilizing the generalized logarithmic law. An analytical equation for the power shall be derived which will show it to be a function of the defined parameters of turbulent air flow in the atmospheric surface layer.

Background material--A generalization of the logarithmic law in the surface layer of the atmosphere was introduced by LETTAU [1949]. Adapting his definition of the atmospheric surface layer to the problem at hand, we write as follows:

Let Q_o be the ground flux of the property q at $z = 0$, and let $\pm \epsilon Q_o$ represent the relative error of flux measurement, then $|\epsilon| << |$. The surface layer is that layer in which $|Q_z - Q_o| \leq |\epsilon Q_o|$ Thus, the change of the turbulent flux of the property in the vertical is negligible. In the present study, the flux will refer to the following two factors: $Q_o = \tau$ is the ground stress, or negative vertical flux of horizontal momentum, when q is the horizontal air flow, while $Q_o = L$ is the vertical eddy heat flux when $q = c_p\theta$, where c_p is the specific heat at constant pressure and θ is the potential temperature.

In addition, Lettau's paper presented an integrated solution for the height variation of wind velocity which reduced to the logarithmic law as the heat flux L approached zero.

In the same year, DEACON [1949] presented a generalized power solution of the vertical distribution of wind velocity near the Earth's surface. Although it was shown that the power varied with thermal stratification, an exact analytical expression for it was not obtained. We shall now show how this can be found.

Notation--The following notation is used.

a	Constant
b	Constant
e	Subscript indicating a value for a layer
i	Subscript indicating a value at a level
$\overset{*}{w}_a$	Adiabatic vertical mixing velocity

z	Height
z_o	Roughness parameter, a geometric characteristic of the terrain over which the air flows
L	Eddy flux of heat
V'	Vertical gradient of mean wind velocity
Y	Non-dimensional function of height, roughness, adiabatic mixing velocity, and heat flux
1	Subscript indicating lower level
2	Subscript indicating upper level

Deacon represented the vertical gradient of mean wind velocity as being equal to a constant times a negative power of the height.

$$V' = az^{-\beta} \dots\dots\dots\dots \quad (1)$$

On the other hand, a different representation of the gradient was illustrated by LETTAU [1949, p. 43, eq. (67)]. He showed that the gradient can be depicted by a constant times a non-dimensional function of height, roughness, adiabatic mixing velocity, and heat flux divided by the height plus the roughness parameter.

$$V' = bY/(z + z_o) \dots\dots\dots\dots \quad (2)$$

The prime objective is to determine those geometric properties of the ground surface as well as the thermal properties of the atmosphere that influence the power β. As a first step, we shall change the boundary condition of the power form of the generalized wind profile by adding z_o, the roughness parameter, to it.

$$V' = a\,(z + z_o)^{-\beta_1} \dots\dots\dots\dots \quad (3)$$

This, in effect, specifies the existence of a definite, finite value for the mean wind-velocity gradient at the ground level, $z = 0$. We can now set up a ratio of the gradient at one level to that of another level. From Deacon (Eq. 1)

From Lettau (Eq. 2)

$$\left.\begin{aligned} V_2'/V_1' &= [(z_2 + z_o)/(z_1 + z_o)]^{-\beta_e} \\ V_2'/V_1' &= [Y_2\,(z_1 + z_o)\big|/\big|Y_1\,(z_2 + z_o)] \end{aligned}\right\} \dots\dots\dots\dots \quad (4)$$

We define β_e to be intermediate in value between β_1 and β_2 such that $\beta_1 \lesseqgtr \beta_e \lesseqgtr \beta_2$. If, in addition, we consider only small differences of $z_2 - z_1$, our way is clear to find an expression for β_e.

Now that we have established ratios of the vertical gradients of the mean wind velocity for two apparently dissimilar formulas which we shall assume are identical in their property representation, we shall equate them by taking an effective value of the exponent given by β_e, for the layer between the two levels we have designated.

$$[(z_2 + z_o)/(z_1 + z_o)]^{-\beta_e} = [Y_2\,(z_1 + z_o)/Y_1\,(z_2 + z_o)] \dots\dots\dots\dots \quad (5)$$

Solving for β_e, we find that

$$\beta_e = 1 - \Big\{[\log_e\,(Y_2/Y_1)/\log_e\,[(z_2 + z_o)/(z_1 + z_o)]\Big\} \dots\dots\dots\dots \quad (6)$$

Now (from Lettau)

$$Y_2/Y_1 = [1 + 2y'\,(z_1 + z_o) + \sqrt{1 + 4y'\,(z_1 + z_o)}]/[1 + 2y'\,(z_2 + z_o) + \sqrt{1 + 4y'\,(z_2 + z_o)}] \dots \quad (7)$$

where $y' = 5300\ L/\overset{*}{w}_a{}^3 = y/(z + z_o)$ [LETTAU, 1949, p. 34, eq. (47a)]. Inasmuch as L equals the heat flux in the vertical and $\overset{*}{w}_a$ is the adiabatic mixing velocity, we may consider y' to be the fundamental thermodynamic parameter of the atmospheric surface layer. (If the surface-layer concept is not used, or if greater accuracy is desired, y' should be replaced by y_1' in the numerator and y_2' in the denominator.) By definition, $y = y'\,(z + z_o)$. For $z_1 = 0$, $y = y' z_o = \epsilon << 1$ as $|y'| = 10^{-3}\ \mathrm{cm}^{-1}$ and $z_o \sim 1$ cm. Therefore y_1 is of the order of 10^{-3} or less. Expanding $\sqrt{1 + 4y_1}$ in series and neglecting terms of second and higher order

$$Y_1 = 2/(1 + 2y_1 + \sqrt{1 + 4y_1}) \approx 2/(1 + 2y_1 + 1 + 2y_1 - \dots) \approx 1/(1 + 2y_1 - \dots) \approx 1 - 2y_1 + \dots$$

or

$$Y_1 = 1 - 2\epsilon + \dots$$

Thus to a good degree of approximation where small values of z_0 are involved, $Y_1 = 1$. Since Y_1 equals unity at $z = 0$, the effective value of β_e from the ground up to a height z_2 is given by

$$\beta_e = 1 - \left\{\log Y_2/\log\left[(z_2 + z_0)/z_0\right]\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

Observation has shown that β_e in (4) is greater than unity for unstable stratification, equal to unity for neutral or adiabatic conditions, and less than unity for a stable atmosphere. This is confirmed by theoretical eq. (6), inasmuch as $Y_2 = 1$ for neutral conditions, $Y_2 > 1$ for stable conditions, and $Y_2 < 1$ for unstable conditions.

The height dependency of the exponent--We shall examine the variation of the exponent in the atmospheric surface layer adjacent to the ground where one can neglect the height variation of the stress and of the eddy heat flux.

Inasmuch as the effective value of the exponent for a layer has been given, we may now seek its value at a designated level. The limiting value of β as z_1 approaches z_2 is readily determinable. In (6)

$$Y_2 = Y_1 + \Delta Y$$

$$z_2 + z_0 = z_1 + z_0 + \Delta z$$

and

$$\begin{aligned}\log_e (Y_2/Y_1)/\log_e \left[(z_2 + z_0)/(z_1 + z_0)\right] &= \log_e \left[1 + (\Delta Y/Y_1)\right]/\log_e\left\{1 + \left[\Delta z/(z_1 + z_0)\right]\right\} \\ &= \log_e \left[1 + (\Delta Y/Y_2)\right]/\log_e\left\{1 + \left[\Delta z/(z_2 + z_0)\right]\right\} \approx (\Delta Y/Y_2)/\left[\Delta z/(z_2 + z_0)\right] \\ &= (\Delta Y/\Delta z)\left[(z_2 + z_0)/Y_2\right] \rightarrow (dY/dz)\left[(z_2 + z_0)/Y_2\right]\end{aligned}$$

but $Y = f(z)$ only, when L and $\overset{*}{w}_a$ are constant; therefore, $dY/dz = \partial Y/\partial z = Y'$.

Thus, by neglecting terms of order higher than one in the series expansion, we attain the limiting relationship

$$\beta = 1 - \left[Y'(z_2 + z_0)/Y_2\right] = 1 - \left[Y' y_2)/(Y_2 y')\right]$$

With the algebraic manipulation of LETTAU [1949, eq. (52), p. 36, and eq. (80), p. 61] the exponent at a given height is found to be

$$\begin{aligned}\beta_2 &= 2 - (1/\sqrt{1 + 4y_2}) \\ &= 2 - \left[1/\sqrt{1 + 4y_2'(z_2 + z_0)}\right] \quad \ldots (8)\end{aligned}$$

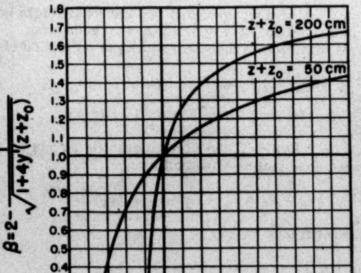

Fig. 1--A theoretical example of the dependence of the exponent upon height and stability in the atmospheric surface layer

At $y = 0$, $\beta = 1$; $y = \infty$, $\beta = 2$; $y = -3/16$, $\beta = 0$; $y = -1/4$, $\beta = -\infty$. For extreme instability, it can be seen that the limiting value of β would be 2. For all practical purposes then, the range of the exponent lies between 0 and 2. At any given level, β increases with decreasing stability. For stable stratification the exponent decreases with height, while for unstable stratification it increases. This is illustrated in Figure 1.

The micrometeorological observations reported by PASQUILL [1949] will be utilized to judge the reliability of the exponent. These observations were chosen because they were the most accurate and representative set of data available and were excellent for evaluating theory.

Observational analysis--If the six levels of the Pasquill data are divided into two groups, one group consisting of the 25-, 37.5-, and 50-cm levels and the other group of the 100-, 150-, and 200-cm levels, every level in the second set is at a height four times that of its corresponding member in the first. From this arrangement of observations, a combination of alternate, successive levels was selected such that a ratio of the differences of the wind speed between levels would be approximately unity for adiabatic stratification.

$$R = (V_6 - V_3 + V_5 - V_2)/(V_5 - V_2 + V_4 - V_1) \quad \ldots\ldots\ldots\ldots\ldots (9)$$

A stability parameter for the air flow was defined as

$$\Phi = \Delta\theta_m / V_m^2$$

where V_m^2 represented the square of the mean wind speeds and $\Delta\theta_m = (\theta_6 - \theta_3 + \theta_5 - \theta_2 + \theta_4 - \theta_1)/3$ was the mean of the differences of the potential temperature at the six levels.

A graph of the ratio of the wind-speed differences R was plotted against the stability parameter Φ, Figure 2. The regression line of R on Φ was computed and R_a, the value of the wind-speed difference ratio for adiabatic or neutral stability, was found to be 0.96.

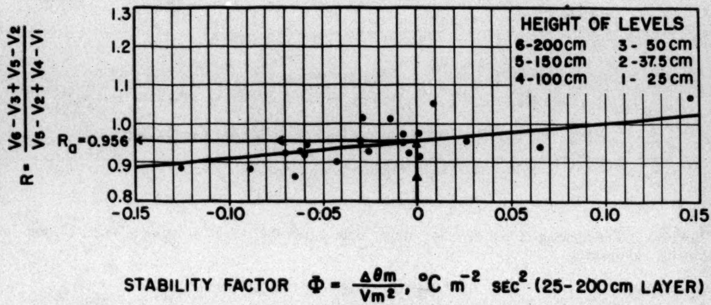

Fig. 2--A graph of the ratio of the wind-speed difference ratio R versus the stability factor Φ, illustrating the determination of the adiabatic value R_a, data from PASQUILL [1949]

The formal mathematical boundary condition $V = 0$ at $z = 0$ specifies that the logarithmic law be written as $V = c \log_e [(z + z_o)/z_o]$, where c is a constant. The quantity z_o is a geometric characteristic of the ground surface. From (9), it then follows that the ratio R_a is a function of z_o.

$$R_a = [\log (200 + z_o) - \log (50 + z_o) + \log (150 + z_o) - \log (37.5 + z_o)]/$$
$$[\log (150 + z_o) - \log (37.5 + z_o) + \log (100 + z_o) - \log (25 + z_o)] = f (z_o)$$

Solving for $R_a = 0.96$, our result is $z_o = -7.0$ cm. However, the desired quantity z_o can be secured more accurately by considering the following combination of velocities.

$$\psi = 2 [V_6 - V_3 + V_5 - V_2 + V_4 - V_1]/[V_6 + V_5 + V_4 + V_3 + V_2 + V_1]$$

Transforming the preceding arrangement into the form (see Fig. 3)

$$\psi_a = 2 [\log (200 + z_o) (150 + z_o) (100 + z_o) - \log (50 + z_o) (37.5 + z_o) (25 + z_o)]/$$
$$[\log (200 + z_o) (150 + z_o) (100 + z_o) + \log (50 + z_o) (37.5 + z_o) (25 + z_o)] - 6 \log z_o$$

and considering that $\psi_a = 0.26$, the solution is found to be $z_o = +0.35$ cm. This result differs from the previous one, not only in magnitude but in sign as well. We can resolve the apparent discrepancy by introducing a height correction factor d into the logarithmic law

$$V = c \log_e [(z + d + z_o)/z_o]$$

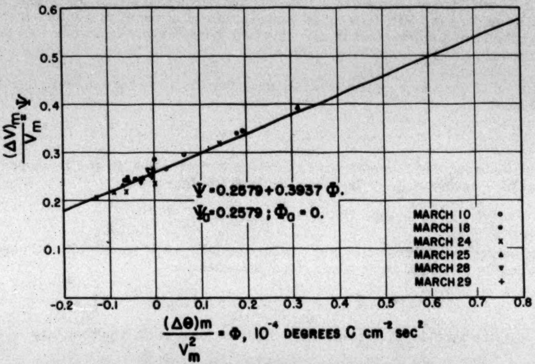

Fig. 3--The variation of the profile contour number ψ with the stability factor Φ, plotted points from PASQUILL [1949]

Let $d + z_0 = D$ = zero-level displacement; then

$$R_a = [\log (200 + D) - \log (50 + D) + \log (150 + D) - \log (37.5 + D)]/[\log (150 + D) - \log (37.5 + D) + \log (100 + D) - \log (25 + D)] = f (D) \quad \text{(9a)}$$

R_a is a single-valued function of D.

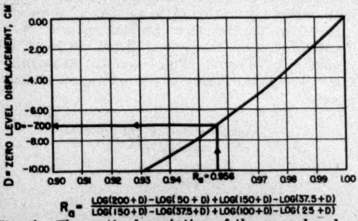

Fig. 4--Theoretical variation of the zero-level displacement D (idealized horizontal surface at which the mean wind is zero) and R_a, the adiabatic value of the wind-speed difference ratio

This dependency is graphed in Figure 4, and D = -7.0 cm when $R_a = 0.96$. Thus we have determined a prime geometric factor involved in the analysis of the observations in the light of the logarithmic law.

The non-dimensional term ψ, introduced above, is called the profile contour number,

$$\psi = (\Delta V)_m / V_m$$

$$(\Delta V)_m = (V_6 - V_3 + V_5 - V_2 + V_4 - V_1)/3$$

Thus the form of the profile was classified numerically. This parameter was plotted against Φ, the stability parameter, and the regression line of ψ on Φ was computed. From this relationship, Figure 3, the adiabatic profile contour number ψ_a was found to be 0.26. With the value of ψ_a, and the zero-level displacement D, one could determine z_0, the roughness parameter of the ground, from

$$\psi_a = 2\,(\Sigma_2 - \Sigma_1)/[(\Sigma_2 + \Sigma_1) - 6 \log z_0] \quad \text{(10)}$$

where $\Sigma_2 = \log (200 + D)(150 + D)(100 + D)$ and $\Sigma_1 = \log (50 + D)(37.5 + D)(25 + D)$.

$$z_0 = \log^{-1} [(\Sigma_2 + \Sigma_1)/6] - [(\Sigma_2 - \Sigma_1)/3] \quad \text{(10a)}$$

$$z_0 = 0.15 \text{ cm}$$

This is a much closer approximation to the roughness parameter than those previously obtained.

Now that z_0 and D have been established, we have succeeded in characterizing the geometry of the bounding surface (z_0) and defining an analytical form of the wind profile which insures zero wind speed at the zero displacement level. This is of considerable importance as the boundary plane influences the growth or decay of turbulence.

The value of β can now be found from (9) following the procedure to be illustrated. According to Deacon

$$V = (a/1 - \beta)\, z_0^{1-\beta} \left[(z/z_0)^{1-\beta} - 1\right] \tag{11}$$

It must be remembered that this equation is semi-empirical in that it was developed to fit the results of a plot of V against $\log_e z$. However, we have (3); therefore

$$V = (a/1 - \beta)\, z_0^{1-\beta} \left\{\left[(z + z_0)/z_0\right]^{1-\beta} - 1\right\} \tag{11a}$$

Extending the height correction d as found for the special case of an adiabatic atmosphere to include any thermally stratified layer of air, (11a) becomes

$$V = (a/1 - \beta)\, z_0^{1-\beta} \left\{\left[(z + d + z_0)/z_0\right]^{1-\beta} - 1\right\}$$

Since $d + z_0 = D$, the zero-level displacement, the final form of the equation may be written

$$V = (a/1 - \beta)\, z_0^{1-\beta} \left\{\left[(z + D)/z_0\right]^{1-\beta} - 1\right\} \tag{11b}$$

Now

$$R = \left[(z_6 + D)^{1-\beta} - (z_3 + D)^{1-\beta} + (z_5 + D)^{1-\beta} - (z_2 + D)^{1-\beta}\right] / \left[(z_5 + D)^{1-\beta} - (z_2 + D)^{1-\beta} + (z_4 + D)^{1-\beta} - (z_1 + D)^{1-\beta}\right] \tag{9b}$$

Thus β could be ascertained for each profile.

After the analysis was completed, it was discovered that of the 25 observations, four were outside the defined atmospheric surface layer. This changed the value of D, z_0, and β somewhat. However, the change was minor and only the variation in the D curve of β versus Ri and β versus Φ is shown (Fig. 5 and 6). The points that are plotted refer to the preliminary analysis.

Table 1--Geometric results of analysis

Item	Preliminary	Final
	cm	cm
D	-7.0	-6.0
z_0	0.15	0.18

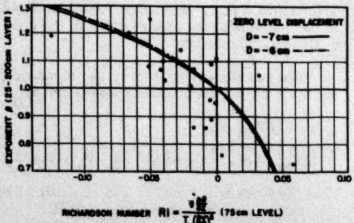

Fig. 5--The theoretical variation of the exponent β in the surface layer with Richardson number Ri; the encircled dots give a semi-empirical value of β for a corresponding value of Ri as reported by Pasquill where the zero-level displacement D is - 6.00 cm; a plot of values for D = - 7.00 cm has not been drawn but the theoretical curve is shown

Theoretical analysis of the power, β--In LETTAU [1949, p. 40, eq. (66h)]

$$V = bZ = b\left\{\log_e\left[(z + z_0)/z_0\right] + \eta\right\}$$

where b is a constant for the profile under examination. Taking a mean value for the layer, $V_m = bZ_m$ and $\Delta V_m = b\Delta Z_m$, thus $\psi = \Delta Z_m/Z_m$. Z and η are defined as non-dimensional functions of height, roughness, adiabatic mixing velocity, and heat flux. An additional factor that is considered in this analysis along with the aforementioned parameters is the zero-level displacement. Applying the displacement concept to the theory of turbulent flow, the mean adiabatic profile contour number is determinable as follows

$$\psi_a = (\Delta Z_a)_m/(Z_a)_m = 0.26$$

where $(Z_a)_m = [\log_e (z + D)/z_0]_m = (1/6) \sum_{n=1}^{6} \log_e [(z_n + D)/z_0]$, and $(\Delta Z_a)_m = (1/3)\left\{ \sum_{n=4}^{6} \log_e [(z_n + D)/z_0] - \sum_{n=1}^{3} \log_e [(z_n + D)/z_0] \right\}$.

It can be shown by theoretical reasoning that $\Phi = \Delta\theta_m / V_m^2$ is a monotonic function of y'. Therefore, the stability parameter was computed for positive and negative values of the argument and a graph prepared. (It would have been simpler to compute ψ and obtain the same result, as a linear relationship exists between ψ and Φ. However, Φ was used because it was necessary for the determination of the Richardson number Ri.) The value of y' derived from Pasquill's experimentally observed Φ was read off the graph. This value of y' was used in computing the ratio

$$Y_2/Y_1 = [1 + 2y'(z_1 + D) + \sqrt{1 + 4y'(z_1 + D)}]/[1 + 2y'(z_2 + D) + \sqrt{1 + 4y'(z_2 + D)}] \quad \ldots. (7a)$$

From (7a), the value of β for the layer from z_1 to z_2 was obtained by modifying (6).

$$\beta = 1 - [\log (Y_2/Y_1)]/[\log (z_2 + D)/(z_1 + D)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

With the aid of (6b), a graph of the theoretical dependence of β on Ri was prepared and is shown as the dot-dash curve of Figure 5 for the case D = -6 cm. In order to permit comparison between this theoretically predicted relation and actual observations, Pasquill's data were used to plot values of β for the layer against the corresponding observed Richardson number. These data are shown in Figure 5 as the encircled dots, but the theoretical prediction is shown as the solid curve.

The forecasting of the variation of β with Ri in the atmospheric surface layer depends upon a theoretical appraisal of the latter. According to Lettau (unpublished paper, The present position of selected turbulence problems in the atmospheric boundary layer, 1951), $Ri = -y/Y(CX)^2$, where $CX = \tau_0/\tau_{0,a}$, the ratio of the ground stress in a non-adiabatic atmosphere to that in an adiabatic atmosphere. This new factor is a non-dimensional function of (y', z, D, z_0), that is, of height, heat flux, adiabatic mixing velocity as well as the geometric terms, zero-level displacement, and roughness parameter, and was evaluated from

$$(CX)^2 = -y' T_m/g Z_m \Phi = -y' T_m (0.3937 \times 10^{-4})/g Z_m (\psi - \psi_a) \quad \ldots\ldots\ldots (12)$$

The theoretical curve indicates a monotonic relationship between β(25-200 cm) and Ri (75 cm) such that

$$\begin{array}{ll} Ri > 0 & \beta < 1 \\ Ri = 0 & \beta = 1 \\ Ri < 0 & \beta > 1 \end{array}$$

Similarly, the variation of the power β with the stability parameter $\Phi = \Delta\theta_m/V_m^2$ for the entire layer was determined, Figure 6. It may be noticed that the two curves, Figures 5 and 6, are very similar. This is because the Richardson number is a unique function of the stability parameter.

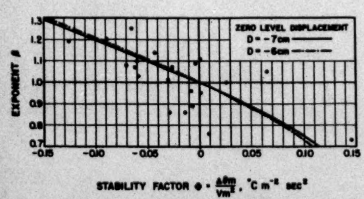

Fig. 6--The theoretical variation of the exponent β, with the stability factor Φ in the 25-200 cm surface layer; the encircled dots give a semi-empirical value of β for a corresponding value of Φ derived from the Pasquill data where the zero-level displacement D is -6.00 cm; a plot of values for D = -7.00 cm has not been drawn but the theoretical curve is shown

The resultant points were scattered somewhat. It is very likely that the scatter in the observed values is due to the following factors:

(1) Horizontal homogeneity was taken for granted by the observer. If this did not exist, then the fact that three anemometer masts were used (the 50- and 25-cm observations were taken on two separate masts) would yield somewhat misleading results.

(2) The immediate surrounding surface may or may not have had the quality of physical heterogeneity. We quote: "Bare soil was visible over a considerable proportion of the pasture and the patchy grass cover was mainly 2 cm long with occasional withered tufts up to 5 to 7 cm and showed no appreciable change in length over the whole period of observation."

Summary and conclusion--The purpose of this paper has been the delineation of the geometrical parameters, that is, (D, z_0) and the thermodynamic parameter y' of the wind profile in the atmospheric surface layer. Moreover, it must be emphasized that the zero-level displacement and the roughness parameter were obtained independent of the thermal stratification.

The numerical comparison between the generalized logarithmic law and the power law forecast the observational results fairly well, principally the effect of atmospheric stability on β.

Acknowledgment--The author is indebted to the members of the Atmospheric Analysis Laboratory, Geophysics Research Division, Air Force Cambridge Research Center, who have contributed to this investigation; Heinz Lettau for his helpful criticisms and valuable suggestions; and Ben Davidson for his review of the manuscript.

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(Communicated manuscript received December 26, 1951, and, as amended, April 9, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

LAMINAR FREE CONVECTION DUE TO A LINE SOURCE OF HEAT

Chia-Shun Yih

Abstract--Closed solutions are given in this paper for the velocity and temperature distributions in the fluid in free convection above a line source of heat embedded in an infinite horizontal plane for Prandtl numbers 2/3 and 7/3. Variations of the kinematic viscosity and of the thermal diffusivity with temperature, as well as the heat generated by viscous dissipation, are neglected. For heat transfer in air under normal conditions the Prandtl number is 0.73, so that the solution for Prandtl number 2/3 may be considered to furnish a fair approximation. Since the Prandtl number for diffusion of water vapor into air is approximately 0.6, the same solution can also be applied to the case of laminar evaporation from a line source of water into air.

Statement of the problem--A line source of heat is placed in an infinite horizontal plane above which the fluid was originally isothermal and at rest. Free convection is induced by the difference of specific weight due to the temperature variation in the fluid. The resulting interdependent velocity and temperature distributions are sought.

The solution--From the equation of state

$$p/\gamma = RT \qquad (1)$$

where p, γ, R, and T are respectively the pressure, the specific weight, the gas constant, and the absolute temperature, one has, for small variations in p and T

$$-\Delta\gamma/\gamma_0 = \Delta T/T_0 \qquad (2)$$

where $\Delta\gamma$ and ΔT are respectively the deviations of γ and T from their datum values γ_0 and T_0.

In a vertical plane perpendicular to the line source, the trace of the line source is taken as the origin, and the vertical and horizontal lines issuing therefrom are taken as the x- and y-axes respectively. Denoting by u and v the velocity components in the x- and y-directions, and by α the thermal diffusivity, the boundary-layer equation of heat diffusion, being linear and homogeneous, by virtue of (2) can be written as

$$u\,\partial\Delta\gamma/\partial x + v\,\partial\Delta\gamma/\partial y = \alpha\partial^2\Delta\gamma/\partial y^2 \qquad (3)$$

Assuming that the pressure distribution is essentially hydrostatic, the boundary-layer equation of motion is

$$u\,\partial u/\partial x + v\,\partial u/\partial y = \nu\partial^2 u/\partial y^2 - g\,\Delta\gamma/\gamma_0 \qquad (4)$$

where ν and g are respectively the kinematic viscosity and the gravitational acceleration. Equations (3) and (4) are to be solved simultaneously with the equation of continuity

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (5)$$

with the boundary conditions for $y = 0$: $v = 0$, $\partial u/\partial y = 0$, $\partial\Delta\gamma/\partial y = 0$; and for $y = \pm\infty$: $u = 0$, $\Delta\gamma = 0$.

Since steady conditions are considered, the quantity

$$G = \int_{-\infty}^{\infty} u\,\Delta\gamma\,dy \qquad (6)$$

must be independent of x and, in fact, is a measure of the strength of the line source. This integral condition can be deduced from (3), integration of which yields, by virtue of (5),

$$\int_{-\infty}^{\infty} u\,(\partial\Delta\gamma/\partial x)\,dx + v\,\Delta\gamma\Big]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \Delta\gamma\,(\partial u/\partial x)\,dy = \alpha\,\partial\Delta\gamma/\partial y\Big]_{-\infty}^{\infty}$$

or, since $\Delta\gamma$ and $\partial\Delta\gamma/\partial y$ both vanish at $y = \pm\infty$, one has

$$(d/dx)\Big[\int_{-\infty}^{\infty} u\,\Delta\gamma\,dy\Big] = 0$$

which gives (6).

Equation (5) permits the use of the stream function ψ, so that

$$u = \partial\psi/\partial y \qquad v = -\partial\psi/\partial x \; \ldots\ldots\ldots\ldots \; (7)$$

At this moment it is adequate to perform a dimensional analysis which will reveal the most adequate transformations to be made in order to arrive at a simple solution. With ψ and $\Delta\gamma$ as the dependent variables, and ρ, μ, G, x, and y as the independent variables, a dimensional analysis yields the relationships

$$\psi/\nu = F_1\,[(\rho^2 G/\mu^3)^{1/3}\,x,\; y/x]$$
$$(\nu/G)\,\Delta\gamma = F_2\,[(\rho^2 G/\mu^3)^{1/3}\,x,\; y/x]$$

the termal diffusivity being omitted for the time being since it will appear through the Prandtl number later, and ρ and μ being the density and dynamic viscosity of the fluid, respectively.

For simplicity one writes

$$C = (\rho^2 G/\mu^3)^{1/3} \; \ldots\ldots\ldots\ldots \; (8)$$

In search of a similarity solution, one tries the following substitutions

$$\psi/\nu = (Cx)^m\,f\,(\eta) \; \ldots\ldots\ldots\ldots \; (9)$$

$$(\nu/G)\,\Delta\gamma = -\,(Cx)^r\,h\,(\eta) \; \ldots\ldots\ldots\ldots \; (10)$$

where

$$\eta = (Cx)^n\,y/x \; \ldots\ldots\ldots\ldots \; (11)$$

Equations (9) and (10) give

$$-\,g\,\Delta\gamma/\gamma_0 = (G/\mu)\,(Cx)^r\,h\,(\eta) \; \ldots\ldots\ldots\ldots \; (12)$$

and, by virtue of (7),

$$u = \nu C^{m+n}\,x^{m+n-1}\,f' \; \ldots\ldots\ldots\ldots \; (13)$$

$$v = -\,\nu\,C^m\,x^{m-1}\,[mf + (n-1)\cdot\eta\,f'] \; \ldots\ldots\ldots\ldots \; (14)$$

where the primes denote differentiation with respect to η. The necessary derivatives of u and $\Delta\gamma$ are easily calculated. Substituting (10), (13), and (14) into (3) and (4), and requiring equal powers in C and x for the terms in each of the resulting equations, one has

$$m = n \qquad m + 3n - 3 = r \; \ldots\ldots\ldots\ldots \; (15)$$

Another condition for the exponents is furnished by (6), which requires that

$$m + r = 0 \; \ldots\ldots\ldots\ldots \; (16)$$

and that

$$\int_{-\infty}^{\infty} f'\,\theta\,d\eta = 1 \; \ldots\ldots\ldots\ldots \; (17)$$

Solution of (15) and (16) gives

$$m = n = 3/5 \qquad r = -\,3/5$$

With these exponents, the equations resulting from the substitution of (10), (13), and (14) into (3) and (4) are

$$f'f' - 3ff'' = 5f''' + 5h \quad \text{(18)}$$

$$hf' + fh' = -\,(5/3\sigma)\,h'' \quad \text{(19)}$$

where $\sigma = \nu/\alpha$ is the Prandtl number.

With the following transformations

$$\xi = 5\eta \qquad \theta = 5h$$

Equations (17) to (19) can be written (primes now denoting differentiation with respect to ξ)

$$\int_{-\infty}^{\infty} f'\theta\, d\xi = 5 \quad \text{(20)}$$

$$f'f' - 3ff'' = f''' + \theta. \quad \text{(21)}$$

$$\theta f' + f\theta' = -\,\theta''/3\sigma \quad \text{(22)}$$

with the boundary conditions

$$f\,(0) = f''\,(0) = \theta'\,(0) = 0 \quad \text{(23)}$$

$$f'\,(\pm\,\infty) = \theta\,(\pm\,\infty) = 0 \quad \text{(24)}$$

Equation (23) is automatically satisfied if f is an odd function and θ an even function of ξ.

Assuming for the moment f to be known, the solution of (22) is evidently

$$\theta = K\,e^{-3\sigma\int_0^{\xi} f\,d\xi} \quad \text{(25)}$$

In view of the boundary conditions, one assumes

$$f = A \tanh B\xi \quad \text{(26)}$$

which yields, from (25),

$$\theta = K\,(\operatorname{sech} B\xi)^{3\sigma A/B} \quad \text{(27)}$$

Equation (27) evidently satisfies the boundary conditions contained in (23) and (24). Substituting (26) and (27) into (21), one finds that (21) is satisfied if

$$7A = 4B + K \qquad A = B \qquad 3\sigma A = 2B$$

which yields

$$\sigma = 2/3 \qquad K = 3A$$

or if

$$7A = 4B \qquad -\,6A = -\,6B + K \qquad 3\sigma A = 4B$$

which yields

$$\sigma = 7/3 \qquad K = 9A/2$$

In the case $\sigma = 2/3$, determination by (20) gives

$$A = B = \sqrt{5}/2 \qquad K = 3\sqrt{5}/2$$

so that (26) and (27) become

$$f = (\sqrt{5}/2) \tanh (\sqrt{5}/2)\xi \qquad (28)$$

$$\theta = (3\sqrt{5}/2) \operatorname{sech}^2 (\sqrt{5}/2)\xi \qquad (29)$$

From (10), (13), and (14), remembering $\xi = 5\eta$ and $\theta = 5h$, one has

$$(\rho\mu/G^2 x)^{1/5} u = (25/4) \operatorname{sech}^2 (\sqrt{5}/2)\xi \qquad (30)$$

$$(\rho^3 x^2/G\mu^2)^{1/5} v = -[(3/2\sqrt{5}) \tanh (\sqrt{5}/2)\xi - (1/2)\,\xi \operatorname{sech}^2 (\sqrt{5}/2)\xi] \qquad (31)$$

$$(x^3/\rho^3 \mu^2 G^4)^{1/5} \Delta\gamma = -(3/2\sqrt{5}) \operatorname{sech}^2 (\sqrt{5}/2)\xi \qquad (32)$$

where

$$\xi = 5(\rho^2 G/\mu^3 x^2)^{1/5} y \qquad (33)$$

The main results for the case $\sigma = 2/3$ are thus obtained.

Similarly, for the case $\sigma = 7/3$, one finds

$$f = (5/2\sqrt{6}) \tanh (35/8\sqrt{6})\xi \qquad (34)$$

$$\theta = (45/4\sqrt{6}) \operatorname{sech}^4 (35/8\sqrt{6})\xi \qquad (35)$$

where ξ is given in (33). Results similar to those contained in (30) to (32) can be easily obtained.

Acknowledgments--This work was done under ONR Contract No. N90nr-82401 with Civil Engineering Department of the Colorado Agricultural and Mechanical College. Thanks are due to C. C. Lin of Massachusetts Institute of Technology for confirming the validity of a similar solution by the writer for the analogous problem of laminar free convection due to a point source of heat. Without this confirmation, the writer would have hesitated to publish both the result for the point source and the present result.

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(Communicated manuscript received December 11, 1951;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

MEASUREMENT OF METEOROLOGICAL VARIABLES BY USE OF MULTIPLE ENERGY-BALANCE INDICATORS

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Abstract--A method is suggested for computing meteorological variables near the ground, including air temperature, radiation, and wind speed. The method involves measuring the temperature of a multiplicity of indicators, each suspended at a given level above the surface of the ground. The temperatures of the indicators are controlled by radiational and convective exchanges of energy between the indicators and their environment. The present status of field tests of the suggested technique is summarized.

Introduction--Sufficiently accurate observations of temperature, dew point, wind speed, and radiation for detailed analysis of the physical processes at work in the lowest layers of the atmosphere have long eluded meteorologists. In the attempt to achieve such measurements, two methods of attack have generally been followed. These are (1) the development of a set of instruments, each capable without correction of measuring one meteorological variable to the desired accuracy; and (2) the development of a set of instruments, each capable of measuring one meteorological variable approximately, and the formulation of techniques for correcting these readings to achieve the required precision. There are fundamental difficulties inherent in both of these lines of approach. It will be the object of the present study to outline a possible third alternative. In this third method, instruments are not designed to measure meteorological variables directly, but rather to supply data necessary for the computation of these quantities from physical considerations.

Accurate measurement of meteorological variables with a single instrument--There are several obstacles to the development of instruments capable, in themselves, of measuring temperature, dew point, wind speed, or radiation in the lowest layers of the atmosphere with the precision required for microclimatological research. The greatest of these, perhaps, is the fact that measurement techniques intrinsically sensitive enough to achieve the required accuracy are generally incapable of completely isolating the effect of any one of these meteorological variables.

Thus, the most sensitive methods of temperature determination now in general use involve resistance or thermoelectric thermometers. In these thermometers, the temperature of the exposed element can be determined with extreme precision; however, errors are inevitably introduced as a result of the fact that the temperature of the element is affected not only by the ambient air temperature but also by radiation and wind speed. In general, the errors in temperature measurement due to the effects of radiation and wind speed are systematic with respect to location, time of observation, and height of the exposed element above the ground. For example, in the determination of vertical temperature gradients the systematic errors introduced make the surface layer of air appear too stable during the night and too unstable during the day. Data containing these errors are of limited value in the study of such problems as the relationship between the turbulent transfer of thermal energy and the stratification of the atmosphere. They are defective, also, as climatic data, since these exaggerate the differences between temperature regimes near the ground and at standard observational heights.

Measurements of wind speed are subject to similar limitations. The most accurate measurements of wind now being made involve the determination of the cooling power of an air stream by the use of some such instrument as the hot-wire or heated thermocouple anemometer. However, if only one instrument is used for each velocity component, assumptions must be made concerning the energy balance of the sensing element that neglect the effects of variations in radiation or ambient temperature. The resulting systematic errors can never be entirely eliminated, and it has been impossible thus far to reduce the magnitude of the systematic errors to such a degree that they may be considered negligible for research purposes.

Use of corrections to increase accuracy of measurements--The use of corrections to improve the accuracy of certain instruments has long been a standard practice. By use of suitable calibrations, stem corrections, etc., even such a simple device as the mercury-in-glass thermometer can be made a highly satisfactory instrument for many purposes of laboratory use for which a considerable degree of precision is required. The most successful instruments for determination of meteorological variables in the lowest layers of the atmosphere depend upon the measurement of the energy balance of a sensitive element. Unfortunately, such measurements do not lend themselves readily to correction when the sensing element is exposed to the free atmosphere.

The reasons for this are almost self-obvious. The lower atmosphere is typically turbulent, and local variations in temperature, dew point, and wind speed may be considerable. In addition, long- and short-wave radiation vary markedly with time. All of these quantities, with the usual exception of dew point, affect the energy balance of the measuring element. Even if the average value of a given meteorological variable is all that is required, the investigator must not only measure a variety of other variables, but must also calibrate his instrument to determine the effect of these other variables, singly and in combination. This herculean task is not usually attempted. Yet it is a necessary step if precision field observations are to be made by the techniques of approximate measurement and subsequent correction.

It should be pointed out in this connection, however, that when the sensitive element is not exposed to the atmosphere these techniques may prove adequate. Thus, it is probable that, with further use and calibration of the dew point hygrometer, the measurement of mean dew point over a given period of time at a point in the lower atmosphere will have been achieved with satisfactory accuracy [THORNTHWAITE and OWEN, 1940; SUOMI, 1948]. Nevertheless, the repetition of this success in the determination of temperature, wind speed, and radiation by methods now in use is unlikely. It is believed that a new approach to these problems of measurement is required.

The general theory of multiple instruments--Even though we assume that it is impossible to design an instrument which responds exclusively to a specified meteorological variable, and even if we further assume that it is impossible to obtain adequate corrections of approximate measurements, it may still be possible to measure the specified meteorological variable without systematic error. This may be achieved through the use of multiple instruments. Generally, it will be convenient to make the instruments of similar, but not identical, design; in any case, all of the instruments must satisfy the following requirements:

(1) The external conditions to which an instrument responds must be completely described by a specified number of parameters. One of these parameters must be the meteorological variable which is to be determined.

(2) The form of the equation relating the instrument reading to the externally imposed parameters must be known.

(3) The instrumental constants which enter into the equation for instrument response must be known or be capable of determination.

If a set of instruments that satisfy these conditions is exposed to the same external conditions, a group of simultaneous equations is obtained in which the meteorological parameters are the only unknowns. It may then be possible to solve these equations to determine the value of the required variable. Once the procedure has been established, the set of instruments may in effect be considered a single instrument, designed to measure the value of one or more meteorological quantities. These quantities may be mean values of a meteorological variable, averaged over a time interval, or they may be statistical measures of fluctuations and correlations within the interval.

The general principle of multiple instruments is certainly not original with the authors. It has often been used in an incomplete manner, neglecting some of the parameters which are of secondary importance in the equations of instrument response. Common examples include the wet- and dry-bulb psychrometer, and the pyrheliometer. ROBINSON [1921] suggested the use of two thermal elements, of different radiational properties but otherwise alike, for measuring the temperature of hot gases enclosed by walls at a lower temperature. He assumed that the coefficients of convective heat transport from the elements would be determined independently. THORNTHWAITE and HOLZMAN [1942] used a two-pan hygrometer for measuring atmospheric humidity. In this case identical pans were filled with different saturated salt solutions; the simultaneous equations describing instrument response were based upon the theory of gas diffusion through boundary layers. A more exact application of the method of multiple instruments was developed by CORRSIN [1949], who used three hot-wire anemometers, heated with electrical currents of different intensity, to measure simultaneous flucutations of wind speed and temperature in a wind tunnel. In Corrsin's study, the effect of radiation was considered negligible. The present problem is to develop a set of instruments which can be exposed in the atmosphere to measure not only wind speed and temperature but also incident radiation.

A multiple energy-balance indicator for determining meteorological variables--To consider the possibility of developing a set of instruments for the simultaneous determination of temperature, wind speed, and radiation, it will be convenient to start with the general equation for the energy balance of an exposed element of as yet unspecified characteristics. The energy equation may be written

Rate of absorption of radiation = rate of emission of radiation
+ rate of heat loss to air + rate of increase of internal energy

This equation assumes that an instrument can be designed for which the conduction of thermal energy to or from each sensitive element through supports and lead wires is negligible. It is believed that this is possible. Although this assumption is not actually necessary to the following theoretical development, the resulting simplification is sufficient to warrant its acceptance unless experimental evidence indicates its inaccuracy.

The particular problem with which the authors were concerned was the determination of temperature, wind speed, and radiation averaged over a period of at least several minutes. Mean values of meteorological variables over an extended time can best be obtained by the use of instruments with considerable lag, which will react slowly to fluctuating external stimuli. This in turn requires elements of high heat capacity.

Further characteristics of the instrument are dictated by the meteorological variables to be measured and by the nature of the energy equation itself. First, it is desirable that the sensitive elements react equally to horizontal winds blowing from any direction. To meet these requirements, disk-shaped elements were designed. Second, to simplify the energy equation, the disks must be essentially isothermal. This has been achieved by casting the disks of pure copper. It has been computed that for a solid copper plate two cm in thickness, the absorption of solar energy at the rate of two cal/cm^{2}min at the top of the plate would result in a temperature difference between the top and bottom of less than 0.04°C. None of the disks used in preliminary field experiments was this thick. Third, to increase the thermal lag, high heat capacities are desirable. The smallest copper disk used was 3/4 inch in diameter and 3/8 inch in thickness. For such a disk almost 30 calories will be required to raise the temperature of the disk one degree Centigrade. The change of temperature during a few minutes' time was therefore usually small. Finally, the temperature of the disk at any moment must be accurately known. The disk itself was designed to be one junction of a copper-constantan thermocouple located in the center of the disk. Fine lead wires, open to aeration for a considerable length, were used. Experience has shown that copper-constantan thermocouples can be calibrated within a few hundredths of a degree Centigrade.

The energy-balance equation of a disk, averaged over a finite period of time, may be written in the form

$$aL + bS = \epsilon T^4 + h\,(T - \theta) + k\,\Delta T/\Delta t \ldots\ldots\ldots\ldots \quad (1)$$

where

a = the absorptivity of the disk for long-wave (terrestrial and sky) radiation
L = the average energy of incident long-wave radiation per unit area of disk
b = the absorptivity of the disk for short-wave (solar) radiation
S = the average energy of incident short-wave radiation per unit area of disk
ϵ = the emissivity of the disk, multiplied by Stefan's constant
h = the coefficient of convective heat transport
T = the average temperature of the disk during the interval
θ = the average ambient air temperature during the interval
k = the product of the specific heat of the disk, its density, and the ratio of volume to area of the disk
ΔT = the change of disk temperature during the interval (generally a small quantity)
Δt = the time interval of observation

(It can be shown that the error introduced by writing T^4 for $(1/\Delta t)\int T_i^4\,dt$, where T is the instantaneous temperature, is substantially less than 0.1 pct.) One equation may be written for each disk. The quantities a, b, ϵ, and k are physical constants for each disk, except insofar as b may vary with angle of incidence of the solar radiation. They can be determined by calibration, or in some cases by methods to be described later. The quantities T and ΔT are accurately measured. Therefore, for a disk of given shape, the energy equation can be expressed in terms of the four unknowns, L, S, h, and θ. Three of the unknowns, L, S, and θ, are determined solely by external meteorological conditions. They are, therefore, the same for all disks of similar shape mounted at the same elevation above a homogeneous surface.

The fourth unknown, h, is defined by the relationship

$$h = \text{rate of heat loss to air per unit area}/(T - \theta)$$

Let u be the speed of a uniform horizontal wind which has the same cooling power as the observed turbulent current. Then, from the usual theory of transfer of head by forced convection [McADAMS, 1942]

$$hd/K = \phi\ (Re, d/W) \qquad (2)$$

In this equation

d = the diameter of the disk
W = the thickness of the disk
K = the coefficient of thermal conductivity of the air
Re = the Reynolds number, defined as ud/ν, where ν is the kinematic viscosity of air

It is to be noted that the Grashof number is omitted in this expression. For the largest disk used (1 1/2 inches in diameter and 3/4 inch in thickness) the Reynolds number equals one thousand for a wind speed of about 0.6 mi/hr. The effect of natural convection may therefore be neglected except under extraordinary circumstances, and the coefficient h may be considered solely a function of size (d), and wind speed (u) for a set of disks of geometrically similar shape (d/W equal to a constant). This function must be determined by calibration, according to standard procedure.

In view of the above facts, it is possible to rewrite the energy equation for any one disk as follows

$$aL + bS = T^4 + f\ (u, d) \cdot (T - \theta) + k\ \Delta T/\Delta t \qquad (3)$$

In this equation there are again four unknowns, L, S, θ, and u. However, all four of these unknowns are now determined solely by external meteorological conditions, and are therefore the same for disks mounted at the same elevation above a homogeneous surface. It remains to show how a number of such equations can be solved simultaneously to determine the values of the meteorological variables.

Solution of the simultaneous energy equations--In order to obtain a set of simultaneous equations in the form (3), it is necessary to expose a number of disks of varying physical characteristics. The physical properties which could be changed from one disk to another are (1) the heat capacity per unit volume (k divided by the ratio of volume to surface area); (2) the radiational type (a, b, and ϵ); and (3) the diameter d. The radiational characteristics can be varied independently by plating the disks. It can be shown that if the disks are similar in any one of these three sets of physical properties, the equations can still be solved for the meteorological variables [COVEY, 1949]. In the present example the disks were all cast of pure copper; therefore, the heat capacity per unit volume is invariant. Radiational type and size were varied.

The following additional notation will be used to simplify presentation of the simultaneous energy equations and their solution

$$\alpha \equiv aL + bS \quad \text{and} \quad \lambda \equiv k\ \Delta T/\Delta t$$

A first subscript (i, j, or k) will be used to indicate radiational type (i, j, and k vary from one to m, and $i \neq j \neq k$); a second subscript (p, q, or r) will be used to indicate size (p, q, and r vary from one to n, and $p \neq q \neq r$). A subscript o in the first or second position indicates invariance of that quantity with respect to type or size, respectively.

With m radiational types and n sizes in all combinations, there will be mn equations of the form

$$\alpha_{io} = \epsilon_{io}\ T_{ip}^4 + h_{op}\ (T_{ip} - \theta_{oo}) + \lambda_{ip} \quad \begin{Bmatrix} i = 1, 2, \dots m \\ p = 1, 2, \dots n \end{Bmatrix} \qquad (4)$$

In these mn equations there are (m + n + 1) unknowns, namely, the α_{io}, h_{op}, and θ_{oo}. It would appear that if m = 2 and n = 3, or if m = 3 and n = 2, the equations could be solved. Actually, the first of these possibilities does not yield a solution. This can be shown by subtracting the equations in pairs to eliminate α_{io}.

$$0 = \epsilon_{io} (T_{ip}^4 - T_{iq}^4) + h_{op} (T_{ip} - \theta_{oo}) - h_{oq} (T_{iq} - \theta_{oo}) + (\lambda_{ip} - \lambda_{iq}) \quad \left\{ \begin{array}{l} i = 1, 2, \dots m \\ p, q = 1, 2, \dots n \\ p \neq q \end{array} \right\} \dots\dots\dots\dots (5)$$

There are now m (n - 1) equations in (n + 1) unknowns, namely, the h_{op} (p = 1, 2, ... n) and θ_{oo}. Equating equations to unknowns after eliminating $\theta_{oo} h_{op}$ and $\theta_{oo} h_{oq}$ between expressions of the form of (5), written for disks of radiational type i and j

$$n = (m - 1)(n - 1) \qquad n = (m - 1)/(m - 2)$$

which yields the unique result m = 3 and n = 2. Therefore, three radiational types and two sizes, a total of six disks in all, are necessary to determine the meteorological variables uniquely. The details of the solution will not be given here. However, the results will be summarized.

$$h_{op} = \frac{\begin{vmatrix} T_{iq} & \epsilon_{io} (T_{ip}^4 - T_{iq}^4) + (\lambda_{ip} - \lambda_{iq}) & 1 \\ T_{jq} & \epsilon_{jo} (T_{jp}^4 - T_{jq}^4) + (\lambda_{jp} - \lambda_{jq}) & 1 \\ T_{kq} & \epsilon_{ko} (T_{kp}^4 - T_{kq}^4) + (\lambda_{kp} - \lambda_{kq}) & 1 \end{vmatrix}}{\begin{vmatrix} T_{iq} & - T_{ip} & 1 \\ T_{jq} & - T_{jp} & 1 \\ T_{kq} & - T_{kp} & 1 \end{vmatrix}} \quad \left\{ \begin{array}{l} p \neq q \\ i \neq j \neq k \end{array} \right\} \dots\dots\dots (6)$$

Since p = 1, 2, there will be two measurements of each coefficient of convective heat transfer; an average can be used as the correct value. The equations also give an internal check, since if significant errors are present the two computed values should differ importantly. The assumption is made that the ratio for h_{op} is not indeterminate. This condition would be approached if the two disk sizes were not significantly different ($T_{ip} \cong T_{iq}$, and $\lambda_{ip} \cong \lambda_{iq}$), or if the radiational characteristics of the disk platings are too much alike ($T_{ip} \cong T_{jp} \cong T_{kp}$). By appropriate design of the disks it is believed that this problem can be eliminated in the vast majority of situations.

The ambient air temperature is given by

$$\theta_{oo} = [1/(h_{op} - h_{oq})] \left\{ \epsilon_{io} (T_{ip}^4 - T_{iq}^4) + (h_{op} T_{ip} - h_{oq} T_{iq}) + (\lambda_{ip} - \lambda_{iq}) \right\} \quad i = 1, 2, 3 \dots\dots (7)$$

where h_{op} and h_{oq} are now known quantities. The three equations must give idential values for θ_{oo}.

The total absorption of energy from incident radiation can now be found from (4) for each type of disk. With α_{io}, α_{jo}, α_{ko}, and the values of the radiational constants of the disk types known, the long-wave and short-wave radiation can be computed from equations of the form

$$\alpha_{io} = a_{io} L_{oo} + b_{io} S_{oo} \qquad i = 1, 2, 3 \dots\dots\dots\dots\dots\dots (8)$$

There are three equations in two unknowns; a check therefore exists on the accuracy of the results.

It is to be noted that it is not necessary to limit the number of disks to six. If more than six disks are used, the additional simultaneous equations will provide the possibility of statistical determination of the meteorological variables. Increased accuracy would certainly follow as a result of improved sampling and the partial elimination of non-systematic instrument errors. In addition, the standard error of the computed values of the variables could be determined.

Measurement of meteorological variables at more than one height--The efficiency of the multiple energy-balance indicator increases if determinations of meteorological variables are to be made simultaneously at more than one level over a homogeneous surface. Since the radiation terms α_{io} do not vary with height, less than six disks are required at additional levels. It can be shown that the number of additional disks required is only two per level. Thus, theoretically, temperature, wind speed, and long- and short-wave radiation could be determined at five levels from a total of 14 disks. Of course, if such a large number of levels were used, it would be advisable to have additional disks to enable the investigator to check the accuracy of his results from statistical considerations. In the tests of the multiple energy-balance indicator now being set up at the Johns Hopkins University Laboratory of Climatology at Seabrook, New Jersey, it is planned to use six disks at each of three levels, for a total of 18 disks. The number of resulting equations is well in excess of the minimum. Statistical procedures will be followed in analyzing the results.

The multiple energy-balance indicator as a thermometer and as an anemometer--It is apparent that temperature is the meteorological variable most accurately and satisfactorily measured by the multiple energy-balance indicator. As shown by (7), it is not necessary to have calibrations of the relationship between the coefficients of convective heat transport and wind speed in order to compute the ambient temperature. In addition, the only radiational constants which enter the equation are the emissivities at long wavelengths. While calibration of the disks to determine their constants is difficult, it can be accomplished with a high degree of accuracy. In addition, the possibility exists (as noted below) of checking these calibrations in the field by making use of the fact that the constants ϵ_{io} do not vary from one run to the next and by computing these constants from additional sets of simultaneous equations. Preliminary tests of copper disks at the Johns Hopkins Laboratory of Climatology indicate that the multiple energy-balance indicator may prove quite successful and accurate as a thermometer.

The accuracy of the determinations of wind speed is open to greater question. The calibration of the coefficient of convective heat transfer as a function of Reynolds number and disk size is not simple. Even if this could be done with absolute precision, the question of the physical meaning of the wind speed obtained from such calibrations must be examined. What actually would be obtained is the uniform horizontal wind giving a specified mean cooling power, and not the mean horizontal wind movement in a turbulent wind field. Vertical eddies and greatly fluctuating wind speeds might cause a significant difference between these two quantities. Nevertheless, it must be noted that for certain physical problems the cooling power of a wind current is of more importance than any arbitrarily defined mean wind speed. In such cases, at least, the usefulness of the multiple energy-balance indicator would be enhanced.

The multiple energy-balance indicator as a radiation meter--One apparent difficulty in the use of the multiple energy-balance indicator as a radiation meter lies in the determination of the radiation constants of the disks. The long-wave absorptivity of the various types of disks (a_{io}) can be determined in the laboratory only with difficulty; the problem is aggravated in the case of the short-wave absorptivity (b_{io}). In the latter calibration the effect of the angle of incidence of the solar radiation should be considered. In addition, the percentage of direct to scattered solar radiation will not be the same on every occasion for which the Sun is at a given elevation. These difficulties, of course, manifest themselves in the calibration of the conventional pyrheliometer; nevertheless, with metallic platings of the disks the problem will be more significant. Deterioration of the disk surfaces in the field must also be carefully avoided.

Under the circumstances, it is important to point out that it should be possible to check laboratory calibrations during field operation. It is to be noted that a_{io}, ϵ_{io}, and b_{io} are independent of level and period of observation, at least for measurements made with a specified solar elevation and similar cloud conditions. By introducing level and period of observation as third and fourth subscripts in the energy equation for each disk, a set of simultaneous equations is obtained which can be solved simultaneously with the radiation constants as additional unknowns. This can be done in several ways. Since the mathematical procedure is involved, it will not be presented here.

It must also be pointed out that the long-wave and short-wave radiation parameters L and S leave much to be desired. The exact value of these quantities must in general be at least slightly dependent upon the diameter-thickness ratio of the disks. In addition, all forms of long-wave radiation are grouped together in L, including sky and ground radiation; all forms of short-wave radiation are grouped together in S, including direct, scattered, and reflected solar radiation. In this sense, the radiation measurements are considerably less useful than pyrheliometer readings. It is, of course, theoretically possible to plate the top and bottom of the disks differently to isolate incoming and outgoing radiation. However, any tests of this procedure must await the results of field investigations on the more simple instrument already described.

Field tests of the multiple energy-balance indicator--The preceding report should be regarded as the presentation of the theoretical background for a promising method of meteorological instrumentation. Field tests have not yet been completed. Many problems remain to be solved such as optimum sizes and platings to be used and method of mounting. Although tentative solutions to these problems have been advanced, field tests may indicate the need for continued experimentation. It may develop that even for optimum sizes and platings the suggested method of meteorological measurement is not feasible under certain conditions. For example, when the wind speed is high, the temperature of all the disks may approach ambient temperature and the values of the coefficients of convective heat transport may tend to become indeterminate. In addition to all of these known problems, unforeseen difficulties will undoubtedly arise.

Certainly, at the present time, all that can be claimed for the method is that it is theoretically sound and that a determined attempt to develop a practical instrument is warranted. The Laboratory of Climatology of the Johns Hopkins University is continuing its work in this direction.

Acknowledgments--Research reported on in this paper has been made possible through the support and sponsorship of the Geophysical Research Directorate of the Cambridge Air Force Research Center under Contract W28-099-ac-378. The work was undertaken while the authors were associated with the Laboratory of Climatology of Johns Hopkins University at Seabrook, New Jersey.

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(Communicated manuscript received November 26, 1951;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

RATE OF DISPERSION IN THE WAKE OF A BARGE AT SEA

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(Contribution No. 567 of the Woods Hole Oceanographic Institution)

Abstract--An analysis is presented of the dispersion of a pollutant discharged at sea from a barge. The pollutant in this case contained ten per cent $FeSO_4$, the iron of which proved to be a convenient indicator of dispersion in the sea since it may be observed to concentrations as low as ten parts per billion. The distribution of iron in the wake is related to the rate of discharge and the horizontal mixing coefficient. This relationship provides a means of estimating the rate of discharge of pollutant necessary to meet specific concentration limits.

Introduction--An opportunity to study the dispersion of pollutants in the wake of a barge was afforded by the discharge at sea of acid-iron wastes from the titanium plant of the National Lead Company. In this paper an attempt is made to explain the distribution of the iron in the wake in terms of the rate of discharge and the horizontal mixing coefficients.

The waste material, consisting of about ten per cent $FeSO_4$ and 8.5 pct H_2SO_4, was carried to sea in a specially constructed barge, the Sayreville, which was towed at a speed of six knots. The waste was pumped directly into the wake through two 12-inch pipes mounted on skegs at keel depth at the after end. These outlets were at a depth of 15 ft when the barge was fully loaded, and at six feet when the barge was empty. Each barge load contained about 3000 tons of waste, which was discharged in periods ranging from 80 to 170 min. The distributions of acidity and of iron in the wake were observed in April and July, 1948, and in January, 1950. The quantities discharged, the iron content, and the rate of discharge for each operation are given in Table 1. The description of the distributions has been published by REDFIELD and WALFORD [1951]. The results indicated that the wake continued to spread horizontally for a long time, but reached its greatest depth almost immediately.

Table 1--Discharge characteristics of various waste disposal operations

Date	Waste discharge	Time of discharge	Iron	Discharge[a] rate
1948	ton	minute	pct	g/cm
April 22	2850	165	2.36	20.0
24	2860	180	2.36	18.4
27	2830	145	2.15	20.6
July 24	2830	125	2.18	24.2
27	2860	95	1.58	22.6
1950				
Jan. 4	3120	80	3.52	67.4

[a] Speed of barge = 6 knots.

Methods--The observations were made in April and July from the R. V. Balanus, and in January from the Fish and Wildlife Service vessel Albatross III. Samples were taken in the wake at various times after discharge. The average speed of the barge, 600 ft/min, was used to interconvert distance and time. A stadimeter was used to determine distances up to 6000 ft. Greater distances were estimated from the time after the barge passed an identifiable spot, such as a turn or a position marked by a free-floating buoy discharging fluorescein dye. Samples at various depths were taken with the sea sampler of SPILHAUS and MILLER [1948]. Every effort was made to obtain these samples in the middle of the visible wake. The water samples were analyzed for reducible iron by the dipyridyl method described by COOPER [1935]. The red color formed was measured using a photoelectric colorimeter [FORD, 1950] with green (Wratten 4010) color filters for concentrations less than 0.5 mg/L. A Duboscq colorimeter with appropriate standards was used for greater concentrations.

From the distribution of iron it is possible to evaluate the magnitude of the mixing coefficients required to give the observed distribution. The decrease of the concentration of iron in the wake results from the turbulence induced by the passage of the barge and the normal turbulence of the sea. Although it is impossible to differentiate between these two types of turbulence from the data available, it is possible to estimate the value of the mixing coefficient which describes the joint effects of these two processes.

Three fundamental assumptions are involved in the analysis of the results:

(1) It is assumed that vertical dispersion is instantaneous. It was observed that the waste reached its greatest depth (35 to 70 ft) almost immediately, and little or no further change in depth could be detected.

(2) It is assumed that horizontal diffusion along the axis of the wake can be neglected. Each unit length of the wake received the same original concentration of waste. The wakes were very long and narrow, and the longitudinal gradients were very much less than the transverse gradients.

(3) It is assumed that the distribution of waste along lines perpendicular to the axis was in accordance with the normal curve of distribution. It was impossible to check this assumption because of the difficulty of determining the position of the samples relative to the center of the wake.

The effect of these assumptions is to treat the wake as though all of the iron discharged were concentrated along the median line of the wake at zero time. The problem is thus similar to that discussed by MUNK, EWING, and REVELLE [1949]. The quantity of iron (y) in a column of water a square centimeter in cross section is given by the usual equation for normal distribution

$$y = (N/\sigma\sqrt{2\pi})\, e^{-1/2[(x-m)/\sigma]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which N represents the grams of iron discharged per centimeter of barge travel, σ is the standard deviation, and (x - m) is the distance, in centimeters, along a line perpendicular to the axis of the wake between x, the point of observation, and m, the location of the median concentration.

In the middle of the wake, where x = m, the above equation reduces to

$$y = N/\sigma\sqrt{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From Fick's equation the following relationship between the standard deviation and the mixing coefficient K was derived by EINSTEIN [1905]

$$d\sigma^2/dt = 2K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This implies that the mixing coefficient is independent of the dimensions of the mixing field σ. RICHARDSON and STOMMEL [1948] and others have shown that this is not true in a fluid medium. Within the range covered by our observations, however, the use of this equation seems justifiable.

The quantities of iron in a column of water one square centimeter in cross section observed in the wake after various periods of time are given in Table 2. If the postulated distribution

Table 2--Quantity of iron in the wake of the discharge barge Sayreville

April, 1948			July, 1948			January, 1950		
Time	$1/\sqrt{t}$	Iron	Time	$1/\sqrt{t}$	Iron	Time	$1/\sqrt{t}$	Iron
sec		mg/cm^2	sec		mg/cm^2	sec		mg/cm^2
15	0.25	51.3	36	0.166	21.2	28	0.189	5.4
44	0.15	28.5	105	0.097	18.5	33	0.174	22.1
75	0.115	6.7	120	0.091	18.3	81	0.111	8.7
113	0.094	6.3	255	0.063	2.1	81	0.111	24.5
143	0.083	6.0	300	0.058	10.6	142	0.084	0.9
157	0.080	12.3	1,200	0.029	2.1	135	0.086	26.0
195	0.071	11.6	1,320	0.028	0.8	218	0.068	16.8
240	0.064	10.2	1,800	0.024	2.4	291	0.059	0.5
240	0.064	7.3	3,900	0.016	0.2	330	0.055	7.1
750	0.036	4.0	14,400	0.008	0.3	600	0.041	8.1
3,300	0.017	1.4	16,200	0.008	0.4	960	0.032	1.6
12,000	0.009	1.0	36,000	0.005	0.15	1,320	0.028	0.1
			41,500	0.005	0.17	1,650	0.025	5.4

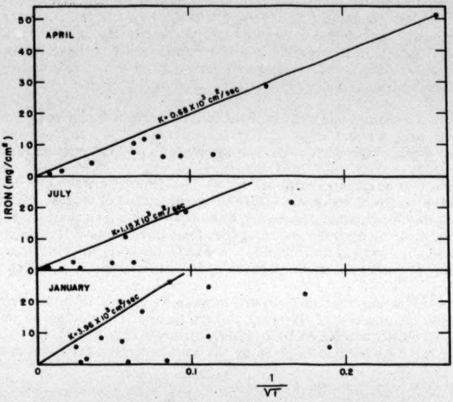

Fig. 1--Relation of iron concentration to mixing time

perpendicular to the axis of the wake applies, the median quantity should be proportional to the reciprocal of the square root of time. The data are presented in this way in Figure 1. The lines drawn in this figure are in accordance with the equation

$$y = (N/2\sqrt{K\pi}) \cdot 1/\sqrt{t} \qquad (4)$$

which is a combination of (2) and the integral of (3) assuming the constant of integration to be zero at zero time. These lines correspond to the minimum value of the mixing coefficient necessary to describe the distribution for each of the sets of observations.

A large proportion of the observed concentrations are less than those predicted by (4). It has been mentioned that it was impossible to determine the exact location of the samples relative to the axis of the wake, though every effort was made to take the samples in the center of the visible axis. The low values are probably attributable to the fact that the ship was not in the center when the samples were taken. As shown in (1), the concentration decreases rapidly as one leaves the center line, that is, when x - m increases.

The most valid observations are those which lie close to the lines drawn in Figure 1. The concentrations selected as giving a reasonably good correspondence with the theory are underlined in Table 2, and the mixing coefficient for each is given in Table 3. The selection is obviously very subjective, but the uncertainty of the location of the samples relative to the center of the wake makes any mixing coefficient computed from the other data meaningless. The average of the mixing coefficients for the selected concentrations was about 2×10^3 cm²/sec in April and July, and nearly 7×10^3 cm²/sec in January.

Several conditions might have contributed to the greater mixing coefficient observed in January. The rate of discharge at this time was about threefold greater than the rates in April and July. If the quantity of iron in the wake is directly proportional to the rate of discharge, as in (2), this should not affect the value of the mixing coefficient. The weather was moderate during all of the operations. At the time of the January observations the wind force varied from 1 to 2. In April and July wind forces of 1 to 3 obtained during the operations. It may be significant that the disposal operations in April and July were located in water ranging up to 185 ft in depth, whereas in January the discharge location was in water where the maximum depth was 80 ft [REDFIELD and WALFORD, 1951]. The most likely explanation, however, may be found in the fact that the vertical stability of the water column was less in January than in April or July. The salinity, temperature, and density (σ_t) of the water at the three times are shown in Figure 2. The cross bars on the density curves show the depth to which the water was contaminated with iron in the wake.

Table 3--Mixing coefficients required to give observed quantities of iron in the wake of the barge

Date	Rate of discharge	Iron	Time	Standard deviation	Mixing coefficient
1948	g/cm	mg/cm^2	sec	cm	cm^2/sec
April	18.4	51.3	15	143	0.7×10^3
	20.6	28.5	44	288	1.0×10^3
	18.4	12.3	157	595	1.2×10^3
	20.6	11.6	195	709	1.4×10^3
	20.6	10.2	240	805	1.5×10^3
	20.6	4.0	750	2,050	3.0×10^3
	20.6	1.4	3,300	5,860	5.6×10^3
	20.0	1.0	12,000	7,950	2.7×10^3
July	24.2	18.5	105	524	1.3×10^3
	24.2	18.3	120	530	1.2×10^3
	22.6	10.6	300	860	1.2×10^3
	22.6	2.4	1,800	3,860	3.9×10^3
1949					
January	67.4	24.5	81	1,090	7.4×10^3
	67.4	26.0	135	1,030	4.0×10^3
	67.4	16.8	218	1,590	5.8×10^3
	67.4	8.1	600	3,300	9.4×10^3
	67.4	5.4	1,650	4,950	7.5×10^3

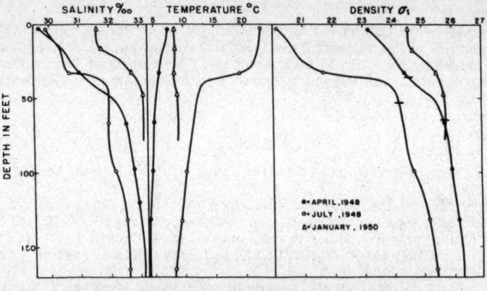

Fig. 2--Salinity, temperature, and density of the water during the three test periods

Discussion--No claim is made for the accuracy of the mixing coefficients computed in this paper. The inaccuracies of sampling, particularly the difficulty of determining whether the samples were collected in the center of the wake, introduce large uncertainties in the estimates.

It might be expected that the mixing coefficient would be very large near the barge because of the turbulence of the wake. There is no clear evidence for any such difference, though there may be a suggestion of it in the July and January data. The mixing coefficients do show a tendency to increase with increasing time, and thus with the dimensions of the mixing field, which also increases with time. This increase would be expected from the results demonstrated by RICHARDSON and STOMMEL [1948]. It should be mentioned that the accuracy of locating the center of the wake was best when close to the barge. It is possible that the larger coefficients obtained for greater distances indicate that these samples were not in the axis of the wake.

The value of these results lies in the fact that they make it possible to estimate the distribution to be expected for similar operations if the rate of discharge is known. To facilitate such predictions Figure 3 has been prepared to show the effects of various mixing coefficients and

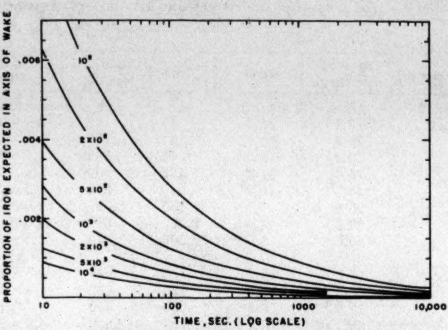

Fig. 3--Curves for forecasting mixing

times on the proportion of the discharged pollutant to be expected in the axis of the wake. This proportion, when multiplied by the discharge rate (g/cm), will give the quantity (g/cm^2) to be expected. Since the rate of discharge is the only variable which can be controlled in the operation, the rate necessary to meet specific concentration limits can be selected. General application of the method is subject to uncertainty since little is known about the value of the mixing coefficients to be expected.

Acknowledgments--The Committee for the Investigation of Waste Disposal of the Division of Biology and Agriculture of the National Research Council sponsored this investigation with a grant of funds provided by the National Lead Company. The comments and suggestions of Alfred C. Redfield, Henry Stommel, and Bernhard Haurwitz concerning this problem are greatly appreciated.

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(Communicated manuscript received July 16, 1951, and, as revised, May 5, 1952; open for formal discussion until March 1, 1953.

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

STATISTICAL DISTRIBUTIONS FOR OCEAN WAVES

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Abstract--Results are presented from the analysis of data obtained by underwater pressure recorders in the Pacific Ocean. Computed parameters describing the statistical distribution of wave heights and wave periods are given and incorporated into one-parameter probability models closely fitting the data. The model for wave-height distributions is found to provide a satisfactory explanation for the ratios among the mean and certain specified wave heights reported by other observers. The degree of relation between the heights or periods of successive waves or between the height and the period of the same wave was found to be relatively low.

Introduction--The statistical characteristics of the ocean surface are of interest in problems relating to the erosion of beaches, the design of marine structures, and the operation of floating craft. Accumulated wave-height data, supplementing wave-forecasting techniques, have provided a means of anticipating the effects of wind-generated waves at a coast line, while wave-period data have been used to track the geographic movement of oceanic storms themselves. Speculation has covered a range from possible useful properties possessed by the order in which waves occur to the superstition that every seventh wave is the highest.

The present paper represents an empirical approach to a search for regularities and limits of predictability in the patterns and distributions of wave heights and periods. The wave systems studied correspond most closely to offshore swell in shoaling water, interest being centered on wave-to-wave variability rather than uniformity. Other work from this point of view has been reported on by BOWDEN [1948], WIEGEL [1949], and SEIWELL [1949].

The study of wave systems has been facilitated by the development of underwater pressure-recording instruments. Interpretation, by means of hydrodynamic theory, of the patterns of heights and periods of waves on the pressure record leads to information about the patterns in the ocean surface. One may, of course, study the pressure record itself without attempting to reconstruct the corresponding surface.

Nature and source of data--The basic data consist of 25 twenty-minute pressure records chosen so as to provide typical, though not strictly random, selections of wave systems from all seasons of the year at five different locations off the shores of the Pacific Ocean at depths ranging from 45 to 90 ft. The pressure-measuring instruments used were the University of California Mark III and Mark V recorders described by CHINN [1949] and ISAACS and WIEGEL [1950]. These

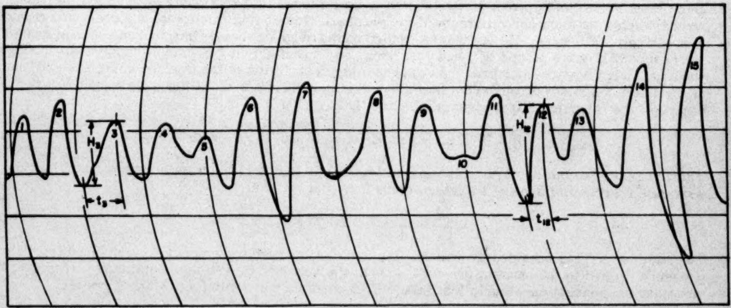

Fig. 1--Section of wave record K, waves 1 to 15, October 11, 1948, 21h 56m - 22h 00m, Point Sur, California

Table 1--Descriptive data for twenty-five ocean-wave records

Location	Depth	Instrument type	Date	Time	Record identification	No. of waves
	ft			h m h m		
Oceanside, Calif.	47	Mark V	July 5, 1949	15 37 - 15 58	A	90
			July 6, 1949	06 57 - 07 17	B	112
			July 6, 1949	16 00 - 16 20	C	89
			July 6, 1949	17 05 - 17 25	D	86
Pt. Sur, Calif.	64[a]	Mark III	Sep. 23, 1948	09 15 - 09 35	E	90
			Sep. 23, 1948	15 15 - 15 35	F	95
			Sep. 23, 1948	21 19 - 21 39	G	93
			Oct. 3, 1948	22 20 - 22 40	H	117
			Oct. 4, 1948	04 18 - 04 38	I	113
			Oct. 5, 1948	10 18 - 10 38	J	125
			Oct. 11, 1948	21 56 - 22 16	K	103
			Oct. 12, 1948	03 56 - 04 16	L	107
			Oct. 12, 1948	09 56 - 10 16	M	105
			Nov. 23, 1947	23 07 - 23 27	N	112
			Nov. 4, 1948	16 48 - 17 11	O	101
			Nov. 4, 1948	22 44 - 23 11	P	103
			June 2, 1949	05 33 - 05 53	Q	128
			June 2, 1949	17 40 - 18 00	R	123
			June 2, 1949	23 42 - 24 02	S	118
Heceta Head, Oregon	53	Mark III	Sep. 8, 1947	16 04 - 16 24	T	130
			Dec. 30, 1947	16 16 - 16 40	U	135
Guam, M. I.	60	Mark III	Feb. 17, 1949	05 17 - 05 38	V	112
			Feb. 18, 1949	03 23 - 03 43	W	120
			Feb. 18, 1949	20 37 - 20 58	X	116
Pt. Arguello, Calif.	83	Mark III	July 22, 1949	22 00 - 22 22	Y	90

[a] Except records N, Q, R, and S, which were taken at 68 ft.

instruments are calibrated by measuring their response to sinusoidal pressure fluctuations of known frequency and amplitude. A general description of the 25 records analyzed is given in Table 1.

A typical section of wave record is shown in Figure 1, illustrating the method of analysis. Each wave record of approximately 100 waves was divided into sections representing individual waves, each maximum and minimum of the pressure trace being counted as the crest and trough, respectively, of a wave. On each wave record each trough-to-crest time increment (designated "pressure half-wave period t_i") and each trough-to-crest height increment (designated "pressure wave height H_i'") were measured. An approximate description of the surface being desired, each of the latter heights was converted into a hypothetical "surface-elevation wave height H_i" by means of the formula of hydrodynamic theory [LAMB, 1945]

$$H = H' \cosh [2\pi d/L]/\cosh [2\pi d/L) (1 - z/d)]$$

where d is the bottom depth, z is the instrument depth, and L is the wave length for a sinusoidal wave train, given in terms of its wave period T by the relation

$$[g T^2 \tanh (2\pi d/L)]/2\pi - L = 0$$

The variable T_i was obtained by doubling the value of the half-wave period t_i measured from the pressure record to the nearest one-half second; the variable H_i was then obtained by graphical means to the nearest one-fourth foot (Mark III record) or one-tenth foot (Mark V record).

In short, the wave systems examined were time profiles at a fixed point and correspond most closely to swell in shoaling water rather than to, for example, systems of breakers, or wind waves

Table 2--Characteristics of height and period distributions of waves in twenty-five ocean-swell records

Record identification	Wave height				Wave period			
	Mean $\bar{H}$	Standard deviation s_H	Skewness coefficient a_{3_H}	Significant height $H_{(1/3)}$	Mean $\bar{T}$	Standard deviation s_T	Skewness coefficient a_{3_T}	Significant period $T_{(1/3)}$
	ft	ft		ft	sec	sec		sec
A	1.53	0.73	0.77	2.35	13.4	4.3	-0.53	15.1
B	0.64	0.36	0.69	1.06	11.1	4.8	0.25	14.8
C	0.92	0.46	0.48	1.45	13.6	3.7	-0.91	15.7
D	0.84	0.44	0.80	1.32	14.4	3.1	-1.18	15.6
E	1.93	1.23	0.60	3.38	13.5	3.1	-0.26	13.9
F	1.85	1.07	0.31	3.05	12.6	2.7	-0.32	13.8
G	2.20	1.35	0.78	3.68	13.0	3.1	-0.41	14.1
H	2.14	1.30	0.62	3.68	10.3	2.1	0.38	10.6
I	2.76	1.60	0.46	4.62	10.6	2.1	-0.13	10.8
J	1.10	0.77	0.70	2.00	9.6	2.7	0.54	8.9
K	4.29	2.24	0.66	6.80	11.6	2.4	0.21	11.5
L	3.18	1.81	0.44	5.22	11.3	2.7	0.24	11.5
M	2.82	1.50	0.41	4.48	11.4	2.5	-0.20	12.1
N	2.60	2.00	2.23	4.75	10.9	3.2	0.37	10.2
O	4.57	2.59	0.48	7.58	13.3	3.7	-0.15	13.8
P	4.58	2.24	0.47	7.20	13.5	3.4	-0.15	14.0
Q	2.69	1.28	0.46	4.12	9.2	1.9	0.16	9.4
R	2.17	1.32	1.19	3.65	9.6	2.1	0.61	9.6
S	2.51	1.28	0.48	3.98	10.0	2.1	-0.05	10.0
T	1.03	0.47	0.53	1.55	9.7	2.0	0.72	9.4
U	0.45	0.32	2.29	0.78	10.9	2.6	0.63	10.1
V	5.15	2.55	0.85	8.08	11.8	2.5	-1.21	12.3
W	4.34	2.07	0.72	6.70	11.0	3.1	-0.33	12.1
X	3.11	1.34	0.45	4.65	11.2	2.5	-0.44	11.8
Y	0.99	0.54	2.08	1.52	14.1	3.8	-0.67	14.7

in deep water. The wave periods studied refer to pressure fluctuations, the wave heights to estimated surface fluctuations.

Individual wave heights--For each wave record the per cent of wave heights not exceeding a given height h, that is, the function known as the "wave-height distribution function F (h)," was computed for all measured values of h. The 25 distribution functions thus obtained were found to resemble one another closely except for centering and scaling parameters characterizing the individual distributions. In arriving at this conclusion there were computed the arithmetic mean given by

$$\bar{H} = (1/N) \sum_{i=1}^{N} H_i$$

as well as the standard (root-mean-square) deviation from the mean given by

$$s_H = [(1/N) \sum_{i=1}^{N} (H_i - \bar{H})^2]^{1/2}$$

where N is the total number of waves in the wave record. These two quantities, which may be considered measures of, respectively, the location of the distribution center and the dispersion or scatter about it, are tabulated in Table 2. A plot of s_H against $\bar{H}$ over the set of wave records appears in Figure 2 together with the corresponding least-squares line. The variables being expressed in feet, the equation of this line is

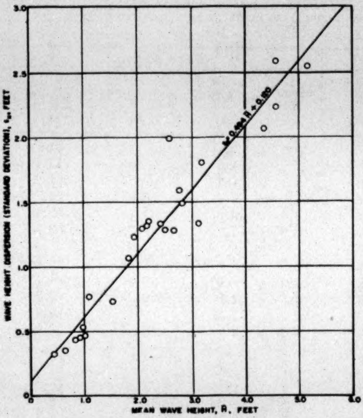

Fig. 2--The relation of wave-height dispersion to mean wave height (25 records)

$$s_H = 0.494\,\overline{H} + 0.120 \quad \ldots\ldots\ldots (1)$$

and it is apparent that it provides a good fit to the data.

Plots of the distribution functions for the 25 wave-height systems showed that the distribution shape was in each case approximated closely by a distribution function of the so-called Gamma type given by

$$\left.\begin{aligned} F(h) &= [1/\Gamma(p)] \int_0^u x^{p-1} e^{-x}\,dx \\ u &= p\,[1 + (h - \overline{H})/s_H] \end{aligned}\right\} \quad \ldots (2)$$

where $\Gamma(x)$ is the Gamma function evaluated for the argument x, and p = 6.25. An interpretation of the parameter p in the general formula for the Gamma-type distribution is given below.

The average relation existing between s_H and $\overline{H}$ suggests the one-parameter family of distribution functions illustrated in Figure 3 and described in the Appendix. Here, given the mean height $\overline{H}$, the distribution function is completely determined, and, accordingly, the per cent of waves below any given height h may be read off the bottom scale, which has been chosen so as to yield straight lines on the graph. It may be observed that the line representing the distribution having an arbitrary mean value $\overline{H}$ may be

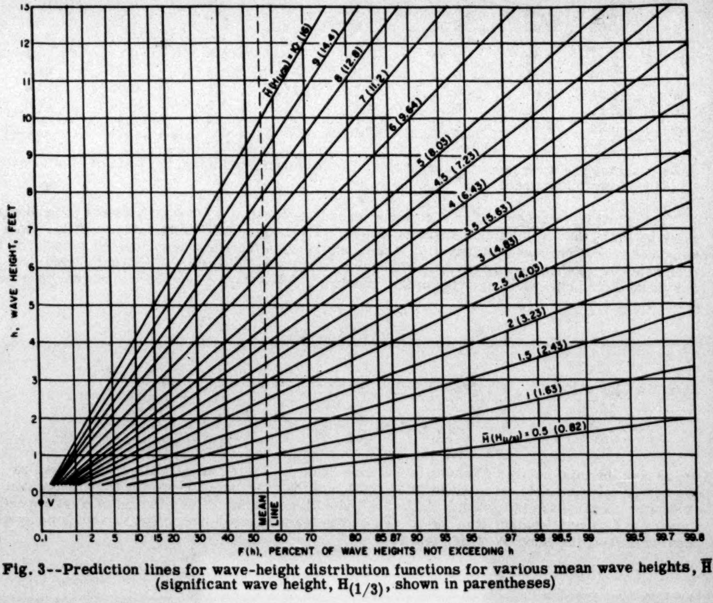

Fig. 3--Prediction lines for wave-height distribution functions for various mean wave heights, $\overline{H}$ (significant wave height, $H_{(1/3)}$, shown in parentheses)

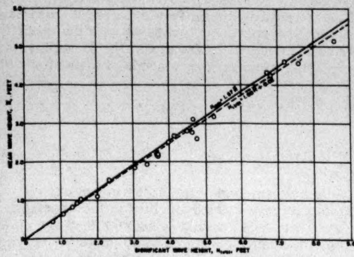

Fig. 4--The relation of mean wave height to significant wave height (25 records)

constructed on Figure 3 by joining the vertex point V with the point on the "Mean Line" having ordinate $\overline{H}$.

An indication of the degree to which the theoretical model distribution family of Figure 3 approximates the data may be seen from the fact that for the set of values of h corresponding to integral numbers of feet, the numerical difference between the value of F (h) observed and the value of F (h) predicted by Figure 3 never exceeds 13 pct, being no more than five per cent, in fact, 87 pct of the time.

This model for wave-height distributions has been applied to explain certain relations observed to exist among the various truncated mean wave heights $H_{(p)}$ calculated for the waves in the upper 100p pct of the height distribution; the latter quantity with p = 1/3 or 1/10 has been used to describe the general magnitude of heights in a wave system. The reciprocal slope of the solid line in Figure 4 is the value 1.57 of the ratio $H_{(1/3)}/\overline{H}$ calculated from the theoretical model, and also reported by SEIWELL [1949] for the Atlantic Ocean, while the 25 plotted points represent the observed values of $H_{(1/3)}$ and $\overline{H}$ for the wave systems of the present study. The equation of the dotted least-squares line minimizing vertical deviations is $H_{(1/3)} = 1.60\ \overline{H} + 0.02$, the variables being expressed in feet. The fit of the two lines to the data, as well as to each other, is seen to be very good. The curves in Figure 3 have been labelled with the value of $H_{(1/3)}$ following in parentheses the corresponding value of $\overline{H}$.

The value 1.29 reported by WIEGEL [1949] for the ratio $(H_{(1/10)}/H_{(1/3)})$ at several Pacific Coast locations is also the value yielded by the distribution model upon which Figure 3 is based.

Individual wave periods--As in the case of wave heights the empirical distribution function for wave periods may be defined and investigated. The distribution curves obtained with the present data were found to approximate a one-parameter family analogous to that for wave heights.

The mean $\overline{T}$ and standard deviation s_T for each of the 25 wave-period distributions is tabulated in Table 1, as well as the standardized (dimensionless) third moment about the mean given by

$$a_3 = (1/N) \sum_{i=1}^{N} (T_i - \overline{T})^3/s_T{}^3$$

The latter quantity may be interpreted as a measure of the degree of asymmetry in the period distributions, positive and negative values corresponding, respectively, to frequency curves skewed to the right and left; it is related to the quantity p of (2) by the definition $p = 4/a_3{}^2$. The mean value of a_3 was found to be -0.11, indicating average near-symmetry for the period distributions. This, together with the fit of these observed distribution functions to the Gamma-type family, suggests the characterization of the average wave-period system by the Gaussian distribution, the latter being the symmetrical member of the Gamma-type family.

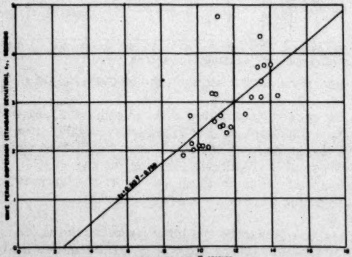

Fig. 5--The relation of wave-period dispersion to mean wave period (25 records)

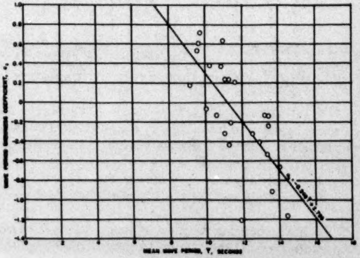

Fig. 6--The relation of wave-period skewness coefficient to mean wave period (25 records)

Figures 5 and 6 show the observed relations between $\overline{T}$ and s_T, and between $\overline{T}$ and a_3, together with the corresponding least-squares lines. It is seen that a moderate degree of association exists among these parameters, the amount of period dispersion increasing with the mean period, and the measure of asymmetry of the distribution decreasing from positive to negative as the mean period increases. The variables $\overline{T}$ and s_T being expressed in seconds, the equations of the lines are

$$s_T = 0.313\,\overline{T} - 0.759 \qquad (3)$$

and

$$a_3 = -0.249\,\overline{T} + 2.795 \qquad (4)$$

The relations between the three period-distribution parameters suggest the model one-parameter family of distribution functions illustrated in Figure 7 and described in the Appendix. Here, given the mean wave period $\overline{T}$, the per cent of waves whose period is shorter than any given time interval t may be read off the bottom scale, which has been chosen so as to make the curves of the family reasonably straight. A check on the adequacy of the theoretical model was made in a manner similar to that for the wave-height model. In the present case, the errors were somewhat larger, the numerical difference between the per cent predicted by Figure 7 and the per cent observed shorter than a given period exceeding ten per cent about five per cent of the time.

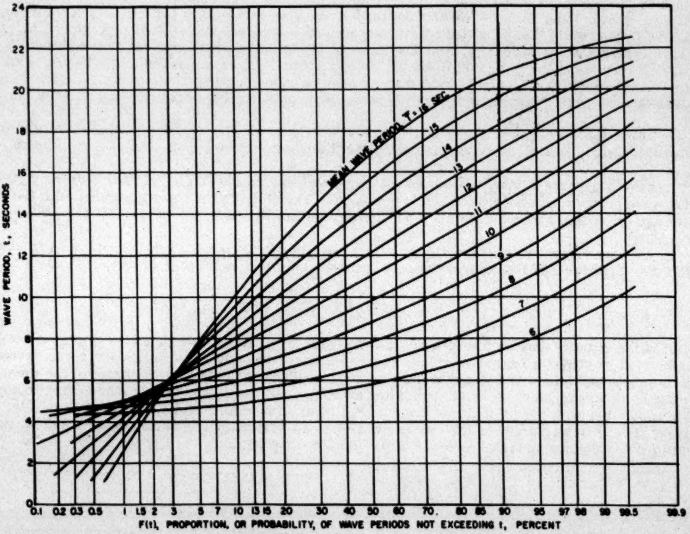

Fig. 7--Prediction curves for wave-period distributions for various mean wave heights

Paired heights and periods--The height and the period associated with an individual wave were found to show little relationship to each other. The typical joint frequency distribution plotted in Figure 8 shows virtual independence, in the statistical sense, of wave height and wave period. Although on some records there was evidence of a slight tendency for the two variables to increase together, possibly the aspect of the observed records which was most characteristic was the tendency for the highest observed waves to occur with near-average periods.

In describing a wave system, when it is desired to emphasize the higher waves, the mean period $\overline{T}$ of all the waves is sometimes replaced by the average period $T_{(1/3)}$ of the waves in the upper one-third of the wave-height distribution. Since the wave periods selected for averaging

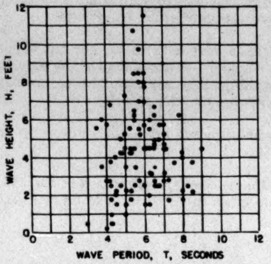

Fig. 8--The relation of wave height to wave period for individual waves, wave record K, Pt. Sur, California

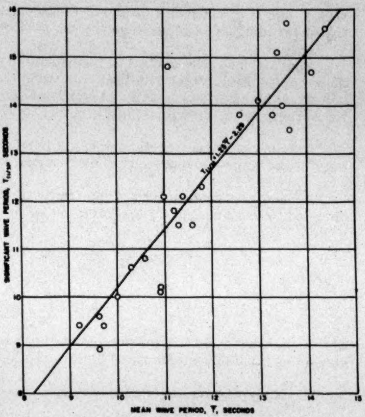

Fig. 9--The relation of significant wave period to mean wave period (25 records)

are made to depend on the corresponding wave heights, the relation of $T_{(1/3)}$ to $\overline{T}$ clearly depends on the statistical relation between wave height and wave period. Complete independence of these two fundamental quantities would lead in the long run to equality of the two averages, while a substantial positive correlation between them would cause $T_{(1/3)}$ to exceed $\overline{T}$ on the average by a substantial amount. The plot of the two variables $T_{(1/3)}$ and $\overline{T}$ in Figure 9 indicates, once more, only a slight degree of positive correlation between wave height and wave period, $T_{(1/3)}$ actually being smaller than $\overline{T}$ in about one wave record in three.

Comparison of wave-height distribution parameters with wave-period distribution parameters indicated little relation among these sets of characteristics. A tendency for systems with the highest mean wave heights to have near-average mean wave periods may be the most characteristic feature of the joint variation.

Successive heights and periods--The heights of two successive waves were found to show a consistent but rather low degree of association, the data from only three or four of the 25 wave records indicating even a moderately close relationship between the two variables. A similar statement may be made concerning the observed periods of two successive waves, the degree of relationship being, on the average, slightly lower in this case. As with the heights, on each wave record consecutive periods appeared to be two variables with a weak tendency to increase together.

Figures 10 and 11 show typical examples of the joint frequency distributions of consecutive wave heights and consecutive wave periods, respectively. It is seen that there would be expected, on the basis of such data, little accuracy in predicting the height or the period of a given wave from a knowledge of the heights and periods of its neighbors. High waves are quite likely to follow low

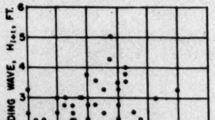

Fig. 10--The relation of height of succeeding wave to height of given wave, wave record F, Pt. Sur, California

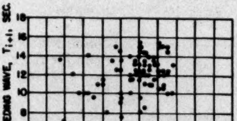

Fig. 11--The relation of period of succeeding wave to period of given wave, wave record V, Guam

waves, and long waves to follow short ones, although there appears to be a slight residual tendency for waves similar as regards height and period characteristics to occur together.

Conclusions--The statistical distributions of individual wave heights and periods appear to show considerable regularity from one wave system to another. A knowledge of the mean wave height or the mean wave period is equivalent to a reasonably close estimate of the dispersion of the corresponding distribution and, in fact, of the distribution function itself.

Only low-accuracy prediction of the height or period of a wave seems possible from knowledge of the heights and periods of neighboring waves.

Appendix--The equations of the family of distribution functions represented in Figure 3 are obtained by replacing in (2) the value of s_H by its value given in (1), leading to the one-parameter system

$$\left.\begin{aligned} F(h) &= [1/\Gamma(p)] \int_0^u x^{p-1} e^{-x}\,dx \\ u &= p\,[1 + (h - \overline{H})/(0.494\,\overline{H} - 0.120)] \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where $p = 4/a_3^2 = 6.25$ is taken to correspond to the average, - 0.80, of the values of the parameter a_3 (defined above) computed for the 25 wave-height distributions, and appearing in Table 2.

Similarly, the family of distributions functions of Figure 7 is represented by the equations

$$\left.\begin{aligned} F(t) &= [1/\Gamma(p)] \int_0^u x^{p-1} e^{-x}\,dx \\ u &= p\,[1 + (t - \overline{T})/(0.313\,\overline{T} - 0.759)] \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

with $p = 4/(-0.249\,\overline{T} + 2.795)^2$. These equations are obtained from (2) with "H" and "h" replaced respectively by "T" and "t", by substituting for the values of s_T and $p = 4/a_3^2$ their values in terms of $\overline{T}$ given in (3) and (4).

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(Communicated manuscript received April 7, 1952;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

THE EFFICIENCY OF DEPTH-INTEGRATING SUSPENDED-SEDIMENT SAMPLING

Ning Chien

Abstract--Because of the physical shape of the depth-integrating sampler, it is impossible to sample down to the bed. Only part of the total sediment load moving across the vertical will be caught by the sampler. Based on the suspended-load theory and bed-load function, this part of the total sediment load is analytically determined. The result is illustrated in graphical form, and its significance and possible errors are pointed out.

Introduction--In the course of developing the technique and equipment used in sediment sampling of streams, the integrating suspended-sediment sampler has gradually gained much popularity and is now being used quite extensively in the field. According to their functions, two types of integrating sampler have been developed, namely, the point-integrating and the depth-integrating sampler. The former is used to obtain a specimen at a specific point in the vertical at which the momentary fluctuations in sediment concentration are averaged; the latter is designed to obtain a specimen from the entire vertical in which the concentrations at different depths are averaged. Only the efficiency of the depth-integrating sampler will be discussed in this paper.

Most depth-integrating samplers in use today consist of cast-bronze streamlined bodies with intake nozzles at the front, and horizontal and vertical guide vanes at the rear [NELSON and BENEDICT, 1950]. They are lowered at a pre-determined rate from the water surface to the stream bed, and during the traverse a sample for the vertical is obtained. The fluctuations of the sediment concentrations at different elevations with time can be averaged if a large number of such samples are collected and mixed together to give a representative sample for the vertical. Because of the physical shape of the sampler, it is impossible to sample down to the stream bed, and the sample is actually integrated from a certain distance α above the bed to the water surface. The value α depends on the shape and size of the sampler, the methods of its operation, and the firmness of the stream bed.

Owing to this practical limitation, not all of the total sediment load carried through the vertical will be caught by the sampler, and the portion caught is designated in this paper as the efficiency of the depth-integrating suspended-sediment sampler. The sampler is otherwise considered as ideal, that is, it is assumed to draw a true sample of the inflowing water-sediment mixture without creating any disturbance of the flow ahead of its mouth.

Vertical distribution of suspended load--Based on the theory of turbulent exchange, the vertical distribution of suspended load in a stream has been established [VANONI, 1946]. For any given particle size D with the settling velocity V_s, its concentration at the distance y from the bed can be determined by the following formula, if the concentration C_a of the same particle at distance a is known

$$C_y/C_a = [(d - y)/y]^z \, [a/(d - a)]^z \quad \ldots\ldots (1)$$

where d is the water depth and

$$z = V_s/(ku_*') \quad \ldots\ldots (2)$$

k is the universal constant (0.4 for pipes) used by von KARMAN [1934] in his universal velocity defect law, and u_*' is the shear velocity with respect to the grain and can be expressed as

$$u_*' = (gS_eR')^{1/2} \quad \ldots\ldots (3)$$

in which S_e = energy gradient of the flow, g = gravitational acceleration, and R′ = hydraulic radius with respect to the grain [EINSTEIN, 1950].

Eq. (1) gives only a relative concentration, and the calculation of the total sediment load is possible only if the reference concentration C_a is known. Furthermore, (1) lets the concentration become infinite at the bed (y = 0), which is practicably impossible. It has been pointed out by

EINSTEIN [1950] that the suspended-load theory entirely breaks down near the bed where the particle size is of the same order of magnitude as the size of the eddies around it. The particle merely settles down between eddies without being lifted up again by the surrounding fluid masses. The thickness of bed layer in which suspension becomes impossible has been found to be about two grain diameters. Within this layer, the particles are pushed forward by the flow. Their weight is no longer supported by the flow, but by the bed itself, and the motion of the particles takes the form of rolling, sliding, or, occasionally, jumping. This part of load is generally called the bed load, in contrast with the suspended load.

It has been found that the bed-load movement follows a probability law which makes the determination of its rate of transport possible. By using this rate of transport of bed load ($i_B q_B$) and the thickness of the bed layer (2D), the sediment concentration at the upper boundary of the bed layer (C_{2D}) can be determined

$$C_{2D} = (Ai_B q_B)/(2Du_B) = (Bi_B q_B)/(2Du_*') \quad (4)$$

in which i_B = fraction of bed load in a given grain size D, q_B = bed-load rate in weight per unit of time and width, u_B = average velocity with which bed-load material moves in the bed layer while in motion, and is proportional to u_*', and A, B = constants.

The constant B is determined experimentally to be 1/11.6, and, therefore,

$$C_{2D} = (i_B q_B)/(23.2Du_*') \quad (5)$$

The same concentration is used as the concentration of the suspension immediate above. Knowing this reference concentration ($C_a = C_{2D}$, a = 2D) and using (1), the sediment concentration at any height above the edge of the bed layer can be determined.

The rate of transport of sediment--Since the horizontal velocity component of the suspended particle is assumed to be the same as that of the surrounding water, the rate of transport of suspended load per unit area and time at elevation y is

$$C_y u_y = C_{2D}\,[(d - y)/y]^z\,[2D/(d - 2D)]^z\,5.75\,u_*'\log_{10}(30.2y/\Delta) \quad (6)$$

in which u_y = velocity of flow at elevation y, and Δ = the apparent roughness diameter. Integration of (6) from the edge of the bed layer to the water surface then gives the total rate of transport of suspended-load.

$$i_s q_s = \int_{2D}^{d} C_{2D}\,[(d - y)/y]^z\,[2D/(d - 2D)]^z\,5.75\,u_*'\log_{10}(30.2y/\Delta)\,dy \quad (7)$$

in which i_s = fraction of suspension in a given grain size D, and q_s = suspended-load rate in weight per unit of time and width.

With appropriate mathematical manipulation, (7) becomes

$$i_s q_s = 5.75\,u_*'\,d\,C_{2D}\,[A/(1 - A)]^z \Big\{\log_{10}(30.2\,d/\Delta) \int_A^1 [(1 - y)/y]^z\,dy + 0.434 \int_A^1 [(1 - y)/y]^z \log_e y\,dy\Big\} \quad (8)$$

where

$$A = 2D/d \quad (9)$$

Substituting (5) into (8) and introducing

$$P = (1/0.434)\log_{10}(30.2\,d/\Delta) \quad (10)$$

$$I_1 = 0.216\,[A^{z-1}/(1 - A)^z] \int_A^1 [(1 - y)/y]^z\,dy \quad (11)$$

$$I_2 = 0.216\ [A^{z-1}/(1-A)^z] \int_A^1 [(1-y)/y]^z \log_e y\, dy \quad \ldots\ldots\ldots\ldots (12)$$

We have

$$i_s q_s = i_B q_B\ (PI_1 + I_2) \quad \ldots\ldots\ldots\ldots (13)$$

The values I_1 and I_2 are functions of A and z, and can be given in graphical form [EINSTEIN, 1950].

Knowing the rates of transport of suspended load and bed load, the total sediment discharge moving across the section is merely the sum of the two

$$i_T q_T = i_s q_s + i_B q_B = i_B q_B\ (1 + PI_1 + I_2) \quad \ldots\ldots\ldots\ldots (14)$$

in which i_T = fraction of total load in given grain size D, and q_T = total load rate in weight per unit of time and width.

The portion of total load caught by the depth-integrating sampler--As pointed out, the depth-integrating sampler draws a sample from a distance α above the stream bed up to the water surface. Using the concentration C_{2D} as the reference concentration and integrating the product $C_y u_y$ from α to d, the total rate of transport of suspended load obtained from the depth-integrating sample is

$$i_{s\alpha} q_{s\alpha} = \int_\alpha^d C_{2D}\ [(d-y)/y]^z\ [2D/(d-2D)]^z\ 5.75\ u_*{}'\log_{10}(30.2\ y/\Delta)\ dy \quad \ldots\ldots (15)$$

in which $i_{s\alpha}$ = fraction of suspension in a given grain size D caught by the depth-integrating sampler, and $q_{s\alpha}$ = suspended-load rate determined from the depth-integrating sample in weight per unit of time and width.

Adopting the same procedure as has been employed in deriving (13) from (7), we have

$$i_{s\alpha} q_{s\alpha} = i_B q_B\ (2D/\alpha)^{z-1}\ [(d-\alpha)/(d-2D)]^z\ (PI_{1\alpha} + I_{2\alpha}) \quad \ldots\ldots\ldots\ldots (16)$$

With

$$I_{1\alpha} = 0.216\ [A_\alpha^{z-1}/(1-A_\alpha)^z] \int_{A_\alpha}^1 [(1-y)/y]^z dy \quad \ldots\ldots\ldots\ldots (17)$$

$$I_{2\alpha} = 0.216\ [A_\alpha^{z-1}/(1-A_\alpha)^z] \int_{A_\alpha}^1 [(1-y)/y]^z \log_e y\ dy \quad \ldots\ldots\ldots\ldots (18)$$

$$A_\alpha = \alpha/d \quad \ldots\ldots\ldots\ldots (19)$$

Eq. (17) and (18) are exactly the same as (11) and (12) if A is replaced by A_α. This shows that the graphical form constructed for (11) and (12) can also be used in determining $I_{1\alpha}$ and $I_{2\alpha}$, only if A_α is used in the place of A.

The portion of total sediment load caught by the depth-integrating sampler is then

$$i_{s\alpha} q_{s\alpha}/i_T q_T = (2D/\alpha)^{z-1}\ [(d-\alpha)/(d-2D)]^z\ (PI_{1\alpha} + I_{2\alpha})/(1 + PI_1 + I_2) \quad \ldots\ldots (20)$$

This formula represents the efficiency of depth-integrating suspended-sediment sampling, and its value differs for different grain sizes.

Figure 1 illustrates the whole thought which leads to (20), and detailed description of the theory should refer to Einstein's paper [EINSTEIN, 1950].

Graphical presentation of the efficiency of depth-integrating sampling--From (20), $(i_{s\alpha} q_{s\alpha})/(i_T q_T)$ is a function of the following six variables: α, the distance above the bed where sampling starts; D, grain size; d, depth of flow at the vertical; $u_*{}'$, shear velocity with respect to the grain; P, varies with the composition of the bed, and ν, the kinematic viscosity of the fluid. It cannot be presented in simple graphical form, unless certain simplification and grouping of variables are made.

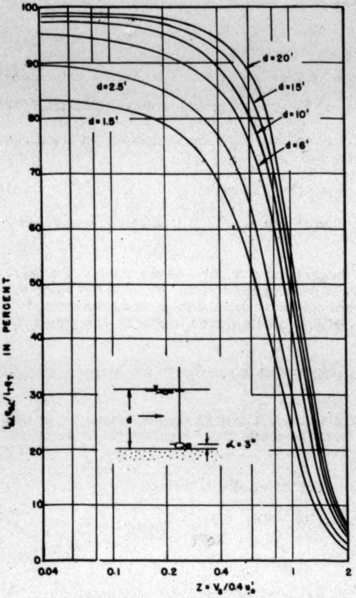

Fig. 1--Sediment-distribution curve across a vertical

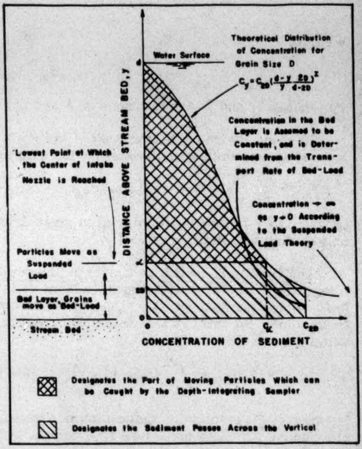
Theoretical Distribution of Concentration for Grain Size D
$C_y = C_{2D}\left(\frac{d-y}{y}\,\frac{2D}{d-2D}\right)^z$
Water Surface
DISTANCE ABOVE STREAM BED, y
d
α
2D
0
Concentration in the Bed Layer is Assumed to be Constant, and is Determined from the Transport Rate of Bed-Load
Lowest Point at Which the Center of Intake Nozzle is Reached
Concentration → ∞ as y → 0 According to the Suspended Load Theory
Particles Move as Suspended Load
Bed Layer, Grains move as Bed-Load
Stream Bed
0
C_α
C_{2D}
CONCENTRATION OF SEDIMENT
▩ Designates the Part of Moving Particles Which can be Caught by the Depth-integrating Sampler
▨ Designates the Sediment Passes Across the Vertical

Fig. 2--Efficiency of the depth-integrating suspended-sediment sampling expressed in terms of the total sediment discharge

For ordinary rivers, P varies from 9 to 12. By a few preliminary trials, it is found that the variation of P does not materially affect the value $(i_{s\alpha}\,q_{s\alpha})/(i_Tq_T)$, as it appears both in the denominator and numerator of (20) in the same order of magnitude. If P is considered as a constant instead of being a variable, an error of five per cent may result, which is acceptable in this case.

If the grain size D is replaced by its settling velocity V_s, which, in turn, combines with the shear velocity u_*' to form a dimensionless parameter V_s/u_*', then for a given α, $(i_{s\alpha}\,q_{s\alpha})/(i_Tq_T)$ is a function of d and V_s/u_*' only. The relationship among them can be represented by a set of curves. The error involved by replacing D and u_*' by V_s/u_*' will be discussed later.

Figure 2 is constructed to illustrate such a relationship. z (= V_s/ku_*', k is assumed to be 0.4) is used as the abscissa instead of V_s/u_*', and the distance α is assumed to be three inches. Table 1 shows the distance between center of intake nozzle and lowest point of the sampler for three different depth-integrating samplers commonly used.

Table 1--Distance from center of intake nozzle to lowest point of sample for various depth-integrating samplers

Name of sampler	Distance, center of nozzle to	
		inch
US DH-48 hand depth-integrating sampler	Lowest point of body	2 5/8
US D-49 depth-integrating sampler	Lowest point of body	4 1/16
US D-43 depth-integrating sampler	Lowest point of body	4 9/16
	Lowest point of vertical tail vane	5 9/16

The distances shown in Table 1 might be too large or too small for the value of α, depending upon whether the sampler is allowed to sink in the bed or pulled up before it touches the bed when taking samples. A distance of three inches is adopted merely for illustrative purpose, and with the value of α actually being determined in the field the efficiency of sampling can be readily computed from (20).

Discussion of the result--Three important features are observed from Figure 2:

(1) The depth of flow becomes an important factor only for shallow streams. The zone near the bed which the sampler cannot reach actually represents a sizable fraction of the total depth in shallow water, but becomes negligible in deep rivers.

(2) The percentage of the total load caught by the sampler rises very rapidly with decreasing z (that is, smaller particle size and larger turbulence). For $z < 0.1$, the particles are so close to being uniformly distributed across the vertical that the efficiency approaches its limiting value $(d - \alpha)/d$. For $z > 2.0$, the grains become so large that they almost exclusively move as bed load in the bed layer.

(3) By using (20) or Figure 2, the total load rate can be determined from the suspended-sediment sampling data. However, for $z > 0.8$, the efficiency of sampling changes substantially with slight change of z. The small values of efficiency also indicates that the motion of grains of that range takes place close to the bed, and the amount of these grains caught by the sampler depends very much on how deep the sampler reaches. A variation of α will then easily introduce deviation to such a degree that it becomes unacceptable if the sampling data are converted to the total sediment discharge.

Two errors are involved in the treatment above. One pertains to the development of the suspended-load theory and the other occurs because D and u_*' are replaced by z in constructing Figure 2.

As pointed out by VANONI [1946] and ISMAIL [1951], the sediment-transfer coefficient is not necessarily the same as the momentum-transfer coefficient. The presence of suspended matter in the water also reduces the von Karman constant k quite considerably. On the other hand, there is reason to believe that the settling velocity of the particle determined in the still water is quite different from that in turbulent water. Owing to these deviations, the results of flume study and river measurement [ANDERSON, 1942] both show that the distribution of the suspended load follows (1) very closely, but that the value of the exponent z given by the theory does not agree with the one which fits the experimental and river data. If a relationship between the measured z_1 and the theoretical z can be established either by analytical approach or by actual measurement, (20) can still be used roughly to determine the efficiency of depth-integrating sampler, only now the value z should be replaced by some other variable z_1 which is a function of z.

Also by replacing D and u_*' by z, certain deviations are introduced. This can be seen readily from the fact that in (20), for the same value of z, V_s (and, therefore, D) can be larger or smaller, depending upon the variation of u_*'. The settling velocities of the particles smaller than 1/16 mm vary approximately as the square of the diameters, whereas, for large particles, the settling velocities vary approximately as the square root of the diameters. For the same variation of u_*', therefore, the larger the value of z, the wider the range of particle sizes will be involved. On the other hand, the particle size occurs in three different places in (20), and their effects should be considered separately:

(1) In the determination of I_1 and I_2, the graphical solution of (11) and (12) shows that the larger the z and the smaller the A (= 2D/d), the less the dependence of I_1 and I_2 on A. The variation of D becomes important, therefore, only for small values of z and for shallow streams. Yet as pointed out above, for small values of z, the range of grain sizes involved is also limited.

(2) In the term $[(d - \alpha)/(d - 2D)]^z$, the variation of D is practically negligible for rivers with large depth.

(3) In the term $(2D/\alpha)^{z-1}$, the variation of D is more important for large values of z.

Concluding from all these considerations, it appears that the discrepancy involved by replacing D and u_*' by z only becomes significant for large values of z and shallow streams. This is verified in the actual construction of Figure 2. Under these two conditions, either the grains are so coarse that they move almost exclusively as bed load, or the river is so shallow that α is not a negligible fraction of the water depth d. This makes the error involved by replacing D and u_*' by z acceptable, as not only the error is small with respect to the total load rate, but a slight change of α will also affect the efficiency of the sampler considerably. Furthermore, this error can always be avoided if, instead of using the simplified graphical form (Fig. 2), the exact solution (20) is used to determine the efficiency of the sampler.

Acknowledgment--The problem was suggested to the writer by H. A. Einstein of the University of California. His guidance and advice are greatly appreciated.

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(Communicated manuscript received November 5, 1951;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

A METHOD FOR APPROXIMATING THE WATER CONTENT OF SOILS

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Abstract--A procedure is described for calibrating fiberglas soil-moisture units, and for preparing the tables required to convert resistance readings into inches of water. The shape and slope of the calibration curves are determined in the laboratory with the units placed in undisturbed soil samples. After the units have been placed in the field, the laboratory curves are translated laterally along the moisture axis by an amount determined through simultaneous field moisture samples and resistance readings. Regression equations characterizing the bulk density of the soil (as a function of moisture content and depth) are taken into account in converting soil-moisture content from a per cent dry-weight basis to inches of water. For thin horizons subject to large errors of measurement, a method is described for correcting the bulk-density regression equation so as to conform with its corresponding calibration curve.

Introduction--In 1948 the Northeastern Forest Experiment Station, in cooperation with the Pennsylvania Department of Forests and Waters, established an outdoor hydrologic laboratory in a small, brush-covered watershed in northeastern Pennsylvania. One study at this laboratory seeks to determine the soil-water relationships in scrub-oak areas. One step in the study was to determine daily changes in the storage of water in the soil mantle. To obviate the time-consuming chore of sampling the soils daily for water content, several batteries of fiberglas soil-moisture units [COLMAN, 1946] were installed in the different soils of the watershed. The resistance of these units to an electric current varies according to the amount of moisture held by the fiberglas. When the units are in the soil, the moisture in the fiberglas tends to be in equilibrium with the soil moisture.

But since soils differ one from another, it is necessary to calibrate each fiberglas unit for the particular soil in which it will eventually be placed. This paper describes the laboratory, field, and mathematical procedures that were used to calibrate the fiberglas soil-moisture units. The final product of the method is a table by which resistance readings can be translated directly into inches of water.

Laboratory calibration of fiberglas units--A preliminary study revealed that valid calibration curves can be obtained only if the original structure of the soil is maintained. A different moisture-resistance relationship is obtained for disturbed samples than would be obtained for the same soil in its original state. Hence, great care was taken to sample the soil in a manner that minimized disturbance to its structure. The soils were sampled with an ordinary bulk-density sampler. (Since the completion of this work, the author has adopted an improved method. The fiberglas units are first inserted into the natural, undisturbed soil. The soil sample (with the inserted unit) is then removed with a bulk-density sampler, and is calibrated in the inner, removable portion of the sampler.) A recent paper by HENDRIX and COLMAN [1951] treats this subject thoroughly.

The soils in the watershed are podzols of the Dilldown, Borie, and Hazleton series (tentative nomenclature). The samples brought into the laboratory for calibration were obtained from each of the visually recognizable horizons and from several points at lower depths. The horizons sampled were: the humus (surface organic horizon), the A_2 (gray, leached horizon), the B_{21} (dark brown layer of deposition), and four additional depths in the B_2 horizon, down to a depth of approximately two feet. (Duplicate samples were taken for use in determining the moisture content of the soil.)

The soil samples were placed in small monel cans [COLMAN, 1947] and were saturated by immersion in distilled water. A fiberglas soil-moisture unit was then inserted in each soil sample. A careful account was kept of the serial number of the unit placed in a particular horizon, since each unit would later be placed in the horizon and depth for which it was calibrated.

After the soil was saturated, the monel can with its encased soil and fiberglas unit was weighed, and temperature and moisture-resistance readings were taken. To facilitate the loss of moisture from the sample, the entire assembly was fanned for one minute, and then placed for about 24 hours in an air-tight chamber maintained at 98 pct relative humidity. At the end of the

24-hour period the cans were weighed, resistances were read, and the samples were fanned, then placed again in the chamber. This procedure was repeated daily. Only one set of readings was taken each day, in order to allow a sufficient amount of time for the moisture in the soil to reach a stage of equilibrium throughout the soil mass. As the soil became drier, the period of fanning was gradually increased to ten minutes. By this procedure the amount of moisture lost by the sample was controlled. The mineral soil lost one per cent (of its dry weight) between successive readings, while the humus lost about seven per cent.

All resistance readings were corrected to a uniform temperature of 60°F [BETHLAHMY, 1951]. For each unit, the corrected resistance readings were then plotted above the moisture per cent they represented. This resulted in a series of preliminary calibration curves, based entirely on laboratory work.

Field installation of fiberglas units--At the site selected for installing the units, a hole, having the smallest necessary diameter, was dug to a depth slightly greater than the point where the lowest unit was to be installed. The several soil horizons were kept separated as they were brought to the surface. In addition, the locations and positions of all rocks were noted, and when put back into the hole they were returned to approximately their former places.

As indicated previously, each soil-moisture unit was installed in the horizon and depth for which it was calibrated. Each unit was forced into the soil on the side of the hole with as little disturbance to the soil as possible. Three color-coded wires (temperature, moisture, and common return) connected each unit to a panel board in a small shelter. The use of color-coded wires minimized the danger of incorrect wiring. To prevent a wire from conducting rain water to any particular unit, each set of three wires was extended to a point deeper than the unit itself, then brought to the surface of the ground on the opposite side of the hole.

Adjustment of calibration curves--After the units were installed, daily readings of resistance were made for each unit. Each time a resistance reading was made, a soil sample, for use in determining moisture content, was collected from the horizon near the unit in question. A table was prepared for each unit, showing resistance measured and the corresponding soil-moisture determination. These were then compared with the resistance-moisture relationships as determined in the laboratory calibration. For a given unit, the discrepancies between the field and laboratory moisture percentages (for a given resistance) were averaged, and the laboratory calibration curve was shifted by that amount in the required direction. Thus, the shape and slope of the calibration curve were determined in the laboratory while its position along the moisture per cent scale was determined in the field. That is, a given resistance on the laboratory calibration curve was adjusted to correspond to the actual field moisture content it represented.

Table 1--Average discrepancy in moisture percentage between field and laboratory calibration curves

Unit number	Site			
	3A	3B	4	8
	pct	pct	pct	pct
1	+27.9	+57.2	-11.2	+21.1
2	+ 1.6	- 3.7	+ 2.8	+22.8
3	+17.8	+ 4.8	- 1.4	- 1.1
4	+18.5	- 0.5	+ 9.3	+ 0.6
5	+ 4.7	- 1.3	+ 0.3	- 1.1
6	- 0.2	- 4.7	- 3.5	- 3.4
7	+ 1.8	- 2.5	- 6.9	- 6.9

Table 1 lists the average discrepancies between the field and laboratory curves for each of seven units at four different sites. For example, the field calibration curve of Unit 2, Site 3A, indicated a soil-moisture content of 1.6 per cent more than that indicated by the laboratory curve. Figure 1 shows the final calibration curves. Sites 3A and 3B are located in a Dilldown soil; Site 4 in a Hazleton soil; Site 8 in a Borie soil.

Determination of bulk density and rock content of the soil--To determine the amount of water stored in the soil mass, it is necessary to know the amount of soil held in a given volume. This means that the bulk density of each horizon must be determined as well as the rock content of the soil.

A number of pits three feet square were dug slightly deeper than the lowest soil unit. The rock content was determined by weighing the rocks found in each six-inch soil layer. The rocks were separated from the soil by sieving the mixture through a coarse sieve (3/4 × 1-3/4 inches). The use of such a coarse sieve would separate all rocks too large to enter the bulk-density sampler. Since most of the rocks were sandstones, the volume they occupied was determined on the basis of the density of sandstone (approximately 150 lb/ft^3). The volume was then apportioned among the different soil horizons. For example, if a six-inch layer of soil contained four inches of A_2 horizon, then 4/6 of the total volume of rocks was assigned to the A_2 horizon. Table 2 lists the average rock content of three typical sites in the watershed.

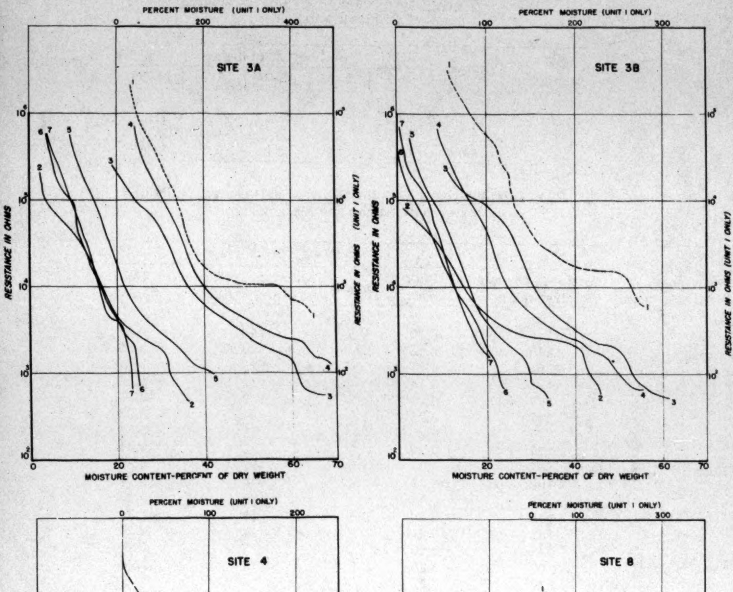

Fig. 1--Calibration curves for soil-moisture units at four sites; at each site the units are installed at increasing depths, Unit 1 in humus, Units 2 to 7 in mineral oil

Bulk densities of the different soil horizons were obtained through extensive sampling of the soil. The sampling was done by horizons and by depth within horizon. Thus, the humus, the A

Table 2--Average rock content (per cent of volume) in soil of Dilldown watershed

Site number	Humus	Soil depth, in inches 0-6	6-12	12-18	18-24	24-30
3	33.1	57.2	50.5	41.1	34.7	27.2
4	18.2	21.0	7.3	10.0	5.8	26.0
8	1.6	10.2	4.2	3.9	6.6	7.8

Table 3--Bulk-density data for Dilldown soil series, Dilldown watershed, humus and A_2 horizon

Humus		A_2 horizon		
Moisture, per cent of dry weight	Bulk density	Depth	Moisture, per cent of dry weight	Bulk density
		inch		
246	0.17	0.5	40	0.51
245	0.13		16	0.93
201	0.24		15	1.04
198	0.16		12	1.06
177	0.19		9	0.90
177	0.23	1.0	28	0.62
177	0.20		24	0.62
177	0.20		22	0.64
167	0.24		19	0.85
162	0.21	2.0	28	0.72
162	0.15		21	0.83
142	0.19	2.5	13	1.30
142	0.21		13	1.36
140	0.25		13	1.27
127	0.23	3.2	12	1.28
124	0.22			
100	0.28			
93	0.21			
83	0.29			
61	0.29			
61	0.29			

horizon, and the B horizon were sampled and analyzed separately. Tables 3 and 4 list the bulk-density data for the most extensive soil type in the watershed.

Analysis of these observations showed that the bulk density of the humus varies with moisture content, whereas the bulk density of the mineral soil varies with both moisture content and depth in inches from the surface of the mineral soil. Regression equations were computed for the three horizons as follows.

$$W = 0.76 - 0.25 \log_{10} M \text{ for humus horizon} \qquad (1)$$

$$W = 1.190 + 0.128\,D - 0.023\,M \text{ for } A_2 \text{ horizon} \qquad (2)$$

$$W = 1.500 + 0.027\,D - 0.016\,M \text{ for B horizon} \qquad (3)$$

in which W is the bulk density, D is depth in inches from the surface of the mineral soil, and M is moisture content as per cent of dry weight. Statistically, all the regression coefficients are significant at the one per cent level (see Table 5).

The equations for the mineral soil indicate that the bulk density depends on the depth of the point in question from the surface of the mineral soil and on the soil-moisture content. Like a living organism, the soil is a dynamic entity that is in continuous motion; it is either expanding or contracting, depending on whether it is gaining or losing moisture. During a storm, the soil gains

Table 4--Bulk-density data for Dilldown soil series, Dilldown water, B horizon

Observations (number)	Depth	Moisture, per cent of dry weight	Bulk density
	inch		
1	2.0	48	0.72
1	4.0	48	0.94
1		33	1.01
1		13	1.70
1	6.0	48	0.94
2		38	1.04
1		33	1.05
1		23	1.05
1		18	1.49
1	8.0	48	1.00
1		38	1.13
1		33	1.30
1		28	1.26
1		23	1.22
3		18	1.40
1	10.0	33	1.34
2		28	1.22
3		23	1.39
1	12.0	28	1.42
4		23	1.42
2		18	1.51
3	14.0	23	1.53
5		18	1.56
1	16.0	23	1.70
4		18	1.60
2		13	1.66
1	18.0	18	1.74
2		13	1.68
1		8	2.20
4	20.0	13	1.84
1	22.0	18	1.87
1	24.0	18	1.82

moisture, expands, and consequently its bulk density decreases. After a storm, when the soil loses moisture by evaporation and transpiration, it contracts, and its bulk density increases.

In the calculations that follow, the depths represented by a particular soil-moisture unit in the mineral soil were considered to remain at a fixed value.

Table 5--Statistical analysis of regression coefficients

Equation	"Student's" t		Degrees of freedom	t_{1} pct
	Depth	Moisture		
	inch	pct		
(1)	. . . .	6.24	20	2.84
(2)	3.58	5.75	12	3.06
(3)	11.25	8.89	53	2.67

Calculation of K coefficients--To convert moisture percentage readings, as given in the calibration curves (Fig. 1), into equivalent inches depth of water, it was necessary to determine the value of a constant (for each horizon) whose product with the moisture percentage yields inches of water. This constant (termed K coefficient) represents the depth in inches of a volume of water equal to one per cent of the net weight of the soil.

The method of calculating the K coefficient for one of the soil horizons (Unit 7, Site 4) is illustrated below.

(1) Depth (from mineral surface) at which unit is installed	24.0 inches
(2) Depth of soil layer represented (distance midway between units above and below)	6.0 inches
(3) Volume of soil under one square foot of surface	0.50 ft^3
(4) Moisture content of soil at time of installation (per cent of dry weight of soil)	16 pct
(5) Bulk density of soil at time of installation	1.89
(6) Rock content (per cent of volume)	16 pct
(7) Net volume of soil = 0.50 (100-16)/100	0.42 ft^3
(8) Net weight of soil = (0.42) (1.89) (62.43)	49.56 lb
(9) Weight of water equivalent to one per cent of net soil weight	0.4956 lb
(10) K coefficient (depth of water occupied by one per cent of net soil weight = (0.4956) (12)/62.43	0.0953 inch

Since this soil section contains 16 pct moisture, the depth of water it contains is: 0.0953 × 16 = 1.52 inches.

The K coefficient calculated above is for a particular depth (24 inches) and moisture content (16 pct); hence, its determination is based on use of the appropriate bulk density (see Eq. 3, above).

$$W = 1.500 + 0.027\ (24) - 0.016\ (16) = 1.89 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

If the soil at this particular depth contained 20 pct moisture, its bulk density would be 1.83; the K coefficient would be 0.0953 × 1.83/1.89 = 0.0923; and the inches of water contained in the soil would be 0.0923 × 20 = 1.85 inches. That is, for a given depth the K coefficients at different moisture-content levels vary directly with the bulk densities. Following this method, K coefficients were calculated for the gamut of per cent moisture contents through which the soil containing a particular unit might pass. Table 6 shows a portion of such a table for Unit 7 of Site 4. To calculate the inches of water contained in the humus, only one K coefficient was calculated, using as parameters the moisture content and depth of the humus when at field capacity. The product of this K coefficient by successive moisture per cents gave the equivalent inches of water.

Table 6--Calculation of K coefficient and inches of water (Unit 7, Site 4)

Item	Per cent of moisture				
	16	17	18	19	20
Bulk density	1.892	1.876	1.860	1.844	1.828
K coefficient	0.0953	0.0945	0.0937	0.0929	0.0921
Inches of water	1.525	1.606	1.687	1.765	1.842

Correction of bulk-density regression equations--In computing the inches of water contained in a particular soil horizon, it was found that a parabolic curve results when "inches of water" are plotted over "moisture percentage." A problem arose when it became evident that the peak of the curve occurred at a point other than the saturation point as indicated in the calibration curves. This was true for the upper horizons of the mineral soil (the leached, A_2 horizon, and the B_{21} horizon of deposition).

Because the A_2 and B_{21} horizons are thin, they were sampled with a small bulk-density sampler. The use of the small sampler introduced errors by compressing the soil sample. In order to solve the problem noted above, it was evident that the volume-weight regression curves would have to be adjusted. This was done by designating a fixed set of conditions that a series of equations would have to meet, solving the equations, and then eliminating the solutions that did not fully meet the requirements.

For example, soil-moisture Unit 3 at Site 4 was installed at a depth of 2.75 inches. From its calibration curve we note that the soil is saturated at 54 pct moisture content. At this moisture content the derivative of "inches of water" with respect to "moisture percentage" should be zero. The first estimates of bulk density and inches of water (using bulk-density Eq. (3) given above) were: 1.2542 gm/cc at 20 pct moisture and 0.8690 inches of water at 54 pct moisture.

The general form of the bulk-density regression equation is

$$W = a + bD - cM \qquad (5)$$

in which W is bulk density; a, b, and c are constants; D is depth in inches from the surface of the mineral soil; and M is moisture content as per cent of dry weight of soil.

However, in the case of Unit 3, we are interested in the bulk-density changes at a particular depth, D, and not in the values of the coefficients a and b. Hence, for (5) we may substitute

$$W = A - cM. \qquad (6)$$

The equation for calculating inches of water has the general form

$$Y = KM. \qquad (7)$$

in which Y is inches of water, K is the K coefficient defined above, and M is moisture content as per cent of dry weight. But for a constant depth, since the K coefficient varies with the bulk density, we may sbustitute for (7)

$$Y = kWM. \qquad (8)$$

in which k is the constant by which the bulk density W must be multiplied to determine the K coefficient. In the case of Unit 3 at Site 4, k = 0.0193/0.86. By substituting the value of W from (6) into (8) we obtain

$$Y = k\ (A - cM)\ M. \qquad (9)$$

Differentiating Y in (9) with respect to M and equating to zero, we obtain

$$dY/dM = 0 = k\ (A - 2cM) \qquad 0 = A - 2cM \qquad (10)$$

Substituting all known values into (6), (9), and (10) we obtain

$$1.2542 = A - 20c. \qquad (11)$$

$$0.8690 = (0.0193/0.86)\ (A - 54c)\ (54). \qquad (12)$$

$$0 = A - 108c \qquad (13)$$

These equations yield three different solutions for A and for c. The different combinations of A and c are then substituted into (6), (8), and (10) to determine the combination which is most suitable. In this particular case, we determine that

$$W = 1.4341 - 0.0133M. \qquad (14)$$

Calculation of the tables of the resistance to water depth--To simplify the analysis of field measurements of soil moisture, a special table was prepared for each soil unit. This table lists the water content in inches of water indicated by the resistance (corrected to 60°F) offered by the soil unit. Table 7 shows a portion of such a table for Unit 7 of Site 4. Such a table is a composite of the information contained in the table exemplified by Table 6 and the calibration curves (Fig. 1).

Table 7--Amount of water in soil horizon indicated by resistance reading (Unit 7, Site 4)

Resistance (at 60°F)	Water depth
ohms	inch
1000	2.32
1500	2.24
2000	2.22
2500	2.18
3000	2.12
3500	2.03

Adequacy of method--Table 8 presents data showing soil-moisture changes during four storm periods. The difference in soil-moisture storage before and after a storm is compared with the rainfall as indicated by automatic rainfall gages in the watershed. Differences between amount of rainfall and soil-moisture change may be attributed to interception, evaporation, transpiration, and differences between rainfall as measured at the gage location and actual rainfall on the soil-moisture site.

Summary--A method of determining calibration curves for fiberglas soil-moisture units is presented. The shape and slope of the curves are determined in the laboratory, but their exact position along the moisture axis is determined by field moisture

Table 8--Calculated soil-moisture storage changes (to 27-inch depth) compared with precipitation records (1949)

Storm period	Site							
	3A		3B		4		8	
	S	P	S	P	S	P	S	P
October 26	6.38	0.34	5.82	0.34	9.77	0.33	9.67	0.44
25	5.67	0.56	4.96	0.56	8.92	0.55	8.88	0.37
Storage change	0.71	. . .	0.86	. . .	0.85	. . .	0.79	. . .
Precipitation	. . .	0.90	. . .	0.90	. . .	0.88	. . .	0.81
October 31	6.79	0.40	5.85	0.40	9.88	0.40	9.73	0.37
30	. . .	0.13	. . .	0.13	. . .	0.13	. . .	0.11
29	. . .	0	. . .	0	. . .	0	. . .	0
28	6.18	0.08	5.50	0.08	9.51	0.07	9.51	0.03
Storage change	0.61	. . .	0.35	. . .	0.37	. . .	0.22	. . .
Precipitation	. . .	0.61	. . .	0.61	. . .	0.60	. . .	0.51
November 4	6.50	0.01	5.70	0.01	9.92	0.02	9.78	0.05
3	. . .	0.63	. . .	0.63	. . .	0.52	. . .	0.50
2	. . .	0	. . .	0	. . .	0	. . .	0
1	6.37	0	5.60	0	9.69	0	9.58	0
Storage change	0.13	. . .	0.10	. . .	0.23	. . .	0.20	. . .
Precipitation	. . .	0.64	. . .	0.64	. . .	0.54	. . .	0.55
November 14	6.67	0	5.90	0	10.28	0	10.39	0.05
13	. . .	1.35	. . .	1.35	. . .	1.27	. . .	1.34
12	. . .	0.05	. . .	0.05	. . .	0.02	. . .	0
11	. . .	0	. . .	0	. . .	0	. . .	0
10	6.00	0	5.33	0	9.37	0	9.28	0.02
Storage change	0.67	. . .	0.57	. . .	0.91	. . .	1.11	. . .
Precipitation	. . .	1.40	. . .	1.40	. . .	1.29	. . .	1.41

S = Calculated soil-moisture storage.
P = Precipitation recorded on nearest automatic rain gage.

sampling. The conversion of resistance readings into inches of water is accomplished by taking into account the continuous changes in bulk density of the soil. An example of the method of calculation is given.

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(Communicated manuscript received November 7, 1951, and, as revised, April 4, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

THE CIRCULATION OF LAKE MENDOTA

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Abstract--The wind-driven circulation of two-layered lakes may be interpreted in terms of kinetic-energy production and horizontal divergence. Evidence from Lake Mendota, Wisconsin, indicates that the epilimnion is circulated directly by the wind, with some piling up of the water downwind. This increased lake level results in depression of the thermocline, which in turn represents a solenoid concentration capable of circulating the hypolimnion.

STARR [1948] set forth some considerations on the production of kinetic energy which provide a basis for the explanation of the wind-driven circulation of lakes. In essence, Starr showed that the rate of production of kinetic energy is equal to the pressure times the horizontal velocity divergence, plus the advection of energy into the system. In addition, there may be kinetic-energy advection into the system through the boundaries, and losses due to work being performed at the boundary by pressure forces and horizontal velocities, and friction. Symbolically

$$\partial(KE)/\partial t = A + S - W - D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where A = advection of kinetic energy across the boundaries, S = the primary production of kinetic energy within the volume, -W = the performance of work by pressure forces at the boundary in virtue of the displacements due to the horizontal velocity components, and -D = the action of frictional forces.

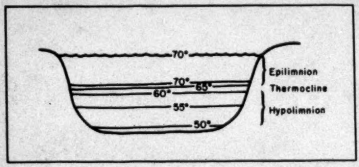


Fig. 1--Simple two-layered lake

Let us consider a lake with a simple form and typical summer temperature distribution such as that delineated in Figure 1. The wind exerts a horizontal stress on the lake surface which drives the water downwind and to the right [SVERDRUP and Others, 1942, p. 494]. Thus water is accelerated away from the upwind shore. This in turn means horizontal divergence in accord with the fact that the water is gaining kinetic energy. Steady-state conditions will be approached when the losses into work and friction equal the production and advection of kinetic energy.

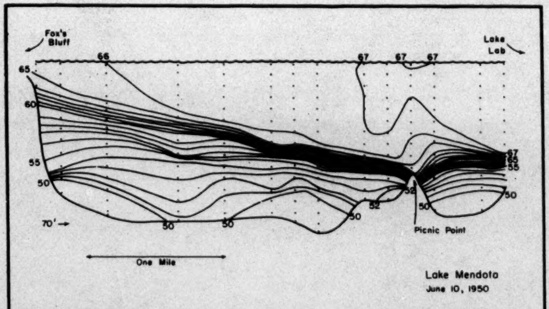


Fig. 2--Northwest-southeast section through Lake Mendota; the wind was from the northwest, producing a typically tilted thermocline

Downwind the wind-driven current converges on the shore and the kinetic energy is converted to potential energy by raising the lake level on that side; work done against gravity. A consequence of the raised downwind surface is an increase in pressure at the level of the thermocline, which in turn depresses that layer; work done against pressure forces. The density difference of air and water is nearly 1.00 while that between epilimnion and hypolimnion is about 0.002. This means that for the extreme cases of thermocline depression, about 50 ft in Lake Mendota, the surface level should change about one-tenth of a foot. This downwind tilt of the thermocline is a very characteristic feature of the thermal structure of stratified lakes (see Fig. 2). Since the thermocline is a concentration of isotherms, their downward tilt in the region of horizontal or upward sloping isobars also represents potential energy in the form of solenoids [HOLMBOE and Others, 1945, p. 300]. Unless opposed by some force (acceleration), the solenoidal acceleration will produce a circulation from the temperature gradient to the pressure ascendant in sense.

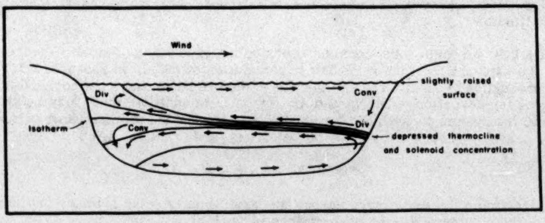

Wind
Div
Conv
Isotherm
Conv
Div
slightly raised surface
depressed thermocline and solenoid concentration

Fig. 3--Structure of a simple two-layered lake after deformation by wind stress

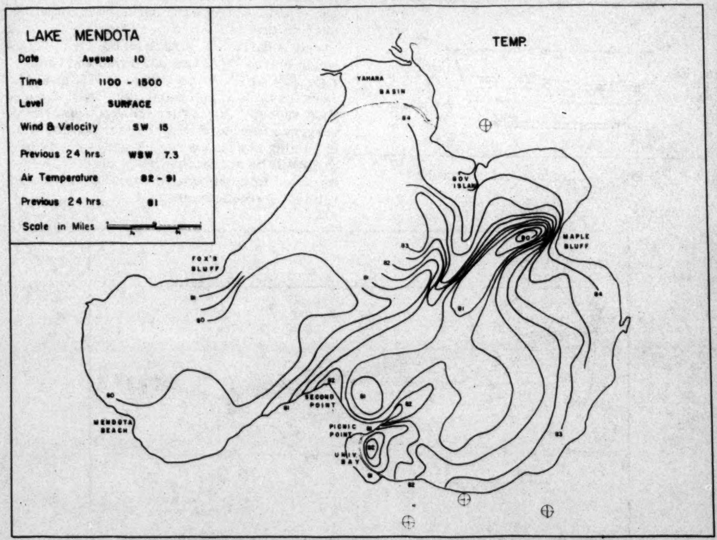

LAKE MENDOTA
Date August 10
Time 1100 - 1500
Level SURFACE
Wind & Velocity SW 15
Previous 24 hrs. WSW 7.3
Air Temperature 82 - 91
Previous 24 hrs 81
Scale in Miles
TEMP.
YAHARA BASIN
84
GOV ISLAND
90
MAPLE BLUFF
83
82
81
84
81
FOX'S BLUFF
81
90
82
SECOND POINT
81
82
90
MENDOTA BEACH
81
PICNIC POINT
82
UNIV. BAY
81
82
83

Fig. 4--Surface temperature distribution on Lake Mendota, August 10, 1949

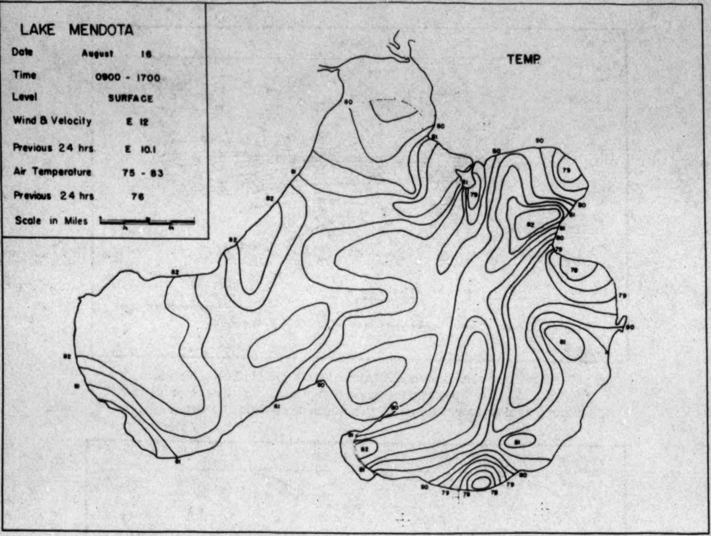

LAKE MENDOTA
Date August 16
Time 0900 - 1700
Level SURFACE
Wind & Velocity E 12
Previous 24 hrs. E 10.1
Air Temperature 75 - 83
Previous 24 hrs 76
Scale in Miles
TEMP.

Fig. 5--Surface temperature distribution on Lake Mendota, August 16, 1949

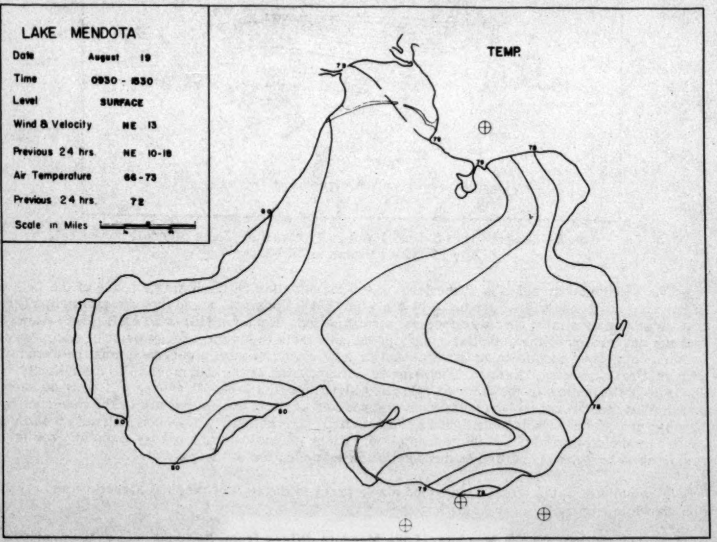

LAKE MENDOTA
Date August 19
Time 0930 - 1530
Level SURFACE
Wind & Velocity NE 13
Previous 24 hrs. NE 10-18
Air Temperature 66 - 73
Previous 24 hrs. 72
Scale in Miles
TEMP.
79
79
79
79
80
78
80
80
79
79

Fig. 6--Surface temperature distribution on Lake Mendota, August 19, 1949

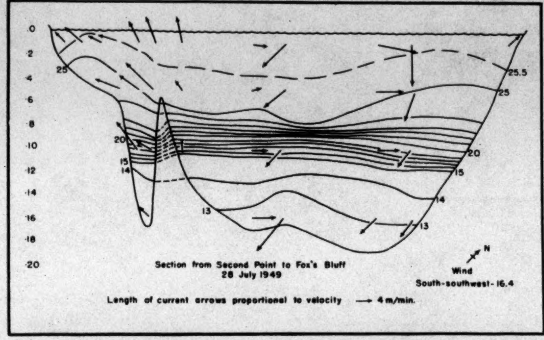
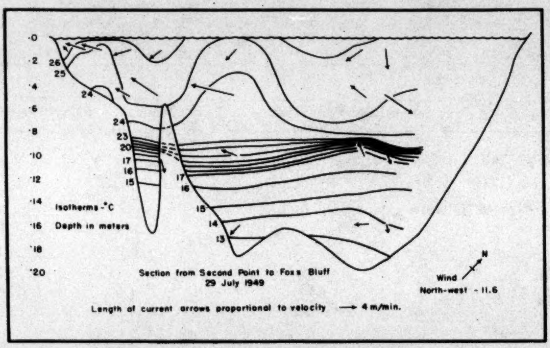

.0
2
4
6
8
10
12
14
16
18
20
25
20
15
14
25.5
25
20
15
14
13
13
Section from Second Point to Fox's Bluff
28 July 1949
N
Wind
South-southwest-16.4
Length of current arrows proportional to velocity → 4 m/min.

Fig. 7--Section from Second Point to Fox's Bluff, Lake Mendota, on July 28, 1949; temperatures are centigrade; the direction of the current vector is referred to the north arrow on the diagram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15
14
13
Isotherms-°C
Depth in meters
Section from Second Point to Fox's Bluff
29 July 1949
N
Wind
North-west - 11.6
Length of current arrows proportional to velocity → 4 m/min.

Fig. 8--Section from Second Point to Fox's Bluff, Lake Mendota, on July 29, 1949; legend as in Figure 7

The solenoidal circulation at the level of the thermocline on the downwind side of the lake is in the opposite sense to that produced by the wind, but it does not extend into the epilimnion for that layer is essentially devoid of thermal stratification. Equation (11) of STARR [1948] shows that the conversion of the potential energy of the solenoids to kinetic energy must be accompanied by horizontal divergence. In an incompressible medium horizontal divergence must be associated with vertical shrinking. Vertical shrinking or compression of the thermocline is a strikingly characteristic feature of the downwind thermal structure of a lake. Conversely, it may be seen that upwind there must be horizontal convergence and vertical stretching within the thermocline. This in turn produces solenoids opposing the vertical stretching, but less concentrated than those caused by the downwind tilt of the thermocline. These solenoids store energy to be utilized in restoring the original condition of the lake following cessation of the wind stress.

In combination, the effects described above produce the typical thermal structure and circulation shown in Figure 3.

The circulation of Lake Mendota--Lake Mendota differs from the ideal lake of the previous

Table 1--Relative frequency (per cent) of current-velocity intervals observed at various depths

Depth	Average current velocity m/min							
	0.1	1-3	3-5	5-7	7-9	9-11	11-13	13
m	pct	pct	pct	pct	pct	pct	pct	pct
1	5	34	18	30	9	3	1	0
5	23	43	12	6	10	3	3	0
10	19	56	9	10	3	3	0	0
15-18	12	41	12	23	0	0	6	6

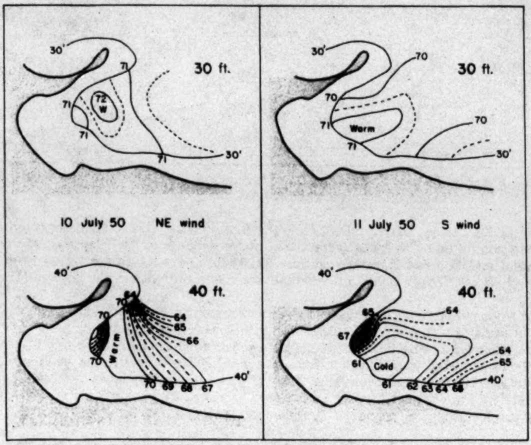

Fig. 9--Temperature (°F) at depth in University Bay, Lake Mendota, on two successive days; the warm core at 40 ft on July 10 is replaced by cold upwelling on July 11

paragraphs in that it is neither simple in outline nor in profile. Furthermore, there is great variation in the roughness of the surrounding terrain, which is reflected in a highly variable wind stress. In addition, the rotation of the Earth must be considered, for the coriolis force for the observed velocities is the same order of magnitude as the pressure gradient and centrifugal forces.

As an example of the resulting complexity, let us consider the rapid change of horizontal temperature pattern with time. On August 10, 1949, the wind had been between west-southwest and west, seven to 15 mi/hr, for over 24 hours and the surface temperature pattern included pronounced tongues of warm water off each point of land, but the warmest water was in general downwind (see Fig. 4). By August 15 the wind had shifted to east ten miles per hour. On August 16 the wind was still from east 12 mi/hr and the warmest water was in the bays rather than off the points (see Fig. 5). The patterns of temperature were hardly less complicated at depth on these days, and tilting of the thermocline produced very strong temperature gradients at depths of 30-40 ft.

Examination of two seasons' records of currents and temperature distributions reveals similar complexity in most cases, but decreasing with increasing wind velocity. Figure 6, for example, shows a less complex pattern than either of the charts in Figures 4 and 5. Nevertheless, it is possible to examine the theory presented in this paper in the light of observational data. One

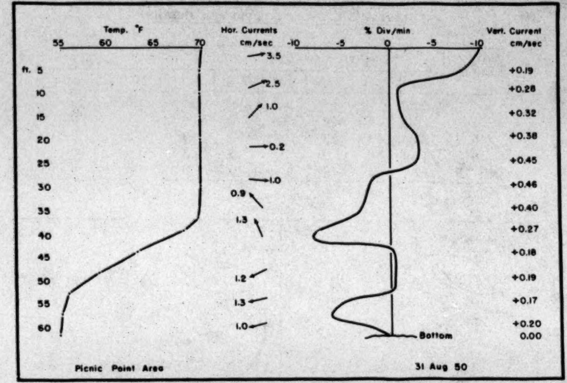

Fig. 10--Vertical distribution of temperature, horizontal and vertical currents, and divergence, Lake Mendota, August 31, 1950

consequence of the theory is that once below the fast-moving directly wind-driven layer near the surface there should be no overall decrease of current velocity with depth. This is shown to be true in Table 1 and Figures 7 and 8. On July 28, 1949, currents as strong as 20 m/min were measured both at one meter below the surface and one meter above the bottom (15 m).

A second consequence of the theory is demonstrated in Figure 9. On July 10, 1950, the wind was from the northeast driving the epilimnion into University Bay and causing the depression of the thermocline indicated by the high temperature intruded into the 40-ft level in the center of the bay. By July 11 the wind was light and from the south. The cessation of wind stress from the northeast that occurred on the evening of July 10 was followed by upward movement of cold hypolimnetic water into the region formerly occupied by unusually warm water. This condition on July 11 was produced by the pressure-temperature solenoids created on the 10th.

Finally, there should be divergence in the surface layers and convergence in the thermocline region on the upwind side of the lake. By means of three concurrent velocity measurements at the apices of a triangle it is possible to measure divergence directly [BELLAMY, 1949]. A series of ten such measurements distributed from surface to bottom, upwind, is presented in Figure 10. It will be noted that the divergence and velocity profiles are nearly as expected. A computation of vertical velocity using the equation of continuity and the divergence data shows a general upward drift of up to 0.45 cm/sec in this place. Figures of half this magnitude are not uncommon for areas as large as one km^2 in Lake Mendota.

No discussion of lake circulation would be complete without reference to the work of Defant, who [DEFANT, 1932, p. 148] computed the circulation of a stratified lake on the assumption that the hypolimnion was driven by friction with the epilimnion. We have rejected this work, however, for several reasons: The thermocline is the region of greatest stability, hence least friction; the shear at the thermocline is usually small; and the hypolimnion in general moves as fast as the epilimnion.

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(Communicated manuscript received February 6, 1950, and, as revised, January 10, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union   Volume 33, Number 5   October 1952

ACCELERATED MOTION OF A SPHERICAL PARTICLE

M. R. Carstens

Abstract--The force components on an oscillating sphere in a fluid medium were studied by experimental means. These force components on the oscillating sphere were separated into the inertial effective force and the dissipative (shear) force. In order to accomplish this experimental program, the writer assumed that the sphere motion was approximately simple harmonic motion, and he analyzed the data as if the motion were exactly simple harmonic motion.

The experimental results were applied to an allied problem of a sphere in an oscillating fluid. The experimental results, as applied to this problem, were found to compare with the data of an experiment performed in Germany in about 1920. Extending the analysis further, the writer presents a simple analysis of the diffusion characteristics of suspended sediment in water.

Introduction--The suspension of sediment in water, the transport of air bubbles in water, and the suspension of mist droplets in air are illustrations of the diffusion of foreign particles in a fluid medium. The foreign particles (the sediment particles, the air bubbles, or the mist droplets) are transported by the movement of the surrounding fluid; therefore, the diffusion process is incompletely understood until the relationship between the fluid and foreign-particle movement is known. The similarities and the differences in the movement of the fluid and foreign particle can be studied either by observing the particle motion in a moving fluid or by determining the force characteristics on a single moving particle and then utilizing these force characteristics in the analysis of more complex motions. This latter scheme is more amenable from the standpoint of experimental investigation.

Symbols--The following symbols are used in this paper:

A	Viscous damping coefficient
a	Fluid displacement
$\dot{a}$	Fluid velocity
$\ddot{a}$	Fluid acceleration
a_0	Half-amplitude of the fluid motion
B	Spring constant
B_1	Spring constant of Spring No. 1 (Fig. 1)
B_2	Spring constant of Spring No. 2 (Fig. 1)
β	Diffusion coefficient ratio ϵ_s/ϵ_m
b_0	Half-amplitude of the upper end of spring No. 1
c	Concentration of the sediment particles
D	Diameter of the cylindrical fluid container
d	Sphere diameter
ϵ_m	Momentum diffusion coefficient
ϵ_s	Sediment diffusion coefficient
F	Resultant force on the sphere surface
k	Virtual mass coefficient
kM	Virtual mass of the sphere
m	Mass of the oscillating system
M	Fluid mass displaced by the sphere
M_0	Mass of the sphere
μ	Dynamic viscosity
N_1	Oscillation parameter $\sqrt{8\nu/\omega}/d$
N_2	Damping parameter $A/3\pi\mu d$
ν	Kinematic viscosity
ω	Circular frequency of the harmonic motion in radians per second
P	External force applied to the oscillating system
α	Phase angle between the sphere and fluid motion
ρ	Fluid density
ρ_s	Sphere density

w Settling velocity of the sediment particles
x Sphere displacement
$\dot{x}$ Sphere velocity
$\ddot{x}$ Sphere acceleration
x_0 Half-amplitude of the sphere
y Displacement of the upper end of Spring No. 1 from the neutral position (Fig. 1)

Analysis of a single oscillating particle--If a solid body undergoes acceleration in a fluid, two distinct types of forces act on the surface of the body. Since the body is being accelerated, the fluid surrounding the body also undergoes an acceleration which, in turn, is manifested as an inertial reaction on the solid body. The second type of force is the dissipative or shear force which is akin to the drag force of steady motion. Thus, the inertial reaction force of the fluid is a function of the body acceleration and the dissipative force is a function of the body velocity.

The total force on a submerged moving body is the resultant of the fluid inertial reaction and the shear force which are functions of the body acceleration and velocity, respectively. Therefore, this total force is a function of the type of body motion. The inertial reaction force and the shear force will be collinear only if the body motion is rectilinear and, in addition, these force components will be in the same direction or will be in phase only if the velocity vector is of the same sign as the acceleration vector. Since the results of an experimental program are to be applied to the mechanics of particle diffusion, the type of motion involved in the experiment should be representative of the motions to be expected in the diffusion process. The rectilinear motion most nearly satisfying the criterions of (a) a possible diffusion motion, (b) elementary relationships between the displacement, velocity, and acceleration, and (c) simplicity of experimental production was found to be simple harmonic motion.

Utilizing the simple harmonic motion of a sphere in an infinite fluid, Stokes [LAMB, 1945, pp. 642-644] was able to derive a formula for the force on this spherical particle due to the presence of the surrounding fluid. He integrated the Navier-Stokes equations after neglecting the convective acceleration terms and obtained the solution

$$F = -kM\ddot{x} - A\dot{x} \quad \text{. . . . . (1)}$$

in which the virtual mass coefficient k is

$$k = 1/2 + (9/4)\,N_1 \quad \text{. . . . . (2)}$$

and in which the viscous damping coefficient is presented in the formula

$$N_2 = A/3\pi\,\mu d = 1 + 1/N_1 \quad \text{. . . . . (3)}$$

The virtual mass coefficient k as given by (2) approaches the value of 1/2 in the limit as N_1 approaches zero. This limit value of 1/2 is identical to the value of the virtual mass coefficient k as a sphere undergoes unidirectional acceleration in an irrotational infinite fluid. The parameter N_1 is constructed such that it is the inverse of the square root of a type of Reynolds number, and hence, when N_1 approaches zero, the Reynolds number of the motion is very large. Conversely, when N_1 is very large, this Reynolds number is small in magnitude, and the viscous damping coefficient A approaches the limit value $3\pi\,\mu d$. This value of A ($N_2 = 1$) is identical to the viscous damping coefficient for steady rectilinear motion of a sphere at a flow condition represented by a low Reynolds number.

The analytical solution as given in (1), (2), and (3) is unusual in that the amplitude $2x_0$ of the simple harmonic motion of the sphere is missing and would surely appear as a variable in an experimental study. For simple harmonic motion of a sphere in an infinite fluid the force on the surface of the sphere is a function of five variables.

$$F = f_1\,(x_0, \omega, d, \rho, \mu)$$

If this surface force is resolved into the component which is the result of the fluid acceleration and into the component which is the shear force, a possible solution of dimensionless variables is

$$k = f_2\,(N_1, 2x_0/d) \quad \text{. . . . . (4)}$$

and

$$A/3\pi\,\mu d = N_2 = f_3\,(N_1,\,2x_0/d) \qquad (5)$$

Even with the choice of simple harmonic motion of the single sphere, the selection of a means to determine separately the values of k and N_2 as functions of the variables N_1 and $2x_0/d$ was confronted. The procedure was to analyze various second-order linear differential equations along with the solutions in order to ascertain whether the constant coefficient of the second-order derivative and the constant coefficient of the first-order derivative could be independently determined. A differential equation and its solution satisfying these requirements is

$$m\ddot{x} + A\dot{x} + Bx = P \sin \omega t \qquad (6)$$

The constant coefficient m of the second-order derivative $\ddot{x}$ is the mass of the oscillating system, which would include the virtual mass of an attached submerged body; and the constant coefficient A of the first-order derivative $\dot{x}$ is the viscous damping coefficient. The solution of (6) for the displacement x is simple harmonic motion after a time t sufficient to allow the decay of the transient. The magnitude of the half-amplitude x_0 of the motion can be altered by successive changes in m, and at the condition of maximum half-amplitude (resonant amplitude)

$$m = B/\omega^2 \qquad (7)$$

and

$$A = P/x_0\omega \qquad (8)$$

Hence, the differential equation (6), with the conditions of (7) and (8) fulfilled at the resonant amplitude, satisfies the prerequisites; namely, the motion is simple harmonic and the magnitude of m and A can be independently determined.

Experimentation--The experimental equipment consisted essentially of two oscillating springs connected by a wire on which the sphere was attached as illustrated on Figure 1. The lower end of Spring No. 2 was fixed, but the upper end was attached to a steel wire (0.010-inch diameter) which was free to oscillate. The sphere was a table tennis ball which was cemented to the steel wire. A short distance above the free surface of the fluid a container was constructed in which mass increments were rigidly held. By this arrangement, the mass of the oscillating system could be altered in increments as small as 0.11 (10^{-4}) slug. The upper end of the alterable-mass container was attached to the lower end of Spring No. 1. The upper end of Spring No. 1 was oscillated with the simple harmonic motion

$$y = b_0 \sin \omega t$$

by means of an adjustable eccentric and a long connecting rod. For this mechanical arrangement of springs, (7) and (8) are transformed into the form

$$m = (B_1 + B_2)/\omega^2 \qquad (9)$$

and

$$A = (B_1/\omega)\,(b_0/x_0) \qquad (10)$$

The magnitude of the impressed amplitude $2b_0$ was measured to the nearest 0.001 inch by a micrometer measurement on the eccentric. A continuous record of the movement of the oscillating system was obtained by means of a spark discharge through a moving recorder paper. A small brass needle placed just below the alterable-mass container was one pole of the spark gap and the recorder was the other pole.

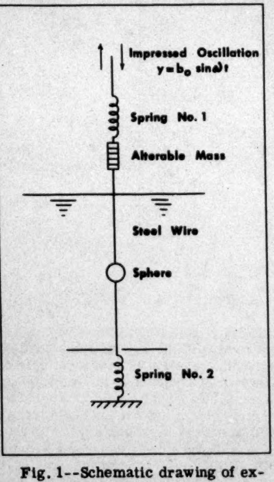

Fig. 1--Schematic drawing of experimental equipment

The equipment was operated in such a manner that the resonant condition of the oscillating system could be determined. A run was started with greater-than-resonant mass on the oscillating system. Successive increments of mass were removed and the corresponding amplitude of the system was recorded for each new mass until the oscillating mass had been reduced well below the resonant mass. The magnitude of the resonant amplitude $2x_0$ was varied by the adjustment of the eccentric; that is, the magnitude of b_0 was varied in order to obtain a range of resonant amplitudes. The series of runs with the sphere in place was followed by a series of runs which were similar except that the sphere was removed. Since the sphere diameter d and the impressed frequency ω were fixed throughout the entire experiment, the variation in the parameter N_1 was accomplished by the utilization of three different fluids. The fluid of least viscosity was water, intermediate viscosity was Eureka white mineral oil, and greatest viscosity was SAE No. 10 lubricating oil.

The experimental data were evaluated at the resonant condition in order to utilize the functional relationships of (9) and (10). For each run the amplitude of the motion was plotted as a function of the mass in the alterable-mass container. From this graph, the resonant amplitude $2x_0$ and the corresponding alterable-mass could be accurately and readily determined. Utilizing this value of the resonant amplitude $2x_0$, the viscous damping coefficient A could be computed from (10) for each run. These values of A were then plotted as a function of the amplitude $2x_0$ for both the series of runs with and without the sphere. Since nothing but the sphere was changed in each series, the viscous damping coefficient A for the sphere alone was the difference in the value of A with the sphere and without the sphere at any particular value of the amplitude. The data are finally presented in the form $N_2 - 1$ as a function of N_1 at various levels of $2x_0/d$ on Figure 2.

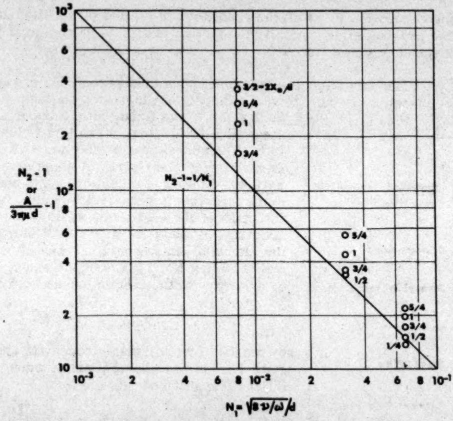

Fig. 2--The damping parameter N_2 as a function of the oscillation parameter N_1

A comparable procedure was necessary to determine the value of the virtual mass coefficient k. It can be seen from (9) that the resonant mass of the oscillating system is dependent only on the value of the spring constants and the frequency of oscillation; consequently, the resonant mass was constant in magnitude for all runs. From the graph of amplitude as a function of alterable mass, the value of the alterable mass at resonance was readily determined for each run. The sum of the resonant-alterable mass plus the mass associated with the steel wire was plotted as a function of amplitude $2x_0$ for both series of runs. The virtual mass of the sphere at any amplitude was then merely the difference of the two functions at that amplitude. From this value of the virtual mass, the virtual mass coefficient k was calculated and is presented graphically on Figure 3 as a function of N_1 at various levels of $2x_0/d$.

Discussion of results--It can be seen from Figures 2 and 3 that the value of N_2 and k are functions not only of N_1 but also of $2x_0/d$. This fact is in contradiction to Stokes' solution as presented in (2) and (3), and is undoubtedly attributable to the neglect of the convective acceleration

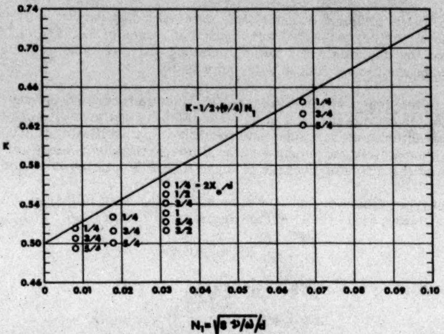

Fig. 3--Virtual mass coefficient k as a function of the oscillation parameter N_1

terms in this analytical solution. This conclusion is justified by two observations. First, the values of k and N_2 are approximately equal to the predicted values when $2x_0/d$ is small and the convective accelerations would be truly negligible. Secondly, the fluid motion was definitely laminar in all runs with SAE No. 10 lubricating oil ($N_1 = 0.0672$) and with Eureka white mineral oil ($N_1 = 0.0315$) at values of $2x_0/d < 1.0$; hence, a majority of the experimentally determined values are in the same flow regime as the analytical solution.

The laminar fluid motion around the oscillating sphere was more stable than the corresponding steady motion of a fluid around the sphere. Any instability of the laminar motion was distinct and could be observed in three different ways. As long as the motion was laminar, no disturbance of the free surface of the fluid was observed, but when the motion was turbulent the vortex motion could be visually observed on the surface of the fluid. Also, as long as the motion was laminar, the sphere would oscillate at all times with rectilinear motion, but in the turbulent region the sphere would have a tendency to oscillate sideways which had to be counteracted by adjusting the tension in the steel connecting wire. These visual observations were substantiated by the definite change in the resistance function at $2x_0/d = 1.0$ for $N_1 = 0.0315$. The implication is that the motion was laminar for values of $2x_0/d < 1.0$, the motion was turbulent for values of $2x_0/d > 1.0$, and the change in flow regime was distinct and abrupt. The corresponding value of the Reynolds number based on the maximum instantaneous velocity is 4000. This result is not entirely unexpected since the boundary layer growth is a function of time in this unsteady motion; consequently, the boundary layer dimensions of the oscillatory motion would be less than the steady-flow boundary layer dimensions.

As an additional check on the stability of the fluid motion, qualitative instantaneous velocities were measured with a constant-temperature hot-wire anemometer with air as the fluid. These measurements indicated only a rare eddy (approximately every 30 seconds) at the values of $2x_0/d$ of 1.1 and N_1 of 0.0345 which corresponds to a maximum Reynolds number of 3600. In addition, the velocities in front and behind the sphere were found to be different in magnitude, with the difference becoming more pronounced as the amplitude was increased.

As long as the motion is stable, that is, laminar, there is no question about the use of a viscous damping coefficient A in the equation of motion (6); however, when the fluid motion is turbulent, the use of an equivalent viscous damping coefficient is subject to question. The effect of the non-linear resistance function of turbulent flow on the motion of the sphere was determined by plotting the sphere displacement as a function of time. This curve was found to be the sine function and no experimental distinction could be found between the sphere motion in laminar and turbulent flow. Consequently, the viscous damping coefficient A as determined for the turbulent flow runs must be considered as the equivalent viscous damping coefficient to be utilized in a laminar-flow analysis. As all the pertinent variables have been included in the experimental determination of this equivalent viscous damping coefficient, no error is involved in its subsequent use in this form.

The effect of the cylindrical boundary was determined by varying only the fluid-container diameter D. The diameter ratios D/d were tested at values of 2.1, 3.6, and 5.4. The effect of the D/d ratio upon the magnitude of k was determined and was found to be negligible if $D/d > 10$. The D/d ratio of the equipment finally selected was 16.

Application to foreign-particle diffusion--The inertia force and the shear force on a spherical particle oscillating in a fluid at rest can be utilized in the analysis of more complex motions, such as the motion characteristics of a spherical particle in an oscillating fluid. The interrelationship between the foreign-particle movement and the fluid-particle movement are necessary to the complete description of the diffusion process of these foreign particles in a fluid.

The simplest type of particle motion in an oscillating fluid is that in which the fluid is considered to be oscillating with simple harmonic motion

$$a = a_0 \sin \omega t$$

The equation of motion for a single particle is

$$M_0\ddot{x} = M\ddot{a} + kM(\ddot{a} - \ddot{x}) + A(\dot{a} - \dot{x}) \ldots\ldots\ldots\ldots \text{(11)}$$

The various forces on the spherical particle are the fluid pressure-gradient force $M\ddot{a}$, the virtual mass effective force $kM(\ddot{a} - \ddot{x})$, and finally the shear force $A(\dot{a} - \dot{x})$. This motion is novel in the respect that the virtual mass effective force and the shear force are motivating forces and hence are positive quantities in (11). After the decay of the transients the solution of (11) for the sphere displacement is

$$x = x_0 \sin(\omega t + \alpha) \ldots\ldots\ldots\ldots \text{(12)}$$

in which the phase angle α between the particle and fluid motion is defined in

$$\tan \alpha = [(1 - \rho_s/\rho)(9/4) N_1^2 N_2] \Big/ \left\{(1 + k)(\rho_s/\rho + k) + [(9/4) N_1^2 N_2]^2\right\} \ldots\ldots \text{(13)}$$

and in which

$$(x_0/a_0)^2 = \left\{(1 + k)^2 + [(9/4) N_1^2 N_2]^2\right\} \Big/ \left\{(\rho_s/\rho + k)^2 + [(9/4) N_1^2 N_2]^2\right\} \ldots\ldots \text{(14)}$$

From (13) it can be seen that the sphere motion lags the fluid motion if the particle material is more dense than the fluid and that the sphere motion precedes the fluid motion if the fluid is more dense. In a parallel manner, it can be seen from (14) that the sphere amplitude $2x_0$ will be less than the fluid amplitude $2a_0$ if the particle has a greater density than the fluid. Conversely, if the fluid has the greater density, the particle amplitude $2x_0$ will be larger than the fluid amplitude $2a_0$. In the limit as ρ_s/ρ and N_1 approach zero, the particle amplitude $2x_0$ will be three times the value of the fluid amplitude $2a_0$.

If the value of the virtual mass coefficient k of (2) and the value of N_2 of (3) are substituted in (14) it follows directly that

$$x_0/a_0 = \sqrt{\left\{[1+(3/2)N_1]^2 + [(3/2\, N_1(1+N_1)]^2\right\} \Big/ \left\{[(2/3)(\rho_s/\rho) + 1/3 + (3/2)N_1]^2 + [(3/2)N_1(1+N_1)]^2\right\}} \ldots \text{(15)}$$

This relationship was presented by **KONIG** [1891] and was experimentally investigated by **WAGENSCHEIN** [1921]. Wagenschein placed a cylinder of fluid on the end of a cantilever beam and suspended a sphere in the cylinder by means of a fine thread. The sphere motion and the cylinder motion then observed as the cantilever beam was oscillated. Eq. (15), as well as Wagenschein's results, are plotted on Figure 4.

The experimentally determined values of the virtual mass and damping coefficients can also be used to determine the numerical magnitude of the amplitude ratio. The amplitude ratio x_0/a_0, when computed from (14) with the experimental values of k and N_2 differs from the same ratio computed from (15) by less than one per cent. This surprising result indicates that the parameter $2x_0/d$ is of negligible importance in computing the value of the amplitude ratio x_0/a_0, even though it is apparent from Figures 2 and 3 that the parameter $2x_0/d$ is of major importance in the determination of either the damping parameter N_2 or the virtual mass coefficient k. The independence of the amplitude ratio x_0/a_0 and the parameter $2x_0/d$ is the consequence of the reduction of the virtual mass coefficient and of the simultaneous increase of the damping parameter N_2 as the parameter $2x_0/d$ increases. A simultaneous decrease of the virtual mass effective force $kM(\ddot{a} - \ddot{x})$ and increase of the shear force $A(\dot{a} - \dot{x})$ of (11) fortunately are combined such that the

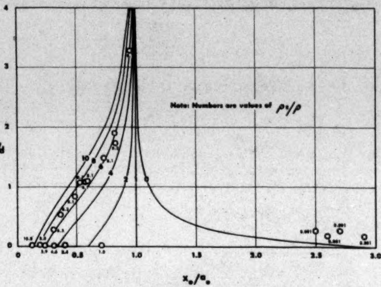

Fig. 4--Amplitude ratio x_0/a_0 as a function of the oscillation parameter N_1

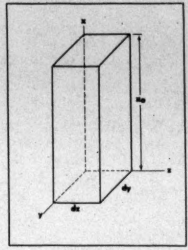

Fig. 5--Fluid element

amplitude ratio x_0/a_0 remains constant. This fact was also observed by WAGENSCHEIN [1921] as he varied the amplitude of the container oscillation with no apparent effect on the amplitude ratio x_0/a_0.

The diffusion of suspended sediment particles can be analyzed as a single spherical particle in an oscillating fluid if the concentration is not great enough to cause an appreciable interference of flow patterns past each individual particle. The steady-state suspended sediment diffusion process is characterized by the mathematical equation

$$wc = -\epsilon_s \, dc/dx \quad \text{.........................} \quad (16)$$

In the derivation of the magnitude of ϵ_s a fluid element of height x_0 and base area dydz is selected as shown on Figure 5. If the fluid motion is specified as

$$a = a_0 \sin \omega t$$

Then from (12) the foreign particle will have a motion described as

$$x = x_0 \sin (\omega t + \alpha)$$

If the concentration of sediment is c at the control surface dydz which is in the yz plane passing through the origin, the amount of sediment passing downward through the control surface in one-half cycle is

$$[c + (dc/dx)\, x_0/2]\, x_0 \, dydz$$

In the next one-half cycle the amount of sediment passing upward through the control surface is

$$[c - (dc/dx)\, x_0/2]\, x_0 \, dydz$$

If the steady-state condition is prevailing, the net amount of sediment transported upward in a cycle must be equal to the amount that settles downward through the control surface.

$$[c - (dc/dx)\, x_0/2]\, x_0 \, dydz - [c + (dc/dx)\, x_0/2]\, x_0 \, dydz = (2\pi/\omega)\, wc \, dydz$$

Simplifying

$$wc = -(x_0)^2 (\omega/2\pi)\, dc/dx \quad \text{......................} \quad (17)$$

It follows from (16) that

$$\epsilon_s = x_0^2 (\omega/2\pi) \quad \text{........................} \quad (18)$$

Since the fluid momentum is associated with the fluid particles, it follows from a similar analysis that

$$\epsilon_m = a_0^2 \, (\omega/2\pi) \quad \ldots\ldots \quad (19)$$

Combining (18) and (19) to obtain a ratio of diffusion coefficients, it follows that

$$\beta = \epsilon_s/\epsilon_m = (x_0/a_0)^2 \quad \ldots\ldots \quad (20)$$

As a result of this present work and WAGENSCHEIN's [1921] observations, (15) can be used to compute the amplitude ratio x_0/a_0 as a function of the oscillation parameter N_1 and the density ratio ρ_s/ρ. The value of β was computed from (15) for quartz sand in water and the function is presented on Figure 6.

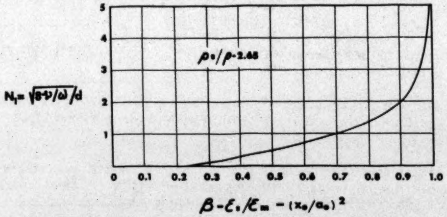

Fig. 6--Diffusion coefficient ratio β as a function of the oscillation parameter N_1 for quartz sand in water

Conclusions--Utilizing a combination of analysis and experimentation, the magnitudes of the inertial and shear forces on a sphere oscillating with simple harmonic motion were determined as functions of the oscillation parameter N_1 and the parameter $2x_0/d$. These results (Fig. 2 and 3) were applied to the case of a sphere in an oscillating fluid. The next step was an analysis of the diffusion coefficient of suspended sediment in water. Significant features of the general result are as follows:

(1) The mechanical system described in this paper is a practical means of determining both the inertia force and the shear force on a sphere oscillating in a fluid. These forces were determined throughout a wide range of maximum Reynolds number, that is, from 250 to 70,000.

(2) For simple harmonic motion of a sphere, the virtual mass coefficient is a function of both the oscillation parameter N_1 and the parameter $2x_0/d$. The virtual mass coefficient k tends to decrease as $2x_0/d$ is increased. At low values of $2x_0/d$ the magnitude of k tended to coincide with the analytical solution of Stokes [LAMB, 1945, pp. 642-644].

(3) In a similar manner it was shown that the value of damping parameter N_2 tended to increase with an increase in the value of $2x_0/d$. The experimental solution was in reasonable agreement with Stokes' solution at low magnitudes of $2x_0/d$.

(4) In the motion of a sphere in an oscillating fluid, the experimentally determined decrease in the virtual mass effective force was offset by the simultaneous increase in the shear force as the amplitude $2a_0$ of the fluid motion was increased. As a consequence, the ratio of the amplitude of sphere and fluid was independent of the value of $2a_0$ with the result that the ratio of the diffusion coefficient of suspended sediment and momentum is also independent of the amplitude of the fluid motion.

Acknowledgment--The material for the paper was taken from a graduate thesis by the author at the State University of Iowa. The experimental work and analysis were guided by J. S. McNown of the Iowa Institute of Hydraulic Research.

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(Manuscript received February 29, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 6, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

DISCRIMINATION OF THORON ALPHA ACTIVITY IN PRESENCE OF RADON

Patrick M. Hurley and Russell R. Shorey

Abstract--The short half-life of ThA is used as a means of discriminating between thoron and radon alpha activity. At low counting rate the count of double alpha pulses due to the rapid decay from Th to ThB indicates the ratio of radium to thorium X in the source solution.

Introduction--Measurements of the small quantities of thorium in most rocks and minerals can be made without difficulty by alpha-particle count on its decay product thoron that is streamed continuously through an ionization chamber from a solution containing its parent elements. The method has been described by URRY [1936] and GOODMAN and EVANS [1941]. In this case a vibrating-reed electrometer was used instead of an electrometer tube for amplifying the alpha pulses.

In this method, if the uranium and thorium content of the mineral sample are of the same order of magnitude, the fraction of counts that are due to radon is insignificant. If, however, the content of uranium in the sample is two orders of magnitude greater than the content of thorium, the radon contribution to the alpha count is large and the method cannot be used unless steps are taken to determine the radon contribution. A method is described below that makes possible a discrimination between alpha particles from radon and from thoron, so that the contribution due to radon can be obtained directly from the information derived during the single count.

The method is based on the short half-life of ThA. In the uranium and thorium series the rapid transition from thoron to thorium B is the only case of its kind in which a short-lived alpha emitter follows an alpha emitter, except for the pair actinon-actinium A. Thus, whenever a thoron atom disintegrates, it is followed almost immediately by a second alpha particle from thorium A (half-life = 0.16 sec). This pulse doublet is easily discernible if the counting rate is kept low by dilution of the source solution. The half-life of actinon (3.92 sec) is sufficiently short so that less than one atom in 10^6 reaches the ionization chamber in the type of streaming apparatus used, so that the contribution of An-AcA doublets is negligible.

Effect of radon--When the content of thorium and uranium is low, but still with a high U/Th ratio (for example, many marine shales), a good chemical separation of ThX and RdTh may be difficult owing to significant losses by adsorption at these low concentrations. Thus it is not always feasible to remove radium from the source solution. If radium and ThX are left in the solution, the effect of the radon on the thoron count will be approximately as follows.

Let

T_1 = mean time that thoron atoms remain in solution (and tube) before entering ionization chamber
T_2 = mean time that thoron atoms remain in chamber before exit from chamber
N_0 = number of emanation atoms produced per unit of time, in solution
N_1 = number of atoms per unit of time entering ionization chamber
N_2 = number of atoms per unit of time leaving ionization chamber
λ = fraction decaying per unit of time
C_{Tn} = alpha count/disintegrations of thoron per unit time (Th-ThA doublet counted as 1)
C_{Rn} = alpha count/disintegrations of radon per unit time (including contribution from Rn decay products)

$$N_{1(Tn)} = N_{0(Tn)}\, e^{-\lambda_{Tn}T_1} \qquad N_{2(Tn)} = N_{1(Tn)}\, e^{-\lambda_{Tn}T_2}$$

Alpha count due to thoron is

$$\left\{N_{0(Tn)}\, e^{-\lambda_{Tn}T_1} - N_{0(Tn)}\, e^{-\lambda_{Tn}T_1}\, e^{-\lambda_{Tn}T_2}\right\} C_{Tn}$$

The ratio of thoron to radon produced in the solution is

$$N_{0(Tn)}/N_{0(Rn)} = N_{ThX}\,\lambda_{ThX}/N_{Ra}\,\lambda_{Ra}$$

so that the ratio of thoron count to radon count is

$$(N_{ThX}\,\lambda_{ThX}/N_{Ra}\,\lambda_{Ra})\,e^{T_1(\lambda_{Rn}-\lambda_{Tn})}\left\{(1 - e^{-\lambda_{Tn}T_2})/(1 - e^{-\lambda_{Rn}T_2})\right\}C_{Tn}/C_{Rn}$$

If the nitrogen streaming rate is ϕ cm$^3\cdot$ sec^{-1} and the volume of the ionization chamber is V cm^3, the value of T_2 is approximately V/ϕ. C_{Rn} can be determined experimentally by counting a known amount of Rn from a stored solution, and T_1 from a deëmanated known radium solution in the bubbler flask. From these values C_{Tn} can be determined from a standard thorium solution.

In the apparatus used the plateau streaming rate was 2 cm^3/sec, V = 700 cm^3. If Tn - ThA doublets were counted as one pulse, and if the effect of ThB build-up was subtracted as background, the alpha count for thoron was 18 pct of the number of thoron atoms produced in the solution. It was thus found that for an equal content of uranium and thorium in the mineral sample, the ratio of counts of thoron to radon was 120. In a sample containing a ratio of Th/U = 0.1 an error of ten per cent would therefore result from not correcting for radon.

Discrimination of radon--Alpha pulses were amplified almost linearly by a vibrating-reed electrometer, and recorded on a moving chart. The time constant (2.1 sec) exceeded the average life of ThA (0.23 sec) so that the doublets were recorded as a single pulse with an amplitude considerably greater than any single alpha pulse. The alpha particles in the ionization chamber usually do not travel their full range without striking the chamber wall; and also the ThA ion is usually attracted to the collecting electrode before breaking down so that its alpha travels through gas only half the time. Furthermore, the random occurrence of pairs of unrelated alphas coinciding closely in time will give indistinguishable high peaks, and the frequency of these is proportional to the counting rate. The spectrum of pulse heights for any given counting rate therefore must be calibrated with known solutions.

Calibrations were made with solutions containing radium alone and thorium X alone, each diluted to give the same net count. The distribution of recorded peak heights for each case is shown in Figure 1. Unknown solutions must be diluted to give the same count so that the incidence of doublets from unrelated alphas remains the same as in the calibration tests. From Figure 1 it is seen that a threshold level of one unit (for these settings) eliminates all but three per cent of the radon alphas, but permits 25 pct of the thoron alphas to be counted. Counts at more than one threshold level are made to improve the discrimination. The absolute amount of thorium may thus be determined, with elimination of error due to radon.

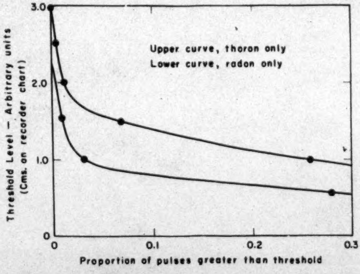

Fig. 1--Comparison of alpha pulses - thoron versus radon; fixed counting rate and amplification; time constant 2.1 sec

Acknowledgment--This investigation was part of a program of research on methods of geologic age determination supported by the Office of Naval Research in a laboratory equipped originally by a grant from the Geological Society of America.

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(Communicated manuscript received March 27, 1952;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

ACTIVITY OF PARICUTIN VOLCANO FROM JULY 1 TO DECEMBER 31, 1951

Carl Fries, Jr., and Celedonio Gutiérrez

(Publication authorized by the Director, U. S. Geological Survey

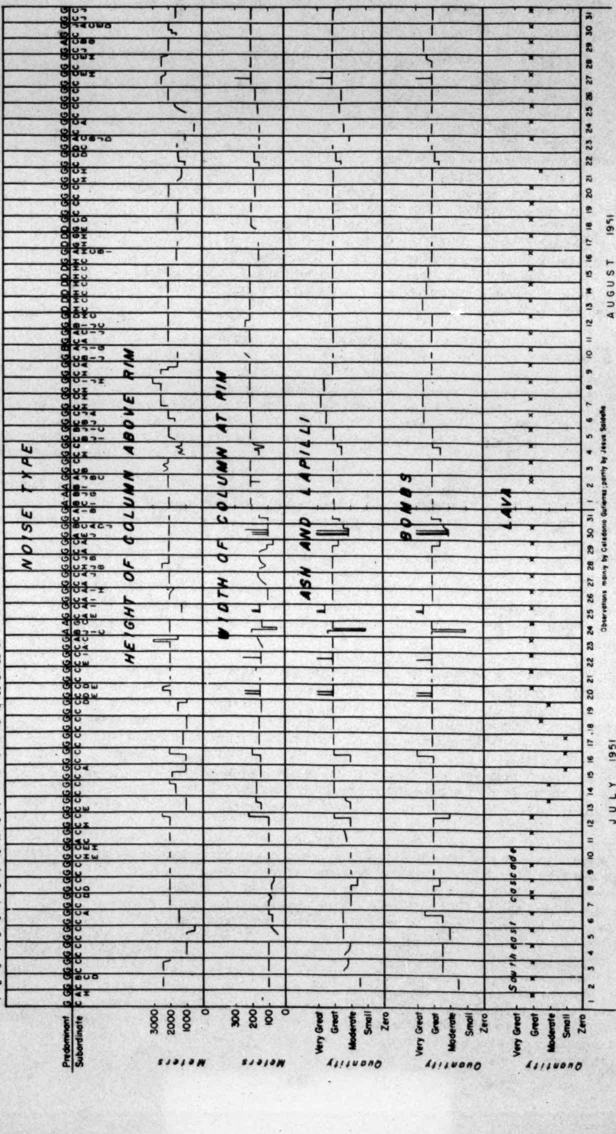
Continuous observation of activity at Parícutin Volcano during the second half of 1951 was carried on mainly by Celedonio Gutiérrez and partly by Jesús Saldaña. The senior author was at Parícutin July 10-12, August 24-27, November 14-15 and 19-22, and December 27-30, for a total of 17 days during the period. Observatory data of Gutiérrez and Saldaña are presented in the diagrams of eruptive characteristics in Figures 1-3, and their records of weather conditions, rainfall, and ashfall are summarized in Table 1. Gutiérrez also mapped the monthly advance of the lava during the period, as shown in Figure 6. The senior author prepared the summaries in Table 2 and measured the changes in horizontal distances and elevations of points on the cone, as given in Table 3, besides interpreting the data, formulating the report, and commenting on the progress of the activity.

In the second half of 1951 the volume of both pyroclastic material and new lava increased appreciably over that of the preceding six-month period, thus interrupting the continuous decline in activity noted over the years since the birth of the Volcano. The volumes of ash and lava, in fact, seem to have more or less recovered their general level of 1949. The period is notable also for the great increase in frequency and intensity of violent explosions or detonations in the crater. As many as 15 or 20 occurred on some days in late December, in contrast to about 45 explosions in the entire second half of 1950. Rainfall was greater than in any corresponding period since 1948.

Activity of the cone--Details of eruptive activity in the crater are presented in Figures 1-3. The lettered noise types on the diagrams in these Figures were described in an earlier report [FRIES and GUTIERREZ, 1951, p. 216]. Eruptions from the crater were probably stronger, on the average, than during any six-month period since the middle of 1949. The eruptive column reached a maximum height of about 3000 m in July and August, 2500 m in September and October, and 2800 m in November and December. The average height of the column was about 2000 m in July and August, 1500 m in September, 2000 m in October, and 1800 m in November and December. Electric discharges in the eruptive column during ash-laden eruptions were noted on five days in July, eight days each in August, September, and October, and one day in November; none was heard in December.

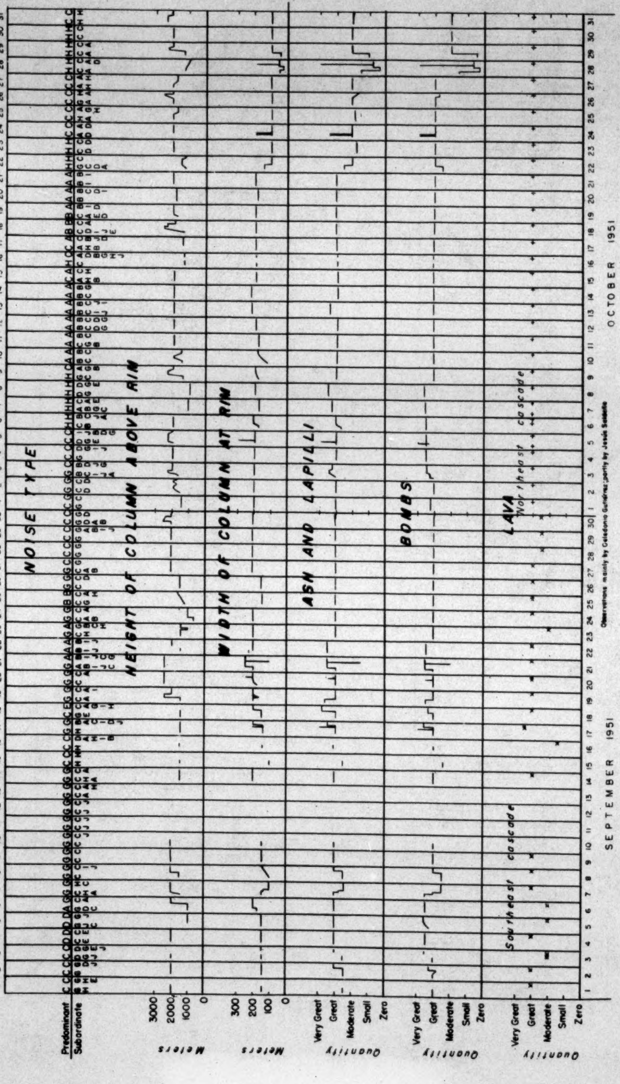
The explosive type of eruption described in some detail in the last activity report [FRIES and GUTIERREZ, 1952, p. 91] has continued to increase in frequency and intensity. These explosions are possibly better called detonations, for they are much like the detonation of a charge of dynamite. Some of them seem to occur down in the throat of the Volcano, and others seem to occur in the crater. They suggest the instantaneous rupture of a metal tank filled with gas under high pressure. The moderately strong explosions, such as shown in Figure 4, peppered the top and base of Nuevo Juatita lava-vent mound with rocks and bombs, and the most intense explosions or detonations hurled rocks as far away as the top of Canicjuata, nearly two kilometers from the crater. Even in the first year of the Volcano's history, rocks rarely reached the top of that hill.

The graphs from the continuously recording barograph in the observatory cabin at Cuezeño station have provided a rough measure of the frequency of these strong blasts. In the second half of 1950, for example, about 45 of them were recorded, of which three were particularly intense. In the first half of 1951 some 80 explosions were counted, of which five were extremely intense. During the second half of 1951, however, nearly 400 explosions were registered, and about 30 of these were particularly strong. In the latter period, the frequency of the explosions was greatest during August, September, October, and the last week of December. In the periods of greatest frequency, the shortest interval noted between explosions was about 15 minutes, although intervals of two to six hours were more common. The explosions were usually preceded by periods of weak gas and bomb eruptions with little ash, and were followed by periods of heavy ash eruptions. The experience of the writers leads them to believe that the explosions are caused by pyroclastic material falling back into the crater and clogging one of the throats. In December the explosions arose from the southwest crater throat, although some of the earlier ones came from the northeast crater throat.



Observations mainly by Celedonio Gutierrez; partly by Jesus Soldaña

Fig. 1--Eruptive characteristics of Parícutin Volcano from July 1 to August 31, 1951



Observations mainly by Celedonio Gutiérrez; partly by Jesús Sotelo

Fig. 2--Eruptive characteristics of Parícutin Volcano from September 1 to October 31, 1951

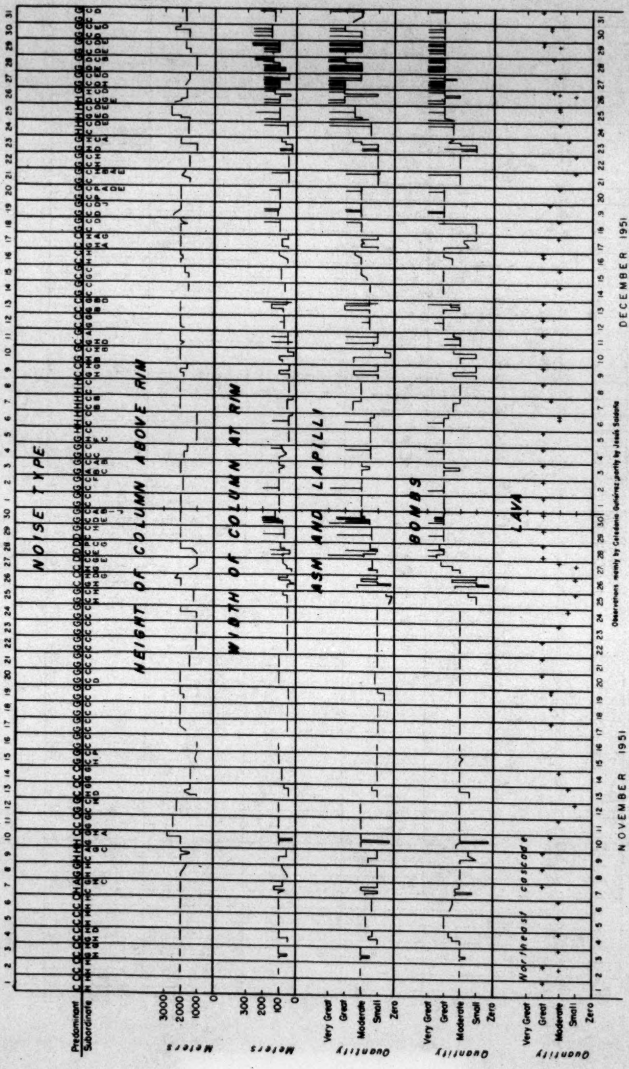


Fig. 3--Eruptive characteristics of Parícutin Volcano from November 1 to December 31, 1951

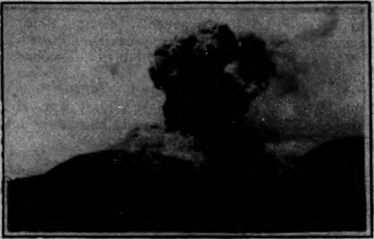


Fig. 4--Parícutin Volcano from Equijuata station on August 27, 1951; a moderately strong explosion peppers flanks of cone and lava-vent mound with rocks and bombs, each of which raises a tiny wisp of dust on hitting the ground

No. 1 eruptive vent, described in the last activity report in these Transactions [FRIES and GUTIERREZ, 1952, p. 91], had become inactive late in June, but on July 2 it again erupted gases intermittently and continued until about July 11, when it ceased to function and was quickly buried. No. 2 eruptive vent, also mentioned in that report, continued to be active and erupted more ash and bombs, on the whole, than the main crater vent. During the early part of the period, eruptions from No. 2 vent came every five to ten minutes; later they slowed down to one every five to 20 minutes, although on some days they were more frequent than that. No constant relation was noted between the eruptions from the main crater and from No. 2 vent, either in timing or intensity. No. 2 vent became the northeast crater throat before the end of the period, by coalescing with the main crater.

Ashfall in the second half of 1951 amounted to 4.7 mm at Cuezeño station and 9.9 mm at Equijuata station (see Table 1). This was a sharp increase over the ashfall at those stations in the same periods of 1950 and 1949, although it did not equal that of the second half of 1948. The increase was due to the activity of No. 2 eruptive vent (northeast crater vent), which supplied the major part of the ash erupted by the Volcano in the second half of 1951. This six-month period seems to be the first one in the history of Parícutin in which the ashfall has shown an appreciable

Table 1--Monthly summaries of weather observations, rainfall, and ashfall near Parícutin Volcano

Month	Cuezeño station							Average direction of upper winds	Equijuata station	
	Temperature		Rainfall		Ashfall		Minimum humidity		Rainfall (total)	Ashfall (total)
	Max.	Min.	Total	Max.	Total	Max.				
1951	°C	°C	mm	mm	mm	mm	pct		mm	mm
July	24.4	9.4	312.3	46.5	tr	tr	58	E	344.0	1.0
August	24.4	8.6	289.3	32.8	1.4	0.5	53	ESE	300.0	2.0
September	26.1	. . .	360.0	54.0	1.7	1.0	51	E	437.5	3.0
October	28.9	. . .	190.0	43.0	1.0	0.5	36	E	186.8	3.6
November	24.4	. . .	36.0	15.8	0.0	0.0	31	E and SW	36.7	tr
December	25.5	0.5	6.2	3.0	0.6	0.2	36	SE and SW	16.5	0.3

Table 2--Rainfall and ashfall at Cuezeño and Equijuata stations for the yearly periods from 1947 to 1951

Period	Cuezeño station		Equijuata station	
	Rainfall (total)	Ashfall (total)	Rainfall (total)	Ashfall (total)
	mm	mm	mm	mm
January-December 1947	1480.5[a]	43.9[a]	2411.4[b]	129.2[b,c]
January-December 1948	1710.8	12.3	1893.2[d]	45.6[c,d]
January-December 1949	1355.2	8.7	1593.9	16.8
January-December 1950	1315.0	7.2	1761.5	11.6
January-December 1951	1510.6	5.8	1600.8	15.1

[a] Corrected over earlier published figures, on basis of original records of Wilcox.

[b] Six months at Jarátiro and six months at Equijuata; probably greater than rainfall at Equijuata for full year.

[c] Jarátiro readings were reduced to agree with the proportion shown on the ash isopach map of 1946 [SEGERSTROM, 1950, pl. 1], or 37.5 pct of Jarátiro readings.

[d] Three months at Jarátiro and nine months at Equijuata.

increase over that of a corresponding six-month period of an earlier year, and it is apparently the first time that the continuous decline in activity has been interrupted. In spite of the increased ashfall in the second half of the year, the total ash recorded at Cuezeño station for the year 1951 was less than the total for 1950 (see Table 2), owing to the very slight ashfall in the first six months of 1951, although at Equijuata station the total for the year was greater than in 1950.

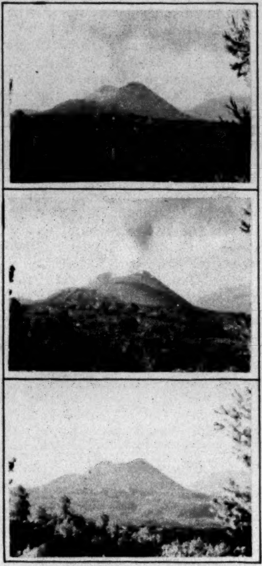


Fig. 5--Parícutin Volcano from Cuezeño station, showing changes in crater rim during second half of 1951; upper picture of July 10 shows low northeast saddle and high, rounded west (right) peak; middle picture of August 27 shows higher saddle and sharper west peak; lower picture of November 20 shows even higher saddle and lower, sharper west peak; note increase in height of left top of lava-vent mound

Form of the cone and crater--At the beginning of the period, No. 2 eruptive vent was separated from the main crater by a steep, firmly cemented pyroclastic ridge, and on July 12 its northeast rim was only 42 m higher than the top of Nuevo Juatita lava-vent mound. The upper view in Figure 5 shows the form of the cone on July 10. No. 2 vent continued to migrate southwestward and to break down the pyroclastic ridge separating it from the main crater, until by August 27 it had coalesced with the main crater (see middle view in Fig. 5). It then occupied the position of the former northeast throat of the crater, before that throat was buried in 1950. By the end of December (see lower view in Fig. 5) the northeast flank of the cone had become rounded out by pyroclastic material from the crater, and the lowest point on the northeast rim of the crater had risen 34 m. The flank of the cone had encroached on the lava-vent mound and had covered the depression between the cone and the mound.

Changes in the position of the crater and the two highest peaks necessitated re-triangulation in order to determine their horizontal distance from Cuezeño station, from which elevation changes are determined. The results of these measurements are given in Table 3, together with the changes in elevation of the points sighted. The crater side of the west peak was gradually eaten away by slumping and explosions (see Fig. 5), and the peak consequently dropped about eight meters in elevation during the period. The east peak rose about seven meters in altitude by accumulation of pyroclastic material, and at the end of the period it was about three meters higher than the west peak. On December 29 the crater rim had a diameter of roughly 280 m from northwest to southeast, and 260 m from northeast to southwest. The latter dimension is more than twice that of March 1951.

Table 3--Horizontal distances[a] from Cuezeño station, and elevations[b] above sea level, of points on Parícutin Volcano

Date	West peak		Lowest point on northeast rim		East peak		Lava-vent mound	
	Dist.	El.	Dist.	El.	Dist.	El.	Dist.	El.
1951	m	m	m	m	m	m	m	m
July 12	5200	2810.1	5110	2717.8	5310	2798.5	4970	2675.7
August 25	5195	2812.5	5135	2735.1	5325	2801.6	4980	2676.6
November 14	5190	2805.5	5170	2753.2	5345	2803.3	4990	2679.0
December 29	5185	2802.3	5195	2751.5	5360	2805.3	5000	2688.6

[a] Estimated accuracy of horizontal distances is ± 10 m.

[b] Elevations determined by alidade from Cuezeño station; estimated accuracy of ± 2 m.

Owing to the highly erratic occurrence of strong explosions, an attempt to climb the cone was not considered advisable during this period. It was felt that anyone caught on top of the cone at the time of such an explosion would have a slim chance of surviving, and the odds of being caught were not in one's favor during most of the period. There were intervals of a week or two at a time, of course, when the danger was not very great, but a method of predicting behavior has not yet been devised. Notwithstanding, it was learned early in January that Carlos García Gutiérrez, of the Mexican Institute of Mineral Resources, and Eduardo Châvez, of the Tepalcatepec Commission, had made the ascent from the east on December 16. These men were not aware of the frequency and intensity of the strong explosions, and the barograph record shows that none had occurred for several days before their ascent. They report that the crater had two main throats aligned northeast-southwest, with a low ridge between. The northeast throat was erupting ash and bombs almost continuously, but the southwest throat was filled and showed only gas eruptions through small holes that pushed their way up through somewhat plastic material on one side of the rounded, filled bottom. This is the throat that was noted from the observatory cabin to produce the intense explosions and to expel only gases otherwise. The condition reported by García and Châvez accords with the writer's opinion that the explosions are due to the stopping up of a throat. Unless conditions should change, the writers recommend that visitors do not ascend the cone without first asking for advice at the observatory cabin.

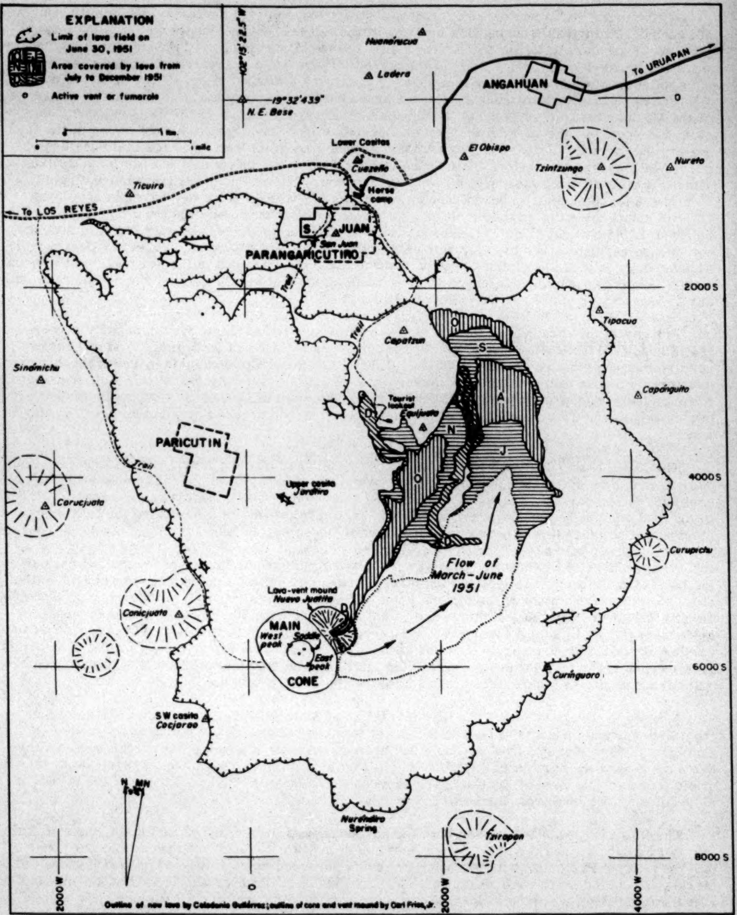
Activity of the lava vents--No. 1 eruptive vent, described in the Parícutin activity report [FRIES and GUTIERREZ, 1952, p. 91], had supplied a small flow of lava in May and had become nearly inactive by the end of June. On July 2, however, gas eruptions again issued from this vent, at intervals of five to ten minutes, and pasty lava was lifted several meters with each eruption. Activity decreased gradually in the following week, but a small flow of lava issued from the vent during July 9-11, after which the vent became stopped up and covered by pyroclastics from No. 2 vent.

The main, Nuevo Juatita lava vent at the northeast base of the Volcano supplied lava continuously through the six-month period. The cascade to the southeast, which had formed in March, continued to operate until the end of September, when an overflow caused a new cascade to descend to the east and northeast. Several overflows occurred from time to time during the following months, but no other new cascade was formed until December 30, when a second northeast cascade began to descend just west of the October-December cascade (see Fig. 6). The pyroclastic material from the crater gradually covered the southwest side of the lava-vent mound and caused the point of lava issue to shift southeastward. Periodic overflows then built up the mound on that side, so that on December 30 the highest point of the mound was about 30 m farther from Cuezeño station than at the beginning of the period (see Table 3 and Fig. 5). The height of the mound increased nearly 13 m during the period. Gas eruptions through this vent were not observed as in earlier periods, although very minor gas escapes could well have missed observation. The bright glow caused by the lava issuing from the vent at night suggested that the flow was somewhat more voluminous during this period than it had been for the past 12 months.

Activity of the lava flows--The flow that began in March had reached a point about 4.5 km from the vent when it came to rest at the end of October because of beheading at its source on October 1. The tip of this flow was only 260 m from the point where the horse trail crosses the San Juan lava at the north end of Capatzun (see Fig. 6). This new flow covered three small islands of old terrain to the east of Equijuata and encroached on the east base of Capatzun; the rest of it moved over older lava from Parícutin.

The flow that began on October 1 moved northeastward to the base of Equijuata, where it split into two tongues around this hill. In the first month the flow had advanced two kilometers over older lava from Parícutin, but thereafter its fronts advanced more slowly. The lava invaded old terrain around the south base of Equijuata and around the knoll on which the tourist lookout is located, cutting the trail to the lookout at one point and affording the tourists a close view of the moving front. At the end of December this flow had three tongues, one to the east of Equijuata, another to the west of Equijuata, and a third in the direction of Curupichu to the east, where it overrode the 1951 lava. It was not certain whether the new cascade that formed on December 30 would behead this flow or whether it would unite with the flow.

The area covered by new lava during the second half of 1951 was about 2,820,000 sq m and was larger than that of any six-month period since the second half of 1948. As the flows were relatively narrow and moved over inclined ground, the average thickness of the lava was not very great and is believed to have been about seven meters. On this assumption, then, the volume extruded in the second half of 1951 was about 20,000,000 cu m, or an average daily volume of 108,000 cu m. This represents an increase of about 9000 cu m a day over the first half of 1951


EXPLANATION
Limit of lava field on June 30, 1951
Area covered by lava from July to December 1951
Active vent or fumarole
0 1 Km.
0 1 mile
102°15'22.5" W
19°32'43.9"
N. E. Base
Huanárucua
Ladera
ANGAHUAN
To URUAPAN
Lower Casitas
Cuezeño
Horse camp
El Obispo
Tzintzungo
Nureto
Ticuiro
To LOS REYES
S. JUAN
San Juan
PARANGARICUTIRO
Trail
2000 S
Tipacua
Capatzun
O
S
A
N
J
Sinánichu
Tourist lookout
Equijuata
Capánguito
PARICUTIN
4000 S
Upper casita
Jardino
Carucjuata
Curupichu
Flow of March-June 1951
Lava-vent mound
Nuevo Juatita
MAIN
West peak
Saddle
East peak
CONE
Canicjuata
6000 S
Curíngueoro
SW casita
Cocjarao
N MN
Nuréndiro Spring
Tzirapan
8000 S
2000 W
0
2000 W
4000 W

Outline of new lava by Celedonio Gutiérrez; outline of cone and vent mound by Carl Fries, Jr.

Fig. 6--Areas covered by Parícutin lava flows from July 1 to December 31, 1951

and 16,000 cu m a day over the second half of 1950 [see FRIES and GUTIERREZ, 1952, Table 4]. The increased lava volume accords with observations of increased flow at the lava vent, and it is also in accord with the increased activity of the eruptive vents, as indicated by the greater ash-fall. Taking into consideration these two factors, the writers feel justified in concluding that the continuous decline in the activity of Parícutin, witnessed up to the middle of 1951, has definitely been interrupted. Whether or not this interruption represents a final spurt in activity before the death of the Volcano remains to be seen.

Weather data--Rainfall in the second half of 1951 was greater at both Cuezeño and Equijuata stations than in any corresponding period since 1948 (see Tables 1 and 2). This is interesting in view of the fact that the first half of 1951 was drier than any corresponding period since 1948. The total rainfall for 1951 at Cuezeño station was also the greatest since the year 1948, although the total at Equijuata station was less than in 1950 and about the same as in 1949. Whether the increased precipitation at Cuezeño might have been caused by the increased eruptive activity in the second half of 1951 is highly doubtful, although the records do not yet extend over a sufficiently long period of time to make this point certain. The 1947 rainfall at Cuezeño was less than that in 1951, although eruptive activity was much stronger in 1947; but the 1951 rainfall at Equijuata was much less than the 1947 rainfall at Equijuata-Jarátiro. Rainfall at Equijuata station during the second half of 1951 was again greater than that at Cuezeño station. On September 12 and 13 a drizzly rain combined with ash from the Volcano's plume to weight down and break branches of trees in the area north of the cone.

Acknowledgments--Observations are being carried on cooperatively by the United States Geological Survey, under the auspices of the Technical Cooperation Administration of the Department of State, and the Instituto Nacional de Investigación Científica of the Mexican Government, and expenses are shared by these two organizations. The writers express their appreciation to José Lauro Ramírez for aid in drafting the illustrations and to Ray E. Wilcox for a critical review of the manuscript.

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(Communicated manuscript received April 17, 1952;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union   Volume 33, Number 5   October 1952

APPARENT DIP - TRUE DIP FORMULA USING A UNIT HEMISPHERE

V. G. Gabriel and Robert J. Miller

Abstract--A hemisphere of unit radius is used in graphical and mathematical derivation by four somewhat different methods of the formula $\tan \psi = \tan \alpha \cos \theta$, where ψ is the angle of an apparent dip, α is the angle of a true dip, and θ is the angle between the direction of an apparent dip and a true dip. Also, some other concepts are derived with the application of the same formula.

Introduction--Several different methods are usually employed for the derivation of the apparent dip - true dip formula which is widely used in structural geology and applied geophysics. However, it is believed that these methods lack clarity necessary for forceful presentation. The introduction of an auxiliary unit hemisphere clarifies and increases considerably the number of methods from which a certain method may be selected for the presentation of the formula.

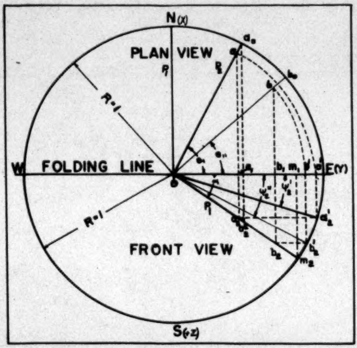

Fig. 1--Plan and front views of a unit hemisphere with the traces of planes P_1 and P_2

Graphical method--Assume a hemisphere of unit radius R with its center located at point O. By taking the folding line as the line passing through the center O of the hemisphere and thus dividing this hemisphere into two equal parts, we can obtain the plan view and front view of the half of the hemisphere. The plan view and front view projections of the portion of the hemisphere will be two semicircles; OENW and OESW, respectively (see Fig. 1).

Assume, also, that a formation, or a plane P_1, whose angle of true dip is equal α degrees, has its true strike coinciding with the line ON. In the front view projection, the intersection of the plane P_1 with the unit hemisphere will coincide with the line Om_2 where the angle EOm_2 is the angle of a true dip, that is, equal to α degrees; the direction of the true dip on plan view will be given by the line OE. Let us pass through point O a number of vertical planes, such as Oa_0 and Ob_0, perpendicular to the plan view, which would intersect both the surface of the unit hemisphere and plane P_1, let us say, at the points a, b, . . . The front view projections of the points a, b, will be located on the line Om_2 and will be a_2 and b_2 respectively; however, the plan projection of point m_2 will be m_1. Let us rotate the lines Oa and Ob into the line OE. The points a and b will coincide with the points a', b' of the line OE respectively. The front view projections of these rotated points will be a_2' and b_2' respectively. By joining the points a_2' and b_2' located on the front view with point O, we obtain the angle EOa_2' and EOb_2' where

$\psi_2' = EOa_2'$ = Angle of apparent dip in Oa_0 direction

$\psi_2'' = EOb_2'$ = Angle of apparent dip in Ob_0 direction

$\theta_1 = a_0OE$ = Angle of the direction of an apparent dip from the true dip

$\theta_{11} = b_0OE$ = Angle of the direction of an apparent dip from the true dip

From the right-angle triangles Oaa_1 and Oa_1a_2 we have

$$\left.\begin{aligned} Oa_1 &= Oa \cos \theta_1 \\ Oa_1 &= Oa_2 \cos \alpha \end{aligned}\right\} \quad \text{(1)}$$

or from (1)

$$Oa \cos \theta_1 = Oa_2 \cos \alpha \quad \text{(2)}$$

But from the triangle $Oa'\,a_2'$ we have

$$Oa = Oa' = Oa_2' \cos \psi_2' = \cos \psi_2' \quad \text{(3)}$$

because Oa_2' is a radius of the unit sphere. By substituting (3) into (2) we obtain

$$\cos \psi_2' \cos \theta_1 = Oa_2 \cos \alpha \quad \text{(4)}$$

From the same triangles Oa_1a_2 and $Oa'\,a_2'$

$$a_1a_2 = Oa_2 \sin \alpha = a'\,a_2' = \sin \psi_2' \quad \text{(5)}$$

Because the points a_2 and a_2' are the front view projections of the point a and its rotated position a'. From (5) we have

$$Oa_2 \sin \alpha = \sin \psi_2' \quad \text{(6)}$$

$$Oa_2 = \sin \psi_2' / \sin \alpha \quad \text{(7)}$$

By substituting (7) into (4) and dropping the subscripts we have

$$\cos \psi \cos \theta = (\sin \psi / \sin \alpha) \cos \alpha \quad \text{(8)}$$

By rearranging (8)

$$(\sin \alpha / \cos \alpha) \cos \theta = \sin \psi / \cos \psi \quad \text{(9)}$$

or

$$\tan \psi = \tan \alpha \cos \theta \quad \text{(10)}$$

Second graphic method--The lines $a'\,a_2'$ and a_1a_2 of the front view projection (Fig. 1) are the Z coordinates of point a located at the intersection of the unit sphere, the dipping plane P_1, and the vertical plane P_2 passing through the line Oa_0, that is, a line of the apparent dip of ψ_2' degrees. Therefore

$$a_1a_2 = a'\,a_2' \quad \text{(11)}$$

But from the right-angle triangles Oa_1a_2 and $Oa'\,a_2'$ we have

$$Oa' = Oa_2' \cos \psi_2' = \cos \psi_2' \quad \text{(12a)}$$

$$a'\,a_2' = Oa_2' \sin \psi_2' = \sin \psi_2' \quad \text{(12b)}$$

$$a_1a_2 = Oa_1 \tan \alpha \quad \text{(12c)}$$

because Oa_2' is a radius of the unit hemisphere. However, from the right-angle triangle Oaa_1 of the plan view, we have

$$Oa_1 = Oa \cos \theta_1 = \cos \psi_2' \cos \theta_1 \quad \text{(13)}$$

because Oa is equal Oa' by rotation and Oa' is equal $\cos \psi_2'$ (from triangle $Oa'\,a_2^{2}$). By substituting (13) into (12c) we obtain

$$a_1a_2 = Oa_1 \tan \alpha = \cos \psi_2' \cos \theta_1 \tan \alpha \quad \text{(14)}$$

By substituting (14) and (12b) into (11) and dropping the subscripts

$$a_1a_2 = \cos\psi \cos\theta \tan\alpha = a_1a_2' = \sin\psi \quad \ldots\ldots\ldots\ldots (15)$$

or from (15)

$$\cos\theta \tan\alpha = \sin\psi/\cos\psi = \tan\psi \quad \ldots\ldots\ldots\ldots (16)$$

Mathematical method--Assume a hemisphere of radius R equals one, a dipping formation or a plane P_1, with angle of dip equals α degrees, and a vertical plane P_2 passing through the line Oaa_o (Fig. 1) where the lines OE and Oaa_o are the lines of the true dip and apparent dip respectively. The line ON is assumed to be a line of the strike of plane P_1.

The equation of plane P_1 is

$$A_1x + B_1y + C_1z = 0 \quad \ldots\ldots\ldots\ldots (17)$$

and the equation of plane P_2 is

$$A_2x + B_2y + C_2z = 0 \quad \ldots\ldots\ldots\ldots (18)$$

because both the planes P_1 and P_2 are passing through a point O (o, o, o).

Planes P_1 and P_2 intersect each other along a line whose plan and front projections are the lines Oa_o and Oa_2^2 respectively. However, this intersecting line should pierce the unit hemisphere at a point, assumed to be a. Therefore, the coordinates of point a located on the intersection of the unit hemisphere, plane P_1, and plane P_2 are

$$\left.\begin{aligned} X_a &= aa_1 = Oa \sin\theta_1 = \cos\psi_2' \sin\theta_1 \\ Y_a &= Oa_1 = Oa \cos\theta_1 = \cos\psi_2' \cos\theta_1 \\ Z_a &= a_1a_2 = Oa_1 \tan\alpha = a'\,a_2' = Oa_2' \sin\psi_2' = \sin\psi_2' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (19)$$

However, plane P_1 is also passing through point N whose coordinates are

$$\left.\begin{aligned} X_n &= 1 \\ Y_n &= 0 \\ Z_n &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (20)$$

and point m_2 whose coordinates are

$$\left.\begin{aligned} X_m &= 0 \\ Y_m &= Om_1 = \cos\alpha \\ Z_m &= m_1m_2 = \sin\alpha \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (21)$$

By substituting the coordinates of points N, a, and m_2, located on plane P_1 into (17), we obtain

$$A_1 + 0 + 0 = 0 \quad \ldots\ldots\ldots\ldots (22a)$$

$$0 + B_1 \cos\alpha + C_1 \sin\alpha = 0 \quad \ldots\ldots\ldots\ldots (22b)$$

$$A_1 \cos\psi_2' \sin\theta_1 + B_1 \cos\psi_2' \cos\theta_1 + C_1 \sin\psi_2' = 0 \quad \ldots\ldots\ldots\ldots (22c)$$

or from (22)

$$A_1 = 0 \quad \ldots\ldots\ldots\ldots (23a)$$

$$-B_1/C_1 = \tan\alpha \quad \ldots\ldots\ldots\ldots (23b)$$

$$-B_1/C_1 = \sin\psi_2'/\cos\psi_2'\cos\theta_1 = \tan\psi_2'/\cos\theta_1 \tag{23c}$$

Therefore from (23)

$$-B_1/C_1 = \tan\alpha = \tan\psi_2'/\cos\theta_1 \tag{24}$$

From (24) and dropping the subscripts

$$\tan\psi = \tan\alpha\cos\theta \tag{25}$$

Second mathematical method--The equation of a unit sphere is

$$X^2 + Y^2 + Z^2 = 1 \tag{26}$$

The point a (Fig. 1) has the following coordinates

$$\left.\begin{aligned} X &= aa_1 = Oa\sin\theta_1 = \cos\psi_2'\sin\theta_1 \\ Y &= Oa_1 = Oa\cos\theta_1 = \cos\psi_2'\cos\theta_1 \\ Z &= a_1a_2 = Oa_1\tan\alpha = \cos\psi_2'\cos\theta_1\tan\alpha \end{aligned}\right\} \tag{27}$$

because $Oa_1 = \cos\psi_2'\cos\theta_1$. By substituting the coordinates of point a into (26) and dropping the subscripts we have

$$(\cos\psi\sin\theta)^2 + (\cos\psi\cos\theta)^2 + (\cos\psi\cos\theta\tan\alpha)^2 = 1 \tag{28}$$

or from (28)

$$\begin{gathered} \cos^2\psi\sin^2\theta + \cos^2\psi\cos^2\theta + \cos^2\psi\cos^2\theta\tan^2\alpha = \cos^2\psi \\ (\sin^2\theta + \cos^2\theta + \cos^2\theta\tan^2\alpha) = \cos^2\psi(1 + \cos^2\theta\tan^2\alpha) = 1 \end{gathered} \tag{29}$$

By substituting into (29) $(1 - \sin^2\psi)$ instead of $\cos^2\psi$ and by developing, we obtain

$$(1 - \sin^2\psi)(1 + \cos^2\theta\tan^2\alpha) = 1 + \cos^2\theta\tan^2\alpha - \sin^2\psi - \sin^2\psi\cos^2\theta\tan\alpha = 1 \tag{30}$$

or from (30), by transposing

$$1 - 1 + \cos^2\theta\tan^2\alpha - \sin^2\psi - \sin^2\psi\cos^2\theta\tan^2\alpha = 0 \tag{31}$$

or from (31)

$$\cos^2\theta\tan^2\alpha - \sin^2\psi\cos^2\theta\tan^2\alpha = \sin^2\psi \tag{32}$$

or from (32)

$$\cos^2\theta\tan^2\alpha(1 - \sin^2\psi) = \cos^2\theta\tan^2\alpha\cos^2\psi = \sin^2\psi \tag{33}$$

and by taking square roots from both sides of (33) we have

$$\cos\theta\tan\alpha = \sin\psi/\cos\psi = \tan\psi \tag{34}$$

Field vector solutions--Equation (34) or its derived form are often used for graphical evaluation of the true dip and true strike of a formation during the field work. To derive a new form, (34) can be expressed as

$$\sin\psi/\cos\psi = \sin\alpha/\cos\alpha\cos\theta \tag{35}$$

or

$$\frac{\text{Line of sine of apparent dip}}{\text{Line of cosine of apparent dip}} = \frac{\text{Line of sine of true dip}}{\text{Line of cosine of true dip}} \times \cos\theta \tag{36}$$

By taking the lines of cosines, that is, the horizontal distances for both the apparent and true dips as equal, let us say, 1000 ft and substituting this assumed value into (36) we obtain

$$\text{Line of sine of apparent dip}/1000 = (\text{Line of sine of true dip}/1000) \times \cos\theta \quad \ldots\ldots (37)$$

or from (37)

$$\text{Line of sine of apparent dip in feet} = (\text{Line of sine of true dip} \times \cos\theta) \text{ in feet} \quad \ldots\ldots (38)$$

or from (38) and (34)

$$\left.\begin{array}{l} A_1 = B\cos\theta_1 \\ A_2 = B\cos\theta_2 \\ \ldots\ldots\ldots \\ A_n = B\cos\theta_n \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39a)$$

$$\left.\begin{array}{l} C_1 = D\cos\theta_1 \\ C_2 = D\cos\theta_2 \\ \ldots\ldots\ldots \\ C_n = D\cos\theta_n \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39b)$$

where

A_1 = Line of sine of apparent dip of ψ_1 degrees, in feet
A_2 = Line of sine of apparent dip of ψ_2 degrees, in feet
. . . .
A_n = Line of sine of apparent dip of ψ_n degrees, in feet
B = Line of sine of true dip of α degrees, in feet
$C_1 = \tan\psi_1$
$C_2 = \tan\psi_2$
$C_n = \tan\psi_n$
$D = \tan\alpha$
$\psi_1, \psi_2, \psi_3, \ldots \psi_n$ = apparent dips
α = true dip
$\theta_1, \theta_2, \ldots \theta_n$ = Angles between apparent dips and the true dip

However, the set of equations given in (39a) are simultaneously satisfied if, by scaling off A_1, $A_2, \ldots A_n$ along their respective directions of apparent dips radiating from a common point O and drawing the perpendiculars at these end points, these perpendiculars will intersect at a common point, let us say, K. Line OK will represent the direction and the magnitude of the true dip expressed in feet per one thousand feet.

The same method of reasoning holds for a set of equations in (39b) where by scaling off C_1, $C_2, \ldots C_n$ along their respective directions of apparent dips and drawing the perpendicular lines from the end points until a new point of intersection is found, let us say, M, a line OM is obtained giving the direction and magnitude of the true dip. In the latter case, the magnitude will be expressed as the tangent of the true dip.

Acknowledgments--The authors wish to express their thanks to A. C. Spreng and W. R. Higgs, both of the Missouri School of Mines and Metallurgy, University of Missouri, Rolla, Missouri, for their helpful suggestions.

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(Communicated manuscript received October 29, 1951;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

THE STRUCTURE OF COLOMBIA

Victor Oppenheim

Abstract--The geology of Colombia involves rock formations of Precambrian to Quaternary ages and their structure reflects all the complex evolutionary stages undergone in their development from the basement Guiana Shield in the east to the Tertiary Andean Cordilleras in the west.

The Colombian Andes proper consist of five distinct and physiographically independent mountain ranges: the Coastal Range, Cordillera Occidental, Cordillera Central, Cordillera Oriental with its continuation, Cordillera de Perija, and the Sierra Nevada de Santa Marta. Each of these elements shows marked structural characteristics and is of different age of uplift. The Guiana Shield and its outliers in the eastern half of the country represent the old continental basement complex. The growth of the rest of Colombia took place west and north of the shield. The largest part of the shield area is covered with thin beds of Tertiary sediments and comparatively little is known of its structural development.

The data available on the Colombian Andes permit an outline of the successive evolution and the structural relations of its component elements. Thus, it can be established that, as in the structure of the Venezuelan Andes, Sierra de Merida, to the northeast [OPPENHEIM, 1937] and the Ecuadorian Andes to the south [OPPENHEIM, 1950b], the predominating tectonic pattern of the Colombian Andes is that of block-faulting and not of overthrusting. This distinctly Andean type of structure is further enhanced in Colombia by the dual constitution of the Colombian Cordilleras -- the Western and Central Cordilleras being of predominantly igneous and metamorphic composition, while the eastern Cordillera Oriental is largely of sedimentary constitution [OPPENHEIM, 1947].

The rise of the Cordilleras does not seem to be due to direct tangential pressures from east to west but, rather, to vertical stresses, or their components. The graben structure of the Magdalena and Cauca troughs and the absence of important regional zones of low-angle thrusting or nappe structures in the Colombian Andes confirm this view.

A hypothetic evolution of the Colombian Andes from island arcs appears suggestive and deserves further study, as it seems to confirm the existing geological and geophysical connections of the Cordilleras with island arcs of the Lesser Antilles, a relation that has been pointed out by HESS [1938], OPPENHEIM [1948], and GUTENBERG and RICHTER [1949].

Introduction

The purpose of the present study is to outline the structural framework of Colombia. Situated in the northwestern corner of the continent, facing the Atlantic as well as the Pacific Ocean, and linked to Central America through Panama, Colombia is of particular significance in the interpretation of the geological evolution of this part of South America.

Several theories have been advanced in the past, interpreting differently the evolution of the Colombian Andes. The many gaps in our knowledge of Colombian geology will, inevitably, make all interpretations tentative. However, some of these may approach the true solution. The present interpretation, which is based on many geological facts or direct field observations, appears to the writer to be the most plausible one.

Besides the available bibliographic material perused, geological observations were carried out by the writer in Colombia over a period of several years. The southeastern part of the country was reconnoitered in 1938 while doing geological work for the Anglo-Saxon Petroleum Company. In 1940 and 1941, acting as Chief of a Geological Commission for the Colombian Government, field studies were made in the Sierra Nevada de Santa Marta, Cordillera de Perija, and the northern part of the country, as well as in the Cordillera Oriental and the Llanos. Cross sections were made of the Cordillera Central and in the Magdalena and Cauca valleys. In 1942, as consulting geologist to the Socony-Vacuum Oil Company, the writer completed the first geological study of the Pacific Coast (Choco) of Colombia, which also included cross sections of the Cordillera Occidental. These field surveys offered basic material for the writer's First Generalized Geologic Map of Colombia which was published by the Ministry of Education and the Colombian Academy of Science in 1943. In the past eight years, as well, the writer has made numerous reconnaissances in different parts of the country which are also being considered in the present study.

Besides the references given at the end of this paper, additional sources of geologic information on Colombia are given by ANDERSON [1945]. Stratigraphy and detailed description of rock formations are referred to in the bibliographies -- only the main structural forms, their interrelation and interpretation, are being considered here.

Tectonic regions of Colombia

The main tectonic provinces of Colombia correspond broadly to the distinct physiographic divisions of the country (Fig. 1) and will be outlined here, with their secondary components, from west to east: The Coastal Range followed by the Atrato - San Juan trough; the Cordillera Occidental followed by the Cauca - Patia trough; the Cordillera Central followed by the Magdalena trough; the Caribbean Coastal Basin; the Sierra Nevada de Santa Marta followed by the Cesar de-

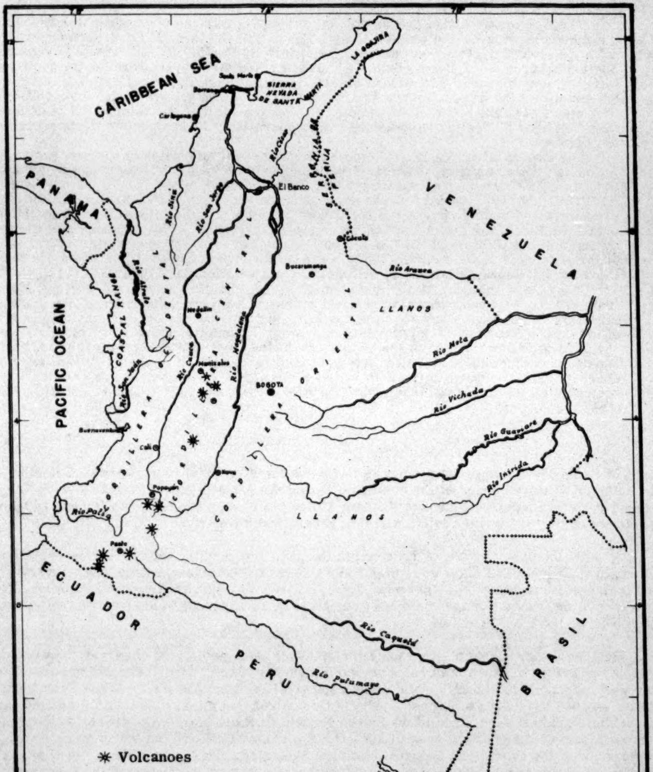
CARIBBEAN SEA
LA GOAJIRA
Santa Marta
Barranquilla
SIERRA NEVADA DE SANTA MARTA
Cartagena
Rio Cesar
SERRANIA DE PERIJA
PANAMA
El Banco
VENEZUELA
Rio Sinú
Rio San Jorge
Cúcuta
Bucaramanga
Rio Arauca
Rio Atrato
COASTAL RANGE
CORDILLERA OCCIDENTAL
CORDILLERA CENTRAL
CORDILLERA ORIENTAL
LLANOS
PACIFIC OCEAN
Medellín
Manizales
Rio Cauca
Rio Magdalena
Rio Meta
Rio San Juan
BOGOTA
Rio Vichada
Buenaventura
Cali
Rio Guaviare
Neiva
Rio Inírida
Popayán
Rio Patía
LLANOS
Pasto
ECUADOR
Rio Caquetá
BRASIL
PERU
Rio Putumayo
RIO AMAZONAS
✳ Volcanoes

Fig. 1--Index map of Colombia

pression; the Cordillera Oriental and the Serrania de Perija; and, to the east of the Andes, the Llanos Basin.

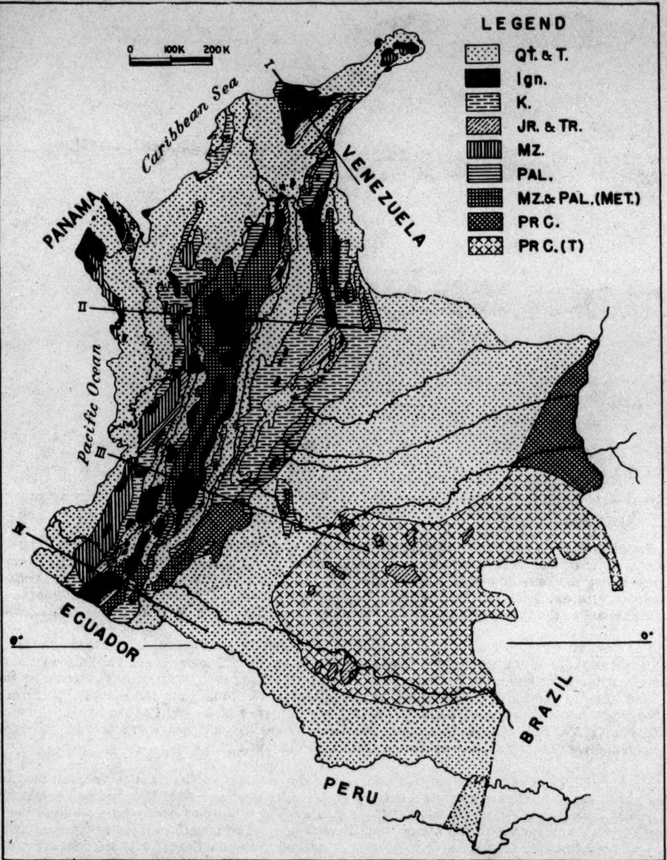
LEGEND
Qt. & T.
Ign.
K.
JR. & TR.
MZ.
PAL.
MZ. & PAL. (MET.)
PR C.
PR C. (T)
0 100K 200K
Caribbean Sea
Pacific Ocean
PANAMA
VENEZUELA
ECUADOR
PERU
BRAZIL
I
II
III
IV
0°
0°

Fig. 2--Geologic elements of Colombia; Qt. and T., Quaternary and Tertiary (undivided sediments); Ign., principal intrusive batholiths and extrusive flows of different ages (granites, diorites, granodiorites, and andesites); K., Cretaceous sediments; Jr. and Tr., Jurassic-Triassic or Triassic, mostly sediments with some intrusives; Mz., Mesozoic, semi-metamorphic schists with intrusives; Pal., Paleozoic sediments from Carboniferous to Cambrian (undivided); Mz. and Pal. (Met.), metamorphic schists and intrusives of presumably Mesozoic to Paleozoic ages; PrC., Precambrian crystalline basement rocks, including intrusives; PrC. (T), Precambrian basement covered with Tertiary sediments in the Llanos

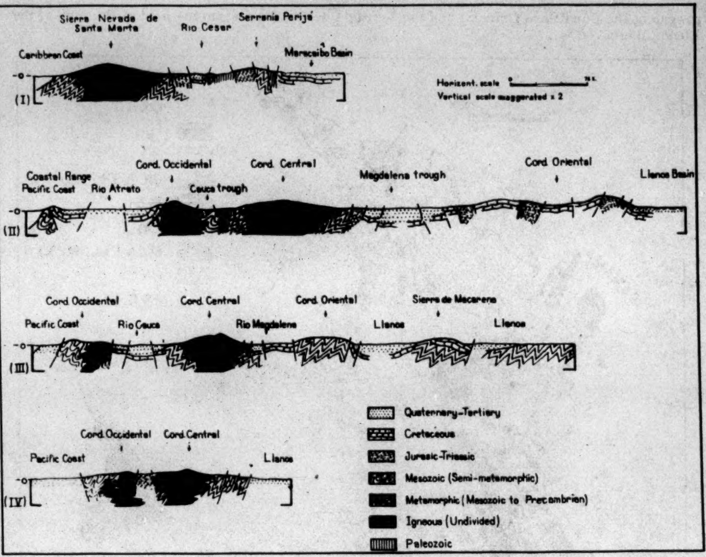
Sierra Nevada de Santa Marta
Rio Cesar
Serrania Perijá
Caribbean Coast
Maracaibo Basin
-0-
(I)
Horizont. scale 0 75 K.
Vertical scale exaggerated x 2
Cord. Occidental
Cord. Central
Cord. Oriental
Coastal Range
Pacific Coast
Rio Atrato
Cauca trough
Magdalena trough
Llanos Basin
-0
(II)
Cord. Occidental
Cord. Central
Cord. Oriental
Sierra de Macarena
Pacific Coast
Rio Cauca
Rio Magdalena
Llanos
Llanos
-0
(III)
Cord. Occidental
Cord. Central
Pacific Coast
Llanos
-0
(IV)
Quaternary-Tertiary
Cretaceous
Jurassic-Triassic
Mesozoic (Semi-metamorphic)
Metamorphic (Mesozoic to Precambrian)
Igneous (Undivided)
Paleozoic

Fig. 3--Generalized geologic sections of Colombia

These distinct structural elements are of different geological age and composition, and are characteristic of this part of the continent which shows great complexity of structural forms. Together with the basement shield to the east, and the Cordilleras of Ecuador, Panama, and Venezuela with which the Colombian Andes are closely connected, these structural elements represent the key to the interpretation of the evolution of northwestern South America (Fig. 2 and 3).

The Coastal Range--The Coastal Range is also known as the Cordillera de Baudo and is the westernmost range of the Colombian Andes. It begins north of Buenaventura and follows the continental outline into Panama. To the east it is separated from the Cordillera Occidental by the San Juan - Atrato depression. The range is not composed of old igneous and metamorphic rocks as thought by some authors [ANDERSON, 1945], nor is it part of the old Pacific border-land, Panadoria [OLSSON, 1942]. The metamorphic rocks of the Coastal Range are largely Mesozoic and contemporaneous with those of the Cordillera Occidental.

The intrusives consisting mainly of gabbro and diorites are of Cretaceous age, but the uplift of the whole range is apparently late Tertiary since in its southern part Miocene sediments are involved in a pattern of folds and normal block faults. The folding evidently took place at the time of the Tertiary emergence of the Atrato - San Juan trough. The trough is lined to the east and west by normal faults. The trend of the range is arched to the northwest with its concave side facing the ocean and is roughly parallel to the Cordillera Occidental. The strike of the range and the formation, as well as the configuration of the coast, do not suggest that a large borderland ever existed to the west. The Coastal Range could, however, have continued some distance south of the Bahia de Buenaventura as a Tertiary fold if the southern coast had not undergone a submergence due, possibly, to an epeirogenic subsidence of this part of the coast in Plio-Pleistocene times [OPPENHEIM, 1949]. The Coastal Range continues northwest into Panama as Serrania del Sapo, where its continuation borders the steep submarine escarpment of the Gulf of Panama. This part of the range consists mostly of Cretaceous intrusive rocks and pyroclastics of a later age. SCHUCHERT [1935] describes several old volcanic craters in Panama where the range appears as the continuation of the Coastal Range of Colombia. He also describes numerous volcanic vents on the islands of the Gulf of Panama, particularly the Pearl Islands. Indications of a later plutonic activity in the range are suggestive and become frequent north of Panama.

Cordillera Occidental--The Cordillera Occidental is a direct continuation of the Cordillera Occidental of Ecuador. To the north it reaches about lat. 8° 30′ N, from where it continues as a chain of low ranges ending on the Caribbean coast west of Barranquilla. The trend of the Cordillera is somewhat arched and follows the outline of the Pacific coast in its southern part, changing its direction to north and northwest in its northern part. The main range is about 40 to 60 km wide at its base and reaches up to 3000 m in elevation.

The Cordillera consists largely of semi-metamorphic black slates and light-colored quartzites of evidently Mesozoic age, with numerous large batholiths extending along its central and western flanks. These batholiths constitute a large part of the Cordilleran massif much in the same way as they do in the Cordillera Occidental of Ecuador [OPPENHEIM, 1950b]. Where exposed, the plutonic rocks of the batholiths appear to be granodiorites, diorites, and gabbro as observed in numerous outcrops. Andesites prevail in the extrusive flows to the south. The igneous rocks of the Cordillera, in the case of some batholiths, appear intruded in the pre-existing strata. Extensive batholiths are exposed and were observed by the writer all along the western front of the Cordillera, where they appear in the deep valleys of the Pacific watershed. These intrusives are the main source of gold and platinum in the mineralization of the Pacific coast of Colombia.

Folded upper Cretaceous marine sediments occur in the northwestern and southeastern parts of the Cordillera, indicating its principal age of uplift as post-Cretaceous in its greatest extent and upper Tertiary in its northern limits in Bolivar, where Miocene sediments are involved in the orogenic folding.

A spur of the Cordillera Occidental trends to the northwest, across the Atrato basin, forming the Cordillera de Darien of Panama, which runs parallel to the Coastal Range; this spur, however, is separated from the main Cordillera by the Atrato depression.

The structure of the Cordillera is of block-faulted nature. To the west the Cordillera is bordered by the Tertiary, San Juan - Atrato depression which connected the Caribbean and Pacific, apparently as late as Pliocene times, when the interoceanic connection was interrupted by the emergence of the depression which followed planes of normal faulting along its outer borders: the eastern foot of the Coastal Range and the western foot of the Cordillera Occidental.

The eastern front of the Cordillera faces the narrow Cauca - Patia trough. This trough is filled largely with continental Tertiary sediments and is bounded by steep faults. According to STUTZER [1934a], it appears as a typical rift valley. The writer has considered the Cauca - Patia trough as a structural continuation of the much wider Inter-Andean basin of Ecuador [OPPENHEIM, 1950b]. GROSSE [1935] mentions thrust faults which he observed in the gorge of the Rio Patia, the faults appearing thrust from the east. These must have been of local occurrence, since thrust faults of a regional character have not been observed in the area of the trough, nor along the western front of the Cordillera.

The Cordillera Central--A physiographic continuation of the Cordillera Real of Ecuador, the Cordillera Central continues in a northeasterly direction to El Banco at about lat. 9°N. This main Cordillera of Colombia is also arched and parallel to the Cordillera Occidental.

The core of this Cordillera is composed of older metamorphic rocks, mainly crystalline schists, and of younger intrusive batholiths which occupy large areas in its central and eastern parts. The principal batholiths occur from north to south in the following areas: at El Banco; the Antioquia batholith in the area of Rio Porce; and the Caldas-Tolima-Huila batholith, extending as a continuous body along the central and eastern parts of the Cordillera. From the few petrographic studies that have been made of these batholiths it is known that the rocks of the Antioquia batholith are predominantly quartz-diorite and monzonite. The Tolima batholith consists mainly of quartz-diorite and grano-diorite. The Huila batholith is made up of principally grano-diorite and granite [SCHAUFELBERGER, 1944]. A considerable increase in silica content of these intrusives as compared to the Coastal Range and Cordillera Occidental is noticeable. The uplift of the Cordillera could be more specifically assigned to the late Paleozoic, and particularly early Jurassic, age of widespread diastrophism in this part of the continent.

The Orodovician graptolites at Cristalina [BOTERO, 1942], as well as the Triassic-Jurassic and marine Certaceous sediments found in the northern and southern extremities and along the flanks of the Cordillera, indicate that the greatest part of the range remained submerged during early Paleozoic and part of Mesozoic times. The Hercynian diastrophism at the close of Paleozoic and beginning of Mesozoic times could have accounted for the uplift of the Cordillera Central. Its rise was associated with intermittent diastrophic activity that probably continued throughout Mesozoic and, to some extent also, Cenozoic times.

The observations made by many students of Colombian geology agree that extensive faults border the western and eastern flanks of the Cordillera Central. The Cauca - Patia graben makes the faulting of the western side of the Cordillera conspicuous. The eastern side of the Cordillera, facing the Magdalena trough, is also extensively faulted. This was amply confirmed not only by geological but also by geophysical observations and in borings for oil. Thus, the ideas of HARRISON [1929] and STUTZER [1934b], who first recognized the graben structure of the Magdalena, are quite correct.

The Magdalena trough which limits the Central Cordillera to the east, like the Cauca - Patia trough which limits it to the west, are both of a graben nature, and the Cordillera, bordered by the two graben, would appear in its broad structure as a great horst, such as could be produced by prevailing vertical stresses, finding their expression in a normal, block-faulted tectonic pattern.

The Magdalena trough and the Coastal Basin--The Magdalena trough separates the Cordillera Central from the Cordillera Oriental. Because of its oil fields and petroleum possibilities, this part of Colombia is one of the better-studied regions of the country. The trough appears bounded by stepped faults extending for some 600 km and forming a down-stepped depression of about 1500 m. The width of the depression varies from 30 to 50 km. Its bottom is mostly flat but hilly in places, and structurally it shows a complex pattern of folding and faulting. The faulting is more noticeable in the southern part of the trough. The blocks forming the walls of the Magdalena graben have been described by some authors as locally thrusted towards the trough [ANDERSON, 1945]. This, however, does not affect the graben character of the depression.

The age of faulting and sinking of the Magdalena trough could be attributed to the Mesozoic, while the emergence must have taken place throughout Tertiary times, since the Tertiary sediments are, in the Magdalena Valley, mostly continental.

The Caribbean Coastal Basin into which the Magdalena trough opens is a large basin bounded by the Santa Marta massif and the Cesar basin to the east, the Caribbean coast to the north, the plunging end of the Cordillera Central, Serrania de Ayapel, to the south, and to the west the hilly ranges forming the continuations of Cordillera Occidental: Serrania de las Palomas, west of Rio Sinu, Serrania de San Jeronimo and Serrania del Carmen east of that river, which eventually reach the coast, to the northwest. These Serranias separate the Sinu basin from the main Coastal Basin. They are of Tertiary age and are very faulted and folded. The whole Coastal Basin is made up of a normal succession of sediments ranging from Cretaceous and Eocene in the west and to Plio-Pleistocene in its central and eastern parts. The uplift of the Coastal Basin may be continuing at the present time, as is suggested by the rising terraces along the Caribbean Coast.

The depth of the basin is of the order of 2000 to 5000 m, as has been indicated by some borings for oil in different parts of the basin. Cretaceous marine sediments appear only, as shown by STOSE [1950], in the western and southwestern parts of the Coastal Basin. The basin existed as such since late Mesozoic times. The eastern part of the basin could have remained largely emerged together with the Sierra Nevada de Santa Marta massif. This could partly account for the continental, upper Tertiary sediments which prevail in this part of the basin. The southeastern part of the Coastal Basin opens into the Rio Cesar depression, south and east of the Sierra Nevada de Santa Marta, with which the Coastal Basin could have been connected in Cretaceous times.

Cordillera Oriental--The Cordillera Oriental, together with its northern extension, the Cordillera or Serrania de Perija, is the longest and widest range of the Colombian Andes. It is of upper Tertiary age and consists mainly of sedimentary formations ranging from early Paleozoic, possibly Cambrian, to uppermost Tertiary. This Cordillera is geologically better known than any other range in Colombia. A fairly detailed stratigraphic description of its formations is given by ANDERSON [1945].

Two large old massifs are involved in the folding of the Cordillera Oriental: the massif of Santander, to the north, which consists of several closely related batholiths, and the massif of Garzon, with its old intrusives forming the southern extremity of the Cordillera near its juncture with the Cordillera Central. The northern batholiths are of granitic composition. Their formation could have been associated with late Paleozoic or early Mesozoic diastrophism, contemporaneous with plutonic activity in the Cordillera Central. The Garzon massif consists of micaschists, gneiss, and granites of Precambrian age. It could represent an extension of the shield area of basement rocks occurring to the east. It is related to the Macarena massif, at the foot of the Cordillera Oriental, which undoubtedly is an outlier of the Guiana basement shield.

The Cordillera is affected by extensive faulting mostly of block-faulted nature. Westward thrusting has also been recorded in some localities facing the Magdalena trough and west of the Cordillera de Perija. This is, however, of moderate, local, and not of general nature. The geosyncline, which apparently emerged during the upper Tertiary, Andean, diastrophism, has evidently undergone earlier diastrophic revolutions which can be traced as Caledonian and Hercynian during its early Paleozoic and early Mesozoic evolutionary cycles.

The Cordillera Oriental extends in a broad arch in a northeasterly direction turning to northwest at about lat. 6°N and to northeast again at its junction with the Serrania de Perija. The Cordillera de Merida of Venezuela is separated from the Cordillera Oriental by the Cucuta depression. This, however, is an independent and not subordinate branch of the geosyncline of the Cordillera Oriental [RUTTEN, 1942]. The structural trend of its sedimentary formations is predominantly northeast-southwest. Regional east-west strikes have also been observed in this Cordillera, as they have been noted in other parts of the country. This east-west trend appears to be quite unusual and contrary to the general northeastern trend of the Cordilleras; a satisfactory explanation of this has still to be found. In the case of the Cordillera Oriental, it might be interpreted as due to the compression caused during the emergence of the geosyncline by the older batholithic bodies at its northern and southern extremities. This could also explain the pronounced eastward arching of the geosyncline. The trend of the structural elements in Colombia were tentatively described by von ESTORFF [1946] and discussed by OPPENHEIM [1946].

Sierra Nevada de Santa Marta--The Sierra Nevada de Santa Marta is a geographically isolated massif consisting of old igneous and metamorphic rocks mainly of Paleozoic and partly Mesozoic age. It is intruded by large bodies of granite and syenite that compose the largest part of the massif. Its highest peaks consist of younger, probably Cretaceous or Tertiary, granite intrusions. A prominent north-south fault separates this massif from the Magdalena valley plains and also from the Cordillera Central. A number of normal faults separate the massif from the Cordillera de Perija to the southeast and the Goajira Peninsula to the northeast [OPPENHEIM, 1942a].

Upon considering the graben structure of the Magdalena embayment, it seems plausible that the Sierra Nevada de Santa Marta is not the isolated massif that it appears to be but, rather, geologically forms part of either the Cordillera Central or the Cordillera de Perija, or both. The widespread occurrences of Jurassic-Triassic formations in the Cesar valley indicates that it was linked with the Cordillera Central and the Serrania de Perija, at least during Mesozoic times [STOSE, 1950]. There are indications that the separation of the Serrania de Perija as such occurred only in Tertiary times. The massif is cut off by extensive faults from the surrounding basins, and its uplift could have been of the nature of isostatic compensation for the subsidence of the adjoining basins to the west, southeast, and northeast.

The Goajira Peninsula appears to be geologically part of the Sierra Nevada massif, yet it also bears close geological relation to the Paraguana Peninsula in Venezuela and the Antillean Islands to the east. The rise of the Goajira Peninsula, which began in late Tertiary times, apparently still continues. The high, elevated sea beaches with a recent or existing fauna indicate its persistent emergence in modern times.

The Llanos Basin--East of the Cordillera Oriental and reaching into the Guiana Shield extend the Llanos of Colombia. This is a vast plain of almost flat-lying Tertiary beds, overlapping to the east the basement rocks of the shield. The geology of the Llanos Basin was first described by OPPENHEIM [1942b] and later by ANDERSON [1945] and RICH [1945].

The Llanos Basin is largely a western extension of the Archean Guiana Shield. The Macarena massif at the foot of the Cordillera Oriental evidently forms a narrow land-bridge of basement rocks under a thin sedimentary cover and separates the Llanos into two distinct basins: the northern and the southern. The predominant basement rocks consist of micaschists, gneiss, and granites. The sedimentary beds of the deep basins surrounding the shield area comprise several thousand feet of Cretaceous and Tertiary sediments. These beds thin out towards the shield and east of the Cordillera Oriental. The basin is warped in broad folds, and its basement is also faulted. Its precise structural pattern could, however, only be established through geophysical explorations. The evolution of the Llanos sedimentary basins was closely related to that of the geosyncline of the Cordillera Oriental. The western edge of the basement shield merged with the eastern boundary of the geosyncline. During and after the emergence of the geosyncline of the Cordillera, adjustments of the marginal blocks of the shield had undoubtedly taken place. The pressure of these could have been responsible for the moderate thrusting observed along the eastern foot of the Cordillera. This, however, would not require a presumed westward drift or excessively great pressures as suggested by some authors.

Relation of eastern and western cordilleras

The geosyncline of the Cordillera Oriental is the most conspicuous structural element of eastern Colombia. Its emergence appears to have involved mainly vertical forces with only secondary tangential components. Bound between the basement shield to the east, and the batholith of the Cordillera Central to the west, its sedimentary formations were block-faulted and folded in the diastrophic process that accompanied the emergence of the Cordillera Oriental. At contacts with the outer walls the strata within the geosyncline had to undergo mechanical adjustments to the resulting pressures. This, coupled with the effects of pressure of the marginal shield-blocks in their tendency to maintain the equilibrium of crustal stresses, could have caused the effects of local thrusting observed in the Cordillera.

The idea of some authors [de CIZANCOURT, 1933] that the rise of the geosyncline of the Cordillera Oriental was due to a westward thrust of the Guiana "foreland" against the "borderland" of the Cordillera Central and Occidental, does not seem to the writer proved by field observations. If such pressure would have been exercised by the great mass of the basement shield, then not only the whole Cordillera Oriental would appear thrusted and exhibit nappe structures of an Alpine type but also the Central and Western Cordilleras would have shown effects of this hypothetic pressure. Wherever observed in Colombia, thrusting does not follow a set direction. It is very moderate and, as mentioned, of only local effect. The prevailing structural pattern is of block faulting and not of the low-angle thrusting of Alpine character which generally does not occur in the Andes.

The fact that the Cordilleras Central and Occidental were not affected by the presumed westward thrusting of the Cordillera Oriental geosyncline is further accentuated by the vertical structural depressions to the east and west of the Cordillera Central proper, as observed in the Magdalena and Cauca graben. The structure of the Cordillera Central indicates that throughout Cenozoic times it remained a static massif, largely unaffected by the rise of the Cordillera Oriental.

We have further seen that the Cordilleras Central and Occidental did not form part of a hypothetical "Pacific land-mass" that was supposed to have exercised an eastward pressure for reasons that have been shown [OPPENHEIM, 1947]. This is partly because there is no evidence of its having existed, and also because the structure of the Cordilleras Central and Occidental indicate that they were predominantly static ranges and emerged mainly through vertical stresses.

The above considerations of the structure of the Colombian Cordilleras seem to point to a different mechanism of orogenesis rather than to the conventional compression forces.

Seismicity and volcanism

Despite the complex and intensely active tectonic history of Colombia, recorded seismicity accompanied by destructive earthquakes has been very limited in modern times. Thus, RAMIREZ and FORERO [1947] in their statistical record of earthquakes in Colombia show that from 1900 to 1946 a total of 54 major seismic shocks were recorded in the country. Some of these, however, occurred near its boundaries with Ecuador, Venezuela, and Panama. The total number of significant earthquakes recorded in the country from 1575 to 1946, as shown by the same authors, is only 102. This last figure evidently indicates only the major shocks strong enough to have affected populated areas. From the above data, it appears that of the total of recorded earthquakes about 50 pct correspond to the Cordilleras Central and Occidental and only 25 pct to the Cordillera Oriental. Many of the destructive shocks seem to have occurred in the region bordering Ecuador, or in areas of junction of the Cordillera Oriental with the Cordillera de Merida of Venezuela. The last destructive earthquake in Colombia, on July 9, 1950, was of high intensity; it caused the partial destruction of several towns and villages, and occurred in this latter part of the Cordillera Oriental.

In contrast to the Cordilleras Occidental and Central, the Cordillera Oriental has no volcanoes or any signs of volcanic activity in recent times. Almost all the volcanoes in Colombia are located in the Cordillera Central; only two are found in the Cordillera Occidental. These are the volcanoes, Chiles and Cumbal, at the border with Ecuador, and here extensive andesite extrusions and lava flows point to a much more intense volcanism of Tertiary and Pleistocene ages. The older intrusives of this Cordillera are related to early Mesozoic, possibly Triassic-Jurassic, diastrophism.

In the Cordillera Central evidence of volcanic activity is widespread. The main volcanic groups center around the recently active volcanoes of Ruiz, Tolima, Huila, Purace, and Galeras. In most cases related to or superimposed on older batholiths, the lavas and ejecta of these volcanoes are of an andesitic nature. At present only two volcanoes are active in Colombia: Galeras and Purace [OPPENHEIM, 1950a], all others being latent or extinct.

Hypothetic relations of the cordilleras with island arcs

The research made to date on the structure, evolution, and geophysical character of the Colombian Andes is insufficient. However, a tentative suggestion based on surface geological observations and some geophysical interpretations [GUTENBERG, 1949] points to certain connections of the Cordilleras with island arcs, particularly with the Lesser Antilles. The alignment of the Cordilleras in curved patterns, their separation from each other by deep tectonic troughs, the batholithic core of the Cordilleras Occidental and Central, as well as of the Sierra Nevada de Santa Marta, and the Goajira Peninsula, all point to their geological relation to the Lesser Antillean island arcs either through the Cordilleras Occidental and Central, and the Goajira, or the Cordilleras of Venezuela [HESS, 1938, 1948].

Thus, the island arcs of the Antilles could tentatively be considered as continuations of the Cordilleran ranges and consequently common factors should be found also in the structure and evolution of these ranges from presumably Paleozoic and Mesozoic island arcs, if such processes took place in the evolution of the Colombian Andes.

Conclusions

The structural study of Colombia indicates that the Guiana Shield is its fundamental basement element, the Cordilleras having evolved west and north of it in an arc-shaped alignment.

The structure of the Colombian Andes proper is predominantly of a block-faulted nature. Low-angle faulting or thrusting is moderate and only occurs locally. Thrusting of a nappe character, of an Alpine type, is not found in the Colombian Cordilleras, which are of a typical Andean structure. Thus, the rise of the Colombian Andes is not due to compressional forces alone but rather to a different mechanism involving mainly vertical stresses.

In Colombia the Central and Western Cordilleras are of batholithic constitution, and when compared to the sedimentary origin of the Cordillera Oriental, confirm the dual character of composition of these cordilleras. This feature appears typical throughout the Andes of South America.

Finally, the relation of the northern extremities of the Cordilleras on the Caribbean coast and their connection, geological and geophysical, with the Antillean island arcs, is very suggestive and deserves further thorough study and investigation.

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(Communicated manuscript received November 20, 1951;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

THE EARTH'S FACE DETERMINED BY THE CORE

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Abstract--The author refers to the old theory of a periodic change in expansion and contraction of the Earth, caused by thermal conditions, especially by periodic accumulation and escape of radioactive heat. To this theory is joined: (1) the hypothesis of rotational differences of core and mantle, the core not being warmed and cooled in the same way as the mantle; (2) the hypothesis of a uniform crystalline (if solid) or semi-crystalline (if nearly fluid) parallel structure of the core, which effects deformations of its spheroid boundary during tangential movements of the mantle. These deformations, in regionally accelerating or retarding the mantle's movement, also produce movements within the crust.

By means of this theory of the Earth's mechanism an explanation is attempted for a geological assemblage of facts, such as (1) the origin of the Mediterranean or Alpide ring of structures, which in general is supposed to have been equatorial in the Tertiary; (2) dislocations of the Earth's axis during Tertiary and other active phases of the Earth's history; and (3) the origin of the Atlantic and Indian Oceans and of the crustal massif of central Asia.

In the second part of this paper, entitled An Actual Phase?, there is an attempt to demonstrate that seismic observations of actual movements in the crust probably in the future will confirm or deny the probability of this theory.

The following pages give the first report of a theory of the Earth's mechanism, based upon a hypothesis of a firm and not very hot macrocrystalline core and a periodic rotation of the 2900-km deep mantle around the core. This periodic rotation is assumed to be caused by a periodic change in accumulation and escape of radioactive heat. The accumulation effects expansion and retardation; the escape of heat effects contraction and acceleration. In this way an assemblage of geological facts is explained by a theory which is based upon geophysics.

American theories of a cold origin of the Earth presuppose the iron of the core to have reached the Earth's center by gravitational exchange movements [UREY, 1949] which produced heat which is increased by radioactivity. These theories permit the concept of a core, which grew up from the Earth's center in a crystalline state of this kind: There are structural collections of a corporeal size, which repeat the crystalline atomic lattice in the same manner as iron meteorites. These are collected again at a higher order of magnitude and so on. This produces a quasi-monocrystalline construction of the whole core. The packing of atoms in these collections becomes denser nearer to the Earth's center. The size of the collections remains the same. Thus the change of density does not disturb the parallel lattice arrangement of the whole. In spite of this lattice-like interior construction, the core grows, always adding new spheroid layers of equal density and always having a spheroid boundary. American geophysicists (B. Gutenberg, among others) suppose that at the 2900-km limit there is a change in the arrangement of electrons inside the atoms, caused more by pressure than by temperature. In a solid core such a change must bring the binding forces of the interior mantle of electrons into interatomic action after the exterior mantle of electrons has vanished. This change may be the reason why transverse seismic waves do not pass through the core. A second change in the arrangement of electrons may be assumed at the boundary of the central body, 1500 km from the Earth's center.

During geological times, when there is more geosynclinal development in the crust than orogenesis, radioactive heat is supposed to be accumulated in the mantle. The accumulation of heat is screened by the crust and causes an expansion of the Earth until a critical phase is reached at which plutonism and volcanism, which come up in the geosynclinal development, effect an escape of heat. The loss of heat causes a contraction of the mantle and an acceleration of its daily rotation. The mantle now will rotate eastward around the core, which is not cooled and contracted because of its distance from the crust and because of sufficient quantities of radioactive material in it, and will grind it off unequally because of its macrocrystalline character. The parts of the surface, which are parallel to a structural plane of the core's crystal, are ground off most. The more a part of the surface is inclined to these planes, the less it is ground off. The differences of the core's radius caused by this process will scarcely exceed one or a very few kilometers. Nevertheless, the balance of gravity will determine the position of the deformation relative to the core's rotation. In other words, the core will tend to this position of gravitational equilibrium. On the contrary, the mantle will tend to bring the core's axis into the position which is the best for sliding movement around the core. Equilibrium position and sliding position will not be the same. Therefore, the mantle, after having reached a considerable acceleration, finally will make the core's axis wander from the equilibrium position to the sliding position. The Tertiary Alpide ring of orogenes represents the equatorial position at the end of the last wandering of this kind. After the contraction of the mantle has finished and its sliding motion has ceased, the core draws the whole Earth back into the core's optimum equilibrium and the axis returns into its normal position. It has this position at the present geological time.

Considering the diminishing velocity of longitudinal seismic waves in passing through the core, the lack of transverse waves, and the conclusions drawn recently by Jeffreys from phenomena of tides and nutation about the core, a core that is more fluid in its behavior than the mantle is more probable than a solid and completely crystalline core. If that is true, a semi-fluid core would also conform to this theory. The upper layers of the core or even the whole core excepting the central body then must be in a semi-crystalline state, with polyhedral crystalline particles suspended in parallel regulation in a liquid and dominating its movements. The mantle in rotating around the core then would remove the uppermost layer of the core. But the flowing movements in this layer would be restrained and accelerated according to the uniform semi-crystalline regulation, which influences all movements of this semi-liquid. In this way the same superficial deformations would be produced at the surface of the core as would be caused by a mantle, which grinds off a solid core.

The difference of equilibrium position and sliding position will be most effective in a core with an icosahedron crystalline lattice, as shown by the comparison of two positions of a regular icosahedron, limited by planes (Fig. 1). Now the meteoric iron of octahedrites has a cubic lattice with centered planes (the closest packing of atoms). This lattice shows two dominant planes: the octahedral plane (111) and the diagonal plane of a half-cube (210). The latter becomes visible especially as the position of inclusions, as J. B. Böggild has shown. (Meteoric iron with a lattice of centered cubes has a different character. The cubic plane and the dodecahedral plane are typical for this lattice.) The two planes (111) and (210) in compensation form an irregular icosahedron closely lined with atoms at its borders (Fig. 2). On the deformed boundary of the core this irregular icosahedron will ultimately be transformed into a regular one by the grinding movement of the mantle. The equilibrium position of the irregular icosahedron is that in which two octahedral planes lie on the poles while the (210)-planes (hatched planes of Fig. 2 and 3) have a position oblique to the equator. On the icosahedron an equatorial "girth trench" (Fig. 1, ABCD) is most favorable for the sliding movement. In the irregular form this trench shows bulgings, which are regulated by the grinding movement.

An icosahedron theory without any crystallographical hypothesis is possible on the condition that the core is fluid. The same icosahedral mechanism, we have demonstrated, then can be based upon a theory of tangential pressures in the mantle, caused by increasing differences of expansion (or contraction) from layer to layer. As the quantity of radioactive material decreases with depth

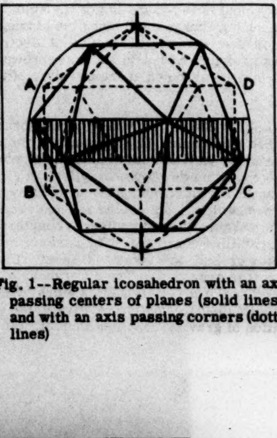

Fig. 1--Regular icosahedron with an axis passing centers of planes (solid lines) and with an axis passing corners (dotted lines)

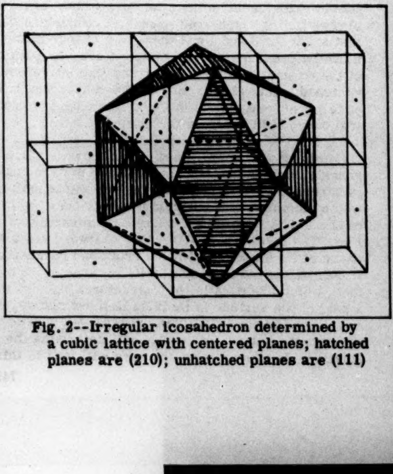

Fig. 2--Irregular icosahedron determined by a cubic lattice with centered planes; hatched planes are (210); unhatched planes are (111)

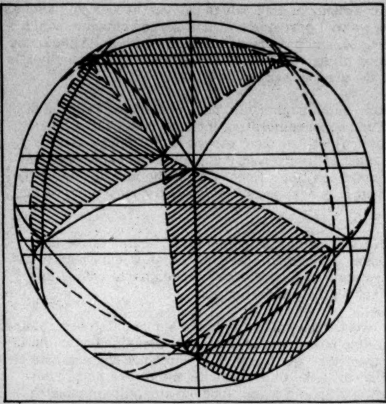

Fig. 3--Globe enclosed by a net of regular icosahedron and by a net of irregular icosahedron; hatched planes correspond to (210), both with an axis passing centers of planes

and probably reaches zero in the core, it is possible that the upper layers of the mantle produce in expanding the same tangential pressure as the lower layers, though they are lifted up by the expansion of the layers bearing them. A fluid core then will permit a deformation of the mantle, by which every layer gets more tangential extension according to its pressure and the lowest layer gets it without a change of the enclosed volume. Pressure of gravity will restrain this deformation to a very small measure and the balance of gravity will constrain it to a most symmetrical and equalized form. Therefore, a spheric icosahedron, which is a most symmetrical body and which has the most nearly equal distribution of the lifted regions that is possible, would be the most probable form of this deformation. At an early period of the Earth's evolution, metals must have been distributed more equally in the mantie than at present. Thermal conductivity, therefore, may have been nearly uniform and small in the whole mantle, while at present it probably increases with depth. The quantity of radioactive material must have been much greater at that time than now. If at that time there were regions in the mantle, which were weakened by convection currents and which yielded to the pressure and if balance of gravity effected an icosahedral distribution of these regions, the deformation then may have originated. It may have diminished later but never disappeared completely. At present, an expansion of the upper layers in effecting deformation may also lift up the lower layers at the yielding regions. A continual contraction of the core would make all that more probable. On this theory the mechanism of rotational differences between the Earth's layers caused first by different expansion, then by different contraction, and a dislocation of the axis would be restrained to movements in the interior of the mantle and relative to the crust.

Now we enclose the Earth globe in a regular icosahedral net. We bring this net into correspondence with the equilibrium position of a deformed icosahedral core. We put one knot of the net

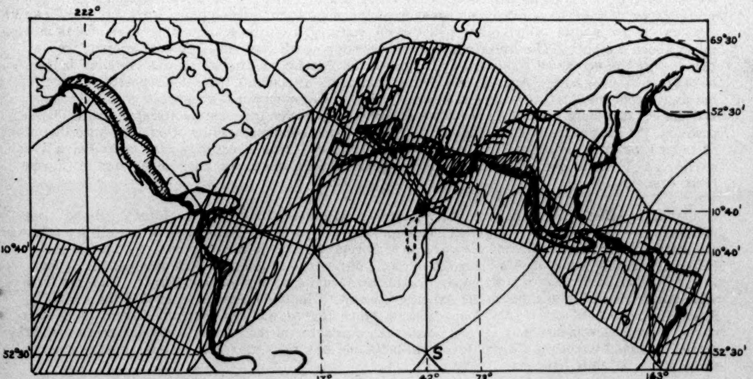

Fig. 4--World map showing the lines of a net of icosahedron which encloses the globe below the arch of Alaska and draw the equator around an axis, which passes through this knot as the arctic pole (Fig. 4, undulated line). Then we will perceive that this second equator, which forms the central line of a "girth trench" (Fig. 4, hatched area), coincides with the Alpide ring of orogenes (Tertiary equator). The inclination of the two icosahedral equators to each other is the same as that of the Tertiary equator and the present equator, 37 1/2°.

The origin of the Tertiary Alpide ring now can be explained. The accelerated rotation of a contracting body, locked by a non-contracted and non-accelerating interior body, will cause a drift in the crust from poles to equator. This drift will be most strong in the middle latitudes. The Tertiary ring, originating this way, has reached its present place after the axis, removed during the contraction, had returned to its normal position. Expansion of the mantle during the following periodic phase of accumulation of heat may even cause the contrary movement of the crust (from equator to poles).

While the axis was wandering to its Tertiary position, the pole-flight impulse in the half globe bearing continents must have been stronger in the south (Africa, Indian Ocean) than in the north (Eurasia) because of the shortening of rotation radius of the displaced parts before the wandering pole and the lengthening of radius behind the wandering pole.

The origin of the Atlantic and that of the crustal massif of Central Asia can also be explained. During contraction and axis wandering, the sliding movement is the most restrained near the intersections of the equator and the eastern borders of the "girth trench," which are the extreme abutments to this movement. Therefore, a vaulting-up of the plastic mantle must take place above these districts and produce a three-axis rotation ellipsoid with a long equatorial axis passing through these points. This position coincides with that of the long axis of an existing equatorial ellipse, one pole at 17°W (Helmert), which at present is only little different from a circle, but may be the remainder of a considerable vaulting-up during the upper Cretaceous and the lower Tertiary. The Middle Atlantic threshold passes the equator near this equatorial pole. A retraction of the Atlantic border continents from this threshold to the west and to the east, which has attenuated the crust between them, is said to have begun at the upper Cretaceous. This retraction now appears as a slipping-off from an Atlantic "mount." After the ellipsoidal vaulting-up had sunk back, this "mount" disappeared, and the ocean bottom sank. During the vaulting-up an amassment of inland ice in these districts would be probable. Indeed, the Atlantic continental borders, which during the Carboniferous must have lain nearer to each other and to the summit of the Atlantic "mount" of a Carboniferous vaulting-up, show marks of inland ice from Carboniferous.

The extreme depth of the ellipsoid is situated at the intersection of the meridian of the little equatorial axis and the Tertiary equator. There we find the center of the Asiatic massif: the Pamir. Our theory says that the upper (continental) layer of the crust during the vaulting-up of the mantle has executed compensative movements to fill the ellipsoidal depth, producing in this way a westward drift of the continents against the layer bearing them. This layer together with a magmatic layer below it have made compensatory movements too, but less, and were carried along to the east by the mantle. During this process the vaulting-up formed a standing wave of this moving mass above the core. The American continents, slipping off from the Atlantic "mount," were driven with the strongest westward drift against the Pacific bottom, which was fastened at its base (Andean orogenes). Analogously continental parts slipped off from the opposite "mount," but away from the Pacific, and produced the East Asiatic meridional orogenes. While the axis and the equator were wandering from the equilibrium position (present position) to the sliding position, the continents at both sides of the Atlantic changed their radius of rotation with the effect of separation at the south and of conjunction at the north. This effect may have contributed to an earlier origin of the southern than of the northern Atlantic and to the later separation of Europe from America.

Another effect during the axis wandering was the dislocation of the meridional ellipse. But the viscous mantle did not change its shape immediately, and the dislocation of the ellipse followed the advancement of the axis and the equator with retardation. In the districts of Asia and the Indian Ocean the result of this retardation was a difference of level on both sides of the equator with a northward descent for the upper crustal layer and a depth near the Pamir. Thus we have an Indian "mount" in analogy to the Atlantic "mount." India descended from it to the north, Australia descended to the east, Antarctica to the south. Africa was restrained from going eastward. These movements may have produced the character of the bottom of the Indian Ocean. The marks of inland ice of the Carboniferous in India and Australia may also be explained in analogy to those at the Atlantic borders.

Captivated continents--The equatorial vaulting-up is not the whole effect of the core's deformation. It seems we must add smaller vaultings-up, produced by regional "standing waves" above the (rounded) "corners" and "edges" of the core, which resist the grinding movement. Thus, we get an explanation of a compression of continental cratogene masses within the icosahedral triangles. This is shown in Figure 4. On this map both American continents, Australia, and Antarctica appear as captivated, each by one triangle, but Africa by two triangles. The Russian and Siberian cratogene is also enclosed and fixed by a triangle. The massif of Central Asia at the east shows stopping folds along the meridional line of its triangle. The icosahedral "corners" appear to separate the continents (corner of Panama, corner of Cape Horn, and the two Atlantic corners) or to be curbstones against a continent which drifts eastward. The corner of Abyssinia is the curbstone against Africa; the corner of Lake Baikal is that against Asia. Moreover, this corner forms the turning point of the whole folding system of Asia.

The Alpide orogene ring shows a serpentine movement of its large folding trains corresponding to the zigzag of the icosahedral "edges," which cross in an oblique direction the "girth trench." The masses drifting from both poles to the equator have evaded the "corners" and in passing each other have pressed forward into the open angles of the "girth trench" triangles. Evidence of this are the folds arranged en echelon from Iran and Pamir to Lake Baikal and the mentioned meridional stopping folds, which finish on Malaka (see Fig. 4). The Red Sea trench and the meridional East African fracture lines appear to be effected by a vaulting-up above the eastern "edges" of the African triangles. Parts of the upper crust (Arabia) were drawn farther, while Africa remained captivated. The Aden Gulf appears as a trace of the Abyssinia corner, which has made a notch into the continent.

A direct seismic examination of the presupposed deformations of the core is very difficult because of the inexactness of the measurements.

An actual phase?--Many recent investigations have confirmed the old suggestion of a westerly rotation of the secular magnetic-field figuration around the Earth with a drift motion of about 0.18° per year [BULLARD, 1948]. ELSASSER [1950] assumes this rotation to be caused by fluid motions and electric currents, which exist simultaneously in a thin layer below the boundary of the core. Elsasser considers thermal convection, produced by radioactive sources of heat in the central body, as a means of generating this fluid motion. A westerly rotation of the mantle around the core would also effect a westerly rotation of the magnetic-field figuration, provided that the crust does not go with the mantle but remains in a fixed position relative to such deformations of the core, as this theory assumes; the core may be solid-crystalline or semi-crystalline fluid. The westerly magnetic drift then may be produced by a fluid motion in a thin layer, which separates core and mantle, or in a layer below the boundary of a semi-fluid core or by the mantle itself.

A westerly movement of the mantle would signify for the present time: expansion of the mantle and contraction of the core, which loses heat in supplying the loss of heat in the mantle, which in the preceding period had effected contraction of the mantle. A new accumulation of radioactive heat

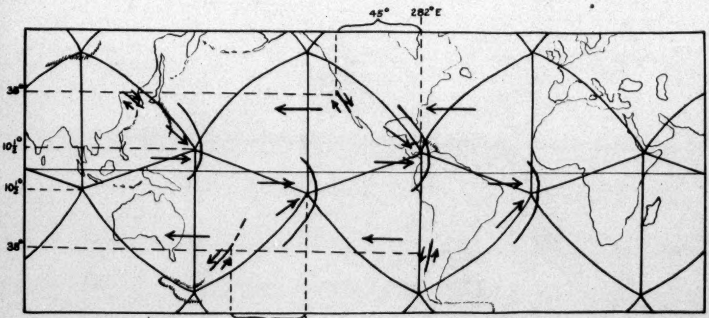

Fig. 5--Crustal movements on Pacific borders related to corner points of icosahedron in the mantle now may increase its expansion. The velocity of the mantle's movement would be considered as insufficient to cause a dislocation of the Earth's axis. The six icosahedral corners, situated near the equator (distance 10.5°, see Fig. 5), which are the strongest points of resistance, would restrain the crust from going with the mantle. The polar areas of the crust would be restrained each by three corners, situated 37.5° from the poles. Between these latitudes (10.5° and 52.5°) the crust would tend to go with the mantle. Stresses therefore would go out especially from the equatorial corners to the middle latitudes (from southeast to northwest in the northern hemisphere, from northeast to southwest in the southern hemisphere). At existing ancient fractures these stresses would effect crustal movements of this kind: on the eastern sides, toward the equator; on the western sides, toward the poles. In the circum-Pacific seismic belts there are fractures in these directions, which show just these movements on both sides. The structural trend lines of California strike out toward the north on the western side, the trend lines of New Zealand and the Tonga groups strike out toward the south on the same western side, while the eastern sides of both lines move toward the equator (see Fig. 5). The situation of the California lines and their distance from the northern equatorial corner of Panama form a pattern of the situation and distance of the New Zealand lines concerning the southern equatorial corner at 222° E.

In other circum-Pacific belts, which have a similar situation, especially that from Marianas Islands to Japan, where the line of deep-focus shocks crossing Honshu reaches to Manchuria, the movements are not as yet known sufficiently. The activity of some seismic belts, which form loops or half-loops near icosahedral corner-points, is remarkable. Such loops are the two Antillean loops, the Macquarie Island loop and the half-loop near Lake Baikal (see Fig. 5). Actual seismic phenomena, which lead to suppose a subcrustal plastic flow, evidently open a less hypothetical field for this theory.

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(Communicated manuscript received April 6, 1951, and, as revised, November 13, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

REPORT OF THE COMMITTEE ON LAND EROSION, 1951-1952

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Whether by design or otherwise, the chief activity in the field of land erosion during the past year appears to have been directed toward obtaining measurements of the rate of sediment movement, and an attempt to relate this rate to the characteristics of the contributing drainage area. The trend in this direction appears to be completely logical since, in the treatment of land erosion, criteria must be developed for selecting the most critically eroding areas and for establishing priorities for treatment among such areas. The measurement of total sediment movement from a given area possibly comes nearer to defining these conditions than any other factor. Likewise, establishment of a relationship between the physical characteristics of a drainage basin and their respective effects on sediment movement constitutes a sound approach to the complex problem of discriminating between the so-called "geologic erosion" on which it is assumed that only minor abatement, if any, can be obtained and "accelerated erosion" resulting from land misuse for which man may be considered responsible.

In compiling sediment rates advantage has been taken of the great amount of data being obtained at sediment gaging stations. Interpretive studies aimed at developing long-term estimates of sediment movement at these localities through careful examination of the contributing watershed are being pursued. These studies are being augmented by measurement of sediment deposition in small reservoirs, mainly stock ponds located on western range lands. Although in some instances the data obtained at stock ponds may be somewhat in error due to uncertainties concerning trap efficiencies and other factors, such studies have the advantage of dealing with a small drainage area, which permits making detailed examinations of watershed characteristics that can in turn be related to measured sediment rates.

Rate of sediment movement in the Midwest--BRUNE [1951] has compiled rates of sediment production at 192 localities listed within 16 physiographic areas. The records are of three types: (1) reservoir sedimentation surveys; (2) suspended sediment measurements; and (3) combined reservoir surveys and suspended-load measurements. With each there is shown the drainage area, length of record, the measured annual rate of sediment production in tons, and the estimated long-term rate of production. Adjustments for size of drainage area have been made, this having been found to be one of the most important factors influencing rates of sediment production at a given point on the stream. Similar adjustments are made for other factors including trap efficiency in reservoirs and type of sampler used in taking suspended-load samples from streams.

Although such an array of measurements would appear to represent a good coverage, extreme variations in the rates of production even within the same physiographic area, together with the wide contrast in physical characteristics, in land use, and in conservation measures exhibited within the various physiographic areas, indicate that the coverage is still inadequate. The variations in sediment rates shown also stress the need for expanded research to determine, for each physiographic area, the most important factors that influence rates of sediment production and the relationships involved. In some measure the method of depicting this relationship has been solved by intensive surveys in the Missouri Basin Loess Hills [GOTTSCHALK and BRUNE, 1950] where sediment production rates have been shown to be related to such factors as degree and length of watershed slopes, land use, crop rotation, extent of use of conservation practices, and size of watershed. Expansion of similar intensive studies to other physiographic areas are needed in order to fully evaluate these same factors under differing conditions of climate and erosion.

In actual rate of sediment production, the Missouri Basin Loess Hills are the worst offenders in the Midwest. Measurements show a spread ranging from 700 to more than 50,000 tons annually per square mile of drainage area. Although the higher figures apply to small basins, rates of 1040 to 5530 tons annually per square mile of drainage area from basins of 2730 and 2800 square miles respectively are shown. In contrast to this, measurements taken in other physiographic areas show rates of only a few hundred tons per square mile annually with a maximum of less than 1000 tons for basins having an area of 1000 square miles or greater. The production rates for smaller drainage areas are also generally much higher in the Missouri Basin Loess Hills than elsewhere, although some small basins in the Upper Mississippi Loess Hills physiographic area approach them with measured production rates exceeding 5000 tons per square mile of drainage area per year as recorded in several basins.

Considered in relation to the sediment load being carried by the major Midwest streams and the associated problem so created, the importance of the measurements cannot be overemphasized. Their value in planning and in fixing priorities for land treatment is also obvious.

Studies on wind erosion in the Great Plains--Wind-tunnel studies and soil analysis on irrigated plots at Scottsbluff, Nebraska, are reported by ZINGG [1951]. Wide contrasts in wind erodibility and clod structures were found on the plots. These varied in large measure with the type of crop rotation used. Results are summarized in Table 1.

Table 1--Erosion, residue, and clod structure following potatoes in various systems

Rotation	Soil loss[a]	Surface residues	Erodible soil fractions 0.42 mm
	lbs/acre	tons/acre	pct
Three-year rotation: potatoes, beets, barley	74,515	0.04	62.3
Three-year rotation: potatoes, beets, barley (manure)	2,720	0.085	53.4
Six-year rotation: potatoes, beets, barley, alfalfa three years	973	0.252	43.6

[a] Amount of soil loss obtained with a portable wind tunnel and a surface drag of 3000 pounds per acre.

Functional relations for estimating conditions of wind erodibility of field surfaces were developed, one of the most important being $X \propto A^{3.5}/R^{0.8}$, where X is the amount of soil eroded, A is the per cent of erodible soil material less than 0.42 mm in diameter, and R is the weight of surface residue. The experiments emphasize very strikingly the importance of proper crop rotation and maintenance of surface residue in keeping soil removal from wind erosion to a minimum.

Wind-tunnel studies of model barriers of different shape and density have demonstrated the variability of air-flow patterns. Conclusions can be generalized as follows: (1) The sharper the barrier, the greater is the degree of protection behind it; (2) rounder shapes have a lesser zone of influence than narrow vertical shapes; (3) porosity of a barrier decreases the degree of protection but extends the zone of influence.

Another interesting and perhaps far-reaching study was initiated to determine the effects of soil conditioners on soil structure and erodibility using CRD-186, known by the trade name of krilium, manufactured by the Monsanto Chemical Co. Short-time results indicate a substantial increase in the state and stability of structure of some soils, and information was obtained on the best method for mixing the conditioner in the soil.

The prospect of improved soil structure, increased infiltration, and other advantages through use of the conditioner has stimulated interest in other localities, and steps are currently under way to test the effect on western range lands in Montana and Wyoming in 1952-53. The areas to be treated contain heavy clay soils derived from disintegration of marine shales. Mainly because of a tight structure and low infiltration, such soils support a sparse vegetation and sediment production is high. The advantages to be gained by possible improvement of soil structure through use of the conditioner, both in forage production and in reduced erosion rates, are so attractive that Government agencies and individual ranchers are initiating experiments in its use, pending the time when it is hoped the conditioner might become plentiful and cheap enough to justify its use on these low-value lands. The experiments will be watched with extreme interest by administrators of western range lands. Hydrologists and users of water will likewise be concerned with the results since if use of the soil conditioner increases rainfall infiltration to a marked extent, it can have a pronounced effect on stream flow.

Rates of sediment movement on western range lands--The investigation of erosion and rate of sediment movement from western range lands has been directed mainly toward determining the effect of sedimentation on existing or planned reservoirs located on the main streams. The large number of stock ponds on range lands affords opportunity for measurements of sediment production at many localities and permits deliniation of areas of high sediment productivity. In some sections the presence of the ponds themselves has exerted a discernible influence on the amount of sediment reaching major stream channels both by reason of trapping of sediment and the storage of water in the reservoir.

In a cooperative study by the U. S. Geological Survey and the Bureau of Reclamation [HADLEY, in preparation] an evaluation of the effect of some 9300 stock ponds located in the Cheyenne River Basin on the amount of sediment reaching the Angostura Dam is under way. The ponds are located in the 9000-square mile drainage area above the dam. A total of 135 ponds have been surveyed for sediment deposition. The rate of sediment movement as measured in the group varied from a half to 4 1/2 acre feet per square mile of drainage area annually, and on the basis of this sample it is estimated that the amount trapped in the 9300 reservoirs contained in the basin is on the order of 5000 acre feet annually. Admittedly not all of this trapped sediment would have reached the Cheyenne River had the reservoirs not been built, since it is evident that a considerable percentage would have been deposited before reaching that point. Measurements on fence lines, trees, and other obstacles show that aggradation of as much as three to four feet has accumulated in places on a number of stream flood plains during the past 25 to 30 years. The average rate of flood-plain deposition may have decreased somewhat since construction of reservoirs, but there is evidence that aggradation is still occurring. Studies are continuing in an effort to determine the combined effect of the stock ponds and the aggradation on the amount of sediment reaching the Angostura Reservoir.

The drainage area above each of the stock ponds used in the sediment study has been examined in detail. Data on the following items were obtained: slope, slope length, drainage density, length of raw and grassed channels, number of raw headcuts, vegetative cover, and condition of the soil as it relates to infiltration. Correlation between the rate of sediment deposition in the reservoirs and these physical characteristics show that only the degree and length of slope, the density of raw channel, reservoir age, and a combined factor covering vegetative cover and condition of the soil have significance. These findings are in general agreement with the studies previously mentioned in the Missouri Basin Loess Hills, but as these lands are used strictly for grazing in contrast to the agricultural use in the latter locality, no direct comparison can be made.

Similar studies using measured deposition in ponds as a measurement of sediment production rates are also under way in the Powder River Basin, Wyoming; the San Rafael Swell area in the Colorado River Basin in Utah, and in the Rio Puerco Basin tributary to the Rio Grande in New Mexico. The studies have not yet progressed to the stage where findings can be reported.

Studies of methods for control of erosion on range lands--During the 1951 field season an intensive investigation was initiated in the Wind River Basin of Wyoming for the purpose of designing treatment programs for abatement of erosion and reduction of sediment reaching Boysen Reservoir. Several tributary basins have been surveyed and plans for control of a designed storm of 50-year frequency are in preparation using detention dams, and water-spreading systems to reduce flood crests to non-eroding stages. An important aspect of the study will be an analysis of the cost-benefit ratio with particular consideration being given to both on-site and off-site benefits. Since this is set up as a pilot study intended to establish principle of design and determination of cost-benefit ratios for treatment of large areas of range lands in the Missouri River Basin, the findings will have great significance for those concerned with erosion abatement in this area.

Gullies in relation to the hydraulic geometry of stream channels--The hydraulic phenomena which cause some gullies to be discontinuous whereas others are continuous have long been disputed by students of the erosion problem. As a result of some research on the hydraulic characteristics of natural stream channels [LEOPOLD and MADDOCK, in preparation], the Geological Survey has developed some tentative methods of studying the mechanics of gully formation. During storm runoff, stream gaging and simultaneous suspended-load data are collected along the length of an active gully. By study of the rates of change with discharge downstream of the width, depth, velocity, and suspended sediment concentration, the instability or lack of adjustment between the channel and the supplied sediment load can be analyzed. There is considerable similarity in the manner in which width, depth, and velocity increase downstream with discharge of a given recurrence interval in most streams and rivers. The preliminary data for gullies, however, show a marked difference in these relations, and the differences are qualitatively explicable in terms of manner in which the suspended-load concentration changes downstream. Hydraulic measurements of flood runoff in gullies appear to offer the possibility of an improved understanding of

discontinuous gullies by utilizing the concept of adjustment of profile and channel shape to the imposed sediment and water which must be carried by the channel.

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(Manuscript received May 5, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 6, 1952; open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

PROGRESS REPORT, SEISMOLOGICAL LABORATORY, CALIFORNIA INSTITUTE OF TECHNOLOGY, FOR 1951

Hugo Benioff and B. Gutenberg

(Contribution No. 595, Division of the Geological Sciences)

Routine observations of local and distant earthquakes

These observations continue under the supervision of C. F. Richter.

Activity of local shocks was low during 1951. The swarm type of activity which recommenced in northeastern Imperial Valley in July, 1950, has continued. A shock of magnitude 5.6 originated near Calipatria on January 23-24, 1951. This generated an air wave recorded at Pasadena [BENIOFF, EWING, and PRESS, 1951]. On December 5, 1951, there was another shock that was felt widely in the same area; the instrumental magnitude did not exceed 4.5. Several other minor groups of these shocks were recorded.

On December 25-26, 1951, a very widely felt shock occurred offshore south of San Clemente Island. The magnitude was about 5.9. On August 15, 1951, there was another of the disturbances under Terminal Island such as took place on November 17, 1949 [BENIOFF, GUTENBERG, and RICHTER, 1950]. There was further extensive damage to oil wells. Seismograms were similar to those of 1949 but smaller, more nearly equal to those of December 14, 1947.

Probably no earthquake of magnitude over 7 1/2 occurred anywhere during 1951. Seismicity was generally low, increasing slightly toward the close of the year.

Station development

Routine station instrumentation changes and relocations were as follows: At Tinemaha, a long-period dual galvanometer recorder has been installed to record N-S and Z components with Benioff seismometers. At China Lake, a 35-mm film recorder has been installed and is recording with a short-period galvanometer connected to a Benioff horizontal-component seismometer and a long-period galvanometer, developed for this purpose by F. E. Lehner, connected to a vertical-component Benioff seismometer. At Mount Wilson, the Wood-Anderson torsion seismometers have been discontinued. Recording at Perris was discontinued on September 11, 1951. Temporary equipment with a short-period paper recorder and a Benioff moving-coil electromagnetic seismometer was put into operation at Big Bear Lake (34°14.3′ N, 116°54.8′ W) on June 21, 1951, under the auspices of the Big Bear Lake Elementary School.

Instrumental development

Development has continued by Benioff on the capacity-transducer seismographs. A horizontal-component pendulum seismometer has been completed. In addition to galvanometer recorders, the vertical instrument is now operating a magnetic tape recorder and an ink recorder. Provision is also made for operation of a Sanborn hot-stylus recorder. The visible writers and the tape recorder operate simultaneously from a single two-stage push-pull amplifier coupled to the diode output of the seismometer. Alternating-current bias for the tape recording-head is taken from the 60 cycle/sec power line. The recording speed of the magnetic tape is 0.5 mm/sec. For playback the tape is speeded up to 15 inches per second corresponding to an acceleration of 750.

Project sponsored by the Geophysical Research Division of the Air Force Cambridge Research Center

Instruments for the study of microseisms--In order to reduce temperature effects, the design of the strain seismographs at Palomar has been modified to provide for quartz tubing in place of steel pipe in the indicator arms. In addition, the outer steel housing-pipe sections are joined with neoprene coupling units to prevent forces from being transmitted to the piers by expansion of the housing. Completion of the tripartite installation was delayed by heavy snow.

The vectorial galvanometer recorder has been rebuilt with changes in the mechanical and optical structures to provide for greater ease and permanence of adjustments. In place of the occulting shutter used in the previous unit for time marks, the new design substitutes momentary increases in brilliance of the recording lamp by means of the discharge of a condenser.

Study of surface waves--Since it is intended to make extensive use of strain seismographs in the study of microseisms, records of surface waves written by the strain seismographs at Pasadena were investigated by Benioff and Gutenberg. For Rayleigh waves (and SV waves) the responses of two mutually perpendicular identical strain seismographs theoretically are always in phase, whereas for shear waves (either Love waves or SH waves) they are opposite in phase and equal in amplitude. Consequently, the combined output of two such instruments is zero for Love waves (and SH waves). For Rayleigh waves, P waves, and SV waves the combined output is unequal to zero and is independent of the azimuth of the wave. Comparison of strain seismograms with pendulum seismograms aids greatly in the study of wave types. The relationship between the responses of strain and pendulum seismographs is especially well adapted to demonstrate the approximately elliptical motion of the ground particles in Rayleigh waves. By studying seismograms written with a vectorial recorder [BENIOFF, GUTENBERG, and RICHTER, 1951] with vertical and horizontal pendulum seismometers or with a vertical pendulum and a horizontal strain seismometer it is possible to determine the actual shape of the Rayleigh wave motion. Preliminary observations suggest that the major axis of this motion may be inclined slightly to the vertical.

Investigation of SV and SH--Similar instrument combinations were used by Gutenberg to study the roles of SV and SH in the S-phase. The agreement between results based on different instrument combinations was good. Best agreement between observed angles of incidence and those calculated from the slope of the travel-time curve was found by assuming, for the calculation, an average velocity of about 3 3/4 km/sec for transverse waves in crustal layers. In teleseismic seismograms no differences in arrival times of SV and SH were found.

Earthquake sequences

The sequence of shallow earthquakes [BENIOFF, 1951] which began April 11, 1925, in the Indian Ocean near latitude 34°S, longitude 58°E, exhibited an elastic strain-rebound characteristic which up to October 23, 1933, was accurately represented by an equation of the form

$$S = a + b \log t \qquad (1)$$

Following the shock of that date the fault was locked strongly, so that no further earthquakes were produced for some 16 years. During this quiescent interval, if strain were accumulating in accordance with (1), it was possible for any given date to predict the magnitude of the earthquake required to reduce the accumulated elastic strain of the fault to zero. Thus in 1949 the accumulated strain was sufficient to generate a 7.5-magnitude earthquake. On October 7, 1949, the sequence became active again with the production of a magnitude 6 3/4 shock. This shock relieved only a small fraction of the strain. However, on December 8, 1951, a magnitude 7.5 shock occurred which relieved all of the accumulated strain to within 0.1 magnitude of the value predicted by the equation. Equation (1) represents an elastic (or recoverable) creep which may be either an elastic after-working following an earlier creep strain accumulation or a forward creep in response to an external force. If the former case is the correct one, this sequence will die out. If the latter obtains, then large elastic and creep strains are stored in the fault blocks outside of the sequence earthquake-generating zone. If these blocks should fracture, a great earthquake of magnitude 8 or larger could be generated.

Benioff has continued his study of seismic sequences in relation to orogeny. The original investigations involved the Tonga-Kermadec and South American regions only. During the year the list of regions has been extended to include Mexico and Central America, the Aleutians, the Kurile-Kamchatka Arc, the Marianas, the Philippines, the Sunda Arc, and the New Hebrides Islands. Evidence from these studies indicates that the principal orogenic structure responsible for each of the great linear and curvilinear mountain ranges and oceanic trenches is a fault. The faults occur either wholly within the oceanic realm or else marginal to the continents. Oceanic faults extend from the surface to depths of 350 to 700 km, the majority falling in the 600- to 700-km depth range. They exhibit an average dip of 60°. Their elastic strain-rebound characteristics show that these faults are composed of two separate mechanical units, a shallow component extending from the ocean bottom to the Mohorovicic discontinuity, which in the vicinity of the fault may be depressed to approximately 60 km, and a deeper component extending from the Mohorovicic discontinuity to the 700-km crustal boundary. The marginal faults occur in dual and triple forms. The dual faults comprise a shallow member extending from the surface to the depressed Mohorovicic discontinuity and an intermediate member extending to a depth of 200-300 km. The average angle of dip is 32°. The marginal triple form is similar to the dual down to the 300-km level. At this depth the angle of dip changes abruptly to 60° to form a third component extending to the 650 ±-km crustal boundary. The elastic strain-rebound characteristics of the marginal faults indicate that the components of these structures also move as separate units, although the two lower elements exhibit evidence for mechanical coupling. In the continental domain, the 300-km level thus represents a tectonic discontinuity not revealed by seismic wave-propagation studies but which is apparently the lower boundary of the continents. In accordance with HUBBERT's [1951] extension of the Mohr stress theory, the 30° dip of the upper two layers of marginal faults may be attributed to a stress pattern in which the greatest principal stress is a horizontal compression. The 60° dip of the oceanic faults and of the lower components of the marginal faults falls within the range of values calculated by Hubbert for normal faults. However, the direction of slip is reversed. Consequently, it must be postulated that these are reverse faults generated by stress patterns in which the greatest principal stress is a vertical tension and the least principal stress is a horizontal compression.

Artificial explosions, wave velocities in the surface layers

Records of atomic explosions in Nevada and of a blast of about 120,000 pounds of DuPont Nitramon, at a quarry of the Minnesota Mining and Manufacturing Company near Corona, California, were investigated by Gutenberg and the results combined with similar data reported previously. Records of different explosions near the same locality show great similarity at a given station. The following average velocities of longitudinal waves in Southern California were found: 5.9 km/sec below the sediments or weathered igneous rocks; a gradual or sudden increase to 6.5 km/sec near a depth of 6 km and to 6.9± km/sec near 10 km; 8.2 km/sec below the Mohorovicic discontinuity at a depth of slightly over 30 km in the lower valley areas, but deeper under at least several mountain regions ("roots of mountains" which have been discussed in previous years).

A "low-velocity layer" somewhere in the depth interval between 15 and 30 km has been inferred from earthquake data. HUGHES and CROSS [1951], working at the University of Texas, have made laboratory experiments with rock samples to find the change in wave velocity with pressure and temperature corresponding to values to be expected in the upper kilometers of the Earth's crust. Their results indicate that for granite, limestone, and shale the rate of increase in velocity with depth decreases under conditions to be expected approaching a depth of six km; a slight decrease of velocity is indicated for dunite and quartzite approaching conditions to be expected at a depth of ten km. Thus there is now some additional experimental support for this hypothetical low-velocity layer. Such a layer would prevent the formation of the $\bar{P}$-phase which is strong in normal earthquake records, but which is doubtful or missing if the source is at the surface.

Waves through the atmosphere

In several instances, sound waves through the air were recorded after explosions. Some seem to have their highest points near 40 to 50 km, but others arriving many minutes later may be conditioned by the low-velocity layer near 80-km elevation and higher temperatures above this level.

Visitors at the Laboratory during 1951

P. G. Gane of the University of the Witwatersrand, Johannesburg, spent six months during the past year doing research at the Seismological Laboratory. A. Galanopoulos of the University of Athens visited the Laboratory for about four months.

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(Communicated manuscript received March 17, 1952;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

ESTIMATION OF MEAN ANNUAL PRECIPITATION FROM SNOW-SURVEY DATA

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Abstract--A high degree of correlation is shown to exist between the mean water equivalent of the snow pack on March 1 at snow courses in the Snake River region and the mean annual precipitation at nearby precipitation stations. Attempts were made to improve the correlation by introducing various additional parameters, but the resulting changes were not significant. In attempting to introduce length of snow-accumulation season as a parameter, a relation between elevation, latitude, and average beginning date of continuous winter snow cover with at least one inch of snow on the ground was developed.

Introduction--In the course of an investigation of topographic influences on mean annual precipitation in the upper and central Snake River regions in the manner discussed by SPREEN [1947], it appeared that the results of the analysis would be seriously impaired by the scarcity of precipitation gages at the higher elevations. Snow surveys, which are generally made at elevations higher than those of most precipitation stations, appeared to be a logical source of supplementary data. The idea of using snow-survey data for estimating annual precipitation is not new and was used by OLTMAN and TRACY [1951] in their study of the Missouri River Basin to determine precipitation at the higher altitudes in Colorado, Wyoming, and Montana.

From the limited data available from snow laboratories (Cooperative Snow Investigations Program sponsored by the Corps of Engineers and the Weather Bureau), it appears that there is good agreement between the total precipitation for the period beginning with the establishment of a continuous winter snow cover and ending March 1 and the water equivalent of the snow pack on that date. The water-equivalent values obtained on the next survey date of April 1 generally do not show such good agreement because some melting usually takes place during March. Consequently, for the sake of greater accuracy, the snow surveys on March 1 were used in deriving a relation between the average water equivalent of the snow pack and mean annual precipitation.

Data used in the analysis--Since there are very few places having both precipitation and water-equivalent measurements, the analysis of the relation between mean annual precipitation and mean water equivalent on March 1 had to be based on snow courses with nearby contemporary precipitation stations at about the same elevation. The first selection of data for the analysis was based on restrictions of distance between snow course and precipitation station to ten miles or less and of difference in elevation to 300 ft or less. It was quickly discovered that there were not enough contemporary stations within these geographic limitations to permit a reliable analysis. The restrictions were then changed to permit the use of stations located within a 20-mile distance and a 400-ft elevation difference and having contemporaneous records for not less than one-third of the period of snow-course records. Twenty-four pairs of stations and snow courses were obtained under these qualifications.

Table 1 gives names, locations, length of record, average water equivalents on March 1, and mean annual precipitation of the snow courses and precipitation stations used in the analysis. In computing mean precipitation for those stations not in operation during the entire period of the corresponding snow-course records, the annual precipitation for the missing years was estimated by first determining the relation between the precipitation for the available portion of the record and that for the corresponding period at neighboring stations. The relation was then used to estimate the missing record. Errors in such estimates are relatively small. The last column in Table 1 gives the number of years for which precipitation was thus estimated. It should be noted that (1) the greatest distance between a snow course and its corresponding precipitation station is 14 miles, the average being slightly over six miles; (2) the largest difference in elevation is 380 ft, with the average being a little under 150 ft; and (3) while one station has as much as 62 pct of its precipitation record estimated, only 16 pct of the total years of record for all 24 stations (398 station-years) consist of estimated amounts.

Relation between average water equivalent on March 1 and mean annual precipitation--Figure 1 shows the mean water equivalent on March 1 plotted against the mean annual precipitation (Table 1).

Table 1--Data used in deriving relation between mean annual precipitation and mean water equivalent on March 1

No.	Snow course	Location		Record	Elev.	Aver. water equiv.	Precipitation station	Distance	Elev.	Annual precip.	Est. record
		Lat.	Long.								
	Idaho	°	°	yr	ft	inch		mi	ft	inch	yr
1	Blackfoot Dam	43.0	111.7	12	6200	7.7	Blackfoot Dam	0	6200	16.2	0
2	Crawford R.S.	44.6	116.0	15	4800	7.5	Alpha	9SW	4800	26.9	0
3	Deadwood Dam	44.3	115.6	14	5500	15.2	Deadwood Dam	2NW	5400	34.1	0
4	Graham Ranch	43.8	114.4	15	6200	11.7	Sun Valley	8SE	5820	18.9	0
5	Island Park No. 2	44.4	111.4	15	6315	14.0	Island Park Dam	4E	6300	29.6	4
6	Kilgore	44.4	111.9	14	6200	9.8	Spencer	14W	5900	19.3	0
7	Redfish Lake	44.1	114.9	14	6600	8.2	Obsidian	9SE	6900	15.6	2
8	State Line	43.6	111.0	15	6400	13.2	Alta, Wyoming	14N	6500	20.7	1
	Montana										
9	West Yellowstone	44.7	110.1	17	6700	9.9	West Yellowstone	1N	6700	21.7	4
	Nevada										
10	Bald Mountain	41.9	119.6	11	6720	3.6	Sheldon	5S	6500	11.4	1
11	Dry Creek	40.9	115.2	17	6500	5.6	Arthur	6SE	6500	16.4	0
12	Fry Canyon	41.6	115.9	17	6800	9.2	North Fork	9SE	6500	10.6	5
	Oregon										
13	Beech Cr. Summit	44.5	119.1	14	4800	6.3	Beech Creek	10NW	4700	19.7	3
14	Eilertson Meadows	44.9	118.1	13	5400	11.5	Rock Creek	3N	5200	21.2	8
15	Olive Lake	44.8	118.6	15	6000	16.7	Olive Lake	3N	5900	32.4	5
	Utah										
16	Silver Lake	40.6	111.6	16	8720	20.4	Silver Lake	0	8700	42.8	0
	Wyoming										
17	Afton R. S.	42.7	110.9	15	6200	4.8	Grover	8N	6100	19.3	2
18	Bechler R. S.	44.2	111.0	12	6400	21.3	Bechler River	3E	6300	42.6	5
19	Bryan Flat	43.3	110.6	15	6250	9.1	Jackson	14NW	6200	16.7	0
20	Deadman Ranch	43.0	110.8	15	6534	9.6	Bedford	6SW	6200	21.9	1
21	East Rim Divide	43.2	110.1	11	7950	9.7	Kendall	9NE	7600	19.0	4
22	Moran	43.9	110.6	32	6800	9.1	Moran	2NE	6700	21.4	0
23	Moran Bay	43.9	110.8	32	6800	15.2	Moose	11S	6600	25.5	8
24	Snake River	44.1	110.7	32	6780	15.1	Snake River	1N	6900	29.9	9

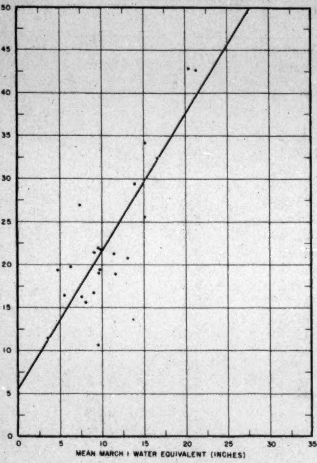

Fig. 1--Relation between mean annual precipitation and mean water equivalent on March 1 in the Snake River region

Least-squares analysis yielded the relation

$$Y = 5.45 + 1.60X \ldots\ldots\ldots\ldots (1)$$

where Y is the estimated mean annual precipitation and X is the mean water equivalent on March 1. The correlation coefficient is 0.85, which is highly significant at the 0.1-per cent level, and the standard error of Y for X = 5, 15, and 25 is 4.4, 4.3, and 5.0 inches, respectively.

It was reasoned that the effectiveness of the mean water equivalent on March 1 as an index of mean annual precipitation might depend on other factors, and attempts were made to improve the correlation by introducing a second parameter. Among those tested were (1) the average beginning date of winter snow cover, (2) the ratio of mean November-February precipitation to mean annual precipitation, and (3) elevation. None of these improved the original correlation significantly.

Average beginning date of winter snow cover--Although the introduction of the average beginning date of continuous winter snow cover as a second parameter in the relation of Figure 1 did not improve the correlation significantly, the relation developed for estimating the average beginning date of winter snow cover is so useful in itself that a detailed description of the method used in deriving it is justified.

Dates of beginning of snow accumulation are not generally available for snow courses, so records of precipitation stations having good snow-on-ground data were used in the analysis. The minimum depth of snow cover was selected as one inch, and the date of beginning is therefore the first day on which there was a snow cover of one inch or deeper which persisted until March 1. Only stations having nine or more years of record with at least 80 pct of the years showing a continuous winter snow cover were used. (A minimum of ten years of record was originally set, but this number was reduced to nine to permit the inclusion of two high-elevation stations with very good records.) There were 47 stations in or near the area of interest that met these qualifications. Table 2 lists these stations together with pertinent data.

There are undoubtedly many factors affecting the beginning date of snow cover, but, since the relation was originally to have been applied to snow-course data, the selection of parameters to be used in the relation had to be restricted to those on which information would be available for the snow-course sites. Of the several parameters tested, elevation and latitude gave the best results. Least-squares analysis yielded the relation

$$X_1 = 696.63 - 64.41\,X_2 - 14.48\,X_3 + 1.330\,X_2X_3 \ldots\ldots (2)$$

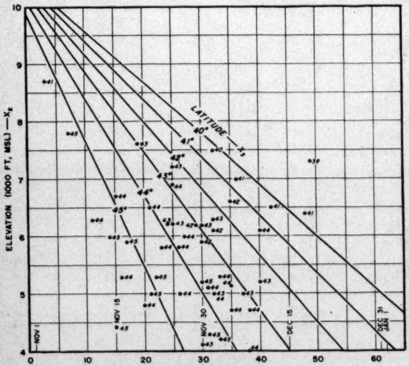

Fig. 2--Relation for estimating mean beginning date of winter snow cover in the Snake River region

Table 2--Data used in deriving relation for estimating average beginning date of winter snow cover

No.	Station	Ratio of record[a]	Aver. date beginning snow cover	Elevation	Latitude
				1000 ft	°
	Idaho				
1	Alpha	27/28	12/5	4.8	44.4
2	Ashton	30/34	12/1	5.1	44.1
3	Atlanta	24/26	11/27	6.0	43.8
4	Blackfoot Dam	24/25	11/25	6.2	43.0
5	Boulder Mine	11/11	11/20	4.8	43.9
6	Cuprum	12/12	12/1	4.3	45.1
7	Deadwood Dam	18/18	11/16	5.4	44.3
8	Dubois	15/18	12/5	5.1	44.2
9	Geneva	10/12	11/29	6.2	42.4
10	Grays Lake	21/22	12/2	6.3	43.1
11	Grimes Pass	39/39	11/26	5.0	44.0
12	Grouse	15/16	12/10	6.1	43.7
13	Hailey	37/39	12/3	5.3	43.5
14	Hill City	21/24	12/7	5.0	43.3
15	Idaho City	30/35	12/8	4.0	43.8
16	Irwin	29/35	12/10	5.2	43.4
17	Little Camas	15/16	12/2	5.0	43.4
18	Loon Creek	9/11	11/22	5.3	44.5
19	McCall	32/33	11/21	5.0	44.9
20	Montpelier	25/31	11/30	5.9	42.3
21	Obsidian	36/36	11/25	6.9	44.0
22	Pine	27/29	11/30	4.1	43.3
23	Prairie	15/16	12/8	4.7	43.5
24	Sheep Hill	30/32	12/2	5.0	43.7
25	Soldier Creek	36/36	11/23	5.8	43.5
26	Sun Valley	12/12	11/26	5.8	43.7
	Nevada				
27	Gold Creek	15/15	12/5	6.6	41.8
28	Kimberly	10/11	12/19	7.2	39.3
29	North Fork	34/37	12/12	6.5	41.5
30	Tuscarora	22/28	12/18	6.4	41.4
	Oregon				
31	Austin	22/25	12/3	4.2	44.6
32	Cornucopia	25/25	11/15	4.4	45.0
33	Ibex Mine	13/13	11/14	6.0	44.8
34	Olive Lake	27/27	11/17	5.9	44.8
35	Rock Creek	27/31	11/30	5.2	44.9
	Utah				
36	Park City	8/9	12/6	7.0	40.6
37	Silver Lake	10/10	11/3	8.7	40.6
38	Soldier Summit	7/9	12/2	7.5	39.9
	Wyoming				
39	Afton	32/37	11/30	6.2	42.8
40	Alta	36/38	11/21	6.5	43.8
41	Bechler River	19/19	11/11	6.3	44.2
42	Bedford	35/36	11/24	6.2	42.9
43	Border	29/38	12/2	6.1	42.2
44	Kendall	25/26	11/19	7.6	43.2
45	Lake Yellowstone	32/32	11/7	7.7	44.6
46	Moran	35/38	11/15	6.7	43.9
47	Pinedale	29/31	11/25	7.2	42.9

[a] Ratio of number of years with continuous winter snow cover to number of years of record.

where X_1 is the estimated average beginning date of continuous winter snow cover, in days after October 31; X_2 is the station elevation, in thousands of feet above mean sea level; and X_3 is the latitude, in degrees. The coefficient of multiple correlation is 0.76, which is highly significant at the 0.1-per cent level, and the standard error of estimate is 6.9 days. The relation is shown graphically in Figure 2.

The other parameters tested were: (1) mean December 10,000-ft free-air temperature estimated from mean upper-air charts, (2) ratio of mean November-February precipitation to the mean annual, (3) aspect, and (4) longitude. The first two showed the most promise, but were so intercorrelated with latitude that their introduction as additional parameters in the relation of Figure 2 did not significantly increase the correlation. Items (3) and (4) showed little, if any, relation to beginning date of snow cover.

Acknowledgments--Max A. Kohler of the Weather Bureau's Hydrologic Services Division made several valuable recommendations during the course of the study and reviewed the manuscript. Several changes were made in the original manuscript following suggestions by H. C. S. Thom and Walter T. Wilson.

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U. S. Weather Bureau,
Washington 25, D. C.

(Communicated manuscript received January 2, 1952;
open for formal discussion until March 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 5 October 1952

DISCUSSION OF

"GRAPHICAL DETERMINATION OF SPECIFIC-VOLUME ANOMALY AND CURRENT"

by E. B. CALLAWAY

[Trans., v. 32, pp. 719-728, 1951]

P. H. Ottosen (Scripps Institution of Oceanography, La Jolla, California)--Ocean surveys in the California Current may expect to encounter temperatures from 3°C to 20°C, salinities from 32.50 $^0/_{00}$ to 34.60 $^0/_{00}$, and $10^5\Delta_{st}$ values from 70 to 390 with depths down to 1000 m. These ranges involve four charts outlined in Figure 1 and one chart in Figure 5 of Callaway's article and subsequently published by the U. S. Navy Hydrographic Office. A single alignment chart can be prepared for the computation of the dynamic depths for the California limits.

The original of Figure 7 is a chart 18 × 24 inches, with the scale for salinity one inch equals 0.10 $^0/_{00}$; the function δ was one inch equals four units of δ, except for the scale marked Δ where the scale was one inch for two units of δ. The values of α_t were plotted at their proper places opposite the δ scales and were obtained from SVERDRUP and Others [1946, Table III, p. 1055]. Next, a table (Table 3) showing the relationship between T, S, and δ was prepared in conformity with the empirical values given in Knudson's Hydrographical Tables. Tables of interpolation by LA FOND [1951] reduced the labor of preparation materially. Within the comparatively narrow limits proposed for the chart, assume that the table can be expressed by the equation

$$f_1(\alpha_t) + f_2(S)\,f_3(T) = f_4(T)$$

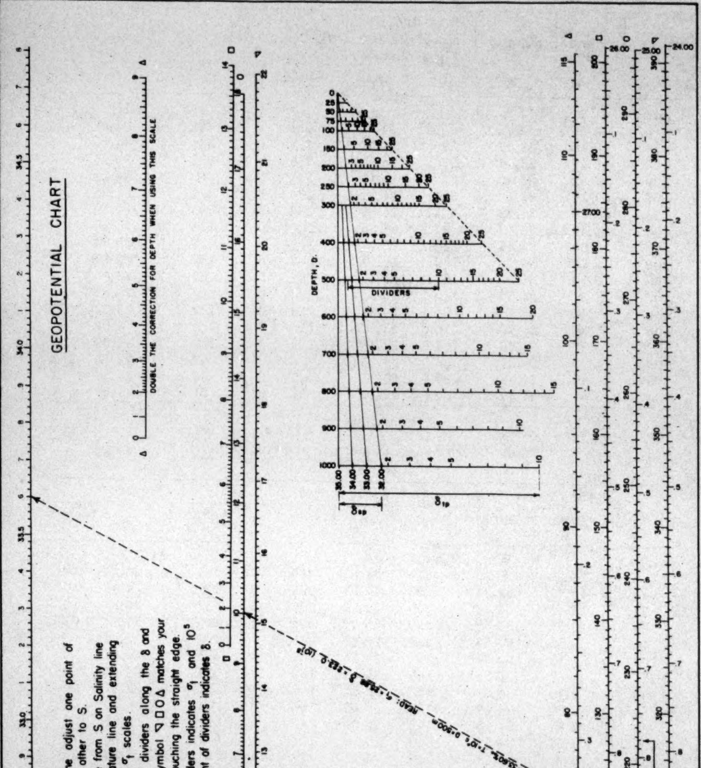
This can be plotted as illustrated by LIPKA [1918, p. 107]. The assumption is warranted by the results of the completed chart (Tables 4, 5, and 6).

Table 3--Relationship between T, S, and δ

Temp. T	δ for various salinities			
	33.00 $^0/_{00}$	33.50 $^0/_{00}$	34.00 $^0/_{00}$	34.50 $^0/_{00}$
°C				
2	164.2	126.2	88.3	50.4
3	171.8	134.0	96.3	58.6
4	180.7	142.6	105.4	67.8
5	190.8	153.3	115.7	78.2
6	201.9	164.5	127.1	89.7
7	214.2	176.9	139.6	102.3
8	227.6	190.4	153.2	116.0
9	242.0	204.9	167.8	130.7
10	257.4	220.4	183.4	146.4
11	273.8	236.8	199.9	163.0
12	291.1	254.3	217.5	180.6
13	309.4	272.7	235.9	199.2
14	328.7	292.0	255.4	218.8
15	348.8	312.2	275.6	239.0
16	369.8	333.3	296.8	260.3
17	391.7	355.2	318.8	282.4
18	414.5	378.1	341.7	305.4
19	438.1	401.8	365.5	329.2
20	462.6	426.3	390.0	353.8
21	487.8	451.6	415.4	379.2
22	513.9	477.8	441.6	405.4

Table 4--Comparison of Figure 7 with H. O. Misc. 15047 and 15078

Temp. T	Salinity S	Figure 7		H. O. Charts		Differences	
		δ	α_t	δ	α_t	δ	α_t
°C	$^0/_{00}$						
3	32.5	208.8	25.92	208.5	25.90	+0.3	+0.02
6	32.5	239.2	25.60	239.3	25.60	-0.1	0
9	33.0	242.0	25.58	242.0	25.57	0	+0.01
12	32.5	327.7	24.68	328.0	24.68	+0.3	0
15	33.0	348.8	24.46	248.5	24.45	+0.3	+0.01
18	33.3	393.0	23.99	392.5	24.00	+0.5	-0.01
3	33.6	126.3	26.79	126.5	26.79	-0.2	0
6	33.6	157.0	26.47	157.0	26.47	0	0
9	33.6	197.3	26.04	197.5	26.04	-0.2	0
12	33.6	246.9	25.53	247.0	25.52	-0.1	+0.01
15	33.6	304.6	24.92	304.6	24.92	0	0
18	33.6	371.0	24.22	371.0	24.22	0	0
3	34.3	73.8	27.35	73.5	27.34	+0.3	+0.01
6	34.5	89.9	27.18	90.0	27.16	-0.1	+0.02
9	34.5	130.3	26.76	130.5	26.74	-0.2	+0.02
12	34.5	180.6	26.22	180.3	26.21	+0.3	+0.01
15	34.5	239.0	25.61	239.0	25.60	0	+0.01
18	34.5	305.3	24.91	305.0	24.90	+0.3	+0.01


GEOPOTENTIAL CHART
SALINITY, S.
TEMPERATURE, T.
DOUBLE THE CORRECTION FOR DEPTH WHEN USING THIS SCALE
DEPTH, D.
DIVIDERS
δ AND σt SCALES
(10)^5 δ
σt
DIVIDERS

DIRECTIONS FOR USE

FIRST: On proper depth line adjust one point of dividers to T, the other to S.

NEXT: Place straight edge from S on Salinity line thru T on a Temperature line and extending across the δ and σ_t scales.

FINALLY: Place the adjusted dividers along the δ and σ_t scale whose symbol ▽ □ ○ △ matches your T line, and lightly touching the straight edge.

RESULT: Lower point of dividers indicates σ_t and 10^5 Δ_{st} and upper point of dividers indicates δ.

Fig. 7.--Geopotential chart

Table 5--Dynamic calculations and comparisons

Depth	Temp. T	Salinity S	H. O. Pub. 614, pp. 23-25 and Fig.24 and 25		H. O. Charts		Figure 7			
			$10^5 \delta$	α_t	$10^5 \delta$	Δ	$10^5 \delta$	Δ	α_t	Δ
decibar	°C	^{o}/oo								
0	18.17	33.46	385.0	24.07	385	0	385	0	24.08	+0.01
10	18.17	33.46	385.3	24.07	385	0	385	0	24.07	0
20	18.17	33.46	385.7	24.07	386	0	385	-1	24.07	0
30	17.92	33.45	380.9	24.12	381	0	380	-1	24.12	0
50	17.06	33.42	364.0	24.31	364	0	364	0	24.31	0
75	15.85	33.42	338.0	24.59	338	0	338	0	24.59	0
100	14.17	33.42	303.8	24.95	304	0	304	0	24.95	0
150	10.62	33.49	234.3	25.69	235	+1	234	0	25.69	0
200	9.10	33.78	189.1	26.17	189	0	189	0	26.17	0
250	8.10	33.95	162.4	26.46	162	0	162	0	26.46	0
300	7.39	34.03	147.1	26.62	147	0	147	0	26.62	0
400	6.40	34.10	130.1	26.81	130	0	130	0	26.82	+0.01
500	5.80	34.20	116.0	26.97	116	0	116	0	26.97	0
600	5.27	34.26	106.1	27.08	106	0	106	0	27.08	0
700	4.90	34.34	96.5	27.18	96	0	97	0	27.18	0
800	4.55	34.40	89.0	27.27	89	0	89	0	27.27	0
1000	3.95	34.43	81.5	27.36	81	0	81	0	27.36	0

Table 6--Dynamic calculations and comparisons

Depth	The Oceans, p. 411				H. O. Charts		Figure 7			
	Temp. T	Salinity S	$10^5 \delta$	α_t	$10^5 \delta$	Δ	$10^5 \delta$	Δ	α_t	Δ
decibar	°C	^{o}/oo								
0	14.22	33.25	315	24.81	315	0	315	0	24.81	0
10	13.72	33.24	306	24.91	306	0	306	0	24.91	0
25	13.71	33.24	306	24.91	306	0	306	0	24.91	0
50	13.35	33.30	296	25.03	295	-1	296	0	25.02	-0.01
75	9.96	33.57	217	25.86	216	-1	216	-1	25.86	0
100	9.38	33.84	187	26.17	187	0	187	0	26.17	0
150	8.82	33.98	169	26.37	169	0	169	0	26.37	0
200	8.48	34.09	157	26.51	156	-1	157	0	26.51	0
250	8.30	34.16	150	26.59	150	0	150	0	26.59	0
300	7.87	34.20	141	26.69	141	0	141	0	26.69	0
400	7.07	34.20	132	26.80	131	-1	132	0	26.80	0
500	6.14	34.26	116	26.97	116	0	116	0	26.98	+0.01
600	5.51	34.35	103	27.12	102	-1	103	0	27.12	0
800	4.65	34.42	88	27.28	88	0	89	+1	27.28	0
1000	3.99	34.44	82	27.36	81	-1	82	0	27.37	+0.01

Corrections for depth were plotted with depth at 100 decibars per inch and the corrections at the same scale as for δ above, one inch equals four units. Values were from SVERDRUP and Others [1946, Tables IVa and Va]. They were plotted from S equals 35.00 ^{o}/oo as zero. This permits δ_{sp} and δ_{tp} to be combined algebraically for the desired depth. With a piece of paper or with dividers extending from the S-value to the T-value, the correction δ_1 can be applied to the basic δ scale. Thus the anomaly of dynamic depth corrected for depth can be directly read on the chart and also α_t.

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U. S. NAVY HYDROGRAPHIC OFFICE, Charts H. O. Misc. 15047 and H. O. Misc. 15078.

Elliott B. Callaway (U. S. Navy Hydrographic Office, Division of Oceanography, Washington 25, D. C., Author's closure)--For limited ranges of the variables involved, there are doubtless many graphical solutions of the problem of specific volume anomaly determination, utilizing both rectangular coordinate systems and alignment charts with non-linear scales, which afford adequate accuracy with fewer charts than the method described.

However, since the adoption of graphical methods in the Division of Oceanography of the U. S. Navy Hydrographic Office, the initiation of a program for the evaluation of a very large quantity of oceanographic data, running into well over a million observations, has rendered imperative the use of more rapid methods. To this end the Hydrographic Office is now employing a card-programmed electronic computer, which performs the necessary mathematical computation of specific-volume anomaly, for a particular observation, in a matter of microseconds, the speed being limited only by the time required in the mechanical process of passing punched cards through the machine.

DISCUSSION OF

"ON THE ORIGIN OF RIVER MEANDERS"
by P. WILH. WERNER

[Trans., v. 32, pp. 898-902, 1951]

T. M. Chacinski and J. R. D. Francis (Department of Civil Engineering, Imperial College, London, England, received April 7, 1952)--In developing his formulas the author assumes that meandering is initiated by forced oscillations and he postulates some cause which will bring about the forced oscillations; for example, a tributary confluence, a sand bar, or the like. So it appears that if the forced oscillations are in fact the cause of meandering, they are not the prime cause but rather the result of some other cause or causes. In his conclusions the author states rather cautiously that his formulas do not take into account the entry conditions. However, early in the paper he states that the first serpentine bend provides the disturbance for the next bend and so on. There is no doubt that this is so, but surely it contradicts the statement about the entry conditions, since the first bend can be regarded as the entry to the second, the second for the third, and so on.

Regarding the influence of the entry conditions, certain experiments on the secondary flows in river bends recently carried out at the Hawksley Hydraulic Laboratory are informative. They concern the motions in a series of bends forming a simplified meandering channel of rectangular cross section six inches wide with five 45° bends of 39-inch radius connected by straights 18 inches long. The entry conditions were controlled at will by a series of bottom and surface vanes. The secondary currents were measured at the walls and bottom of the channel and also throughout the cross section.

According to the entry conditions, many different kinds of motion occurred in the bends, though usually they conformed to one or other of the patterns shown in Figures 2, 3, and 4. I is the inner wall, and O the outer.

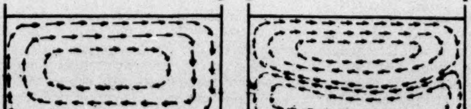

Fig. 2--Patterns resulting from parallel entry to first bend; (a) at mid-section of first bend; (b) at mid-section of second bend

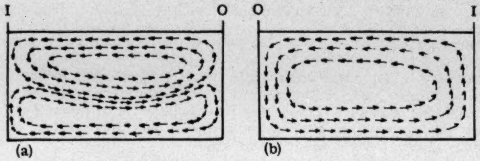

Fig. 3--Patterns resulting from a weak spiral flow at the entry to first bend; (a) at mid-section of first bend; (b) at mid-section of second bend

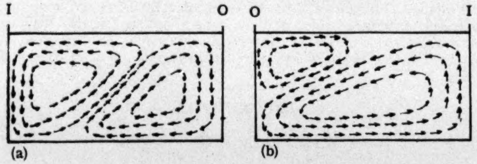

Fig. 4--Patterns resulting from a strong spiral flow at entry to first bend; (a) at mid-section of first bend; (b) at mid-section of second bend

It is seen that only when the entry is parallel does the first bend display the usual helicoidal flow. The second bend then displays two long helices one on top of the other, and the top helix is actually the continuation of the one in the first bend. For parallel entry, the third and fifth bends behave exactly as the first bend, and the fourth bend as the second bend.

By introducing an artificial secondary flow at the entry to the first bend it is possible to change the secondary pattern to that shown in Figure 3, a change which is almost immediately reflected in all subsequent bends. For example, there is no difficulty in making the first bend display the same secondary as the second bend did for parallel flows, in which case the second bend acts as the first, and so on.

If the artificial secondary at the entry is of great strength, then the first bend loses its normal character with two helices one on top of the other; instead, two helices appear in parallel, one in the sense due to the bend, and the other in the opposite sense. And in this case, when the channel is conveying sand there appears to be a strong tendency to form a diagonal bar across the exit from the bend. The sketches show the most frequent or normal cases, but the number of intermediate patterns is infinite, depending upon the secondary at entry which may form one helix, two symmetrical helices in parallel, or any other asymmetrical combination.

The experiments, so far as they have progressed, have established: (1) The secondary flow in bends is sensitive to any secondary flow existing at entry. (2) The secondary flow persists for a great distance before being dissipated into parallel flow. (3) The entry conditions control the secondary along the whole length of a meandering channel of uniform cross section. (4) An infinite number of secondary patterns can exist. The most frequent patterns are either a single helix, or two helices one on top of the other, when the bends are connected with short straights.

As far as we know, (2), (3), and (4) have not been described before. In view of them it seems evident that meandering may be caused by secondary flow at the entry to the channel, or by any asymmetry in the entry; this does not prove that if the entry were perfectly parallel, and the bed perfectly homogeneous, and the turbulence isotropic, a straight channel would remain straight. In fact, the majority of research workers on meandering channels report that any straight channel in a movable bed will meander after a short time, and they also state that from the very beginning the bed material moves diagonally to the channel axis along some curved path. In only a few isolated cases has it been reported that a straight channel remained straight.

However, it seems unnecessary to postulate any additional cause for meandering beyond the existence of some asymmetry or secondary flow at the entry to any section, and that is probably the prevailing state of affairs in nature. Actually, it is an observed, but still not well understood, fact that a double or multiple helix often exists in a straight channel, so that the statistical probability of the existence of helical flow at any entry must be great.

In the past most of the experiments on meandering seem to have suffered on two points: First, an absence of effective control of entry, and second, an insufficient recognition and measurement of the secondary currents, which are indeed very difficult to observe in a channel with a movable bed. Paradoxically, practically all observers verbally stress the importance of the entry and report the existence of the secondary. However, in fact, in most cases, the entry is experimentally ignored, and the secondary is dismissed on some pretext; for example, as being too weak to carry the bed load across the channel. But, in fact, although the secondary in a normal bend with two helices in horizontal or in vertical does not carry the bed load across the channel in that same bend, the bed load may cross the channel in the connecting straight or in some section farther down, exactly as has been observed very often in meandering models, and in natural rivers.

We believe that very many of the other observed facts can be simply and adequately explained if the secondary is taken into account, although in the later stages of meandering other factors, including those mentioned by the author, influence the mechanism of meandering. In the initial stages, however, the entry flow and inevitable bend secondary seem to be the controlling factors. A detailed account of the experiments mentioned is being prepared, and will be published in due course.

C. J. Posey (State University of Iowa, Iowa City, Iowa, received May 21, 1952)--It seems illogical to look for the origin of river meanders in conditions existing in a perfectly straight stretch of river. Although laboratory experiments may be started with a straight channel molded in erodible material, natural rivers are practically never straight, and most meandering rivers were probably never straight at any time in their geologic history. Every curve that exists at the time the stream is first able to meander will have, by virtue of the phenomenon of transverse oscillation, an influence that extends downstream in lessening amount. The river responds to the algebraic sum of all such oscillatory effects.

The factor of valley slope should also be carefully examined for clues as to the origin of meanders. Did the river or its pre-glacial ancestor build the alluvial plain? Was the plain subsequently tilted? Was the area to be occupied by the plain delimited? The tortuous meanders of the River Jordan, cut in an alluvial plain bounded by parallel walls, bear little similarity to those of a stream crossing delta-shaped alluvial deposits.

Measurements on a small tributary of the Cache la Poudre (Escobar, Daniel, A study of stream meanders, unpublished Master's Thesis, State University of Iowa, April, 1944) throw light on some of these questions, and also upon other questions of importance in stream measurements. This tributary descends the glacial deposits on the lower sides of a steep glacial valley in tortuous curves which, if not true meanders, certainly resemble them, and emerges onto its own delta, where its curves are less sharp and more regular. The slope based on talweg distances shows a very gradual lessening throughout with no discontinuity at the beginning of the delta.

Certainly much additional data of improved accuracy will have to be gathered and studied (Nuguid, Cezar P., A study of meanders, unpublished Master's Thesis, State University of Iowa, August, 1950) before this fascinating phenomenon can be adequately described by mathematical formulas.

P. Wilh. Werner (Vattenbyggnadsbyrån, 29 Humlegårdsgatan, Stockholm, Sweden, Author's closure)--Helicoidal flow, including extended spiral motion and double spirals, has been studied for a long time. It is discussed at some length even by de GEER [1911], who in his investigation of the Klarälv River in Sweden refers to several preceding papers on the subject. Now the discussion by Chacinski and Francis seems to convey the meaning that helicoidal flow, caused by the entrance conditions, is the one controlling factor in meandering. This view touches an old controversy. In the writer's opinion, it is an exaggeration, and does not agree with the Vicksburg experiments, in which a careful attempt was made to analyze, by separation, the various influencing factors, including the angle of attack.

In this connection, it is of value to note MATTHES' [1941] study of the problem, from which we may quote:

"Helicoidal flow--This takes place in distorted models, and in channels which are deep relative to their width. Its well-known occurrence there has created the impression that helicoidal flow also takes place in meandering rivers. The writer's studies indicate that streams engaged in meandering in alluvial deposits are notoriously shallow in cross section and are devoid of helicoidal motion. In fact, helicoidal flow is almost physically impossible of occurrence in them. No evidence of helicoidal motion has been found in the meandering portion of the Mississippi River, nor in lesser meandering streams examined by the writer."

There need not be any doubt, however, that the entrance conditions affect the meander pattern. This is evidenced also by the Vicksburg tests, and also clearly stated in the paper although, so far, due to lack of adequate data, not incorporated in the formulas. It is hoped that the experiments referred to by Chacinski and Francis will be published soon and in full, to promote further studies.

Posey rightly stresses the point of complexity of the meander phenomenon. However, the writer cannot agree with Posey's statement as regards the procedure of solving the problem. In the writer's opinion, a problem of this nature is most suitably attacked by cooperation of theoretical considerations and practical investigations, including model tests. Moreover, even though a meander pattern in natural rivers may be the sum of a number of oscillatory effects, it would seem logical first to reduce the elements to their simplest form in order to arrive at the fundamentals of the phenomenon.

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Transactions, American Geophysical Union Volume 33, Number 5 October 1952

REVIEWS, ABSTRACTS, AND BIBLIOGRAPHY

REVIEWS AND ABSTRACTS

CLARK, DAVID, Plane and Geodetic Surveying, v. 2, 4th ed., revised and enlarged by James Clendinning, Constable and Co., London, 582 pp., 1951.

This fourth and revised edition is a text on geodetic surveying which deserves a wider use in the United States.

In its first two chapters comprising 140 pages, there is given a sound treatment of geodetic astronomy preserving an excellent balance between theory and practice. The first chapter, Field astronomy - Introductory, provides a concise presentation of the useful background mathematics and formulas, careful definitions of the elements of the celestial triangle, and a discussion, complete with detailed examples, of the use of almanacs and ephemerides. Minor flaws were noted. Bessel's refraction values, see page 19, are recognized as being in need of revision. They are consistently higher than Pulkovo values. The statement on page 21 that the rate of rotation of the Earth is perfectly uniform, while acceptable for most purposes, is considered out of place in a modern text on geodesy. The detailed presentation of interpolation on page 27 is marred by a reversal of sign in the definition of $\Delta'_{-1/2}$. Some confusion is introduced unnecessarily into the discussion of the equation of time on page 24, because Figure 19 depicts the correction in the form mean minus apparent. More important, from the point of American practice, is the absence of a discussion of the format of the American Nautical Almanac, in which sidereal hour angles and Greenwich hour angles are listed, rather than the conventional right ascension.

In the second chapter, Field astronomy - Observations, the usual discussion of instruments and instrumental adjustments and errors is supplemented by sections on the de Graaff-Hunter shutter eyepiece and on the prismatic astrolabe. The standard methods for determining time, azimuth, latitude, and longitude are discussed, distinction being made between primary and secondary methods. A very desirable feature is that the derivation of formulas for correction terms is either given in full or the method of derivation indicated.

Also, two methods for simultaneously determining latitude, longitude, and azimuth attributed to de Graaff-Hunter have been added in the revision. The first of these, observation of the times of transit and the altitudes of two stars when crossing the same vertical plane, seems to be essentially identical with a method mentioned earlier in the chapter for which reference is given to an article by McCaw in the Empire Survey Review. McCaw, however, derives formulas valid even when the azimuth of the vertical plane is large. In passing, it might be mentioned that the formulas for the solution of this problem were given in 1890 by W. F. Wislicenus in the Astronomische Nachrichten, v. 124, no. 2958. Wislicenus believed that his results were new, but Niethammer (Die simultane Bestimmung der Zeit, der Polhöhe und des Azimutes zweier Richtungen, Verh. der Naturf. Gesell. in Basel, v. 54, p. 327, 1943) suggests that Wislicenus was not familiar with the work of Daniel Bernoulli.

Other interesting features are discussions of the line of position method and of star identification from the measured rate of change of altitude with time. For the last, in the simple formula given, namely, $\sin A = -(1/15)\,\Delta h/\Delta t \sec \phi$, where Δ is in seconds of arc, and Δt is in seconds of time, A should be reckoned clockwise from south, and not, as the author implies, from north.

For some reason, the section on time signals omits reference to the continuous time-signal services maintained by the National Bureau of Standards.

The next four chapters, pages 141-434, cover triangulation, traverse, and leveling in considerable detail. In Chapter III, Geodetic surveying, we find a general exposition of the nature and scope of geodesy followed by detailed discussions of triangulation, traverse, and reconnaissance from the field man's point of view. A desirable feature is the description of the different procedures used in a number of countries, and particularly in the United States. The various instruments and accessories, such as theodolites, signals, tapes, both European and American, are described and compared, and since this is a recent book, discussions of practically all the current models will be found here. Where necessary, error and precision are discussed, but the primary consideration of probability, errors, and adjustments is left to the next chapter.

Chapter IV, Survey adjustment, cannot be expected to replace a treatise on least squares such as Wright and Hayford, but it gives the minimum amount of necessary theory and a clear and careful practical discussion of the formation and solution of normal equations for both conditioned and unconditioned observations.

The section on probable error leaves something to be desired. Working formulas are given only for the probable error of an observation, and not for the probable error of an indirectly observed quantity. Furthermore, the expressions given on page 280 for probable error of an adjusted observation are in error, in that the factor of $[\omega]$ or the sum of the weights in the denominator of the square root should be replaced by q, the number of observations. The effect of this error becomes very plain in the example, pages 423-425, of the adjustment of a level net. Here the probable error of an adjusted observation of unit weight should be ± 0.0095 rather than ± 0.0073. The formulas given will be correct only when the unit of weight is chosen so that the sum of the weights equals the number of observations. If the formula is used as it stands, the computed probable error per mile will depend on the arbitrary choice of the unit of weight.

The methods developed are applied to the adjustment of triangulation and traverse, with the application to the adjustment of leveling reserved for a later chapter.

Getting away from the statistical techniques of adjustment, the theory of the geometry of the ellipsoid of reference is taken up in Chapter V.

We find the two-term expression for Legendre's Theorem, the formula for correcting from spherical to geodesic angles used in electronic surveying, and a consideration of the relative magnitudes involved. Brief developments of the radii of curvature and of meridional distance are given, followed by an outline of the geodetic method for the determination of the figure of the Earth and a datum. This is followed by a detailed treatment of the formulas normally used for the computation of geodetic positions and for the inverse problem. There is no discussion of gravimetric methods for the determination of datum.

The Transverse Mercator coordinates are developed from the rectangular spherical or Cassini coordinates, a method which avoids the usual complex variable treatment of conformal mapping; the Lambert conformal conic projection is developed by scale modification of the ordinary conic projection. The transformation formulas for both projections are given to more terms than are normally needed. One valuable feature is a detailed example of the computation of a triangulation net in terms of Transverse Mercator coordinates. No mention is made of the oblique Mercator (or skew orthomorphic on the ellipsoid), a type of projection which is beginning to be of importance.

In Chapter VI, Geodetic leveling, we find descriptions of the principal geodetic levels in use today, the various types of rods, the procedures to be followed in field work, and a lengthy discussion of the propagation of error in spirit leveling.

The treatment of orthometric and dynamic heights is a little disappointing, not only because of the absence of derivations, but because, although there is a start towards the kind of explanation that is helpful to the beginner, the subject is then dropped. A sample adjustment of a level net by condition equations is given, and, as noted previously, the probable error of an adjusted observation is computed from the erroneous formula on page 280.

There is no discussion of the design of bench marks to minimize seasonal vertical displacement, nor of the problems of regional subsidence.

The chapter closes with a discussion of trigonometric leveling, including the derivation of the basic formulas, the errors involved, and the adjustment of networks.

The next three chapters, Topographical and reconnaissance surveying, Map construction, and Radar ranging and triangulation, are more broadly descriptive than most of the earlier chapters.

Some of the topics treated which might deserve special mention are the extensive treatment of barometric leveling, the sections on stereo-photographic surveying both on land and by airplane, and the sections on map classification and choice of projection. The electronic surveying methods are not discussed in detail, but the principles are explained and the distinctions among the various systems are made clear.

There are five appendices. The first, on Mechanical computing by Comrie, puts forward the advantages of desk machine computation with natural trigonometric functions over the older methods using logarithms, and emphasizes the necessity for the use of formulas suited to machine computation. In future revision of this book, it might be desirable to include a discussion on automatic computing machinery.

Appendices II and III are brief, giving the derivation of a formula for sag-correction and of a theorem in spherical trigonometry, respectively.

Appendix IV is devoted to a discussion of the Bergstrand geodimeter which is a device for measuring base lines by means of modulated light waves. Up till now, no use of this apparatus has been made in the United States, but in the near future it will probably be used extensively to replace the classical but time-consuming method of base measurement.

Appendix V takes up the problem of the computation of the exceptionally long lines encountered in using the recently developed electronic surveying techniques. The formulas recommended are those of Jordan, modified by J. E. R. Ross, and are given for both the direct and the inverse problem. There is no sample computation. It is surprising that no mention is made here of Helmert, in whose Theorien der höheren Geodäsie is found a most detailed development of formulas similar to those of Jordan.

D. A. Rice of the U. S. Coast and Geodetic Survey has arranged the Helmert formulas for the inverse problem in a compact form, using only three short auxiliary tables which are printed on the back of the form. Despite their dissimilar appearances, these formulas go back over 125 years to Bessel, who put them into substantially their present form.

The type and format of the book are clear and readable, and it is reasonably free from typographical errors, although a few were noted. The "Pierce" referred to near the top of page 281 should be "Peirce," admittedly an unorthodox spelling; in the footnote on page 539, read Aslakson for Aslaskan; and near the bottom of page 540, read Bergstrand for Bergshand.

Despite the several flaws mentioned, this book is highly recommended. An outstanding aspect is its liberal use of and reference to the periodical literature in the field. It is evident that considerable effort has been expended, and successfully, in creating a text which is current.

A. D. SOLLINS

BARTH, TOM. F. W., Theoretical Petrology, a Textbook on the Origin and the Evolution of Rocks, John Wiley and Sons, New York; Chapman and Hall, London, 387 pp., 1951.

In recent years there have been published several advanced and up-to-date treatises on igneous petrology, on metamorphic rocks and metamorphism, and on sedimentation and sedimentary petrology, but Barth's is the only recent textbook to undertake the advanced treatment of all three major groups of rocks. In the author's words the book "attempts to give a complete and logical presentation of the principles and underlying theory of the petrology of the rocks of the Earth's crust." He expresses the hope that the book will make it easier to understand and to teach the fundamental physico-chemical principles of petrogenesis.

The introductory Part I (36 pp.) broadens the scope of the book still more. It deals with the origin, heat economy, strength and structure of the Earth, and general geochemical cycles.

Sedimentary rocks are treated very briefly (22 pp.) in Part II from an unusual point of view. Not only is no systematic classification given, but only ten names of sedimentary rocks are mentioned in the entire section. The author purposely limits this part of the book largely to "subjects of fundamental petrological and geochemical significance" and so devotes most of the space to the following topics: Sedimentary differentiation and the Physical chemistry of sedimentation. At the end of Part II the various special types are described very briefly.

As might be expected from the author's background, experience, and interests, by far the largest section of the book, almost half, is devoted to igneous rocks. Here again the primary concern is with theoretical petrologic and geochemical considerations, but a short preliminary section is devoted to descriptive classifications of igneous rocks. Then follow sections on the Physical chemistry of the (rock-forming) minerals, Magmas and their locales, and Rocks as products of defined processes.

The last third of the book (Part IV) is devoted to Metamorphic rocks. The emphasis is still on principles, but most of the well-recognized types of metamorphic rocks are mentioned in the course of the discussion of the various kinds of metamorphism. The facies concept is dealt with at considerable length both theoretically and descriptively, the latter by giving examples from well-studied areas of metamorphic rocks where the facies concept has been applied by various individuals.

Barth has done a fine job of what he set out to do, especially with the igneous and metamorphic rocks. The book is comprehensive in its treatment of petrogenic theory, and the parts of geochemistry, crystal structure, thermodynamics, etc., that are applicable thereto. Conciseness is in some places achieved at the expense of clarity and readability, but the advanced student who is well grounded in classical petrology and physical chemistry, including thermodynamics, will have no difficulty with the continuity or logic of the presentation.

Theoretical Petrology is broad not only in scope, but also in viewpoint. It attempts to give the current status and trends of petrology without regard to personal, institutional, or regional ideas or prejudices, and very nearly succeeds in doing so. It is heartily recommended as a textbook for courses in advanced general petrology or as a reference work.

The field of theoretical petrology has for some years been largely dominated by the writings of those whose viewpoint has been, to a large extent, that of the laboratory worker. In recent years, there has been a swing toward the other extreme of neglecting the data furnished by laboratory studies. It is interesting to observe how Barth, who was for some years a staff member of the Geophysical Laboratory, Carnegie Institution of Washington, and who has been a prominent advocate of some of the newer concepts of granitization, treats the subject of theoretical petrology without doing violence to either point of view.

There are, of course, ideas expressed in this book to which more conservative petrologists will take exception. Perhaps the principal one is the notion that great masses of material have migrated over very long distances as clouds of free molecules and atoms moving through solid rocks along grain boundaries and even through crystalline lattices. The greatest specific distance mentioned, however, over which diffusion appears to have been effective is six meters (Fig. 122, p. 319).

The evidence cited on page 318 for the origin of certain pegmatites by diffusion has also been used by other authors as evidence that the pegmatites were formed by crystallization of a magmatic residue. Concentration of large amounts of rare elements by diffusion through great distances is a difficult process to understand, to say the least. It might also be pointed out that experimental work by ROBERT JAGITSCH [Untersuchung der Reaktionskenetik im System Na_2O-Al_2O_3-SiO_2, Zs. Naturforsch., v. 4a, pp. 97-101, 1949; Geologische Diffusionen in kristallisierten Phasen, Arkiv. Mineral. Geol., v. 1, pp. 65-93, 1949] and others indicates that even at temperatures considerably above those prevailing near the surface of the Earth, diffusion through solids is too slow to allow for important transfer over great distances during geologic time. For example, in the favorable case of the diffusion of Na_2O Jagitsch calculates from his experiments that in 10^8 years diffusion would be effective over a distance of 1000 cm at 700°C; only 1.8 cm at 500°C. It is highly improbable, of course, that such temperatures would be maintained for anything like that length of time at any given place in the Earth's crust.

Evidence is accumulating that the uranium-vanadium deposits of the Colorado Plateau are hydrothermal in origin and not reworked sedimentary deposits, as indicated on page 56.

The statement in paragraph 3, page 153, that the time required for crystallization of a large magma chamber is known is probably too strong. Some other geologists do not agree with Larsen's assumptions and calculations.

Ordinary basalt is commonly taken as the parental magma in discussion of crystallization differentiation, not olivine basalt, as indicated on page 177.

It should be pointed out that most petrologists would not consider the "peridotite dike" mentioned in paragraph 6, page 224, a peridotite at all. If this dike is in reality a lamprophyre, the whole inference to be drawn is changed.

Table 59 on page 227 gives the impression that anorthosite is much more limited in its occurrence, both geographically and in geologic time, than is actually the case.

In paragraph 3, page 221, it is stated that crystals of quartz five meters long are not unusual in pegmatites and on page 318 that in many occurrences these single crystals may attain lengths of ten meters. The largest pegmatite quartz crystal ever described is less than three meters in length, and the largest quartz crystal ever found (in a vein; not yet described in print) is slightly under six meters long.

A few questions of usage, spelling, and definition should be mentioned. Cyanite should now be spelled kyanite (pp. 256-259). Reichardtite (p. 299) is erroneously spelled reichardite; anyway, the synonym, epsomite, is in more common usage.

Readability, clarity, and ease of reference of many parts of the book could be enhanced by some comparatively simple changes and additions in another printing or edition. For example, some 23 figures and at least three tables are not mentioned in the text.

Figures 31-34 are discussed in the text, but the average student would derive much more benefit from them if there were included a detailed description of just how the various points, compositions of crystals and liquids, etc., are determined instead of simple statements of what they are.

If Figures 36 and 65 (pp. 126 and 177), which represent the same system, had the same orientation on their respective pages, comparison and interpretation would be facilitated.

Bibliographic material is very badly handled. Some of the lists of literature are given in the text at the beginning of a section, others as long footnotes, and some are appended separately at the ends of sections. Still other references are given individually as footnotes, or parenthetically in the text. The result is that one does not know where to look for a reference and many of them might as well not be given at all. The reviewer was unable to find some of the references in any of the literature lists, or elsewhere in the book; for example, on page 162, Logario [1887], Teall [1888], Brögger [1890], and Bowen [1915, 1921]. For very few of the references are page numbers given, even for articles in journals.

The index (18 pp.) is longer than average for 369 text pages, but it could be improved in several respects. For example, many entries, such as Barth, Bowen, Calcite, Goldschmidt, Olivine, are followed by long lists of pages, with no indication of the subject matter. It would not be difficult, and would add very materially to the usability of the index, to break these entries down to individual page references by subjects, in the same manner that others are treated, Earth, Metamorphism, Mineral facies, etc. Many of the entries are incomplete. For example, anatexe granites are also discussed on page 255, and anorthositization on page 363. Carbonate sediments, crystallization, feldspar, pyrite, quartz, and other subjects are mentioned on many pages not indicated in the index.

Crystallization differentiation, page 161; Fracture cleavage, page 294; and other subjects should be added to the index. The entry Anorthoclase refers one to Sanidine, but on the page where sanidine is discussed anorthoclase is not mentioned.

The following typographical errors were noted in the index:

Adams, L. A., should read Adams, L. H.
Altvatergebirge, Silecia, should read Altvatergebirge, Silesia.
Argellite should read argillite.
Cancrinite, 89, 200, 203; for 200 read 262.
Clausius-Clapeyron equation, 21; for 21 read 91.
Gibson, E. R., should read Gibson, R. E.
Malachite, 202; for 202 read 262.
Margarite, 260, is out of sequence and so is on the wrong page.
Marble, 260; for 260 read 358.
Omphasite should read Omphacite.
Phosphate deposits, 27; for 27 read 57.
Santiaquito should read Santiaguito.
Uroa should read Urao.

Of the numerous typographical errors noted in the text, the following are perhaps important enough to warrant mention in a review:

In the tables on pages 12 and 25 the symbol for xenon should be Xe instead of X.

Page 75, paragraph 2 from bottom, line 6, mg should read fm.

From the equations near the bottom of page 83 and at the top of page 85 the subscripts 160 should be deleted.

Page 99, paragraph 2, line 5, ternary should read quaternary.

Page 99, Fig. 18, point E is incorrectly marked. See text, paragraph 1, page 100.

Page 151, Fig. 50, Log n should read Log η.

Page 277, first equation, and page 289, equation (9), CO_2 should read $3CO_2$.

Page 288, first equation, molar volumes of $CaCO_3$ and SiO_2 are interchanged.

Page 288, second equation, value for ΔV should be negative (-18).

Page 294, line 4, igneous should read sedimentary.

Page 295, line 3 from bottom, elastically should read plastically.

Page 299, second equation should have $+MgCl_2 + 7H_2O$ added to its right side and $2MgSO_4 \cdot H_2O$ should read $2[Mg_2SO_4 \cdot H_2O]$. Also on the left side $2KClMgSO_4 \cdot 3H_2O$ should read $KCl \cdot MgSO_4 \cdot 3H_2O$.

Page 299, equation in footnote, $2KCl \cdot MgSO_4 \cdot 3H_2O$ should read $2[KCl \cdot MgSO_4 \cdot 3H_2O]$. Also $+7H_2O$ should be added to the right side.

Page 326, under (4), it replaces Al_2O_3 should read Fe_2O_3 replaces Al_2O_3.

Page 337, first equation $KAlSi_3O_3$ should read $KAlSi_3O_8$.

Page 339, Fig. 130, lower right, Anthophyl should read Anthophyllite.

Equation (8), page 343, and equations (9) and (10), page 346, should all have $+H_2O$ added to the right side.

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NOTES AND PERSONALIA

NOTES

Fellowship program of the National Science Foundation--The graduate fellowship program of the National Science Foundation will continue in 1953-1954 along the same basic lines established during the current year. About 600 fellows are now pursuing predoctoral and postdoctoral studies. Application forms may be secured from the National Science Foundation, Washington 25, D. C. Completed applications must be returned by January 5, 1953.

International Bibliography of Geodesy--Work on the International Bibliography of Geodesy has been resumed. Three volumes, published previously, cover the period 1926-1937. The fourth volume, covering the period 1938-1940, is now being printed, and the fifth volume for 1940-1945 is in the advanced stages of preparation. Although this is intended especially for geodesists, Part 8 of each volume lists papers involving constants and properties of the Earth as a whole, and has broad interest in geophysics. Inquiries and orders should be addressed to Association Internationale de Geodesie, U.G.G.I., 19 Rue Auber, Paris 9^{e}, France.

Meeting of the Carolina Geological Society--This Society will hold a field conference in the Great Smoky Mountains, November 1 and 2, 1952, with headquarters at Gatlinburg, Tenn. For further information, write to Philip B. King (Program Chairman), U. S. Geological Survey, Gatlinburg, Tenn.

Department of Meteorology and Oceanography at New York University--The Department of Meteorology of New York University has now been designated as the Department of Meteorology and Oceanography. Gerhard Neumann, who was Visiting Associate Professor of Oceanography in 1951-1952, has now been appointed as Associate Professor of Oceanography.

Seminars of the Geophysics Research Directorate, Air Force Cambridge Research Center--These seminars are continuing through the autumn months. On October 31 the subject will be Magneto-hydrodynamics, and will be conducted by Arthur L. Aden. The seminars are held in the Seminar Room of the Directorate at 415 Summer Street, South Boston, Mass., at 15h 00m.

New index maps showing progress of mapping in the United States--The 1952 edition of index maps showing the status of topographic mapping, aerial mosaics, horizontal control, and vertical control is now available through the Map Information Office, U. S. Geological Survey, Washington 25, D. C.

Gerlands Beiträge zur Geophysik--Earlier plans for the resumption of regular publication of this journal [see Trans., p. 151, v. 31, 1950] were disrupted, but it is now announced that numbers 1 and 2 of volume 62 will appear shortly. G. Krumbach, Direktor, Zentralinstituts für Erdbebforschung in Jena, is the editor.

PERSONALIA

Archie Blake is now head of the IBM Computing Section of the Cornell Aeronautical Laboratory at Buffalo, N. Y.

Robert I. B. Cooper of the Department of Geodesy and Geophysics of the University of Cambridge is to devote the academic year 1952-1953 to work at the University of Toronto.

J. Cl. De Bremaecker has completed his studies for a Ph. D. degree at the University of California, Berkeley. He will spend a month at the Hawaiian Volcano Observatory and a month at the Earthquake Research Institute, Tokyo. After two months in Belgium he will go to the Belgian Congo to establish three seismographic stations in Kivu and Ruanda Urundi for the Institute for Scientific Research in Central Africa, with headquarters at Costermansville, Belgian Congo.

Dan K. Hamilton, formerly with the U. S. Geological Survey, has now been engaged as a geologist with the American Overseas Petroleum Co. with headquarters in New York.

William A. Lamont is now on seismic work with the Geophysical Section of the Atlantic Refining Co. with headquarters at Dallas, Texas.

George F. McEwen, Professor of Oceanography at Scripps Institution of Oceanography and Past President of the Section of Oceanography, AGU, became Professor Emeritus of Oceanography of Scripps Institution of Oceanography on July 1, 1952. He has been associated with Scripps for over 40 years, and will continue his researches in his capacity as Professor Emeritus.

John O. Nigra has resigned his position as research geologist with the New York Office of the Arabian American Oil Company and has established an office as Consulting Geologist at Los Angeles, California.

Robert T. Novotny has entered upon 30 months of geological investigations for the C. J. Longyear Co. in Portuguese East Africa.

Samuel F. Turner, formerly District Engineer with the U. S. Geological Survey with headquarters in Tucson, Arizona, has opened an office as Consulting Ground-Water Geologist, Engineer, and Geophysicist with headquarters in Phoenix, Arizona.

Donald W. Van Tuyl, formerly with the U. S. Geological Survey, has taken up duties in the Natural Resources Department in the Chamber of Commerce of the United States.

Dana Coman, Scientific Adviser to the General Staff of the U. S. Army, died at Fort Churchill, Canada, on January 29, 1952, in an igloo where he was testing cold-weather suits. He was born in Wellsville, New York, in 1895. He took his Master's degree from Harvard University in 1920 and his degree of Doctor of Medicine from Johns Hopkins University in 1924, and an honorary doctorate from Florida Southern College in 1940 where he served as Professor of Nutrition from 1939 to 1946 when he took up his duties with the General Staff. He was on the Byrd Expedition in 1928-1930, the Ellsworth Expedition in 1934-1935, and Leader of the Oceanographic Mid-Pacific Survey of 1936-1937, and was the recipient of many honors and awards. He had been a member of the American Geophysical Union since 1950.

John S. Dodds, Professor of Civil Engineering at Iowa State College, retired, died November 2, 1950. He was born in 1885 at Northfield, Minnesota, and took his baccalaureate degree from Iowa State College in 1912 and his professional degree in civil engineering in 1917. He served as Director of the Surveying Camp of Iowa State College from 1923 until his retirement, as Chairman of the Iowa-Nebraska Boundary Commission, and as a member of the Iowa Board of Engineer Examiners. He filled various posts in state and national societies, his last work being a compilation of state licensing practices for surveyors for the National Council of State Boards of Engineering Examiners. He had been a member of AGU since 1935.

David J. Lewis, former member of the House of Representatives from Maryland, died August 12, 1952, at Cumberland, Maryland, at the age of 83. He was born near Osceola, Pennsylvania, had no formal schooling, and was employed in coal mines from the age of 9 to 23. He studied law and Latin, and was admitted to the bar in 1892. He served in the Maryland Senate, and from 1911-1917 and 1931-1939 served in the House of Representatives of the United States. It was the bill which he advocated in the House that resulted in the inauguration of parcel post; he was frequently referred to as the father of parcel post. Science was his avocation. He was a member of AGU from 1943 to 1948.

Allyn C. Swinnerton, Professor of Geology at Antioch College and Chairman of the Department of Earth Sciences, died suddenly on July 6, 1952, in Springfield, Ohio. He was born in New Berlin, New York, in 1897. He took his baccalaureate degree from Williams College in 1915, and subsequently his master's degree and doctorate from Harvard University. In 1922, he joined the faculty of Antioch College and served as Assistant to the President from 1932 to 1942. He was perhaps the first teacher in the country that taught an integrated course in the Earth sciences. Incidental to his work, he fulfilled various missions with agencies of the Federal Government, such as the U. S. Geological Survey and the Tennessee Valley Authority, and during World War II he served as an officer in the Signal Corps and was awarded the Legion of Merit. In this work he was especially active on the project of developing artificial quartz crystals, which he has continued in association with the Signal Corps as Director of the Quartz Research Project at Antioch College. He had been a member of AGU since 1935, and had served on various committees.

SPECIAL ANNOUNCEMENTS

AGU Elections in 1953--June 30, 1953, ends the current triennium of the American Geophysical Union, and elections will be held in early 1953 to elect officers of the Union and the Sections for the triennium ending June 30, 1956.

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Election procedures were established by the Executive Committee of the Union and by the Sections in 1946 [see Trans., v. 27, pp. 910-914, 1946], modified somewhat for Seismology and Hydrology in 1949 [see Trans., v. 30, p. 927, 1949], and in 1952 for the Section of Geodesy (one nominee for each office is now permitted instead of two). The following is the time-table for the election.

See also p. 94 Dec 49

November 7, 1952--Reports of Committees on Nominations of Union and Section officers to be filed, and, in connection with the Union Nominating Committee, the Executive Committee will perform its functions under item 3 of the procedure established in 1946. The names of "official nominees" of the Union and the Sections will be published in the December, 1952, issue of the Transactions.

February 10, 1953--All nominations by petition for Union and Section officers must be in the office of AGU, including written consent of the petition nominees, by February 10, 1953.

March 1, 1953--Ballots carrying the names of official and petition nominees for officers of the Union and the Sections will be mailed to members not later than March 1, 1953. A brief biographical sketch relating to each nominee will accompany, but not be a part of, the ballot.

March 31, 1953--Balloting will close March 31, 1953. Ballots must be postmarked not later than that date, and be received not later than April 4, 1953, in order to be counted.

May 4-6, 1953--Announcement of the results of the election will be made at the Business Sessions of the Union and the Sections at the Annual Meeting, May 4-6, 1953. Results will also be published in the June, 1953, issue of the Transactions.

Meeting in St. Louis, Mo., December 26-31, 1952--The American Association for the Advancement of Science, with which the American Geophysical Union is now affiliated, will hold its annual meeting in St. Louis, Mo., December 26-31, 1952. The two sessions of the Association that bear on geophysics will be co-sponsored by the American Geophysical Union, one being a Symposium on Tornadoes and Tornado Forecasting on the morning of December 30 being arranged by E. M. Brooks, and the other a general session on Geophysics on the afternoon of December 30 being arranged by Ross R. Heinrich. Inquiries concerning these sessions should be addressed either to Professor Brooks or Professor Heinrich, Institute of Geophysical Technology, 3621 Olive St., St. Louis University, St. Louis 8, Mo.

Meeting in California, February 6-7, 1953, and activity in the Pacific Southwest Region--The Annual Regional Meeting of the Pacific Southwest Regional Branch (Twenty-Ninth Regional Meeting) will be held at Stanford University, February 6-7, 1953. The Committee appointed by the Regional Committee for the meeting consists of David K. Todd (Chairman), College of Civil Engineering, University of California, Berkeley 4, California; Howard H. Engelbrecht, U. S. Weather Bureau, 557 Federal Office Bldg., 50 Fulton St., San Francisco 2, California; R. K. Linsley, 2590 Ross Road, Palo Alto, California; and E. M. Vernon, U. S. Weather Bureau, San Francisco International Airport, San Bruno, California. Inquiries concerning the meeting and word of plans to take part in the program should be addressed to one of these members.

The Regional Committee has been changed through the resignation of R. H. Simpson who has been transferred from Hawaii to Washington, D. C., and the addition of Chas. M. Woffinden who has been elected Chairman of the newly organized Hawaiian sub-branch.

The By-Laws of the Pacific Southwest Branch were published in the Transactions of August, 1952, pages 629-630.

Meeting in Minneapolis in late summer of 1953--While no late word is available regarding the meeting planned to be held in Minneapolis in early September, 1953, this will serve as a reminder of that meeting. See Transactions of August, 1952, page 638, for the preliminary announcement and the members of the Committee, and watch the subsequent Special Announcements for later word.

Thirty-Fourth Annual Meeting, Washington, D. C., May 4-6, 1953--An announced in the August issue of the Transactions, the Thirty-Fourth Annual Meeting will be held in Washington, D. C., May 4-6, 1953 (Monday to Wednesday). Plans for the meeting are under way, and it is anticipated that all Sections will have sessions, as well as the Union as a whole. Place this meeting on your calendar and make plans to come.

This announcement serves also as an invitation for members and others to send word of plans for papers to be presented. Titles of papers with three copies of the abstract should be sent as soon as possible, and not later than March 10, 1953, with advice concerning the time required (most papers are allowed from ten to twenty minutes) and the projection equipment needed. Send this word to the office of AGU at 1530 P Street, N.W., Washington 5, D. C., or to the Secretary of the appropriate Section, as follows:

Geodesy	Frank L. Culley	U. S. Coast and Geodetic Survey Washington 25, D. C.
Seismology	James T. Wilson	Department of Geology University of Michigan Ann Arbor, Michigan
Meteorology	Woodrow C. Jacobs	4302 Kentbury Drive Bethesda 14, Md.
Terrestrial Magnetism and Electricity	L. R. Alldredge	306 William James Hall Harvard University Cambridge, Mass.
Oceanography	Richard H. Fleming	Department of Oceanography University of Washington Seattle 5, Washington
Volcanology	Felix Chayes	Geophysical Laboratory 2801 Upton Street, N.W. Washington 8, D. C.
Hydrology	Ray K. Linsley or	Dept. of Civil Engineering Stanford University Stanford, California
	Ralph N. Wilson (Program Chairman)	3900 Hamilton St., Apt. E-101 Hyattsville, Maryland
Tectonophysics	Wilbur S. Burbank	3346 Runnymede Place, N.W. Washington 15, D. C.

As the Section Secretaries reserve the right to close filled programs in advance of the date indicated, it is desirable that word concerning papers be sent in early.

Complete manuscripts of papers should be forwarded in triplicate to the office of the Union by April 15, 1953, in order that news releases may be prepared. Indicate whether the paper is being submitted for the program only or also for consideration for publication in the Transactions. (Papers for publication only are also welcome for consideration for the Transactions.)

If a paper is written in collaboration, please indicate by whom it will be presented. Arrangements can usually be made for presentation of papers by authors who are unable to attend.

The Chairman of the Committee on Meetings (J. P. Marble, 321 U. S. National Museum, Washington 25, D. C.) invites suggestions and comments that will be helpful in planning this meeting. Especially invited are comments relating to papers and symposia bearing on more than one Section.

Make hotel or room reservations directly with the hotel of your choice at an early date. Failing on the first attempt, bear in mind that advance bookings not taken up are cancelled two weeks in advance, so a follow-up request in mid-April may bring the desired response. We are making block booking at two or three hotels again, an announcement concerning which will appear in the Special Announcements of the February, 1953, issue of the Transactions.

John Putnam Marble, Chairman
Committee on Meetings

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TRANSACTIONS
AMERICAN GEOPHYSICAL UNION

Volume 33 DECEMBER 1952 Number 6

GEOMAGNETIC SECULAR-CHANGE IMPULSES

George B. Walker and Patrick L. O'Dea

Abstract--Secular-change impulses in magnetic declination, dip, and horizontal intensity, derived from world-wide observatory data, tend to congregate around certain years (epochs). This tendency is examined statistically. As indications of the character of secular change, the impulses are well suited for use in the development of isoporic charts. Using a selected group of observatories, precise impulses were determined. These promise to be of value in investigations of the nature of secular change.

Introduction--One of the more difficult problems encountered in the compilation of world geomagnetic charts is the reduction to a particular epoch of world-wide magnetic observations spread over several decades. Secular change at field stations is dependent generally on both time and geographic position, but is too complex to be expressed as a formula that could be applied over a wide area. Consequently, secular-change values can be obtained for field stations only after much laborious preliminary scaling and computation.

As long as the secular change of a magnetic element maintains a consistent, steady pattern, there is no special problem involved; but when the rates of change depart from their former values and establish a new pattern, it becomes necessary to adopt some expedient for dealing with the shift. The principles involved in the so-called impulse method have been in use by the U. S. Coast and Geodetic Survey for a number of years in the reduction of field-station data entailed in the construction of magnetic charts. An isolated instance involving one such shift was that resolved by Howe and Knapp [KNAPP, 1942, pp. 59-60]. About the same time WILLIAMSON [1943] developed and applied the impulse method extensively. This was one aspect of the wartime accomplishment of compiling isogonic patterns of the entire world for use by the aeronautical arms of the military establishment [WILLIAMSON, VESTINE, and JOHNSTON, 1943; SWAINSON, 1944]. Some areas were processed more than once in this program. More recently, in planning the postwar program leading to the 1950 isogonic chart of the world, HOWE [1948] introduced the simplifying modification of adopting identical epochs for impulses in all parts of the world. The successful execution of this program has already been reported [WEBER and ROBERTS, 1951].

This paper is intended to examine some aspects of the impulse method as so developed and modified. The means of determining epochs and the degree of concentration of impulses in various years are shown. On the latter point, results of a detailed study of 20 selected observatories are presented. The adaptability of secular-change impulses to certain forms of geomagnetic research is briefly evaluated.

Description--The impulse method makes use of the fact that observatory annual mean values of a magnetic element plotted against time form a curve which usually exhibits several rather abrupt changes in slope. At these points of abrupt change, the secular-change rate is said to have received an impulse which changes the direction of the curve. Whether the Earth's field actually undergoes changes in the secular-change rate as abrupt as the method assumes (within about 0.1 year) is outside the scope of this discussion. By treating the data as if such changes do occur, the magnetic cartographer is provided with a tool of great practical importance.

Between points of impulse on the observatory curves, the secular-change rate is assumed constant and is represented by straight lines (Fig. 1). These lines intersect at dates of impulse, and the difference between the slopes of two intersecting lines is a measure of the magnitude of the impulse defined by the point of intersection. The assumption of constant rates between dates of impulse curtails sharply the amount of computation in the reduction of field data.

When this process is applied to the curves of a large number of observatories, it becomes evident that the impulses occur much more frequently in certain years. Figure 2 shows the number of impulses occurring in declination, dip, and horizontal intensity at a large number of observatories in the period 1900-1945. It shows graphically the tendency for impulses to concentrate around certain years about five years apart. This natural tendency to group together is utilized

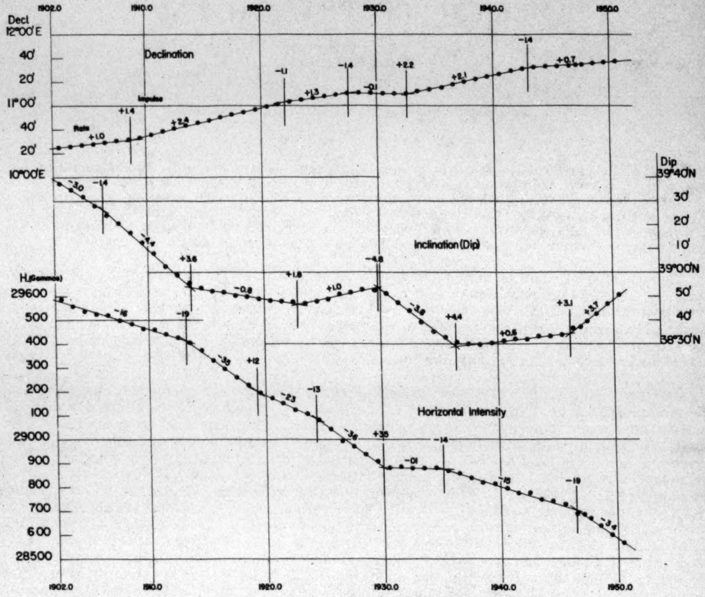

Fig. 1--Rates and impulses determined for Honolulu Magnetic Observatory

in the selection of epochs. The high points on the graph are selected as epochs since they mark years when many observatories show natural changes in their rates of secular change.

Because large areas of the Earth are without adequate observatory coverage, repeat-station data must be employed if we are to gain a reasonably detailed picture of the isoporic pattern, centers of foci, etc. The impulse magnitudes provide secular-change information in a form which is very convenient for incorporating repeat-station data. Using charts of iso-impulse lines, values are obtained from which secular-change rates can be determined even for repeat stations where only two observations (years apart) were made.

Selected observatories--We had on hand the dates of impulse determined for three magnetic elements at seventy-odd observatories. To what extent are these dates dependent on personal opinions? If dates of impulse were redetermined independently for some of these observatories and compared with the dates already on hand, we might have an answer to this question. This was done for 20 observatories, well distributed geographically, for which several decades of data were available. Table 1 gives a summary of the results.

Table 1--Analysis of discrepancies between independently scaled impulse dates

Difference	Magnetic element			Totals
	D	I	H	
year	pct	pct	pct	pct
0	23	38	37	33
1	30	40	32	34
2	18	5	7	10
over 2	1	1	2	1
X	28	16	22	22

A total of 391 comparisons were made. In one-third of the cases the impulses occurred in the same year. In another third the impulses occurred in adjacent years. Just over 11 pct showed a difference of two or more years between impulses in approximately the same position on the curve. The X category, in which 22 pct of the cases fell, is the special case where one determination showed an impulse and the

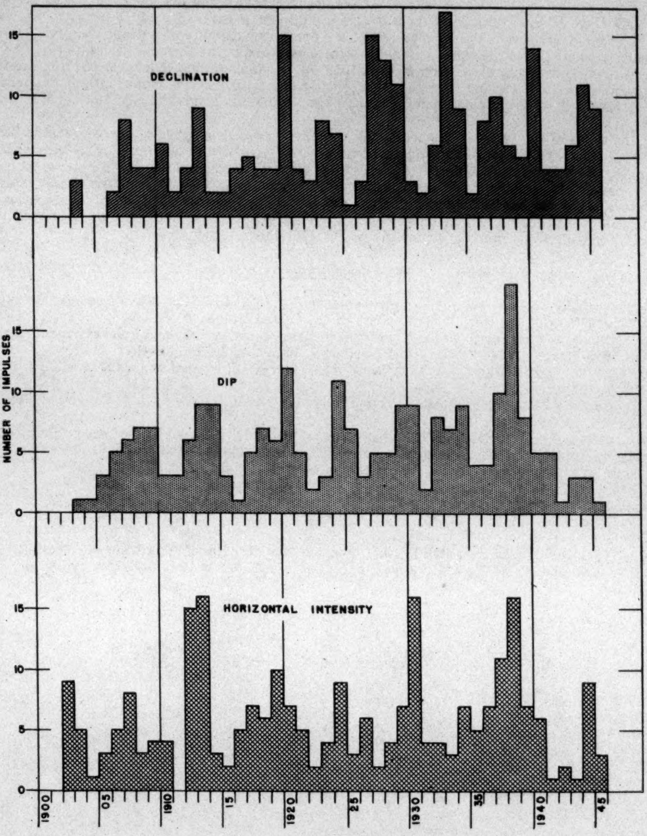

Fig. 2--Distribution of impulse dates at approximately 80 observatories

other showed no impulse in the vicinity. In other words, the same portion of a curve was represented, once by a single straight line, and once by two straight lines intersecting at an angle near 180°. Such cases present no serious difficulty since any differences introduced tend to be self-compensating and would be well within the allowable error. Declination impulses seem to show the greatest deviation. This may be due, at least in part, to the fact that the first declination impulses were determined about two years ago, and in the meantime the technique may have been more completely standardized. The similarity between I and H results also attests to this fact. It was concluded from this comparison that the impulse method is relatively free from personal bias. Comparisons between rates and between impulse magnitudes were deemed unnecessary since these quantities are determined within narrow limits by dates of impulse.

Conclusions--It was hoped when this investigation was begun that these impulses would demonstrate some of the attributes normally associated with secular change. Naturally, observatories in the same area showed impulses of the same approximate magnitude and sign, but other relationships were sought also. Impulse magnitudes were plotted against latitude, times of impulse were plotted against latitude, and various other combinations of magnitude, time, and position were examined, and in each case the results were inconclusive. Apparently the approximate values obtained by the impulse method mask some of the more subtle aspects of secular change.

The value of the impulse method in the construction of isomagnetic charts seems to be recognized. It is another step in removing magnetic cartography from the status of an individualized art to that of a standardized technique. Can this method be adapted to other forms of geomagnetic research? It seems that a tool of this type would be useful in a situation where comparisons between large numbers of observatory curves are necessary. The approximate values thus obtained would probably be sufficiently accurate to indicate trends and could be used to determine the most promising avenues of investigation.

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U. S. Coast and Geodetic Survey,
Washington 25, D. C.

(Manuscript received May 5, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 5, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

GRAVITY ANOMALIES AND DEFLECTIONS OF THE VERTICAL ABOVE SEA LEVEL

R. A. Hirvonen

Abstract--By the aid of Poisson's integral formula, the gravity anomalies, the potential of the disturbing masses, and the deflections of vertical above the sea level can be computed if the gravity anomalies at the sea level are known. This means a generalization of the well-known formulas of Stokes and Vening Meinesz. Some practical applications on the basis of presently available gravity observations are given.

STOKES [1849] published a formula by which we may compute the distance between the geoid and the spheroid at any point when the anomalies of gravity at the geoid are known everywhere around the Earth. **VENING MEINESZ** [1928] modified this formula so as to give the deflection of the vertical at sea level on the basis of the gravity anomalies. Vening Meinesz also showed how to compute the gravity anomalies and the deflections of the vertical for stations which are not situated at the geoid but at any altitude above it. Both Stokes and Vening Meinesz used spherical harmonics to develop their formulas. **PIZZETTI** [1911] derived Stokes' formula using, not spherical harmonics, but Poisson's integral formula. This formula expresses the value of any harmonic function at a point outside of a closed surface when the values of this function are known at all points of the surface. Accordingly, the formula obtained by Pizzetti can be used for the points above sea level as well as for the special case of Stokes.

In recent years, gravity anomalies and deflections of the vertical at high altitudes have become of practical significance because of the development of modern devices capable of reaching these altitudes. Therefore, the practical application of Vening Meinesz's and Pizzetti's high-altitude formulas, which seem to have been long forgotten, derserve some consideration. The strict theoretical refinements and other details of these questions will be omitted in this paper.

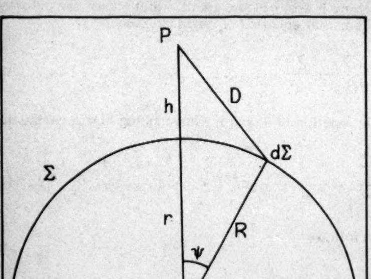


Fig. 1--Diagram illustrating Poisson's integral formula

Poisson's integral formula for a sphere (see Fig. 1) is

$$H_P = \frac{r^2 - R^2}{4\pi R} \int_{\Sigma} \frac{H_\Sigma}{D^3} \, d\Sigma \; \ldots\ldots\ldots \; (1)$$

where H is a harmonic function, Σ a sphere with radius R, P a point at distance $r > R$ from the center of the sphere, and

$$D = \sqrt{R^2 + r^2 - 2Rr\cos\psi}$$
$$= \sqrt{h^2 + 4Rr\sin^2\frac{\psi}{2}} \; \ldots\ldots \; (2)$$

the distance of P from any surface element dΣ of the sphere.

Now the geoid is not exactly a sphere, but is sufficiently close to it for our purpose. The gravity anomaly Δ, on the other hand, is no harmonic function, but we can put $H = r\Delta$. Then the anomaly at height h (see Fig. 2) can be computed on the basis of anomalies Δ_0 at sea level by the integral formula

$$\Delta_P = \frac{r^2 - R^2}{4\pi r} \int_{\Sigma} \frac{\Delta_0}{D^3} \, d\Sigma \; \ldots\ldots \; (3)$$

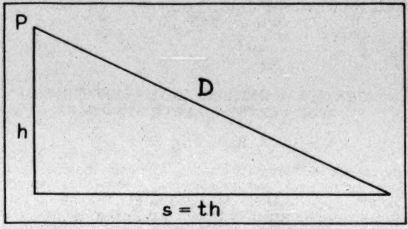


Fig. 2--Diagram illustrating anomaly at height h

Let h = r - R denote the altitude of P. For h = 0 we must get $\Delta p = \Delta_0$, which means that the element $d\Sigma$ with $\psi = 0$ yields the only contribution. For increasing values of h, the closest surroundings exert some influence, but the total result generally decreases. My experiments show that the total result begins to be negligible before the curvature of the Earth begins to play any role. In other words, we can always use the approximate formula

$$\Delta_p \approx \int_0^t \Delta_t \frac{t \cdot dt}{(1+t^2)^{3/2}} \quad \ldots\ldots (4)$$

where

$$t = s/h \quad \ldots\ldots (5)$$

s being the horizontal distance from the point P and Δ_t the mean anomaly at this distance.

Now let T denote the potential of the disturbing masses of the geoid, which cause the deformations from the spheroid. We then have the fundamental equation of physical geodesy

$$\Delta = -\frac{\partial T}{\partial r} - \frac{2T}{r} \quad \ldots\ldots (6)$$

which expresses the anomaly Δ as a differential function of T and r. Introducing now a harmonic function

$$H = 2T + r\frac{\partial T}{\partial r} = \frac{1}{r} \cdot \frac{\partial}{\partial r}(r^2 T) \quad \ldots\ldots (7)$$

we can express both Δ and T by the aid of H as follows

$$\Delta = -\frac{H}{r} \qquad r^2 T = \int rH\, dr \quad \ldots\ldots (8)$$

Poisson's integral formula gives now

$$r^2 T = \frac{1}{4\pi R} \int_\Sigma H_0 \int \frac{r^3 - rR^2}{D^3}\, dr\, d\Sigma \quad \ldots\ldots (9)$$

Working out the indefinite integral where the sphere Σ with the radius R is replaced by a sphere σ with radius 1, we get

$$T = \frac{R^2}{4\pi r^2}\int_\sigma \Delta_0 \left\{ \frac{2r^2}{D} - 3D - 3R\cos\psi \log_e(r - R\cos\psi + D) + \text{const.}\right\} d\sigma \quad (10)$$

The indefinite integral involves an arbitrary constant. Tentatively, the constant will be taken so that T = 0 when r = ∞, which makes const = 0. Then it can be shown that if T from (10) is substituted into (6), (3) will result.

Denoting by N the difference of the actual and the normal surfaces of the same constant potential, we have

$$N = \frac{T}{\gamma} = \frac{T}{G}\cdot\frac{r^2}{R^2}$$

$$= \frac{1}{4\pi G}\int_\sigma \Delta_0 \left\{ \frac{2r^2}{D} - 3D - 3R\cos\psi \log_e(r - R\cos\psi + D) + \text{const.}\right\} d\sigma \quad (11)$$

where γ is gravity at P and G is gravity at sea level and can here be considered as a constant.

There is now a question of minor practical significance which must be mentioned here in order to allow for the established theory. When r = R, (11) does not lead to the original Stokes' formula. This is due to the fact that Stokes supposed (1) the mean anomaly on the Earth to be zero, and (2) that the anomalies must necessarily be "balanced." Using spherical harmonics we can express these requirements making the harmonics of order 0 and 1 disappear. Whether the mean anomaly actually disappears depends partly on the normal formula of gravity, and can always be corrected later. The harmonics of the first order should be absent in every case, for even in the case that the disturbing masses should not be balanced, the Bruhn's term or the last term in (6) restores the balance in the gravity anomalies. If the observed gravity field does not fulfill this requirement, there must be errors in the observations, reductions, or in the filling in of the gaps, and a proper adjustment of the field should be made.

Dealing with a potential field, where the spherical harmonics of orders 0 and 1 are absent, not only T but r^2T is zero when r = ∞. Thus the constant in (10) must be altered and we get

$$N = \frac{1}{4\pi G}\int_\sigma \Delta_0 \left\{ \frac{2r^2}{D} + r - 3D - R\cos\psi\left(5 + 3\log_e\frac{r - R\cos\psi + D}{2r}\right)\right\} d\sigma \quad (12)$$

which for r = R leads to Stokes' original formula. If the gravity field contains no harmonics of orders 0 and 1, both (11) and (12) give identical results. But if we have taken no precautions to get rid of these harmonics of the gravity field, (12) makes them disappear from the potential and N, whereas (11) maintains them. Because we must integrate over the entire Earth, these terms appear rather exaggerated. The practical applications of Stokes' formula, made by myself [HIRVONEN, 1934] and by TANNI [1948], implied gravity fields where these terms contribute to N nearly as much as the really important terms of higher order. Therefore, when computing just N, (12) is preferred.

For the anomalies at the high altitudes we obtain on the basis of (6) and (10), when the new constant is used

$$\Delta_P = \frac{1}{4\pi}\int_\Sigma \Delta_0 \left\{ \frac{r^2 - R^2}{rD^3} - \frac{1}{r^2} - 3\frac{R}{r^3}\cos\psi \right\} d\Sigma \ldots\ldots\ldots (13)$$

a formula developed by Vening Meinesz. When compared with (3), this formula shows two small terms intended to get rid of the spherical harmonics of orders 0 and 1. Therefore, for r = R, (13) does not give exactly Δ_0 as should be expected. The new terms, however, have no practical significance because the integration in this case can be -- and considering the gaps of the gravity field, must be -- confined to the nearest surroundings where the first term has a dominating effect. Therefore, we can use the simplified formulas (3) and (4) for our practical applications.

For the meridional component of the deflection of vertical we have

$$\xi = -\frac{R}{r}\frac{\partial N}{\partial x} \quad \ldots\ldots (14)$$

where x denotes the usual rectangular horizontal coordinate at sea level below the point to be computed. Formula (12) now gives

$$\xi = \frac{R}{4\pi r G}\int_{\sigma}\Delta_0\left\{\frac{2r^3}{D^3} + \frac{3r}{D} - 5 + \frac{3R\cos\psi(r+D)}{(r - R\cos\psi + D)D} - \right.$$
$$\left. - 3\log_e\frac{r - R\cos\psi + D}{2r}\right\}\sin\psi\cos\alpha\, d\sigma \quad \ldots (15)$$

where α denotes the azimuth. We could get a shorter formula using (11) instead of (12). But if the field of Δ_0 shows no spherical harmonics of orders 0 and 1, both formulas give identical results. In the practical applications we must confine ourselves to small values of ψ, and then the first term $2r^3/D^3$, which is common to both (15) and the shorter formula, has a dominating effect.

For r = R, (15) gives the well-known formula of Vening Meinesz

$$\xi = \frac{1}{4\pi G}\int_0^{\pi}\int_0^{2\pi}\Delta_0\left\{\frac{1}{\sin\frac{\psi}{2}} + 3 + 8\sin\frac{\psi}{2} - 32\sin^2\frac{\psi}{2} - 12\sin^3\frac{\psi}{2} + 32\sin^4\frac{\psi}{2}\right.$$
$$\left. - 3\sin^2\psi\log_e\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)\right\}\cos\alpha\, d\alpha\, d\psi \quad \ldots\ldots (16)$$

It is easy to see that the first term $1/\sin(\psi/2)$ affects the results most. In practice it seldom is possible and hardly even necessary to go farther than to $\psi = 5°$. At this limit the combined contribution of the other terms is about 15 pct and in the whole integral much less. SOLLINS [1947], however, has published complete and very accurate tables for the functions appearing in (16), which is valid at sea level.

Computing deflections of vertical for higher altitudes we need similar tables for the function which appears in (15). Each altitude requires separate tables and, at present, their preparation seems unreasonable. Since great accuracy is not necessary, we might be satisfied by the following approximate device.

Suppose that we, in all cases, first compute the deflection at sea level, using the tables of Sollins or the special template of RICE [1949, 1951] derived therefrom, which renders further use of the tables unnecessary. Then we reduce the contribution of each zone for higher altitudes considering only the first terms of (15) and (16). The coefficient of reduction from r = R to r > R is approximately

$$f = \frac{8r^3}{D^3}\sin^3\frac{\psi}{2} \approx \left(\frac{t}{\sqrt{1+t^2}}\right)^3 \quad \ldots\ldots (17)$$

Having a table of this function we can for every zone radius compute t and estimate the mean value of f within the zone.

Now the zones of Rice have a general property that s_2/s_1 is almost constant for the inner zones, varying from 1.19 to 1.17. It is advisable to design the table of f so that $t_2/t_1 = \sqrt[4]{2} = 1.1892$, which then simplifies the computations with the template of Rice.

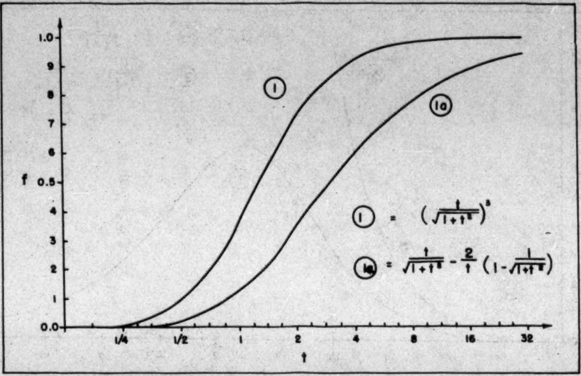


Fig. 3--Graphs of coefficients of reduction (deflection of the vertical)

Curve 1 in Figure 3 represents the function f. It can be seen that the zones nearer than s = h/4 have no significant effect at altitude h, and that the zones beyond s = 10 h have nearly the same effect at the altitude h as at sea level. The reduced effect for the about 23 zones between these limits can be computed very rapidly by the aid of curve 1. Thus, in case the mean anomalies in the compartments of Rice or of other templates have been estimated, and the deflection of the vertical at sea level computed, the deduction of the corresponding result for any number of higher altitudes can be performed very easily.

For small values of h it can happen that the innermost zone, where only the constant gradient of gravity has been estimated, has an effect which is not negligible. The effect of this zone at sea level, which has a radius s_o, is

$$\xi_o = -0''.105 \frac{\partial \Delta_o}{\partial x} s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

whereas at the altitude h we get

$$\begin{aligned}\xi_h &= 0''.105 \frac{\partial \Delta_o}{\partial x} \left\{ - \frac{s^2}{\sqrt{s^2+h^2}} + 2h\left(1 - \frac{h}{\sqrt{s^2+h^2}}\right) \right\} \\ &= -0''.105 \frac{\partial \Delta_o}{\partial x} s \left\{ + \frac{t}{\sqrt{1+t^2}} - \frac{2}{t}\left(1 - \frac{1}{\sqrt{1+t^2}}\right) \right\} \quad \ldots\ldots\ldots \quad (19)\end{aligned}$$

where the factor between the braces can be taken from curve 1a.

While computing the anomaly for high altitudes, the same zones may be used again. The mean anomalies of different compartments -- thus before the multiplication by cos α or sin α -- must be united to zone averages Δ_t, and these multiplied by the coefficients which are taken from curve 2 in Figure 4. For the innermost zone, notwithstanding the gradient, the mean anomaly must be multiplied by the coefficient taken from curve 2a of Figure 4.

It may be useful to point out that all our auxiliary functions, which are given by the curves, contain the expression $1/\sqrt{1+t^2}$, which can be found in any trigonometric table as cos v, using t = tan v as an entry. Also, the function of curve 1 can be found by cubing the value of sin v found by using t = tan v as an entry.

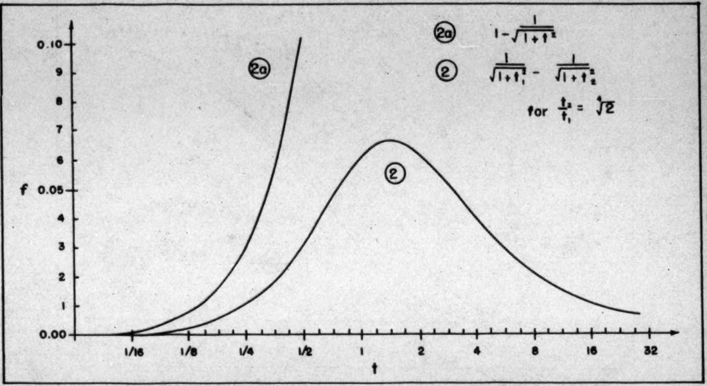


Fig. 4--Graphs of coefficients of reduction (anomalies)

Examples--At first we will take two simple idealized examples, in order to get a comparison with the true values, obtainable by other exact formulas.

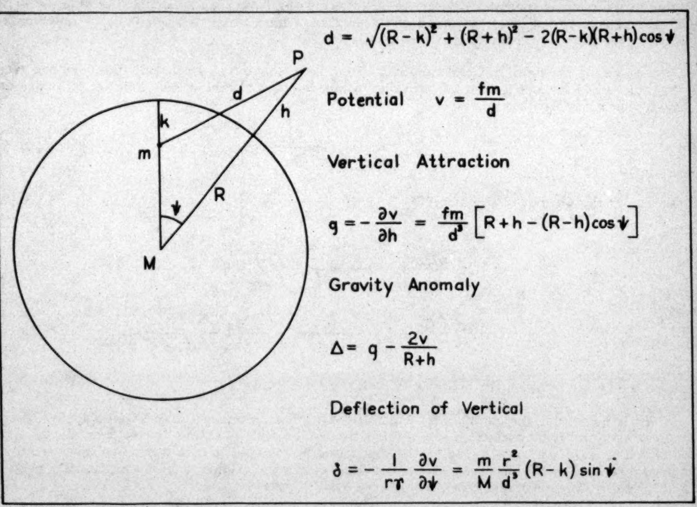
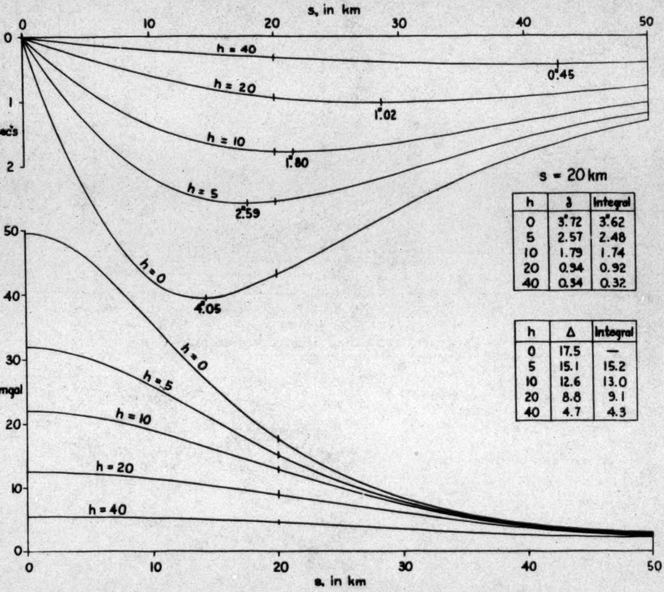

P
d
h
k
m
R
ψ
M
$d=\sqrt{(R-k)^2+(R+h)^2-2(R-k)(R+h)\cos\psi}$
Potential $v=\frac{fm}{d}$
Vertical Attraction
$g=-\frac{\partial v}{\partial h}=\frac{fm}{d^3}\left[R+h-(R-h)\cos\psi\right]$
Gravity Anomaly
$\Delta=g-\frac{2v}{R+h}$
Deflection of Vertical
$\delta=-\frac{1}{r\gamma}\frac{\partial v}{\partial\psi}=\frac{m}{M}\frac{r^2}{d^3}(R-k)\sin\psi$

Fig. 5--Diagram and formulas indicating effects of a mass m at depth k on point P

We assume that the Earth is a non-rotating sphere with radius R and mass M and consisting of homogeneous spherical layers. Suppose that we have a disturbing mass point m at a depth k below this normal surface. At a point P, situated at height h and at angular distance ψ from m, we have the exact formulas shown in Figure 5.

We shall first compute gravity anomalies Δ_0 at sea level around the Earth, then apply our integral formulas, and check the results by the formulas in Figure 5.

Example 1--Let k be 20 km and $fm/k^2 = 50$ mgal. The gravity anomalies Δ and the deflections of the vertical δ at distances s and at altitudes h are given by the curves in Figure 6. The integral formulas have been applied for s = 20 km only. The comparison of the results is shown by the table in Figure 6. The last zone considered in the integration had an outer radius of 92 km.



s = 20 km

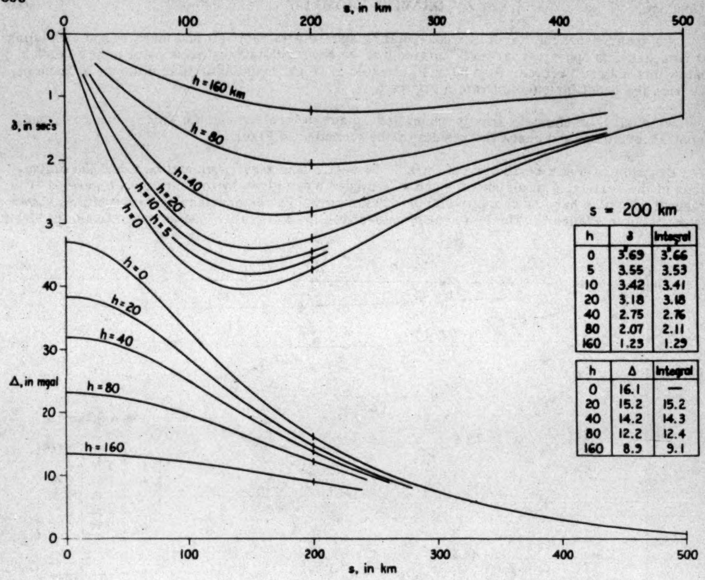
h	δ	Integral
0	3".72	3".62
5	2.57	2.48
10	1.79	1.74
20	0.94	0.92
40	0.34	0.32

h	Δ	Integral
0	17.5	—
5	15.1	15.2
10	12.6	13.0
20	8.8	9.1
40	4.7	4.3

Fig. 6--Diagrams and tables illustrating Example 1, k = 20 km

Example 2--Let k be 200 km and $fm/k^2 = 50$ mgal. The values of δ and Δ have been calculated for s = 200 km. The results are shown in Figure 7. The last zone considered had an outer radius of 8°.62. We conclude that the accuracy of our integral formulas is sufficient for all practical purposes.

Example 3--Now we shall take an example with a real observed gravity field. I have chosen the station Little Rock Longitude, for which Rice has computed the components of deflection of the vertical at sea level. Unfortunately, Rice has not published the details but only the effect of the innermost zone, the total effect, and a diagram of the increasing of this effect through the successive zones. Estimating the effect of single zones by the aid of this diagram and using our reduction coefficients for higher altitudes, I have obtained the results shown in Figure 8.

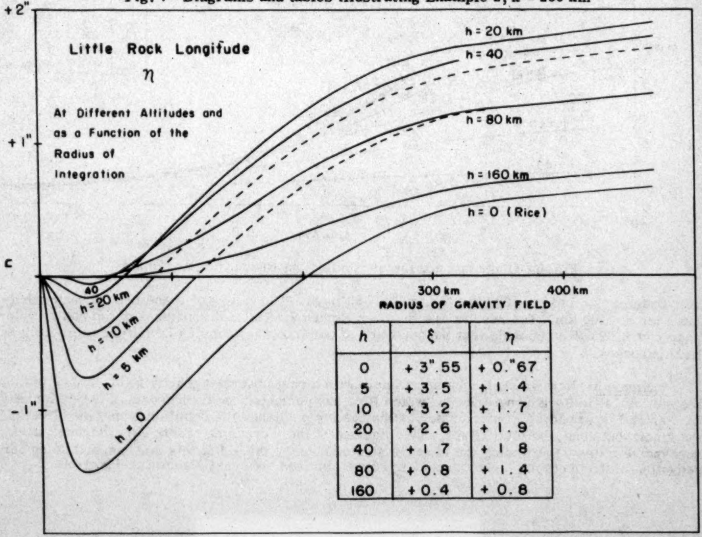


s = 200 km

h	δ	Integral
0	3".69	3".66
5	3.55	3.53
10	3.42	3.41
20	3.18	3.18
40	2.75	2.76
80	2.07	2.11
160	1.23	1.29

h	Δ	Integral
0	16.1	—
20	15.2	15.2
40	14.2	14.3
80	12.2	12.4
160	8.9	9.1

Fig. 7--Diagrams and tables illustrating Example 2, k = 200 km



h	ξ	η
0	+ 3".55	+ 0."67
5	+ 3.5	+ 1.4
10	+ 3.2	+ 1.7
20	+ 2.6	+ 1.9
40	+ 1.7	+ 1.8
80	+ 0.8	+ 1.4
160	+ 0.4	+ 0.8

Fig. 8--The results for Example 3

This example was chosen to show that the deflection of the vertical can, in exceptional cases, be considerably greater above than at sea level. Here such an exceptional case was found for the η-component, whereas the ξ-component shows the usual behavior, that of a general decrease with the altitude.

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(Manuscript received April 30, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 7, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

THERMAL TIDES IN THE ATMOSPHERE

Jerome Spar

Abstract--Margules' theory of thermally induced diurnal and semidiurnal pressure oscillations is extended to the three-dimensional atmosphere by the method of vertical integration. The numerical solutions obtained by Margules for the surface pressure oscillations are retained in the present theory by introducing a suitably defined atmospheric "depth." A method for calculating the surface pressure oscillation resulting from periodic heating and cooling of layers high in the atmosphere is suggested.

Introduction--Harmonic analysis of the daily variation of atmospheric pressure reveals the existence of two dominant waves, one with a period of 24 hours and one with a period of 12 hours [WILKES, 1949]. The former is generally believed to result from the diurnal heating and cooling of the atmosphere. The origin of the latter, which is the dominant component in low latitudes, is still in doubt although most meteorologists accept the theory that the atmosphere has a free period very close to 12 hours [PEKERIS, 1937] and resonates in response to the 12-hourly component of either the solar tidal potential or the daily temperature wave [WILKES, 1951] or both [CHAPMAN, 1924].

The theoretical investigation of thermally induced pressure oscillations in the atmosphere was initiated by **MARGULES** [1890] in a paper in which the vertical coordinate was ignored. The theory presented below follows that of Margules very closely but the restriction to two dimensions is partly eliminated by the method of vertical integration. Mathematically the problem remains two-dimensional. However, the integration method makes it possible to determine the effect on the surface pressure oscillation of temperature waves at different elevations in the atmosphere.

An important aspect of the problem which cannot be treated by the present method is that associated with the phase shift with height of the pressure and temperature waves [BJERKNES, 1948].

Fundamental equations--Terms containing the vertical velocity may be neglected in the equations of motion but must be retained in the continuity equation since the vertical divergence is of the same order of magnitude as the horizontal divergence. The atmosphere is assumed to be inviscid. The equations of motion and of continuity may then be written

$$\bar{\rho}\, d\bar{u}/dt + 2\,\omega\bar{\rho}\,\bar{v}\cos\theta = -\partial\bar{p}/r\sin\theta\,\partial\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\bar{\rho}\, d\bar{v}/dt - 2\,\omega\,\bar{\rho}\,\bar{u}\cos\theta = -\partial\bar{p}/r\,\partial\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$0 = \partial\bar{p}/\partial z + \bar{\rho}\, g \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\partial\bar{\rho}/\partial t + (1/r\sin\theta)\,[\,\partial(\bar{\rho}\,\bar{u})/\partial\lambda + \partial(\bar{\rho}\,\bar{v}\sin\theta)/\partial\theta] + \partial\bar{\rho}\bar{w}/\partial z = 0 \quad \ldots\ldots\ldots (4)$$

where λ and θ are longitude and colatitude, $\bar{u}$ is the eastward component of motion, $\bar{v}$ is the equatorward component of motion, $\bar{w}$ is the upward component of motion, $\bar{p}$ is pressure, $\bar{\rho}$ is density, and g is the acceleration of gravity. The radial distance from the center of the Earth is r, which is considered constant and equal to the mean radius of the Earth. The angular velocity of the Earth is ω, and T will be used later to denote temperature.

The subscript zero will be used to indicate the value of a quantity in the undisturbed state and quantities without bars or subscripts are perturbation quantities. If x represents any of the variables

$$\bar{x} = x_0 + x$$

It will be assumed that in the undisturbed state there is no motion and all horizontal gradients are zero. Products of perturbation quantities or of their derivatives are neglected. The following perturbation equations are then obtained.

$$\rho_o \, \partial u/\partial t + 2\,\omega\,\rho_o\, v \cos\theta = -\,\partial p/r \sin\theta\,\partial\lambda \quad \ldots\ldots (1a)$$

$$\rho_o \, \partial v/\partial t - 2\,\omega\,\rho_o\, u \cos\theta = -\,\partial p/r\,\partial\theta \quad \ldots\ldots (2a)$$

$$\partial\rho/\partial t + (1/r \sin\theta)\,[\partial(\rho_o u)/\partial\lambda + \partial(\rho_o v \sin\theta)/\partial\theta] + \partial\rho_o w/\partial z = 0 \quad \ldots\ldots (4a)$$

The last three equations can be integrated with respect to height with the result

$$\partial U/\partial t + 2\,\omega\, V \cos\theta = -\,\partial K/r \sin\theta\,\partial\lambda \quad \ldots\ldots (1b)$$

$$\partial V/\partial t - 2\,\omega\, U \cos\theta = -\,\partial K/r\,\partial\theta \quad \ldots\ldots (2b)$$

$$(1/g)\ \partial p_s/\partial t + (1/r \sin\theta)\,[\partial U/\partial\lambda + \partial(V \sin\theta)/\partial\theta] = 0 \quad \ldots\ldots (4b)$$

where the subscript s denotes the surface of the Earth and

$$U = \int_s^\infty \rho_o\, u\, dz$$

$$V = \int_s^\infty \rho_o\, v\, dz$$

$$K = \int_s^\infty p\, dz$$

The vertical velocity is eliminated in (4b) by virtue of the fact that

$$\rho_o\, w = 0 \ \text{ at } \ z = s \ \text{ and } \ z = \infty$$

if the surface is horizontal. The first term in (4b) follows from (3).

If the gas law

$$\overline{p} = R\,\overline{\rho}\,\overline{T}$$

is introduced into (3), we have

$$(1/\overline{p})\,\partial\overline{p}/\partial z = -\,g/R\overline{T} \quad \ldots\ldots (3a)$$

where R is the gas constant for air. We may now differentiate (3a) with respect to time t, and integrate with respect to z between the limits s and the variable limit z to obtain

$$(1/\overline{p})\,\partial\overline{p}/\partial t - (1/\overline{p}_s)\,\partial\overline{p}_s/\partial t = (g/R)\int_s^z (1/\overline{T}^2)\,(\partial\overline{T}/\partial t)\, dz \quad \ldots\ldots (3b)$$

Upon introducing the perturbation quantities we obtain

$$(p_o/p_{so})\,\partial p_s/\partial t = \partial p/\partial t - (g\,p_o/R)\int_s^z (1/T_o^2)\,(\partial T/\partial t)\, dz \quad \ldots\ldots (3c)$$

The last equation can now be integrated with respect to z between the limits s and ∞ with the result

$$\partial p_s/\partial t = (1/h_o)\,[\partial K/\partial t - \partial Q/\partial t] \quad \ldots\ldots (3d)$$

where

$$h_o = (1/p_{so})\int_s^\infty p_o\, dz \quad \ldots\ldots (5)$$

and

$$Q = \int_s^\infty (g\,p_o/R)\int_s^z (T/T_o^2)\, dz\, dz \quad \ldots\ldots (6)$$

In an isothermal atmosphere

$$h_0 = R\,T_0/g$$

the height of the homogeneous atmosphere.

Upon substituting (3d) in (4b) we obtain

$$(1/g\,h_0)\,[\partial K/\partial t - \partial Q/\partial t] + (1/r\sin\theta)\,[\partial U/\partial\lambda + \partial(V\sin\theta)/\partial\theta] = 0 \;\ldots\ldots\ldots\; (7)$$

The 24-hour oscillation--The temperature perturbation will be considered to consist of a sine wave which follows the Sun with a period of one day. In the absence of a phase shift with height, the integrated temperature perturbation Q may also be represented as such a simple wave. Thus

$$Q = H(\theta)\sin(\omega t + \lambda)$$

The corresponding solutions for K, U, and V may be written

$$K = L(\theta)\sin(\omega t + \lambda)$$

$$U = M(\theta)\sin(\omega t + \lambda)$$

$$V = N(\theta)\cos(\omega t + \lambda)$$

If the expressions above are substituted in (1b), (2b), and (7), we obtain

$$(r^2\omega^2/g\,h_0)\,(L - H) + (1/\sin\theta)\,[dG/d\theta - (2\cos\theta/\sin\theta)\,G - L/\sin\theta] = 0 \;\ldots\ldots\ldots\; (8)$$

where

$$G = (r\,\omega/g\,h_0)\,N\sin\theta$$

Equation (8) is completely analogous to the one derived by Margules for the two-dimensional atmosphere, and we may adopt his numerical solution with only slight modifications. The constant, $r^2\omega^2/g\,h_0$, replaces Margules' constant, $r^2\omega^2/R\,T_0$. Margules solution is therefore valid only for an isothermal atmosphere.

To illustrate a simple case let us assume, as Margules did, that the amplitude of the temperature perturbation is of the form

$$H = C\sin\theta$$

where C is the amplitude of Q at the equator. The integrated pressure perturbation K may be in the form of an infinite series

$$K = C\sin(\omega t + \lambda)\,[\,\beta_1\sin\theta + \beta_3\sin^3\theta + \beta_5\sin^5\theta + \ldots]$$

The surface pressure perturbation may be written

$$p_s = P(\theta)\sin(\omega t + \lambda)$$

From (3d) it follows that

$$P = (C/h_0)\,[(\beta_1 - 1)\sin\theta + \beta_3\sin^3\theta + \beta_5\sin^5\theta + \ldots]$$

The coefficients β_i for i = 1, 3, 5, 7, 9 were computed by Margules by the method of continued fractions and are reproduced in Table 1 for $r^2\omega^2/g\,h_0$ equal to 2.5 and 2.7352, corresponding to h_0 equal to 8.75 and 8.01 km respectively. (The mean value of h_0 for the whole year and both hemispheres is about 7.5 km [SPAR, 1950].)

The amplitude P of the surface pressure oscillation computed from the first five terms of the series is shown as a function of colatitude in Figure 1. Since the solution is symmetrical about the equator, only the distribution in one hemisphere was drawn. The ordinate is $P\,h_0/C$. The corresponding amplitude of Q for the annual temperature oscillation has been found to be of the order

Table 1--Coefficients of the series for the calculations of the diurnal surface pressure oscillation

h_0	β_1	β_3	β_5	β_7	β_9
km					
8.75	1.119	-0.448	-0.326	-0.090	-0.013
8.01	1.146	-0.423	-0.370	-0.106	-0.018

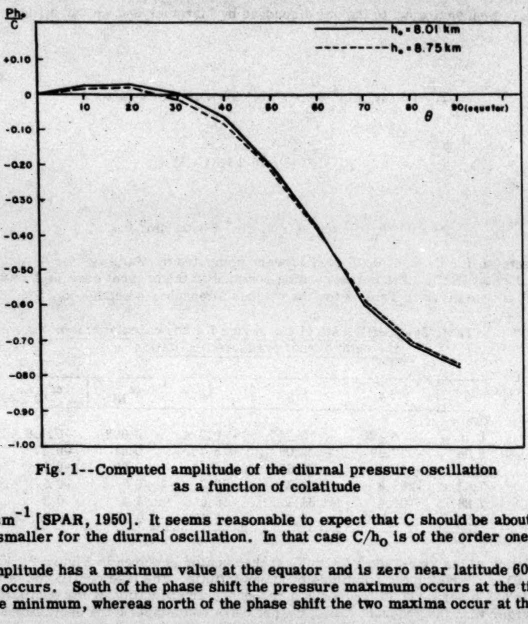

Fig. 1--Computed amplitude of the diurnal pressure oscillation as a function of colatitude

10^{10} dyne cm^{-1} [SPAR, 1950]. It seems reasonable to expect that C should be about one order of magnitude smaller for the diurnal oscillation. In that case C/h_0 is of the order one millibar.

The amplitude has a maximum value at the equator and is zero near latitude 60° where a phase shift occurs. South of the phase shift the pressure maximum occurs at the time of the temperature minimum, whereas north of the phase shift the two maxima occur at the same time.

No attempt has been made to verify the distribution shown in Figure 1 since the model is obviously not a good approximation to reality. The daily temperature variation depends to a greater degree on continentality than on latitude. Therefore, the assumption of zonal symmetry cannot be expected to give the observed distribution of amplitudes.

The 12-hour oscillation--The response of the atmosphere to the 12-hourly component of the daily temperature wave can be investigated by writing the temperature perturbation in the form

$$Q = H'(\theta) \sin 2(\omega t + \lambda)$$

The periodic solutions desired are then

$$K = L'(\theta) \sin 2(\omega t + \lambda)$$
$$U = M'(\theta) \sin 2(\omega t + \lambda)$$
$$V = N'(\theta) \cos 2(\omega t + \lambda)$$

After substituting these expressions in (1b), (2b), and (7), we obtain

$$\sin^2\theta\, d^2L'/d\theta^2 - \sin\theta\cos\theta\, dL'/d\theta + L'\left[(4\,r^2\omega^2/g\,h_0)\sin^4\theta + 2\sin^2\theta - 8\right] = (4\,r^2\omega^2/g\,h_0)\,H'\sin^4\theta \quad \ldots\ldots\ldots\ldots\ldots (9)$$

We will assume, as Margules did, that the amplitude of the temperature perturbation is of the form

$$H' = C'\sin^2\theta$$

The problem is then analogous to the one discussed by Margules and we can utilize his numerical solution.

Let

$$K = C'(\alpha_4\sin^4\theta + \alpha_6\sin^6\theta + \ldots)\sin 2(\omega t + \lambda)$$

and

$$p_s = P'(\theta)\sin 2(\omega t + \lambda)$$

Then

$$P' = (C'/h_0)(-\sin^2\theta + \alpha_4\sin^4\theta + \alpha_6\sin^6\theta + \ldots)$$

The coefficients α_i for $i = 4, 6, 8, 10$, and 12 were computed by Margules for several values of the constant $4\,r^2\omega^2/R\,T_0$. The corresponding constant in the present case is $4\,r^2\omega^2/g\,h_0$. The values of α_i are given in Table 2 for the various atmospheric depths, h_0.

Table 2--Coefficients of the series for the calculations of the semidiurnal pressure oscillation

h_0	α_4	α_6	α_8	α_{10}	α_{12}
km					
8.70	- 6.196	- 3.247	- 0.724	-0.092	-0.008
8.04	- 37.99	- 23.06	- 5.75	-0.81	-0.07
8.00	- 55.00	- 33.68	- 8.46	-1.20	-0.11
7.93	-247.8	-154.2	-39.2	-5.6	-0.5
7.86	101.8	64.3	16.5	2.4	0.2
6.55	8.270	5.919	1.662	0.260	0.26

The variation of the amplitude P' with h_0 at the equator is shown in Figure 2. For a value of h_0 between 7.93 and 7.86 km the amplitude becomes infinite. The free oscillation of an atmosphere with this depth has a period of 12 hours and a resonance amplification of the oscillation occurs. When h_0 is larger than the resonant value, the pressure and temperature oscillations are 180° (that is, six hours) out of phase; when h_0 is less than the resonant value, the two oscillations are in phase. The mean annual value of h_0 is about 7.5 km. However, it would be unwise to conclude anything about the nature of the real pressure oscillation from this fact in view of the crudity of the physical model.

In Figure 3 is shown the variation of the amplitude with latitude for h_0 equal to 7.86 km. As in Figure 2, the ordinate is the dimensionless quantity $P'h_0/C'$.

The influence of upper-level temperature waves on the surface pressure oscillations--The amplitude of the surface pressure oscillation depends upon the quantity H (or H'). The latter is defined by (6) if T is now interpreted as the amplitude of the temperature wave at the level z. The contributions of temperature waves in various layers of the atmosphere to the surface pressure wave can easily be calculated from (6).

To illustrate the method of computation we will assume that at the equator at every level in the atmosphere the amplitude of the temperature oscillation is 1°A. Consider a layer between

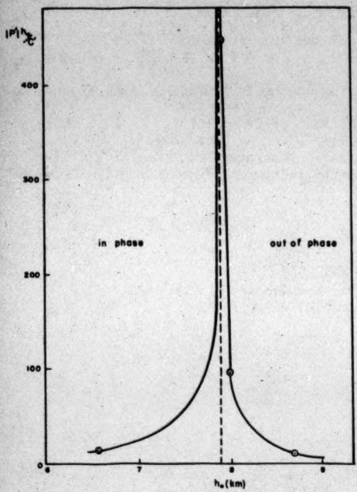

Fig. 2--Variation of the amplitude of the semidiurnal pressure oscillation with the equivalent depth h of the atmosphere

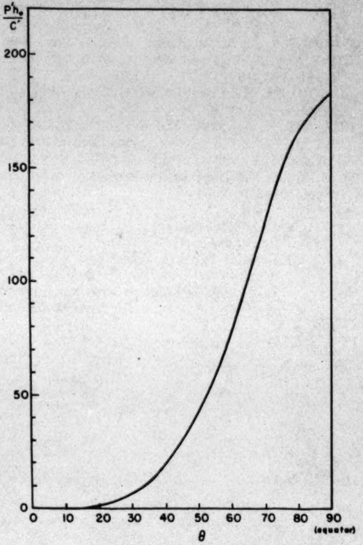

Fig. 3--Computed amplitude of the semidiurnal pressure oscillation as a function of colatitude

sea level (s = 0, p_{os} = 1000 mb) and 5000 ft with an isothermal equilibrium temperature distribution (say, T_o = 290°A). The contribution of this layer to the surface pressure wave is then proportional to

$$C_1 = (g/R\ T_o^2)\int_{z=0}^{z=h} p_o\, z\, dz$$

$$= (R\ p_{os}/g)\,[1 - e^{-gh/R\,T_o}\,(gh/R\,T_o + 1)]$$

For g = 981 cm $\sec^{-1}$ and R = 2.87×10^6 cm^2 $\sec^{-2}$ deg^{-1}, C_1 is found to be 4.5×10^7 dyne cm^{-1}. For h_o equal to 8 km, the corresponding amplitude of the 24-hour pressure oscillation at the surface is 0.04 mb (see Fig. 1).

The contributions of higher layers can be calculated in similar fashion. For example, consider an isothermal layer between the levels 35,000 and 60,000 ft. The pressure at the bottom of the layer is about 250 mb and the mass of the layer is almost the same as that of the layer in the previous example. Let the temperature of the layer be 220°A. Then the contribution of this layer to the 24-hour surface pressure wave is found to be 0.26 mb for h_o equal to 8 km.

Although the daily temperature variation is probably larger in the layer 0-5000 ft than it is in most higher layers of the atmosphere, the possibility exists that it may be equally as large in the ozonosphere. If this is the case, the previous calculation would indicate that temperature waves in the high atmosphere may provide important contributions to the surface pressure oscillation. Unfortunately, no data exist for the direct determination of the daily temperature wave in these high layers.

Acknowledgments--This study was made possible through support and sponsorship extended by the Geophysical Research Directorate of the Cambridge Field Station, A.M.C., U. S. Air Force, under Contract No. AF 19(122)-49. It does not represent recommendations or conclusions of the sponsoring agency.

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(Communicated manuscript received January 9, 1952;
open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

MODEL EXPERIMENTS IN THE FORMATION OF EDDIES BY LATERAL SHEAR

Lester F. Hubert

Abstract--An apparatus has been constructed to contain a shell of fluid (air) in the shape of a 70° spherical segment extending from the pole (axis of rotation) to 55° latitude, and to produce lateral shear and rotation in the fluid. The shear was produced by rotating the fluid between 70° and 90° latitude (the polar cap) at a different rate from the fluid between 55° and 70° latitude (the zone).

Experiments show that this lateral shear can produce large organized eddies in a rotating fluid shell under conditions that can be interpreted as being dynamically unstable and that the eddies are totally absent under conditions of dynamic stability. (Dynamic instability exists when the angular momentum of the fluid decreases with increasing radial distance from the axis of rotation.) In addition to lateral shear, the experiments indicate that an organized transport across the shear zone is also a necessary condition for eddy formation.

Introduction--The experiments described here constitute part of a U. S. Weather Bureau project to investigate certain phases of hydrodynamic analogues. Their purpose is to investigate the efficiency of shear in producing large-scale eddies in a thin fluid shell. If this is considered to be a model of the free atmosphere, then the horizontal scale of the eddies under consideration corresponds to the cyclones and anticyclones that appear on synoptic charts. No attempt was made to simulate the vertical motion of the atmosphere.

The apparatus illustrated in Figures 1-3 consists of two concentric boundary surfaces that are segments of concentric spherical surfaces subtending an angle of 70°, that is, segments centered at the pole and extending to the 55° latitude circle. The outer boundary surface is transparent while the inner boundary surface is metal. Zonal motion of the fluid shell contained between these surfaces is produced by frictional drag of the rotating boundary surfaces. Figure 2 shows how the driving surfaces are divided around the 70° latitude circle so that two portions of the fluid shell can be driven at different angular velocities to create a line of shear along that latitude. Figure 3 shows the assembled apparatus and the two motors that provided independent drive for each section. Figure 4, a schematic cross section along a meridional plane, illustrates the essential features and dimensions of the model.

Historical--Experimental work of this general nature has been performed by many others, although no other work, to the best of this author's knowledge, has dealt primarily with shear-produced eddies tangent to a spherical surface.

AHLBORN [1924] rotated a sphere in a large tank of fluid and obtained a meridional circulation superimposed on the zonal motion from pole to equator near the surface of the rotating sphere with a return flow some distance away from the moving body. TERADA and HATTORI [1926] rotated a sphere inside of a spherical shell and described the same type of meridional circulation. This general type of circulation is referred to as "Sekundärstromungen" and discussed by PRANDTL [1942, pp. 130-131], and an experiment involving the same principle is described by HELMHOLZ [1876]. Such secondary circulation is brought about by the greater centrifugal force acting on the fast-moving fluid adjacent to the driving surface as compared to the force acting on the fluid more distant. In connection with the apparatus used here then, this secondary circulation is expected wherever there is radial shear and is not expected under conditions of no radial shear. This is verified and most clearly illustrated by Experiments 1 and 6.

TAYLOR [1922] investigated instability resulting from the shear produced by two concentric rotating cylinders. The points of similarity of that work to the experiments discussed here will be mentioned in the section below headed "Interpretation."

Experimental procedure--Tobacco smoke was found to be most convenient as a tracer material since it could be generated continuously and injected into the working space at intervals. After a period of trial and error to develop the best method of lighting the apparatus and of introducing the smoke, all experiments follow the same procedure. The smoke generator was started,

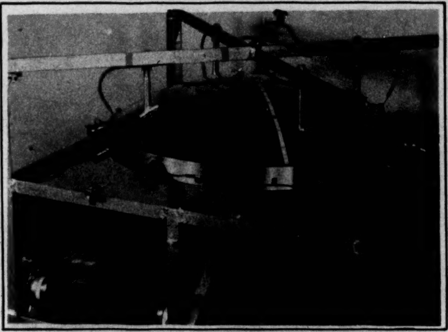

Fig. 1--Assembled apparatus showing boundary surfaces, smoke input tubes, and interior lights

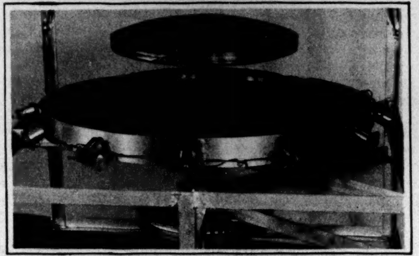

Fig. 2--Inner driving surfaces

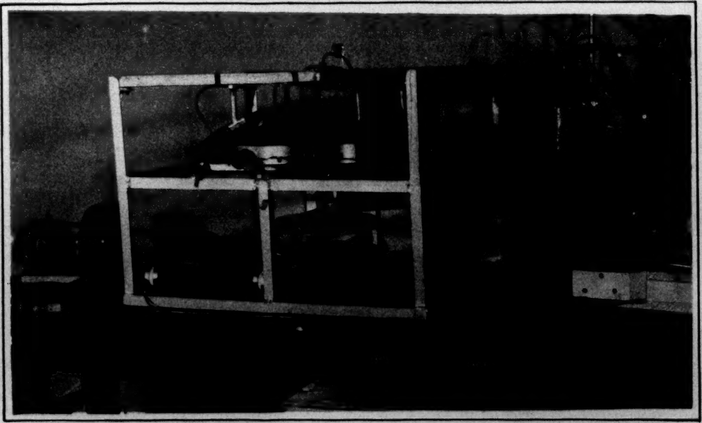

Fig. 3--Complete apparatus, smoke generator (right background), and motor drive

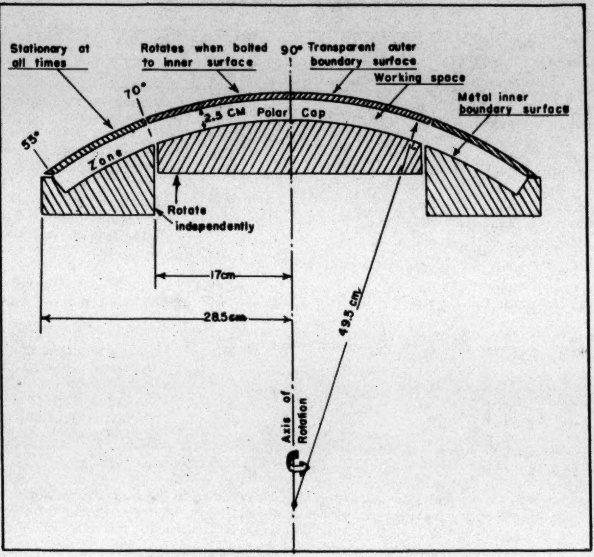

Fig. 4--Schematic cross section of model

the interior lights of the model turned on, and the smoke injected slowly through the lucite dome at four points around the 57° latitude circle. The smoke gradually filled the periphery of the working space from the 55° latitude boundary wall to about 60° latitude. The smoke source was then shut off, allowing the fluid and tracer to come to rest. Because of the high specific gravity of the smoke, the boundary between the clear air and the smoke remained sharp and motionless with no perceptible diffusion. The voltage regulators were set to provide the desired rate of rotation and the motors started.

A certain interval of time elapsed before the working fluid and tracer settled into a given flow pattern, but the interval was always less than a minute. The streamlines delineated by the smoke were always symmetrical about the axis of rotation (north pole) and, once established, the pattern remained the same as long as it was visible. Furthermore, the same pattern was reproduced each time the same conditions of shear and velocity were repeated.

Discussion of experiments--Table 1 represents the principal results of the experiments. It is a summary of the six possible combinations of relative motions. The results remained unchanged as long as the relative motion between the portions of the fluid was maintained, even though the direction of rotation was reversed. The rate of rotation varied from six to 25 rpm and each experiment was repeated many times throughout this range, but only in Experiment 5 did the results depend largely upon speed. The Table therefore indicates only relative directions and speeds. For convenience, meteorological terminology is adopted. For example, shear along the radius (of the sphere) is "vertical shear" while the shear along the 70° circle of latitude is called "horizontal shear." Zonal motion is east-west and meridional motion is north-south.

Experiment 1--Although no perceptible pattern of eddies was generated, the smoke initially injected south of 70° diffused northward at a rate considerably greater than molecular diffusion, apparently due to eddy diffusion in transient eddies no larger than, say, one degree of latitude (1° = 8.7 mm).

Table 1--Results of the experiments

Exp. No.	Surface	Polar cap (70°-90°)	Zone (55°-70°)	Vertical shear		Horizontal shear (along 70°)	Results
		Rotating	Rotating	Polar cap	Zone		
1	Outer	Yes[a]	No	None	None	Yes	No eddies visible; no meridional circulation.
	Inner	Yes[a]	No			Yes	
2	Outer	Yes[a]	No	None	Yes	Yes	No eddies visible; meridional circulation in zone.
	Inner	Yes[a]	Yes			Yes	
3	Outer	No	No	Yes	None	Yes	Small eddies 5° to 6° of latitude in diameter, but not through entire depth of fluid; meridional circulation in polar cap.
	Inner	Yes	No			Yes	
4	Outer	No	No	None	Yes	Yes	No eddies visible; meridional circulation in the zone.
	Inner	No	Yes			Yes	
5	Outer	No	No	Yes	Yes	Yes	Vigorous eddies when minimum shear conditions met; meridional circulation in polar cap and zone.
	Inner	Yes[b]	Yes[b]			Yes	
6	Outer	No	No	Yes	Yes	None	No eddies visible; vigorous meridional circulation in polar cap and zone.
	Inner	Yes[a]	Yes[a]			None	

[a] Rotating as a unit. [b] Rotating at different rates or directions, or both.

Experiment 2--The results of this experiment differed from the first in that a secondary circulation (meridional) was superimposed on the zonal motion in the zone. Smoke diffusion into the polar cap again suggested the presence of small transitory eddies, but no organized vortices appeared.

Experiment 3--Elliptical eddies were formed about six degrees of latitude along their east-west axes and four degrees along their north-south axes. Always eight in number, they moved regularly in the direction of rotation of the polar cap, were generated throughout the entire speed range, and appeared to involve only a small layer of the working fluid near the outer boundary surface. The sense of the vortex rotation (always the same in all eight) was the same as the sense of the shear.

Experiment 4--Compare the conditions of this experiment with the preceding one. Notice that the vertical shear (thus the secondary circulation) is now confined to the zone instead of to the polar cap. As in Experiments 1 and 2, diffusion of the tracer indicates small transitory eddies.

The results of the experiments up to this point show that organized eddies can be generated by horizontal shear but that an additional necessary condition is a meridional component in the polar cap -- the fluid nearest the axis of rotation.

Experiment 5--When there was horizontal shear along the 70° latitude circle and vertical shear throughout all of the working fluid, the most vigorous eddies were generated. Eight elliptical eddies, with their major (east-west) axes and minor (north-south) axes respectively ten to 15° of latitude and eight to ten degrees of latitude in length, were formed under various combinations of shear and speed. Certain minimum conditions have to be met, however, before there is any departure from zonal flow:

(1) The driving surfaces must attain a minimum tangential speed difference along the 70° circle of about 45 cm sec^{-1}. (All speeds mentioned are based on measurements that are accurate to only plus or minus five per cent. No measurements of the fluid velocities were made.)

(2a) If the zone and polar cap are rotated in opposite directions, the angular speed of the polar cap must exceed the angular speed of the zone by approximately one revolution per minute. (b) If the polar cap and zone are rotated in the same direction, the polar cap must rotate at the greater speed. If the zone is turned faster than the polar cap, no vortices will form even though the critical speed difference of 45 cm sec^{-1} is greatly exceeded.

Uniformly spaced around the 70° circle, each vortex rotated in the same direction and in the sense of the horizontal shear. The tracer material was concentrated in the vortices, so it was not possible to observe the characteristics of the motion between adjacent eddies.

An interesting effect appeared when eddies were produced under conditions (2a). Once formed, they persisted even though the relative speeds of the zone and cap were gradually changed so that condition (2a) was not fulfilled. The vortices generated migrated in the direction of the polar cap with approximately ten to 20 pct of the speed of that driving surface. As the speed of the zone was increased or that of the polar cap decreased, the eddies continued to move in the same direction but at a lesser rate, even when the angular speed of the zone was equal to that of the polar cap. Further, when the angular speed of the zone exceeded that of the polar cap about 3/4 rpm, the eddies became stationary, but when the zone was turned faster than the cap by about one revolution per minute, the eddies disappeared and the motion returned to zonal flow. Eddies reappeared only when condition (2a) was again satisfied.

Experiment 6--Under conditions of vertical shear alone, the secondary circulation is best illustrated (see Fig. 5). The smoke tracer that initially occupied the working space south of 60° commenced to spiral northward in a relatively thin layer adjacent to the outer surface. The advancing edge of smoke moved north to 85° where it descended to the inner surface and moved south to complete the closed circulation.

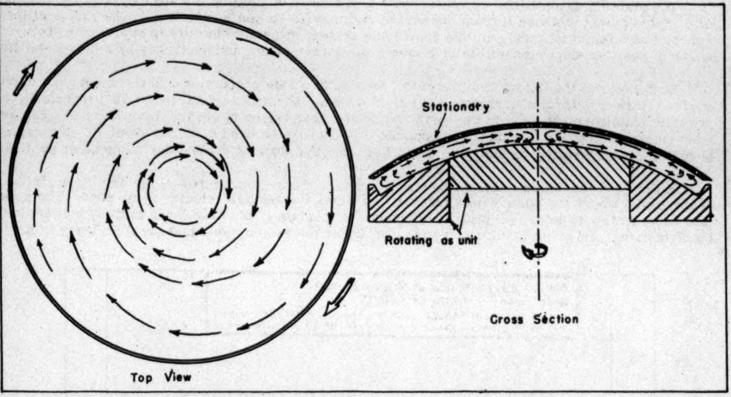

Fig. 5--Streamlines in the case of no lateral shear

It should be pointed out that in Experiment 5 when eddies were formed in the shear zone, the meridional circulation discussed above existed except in the band that contained the eddies. In this zone the character of the meridional circulation was completely changed because the vortex motion took place throughout the entire thickness of the fluid. Outside of the region of traveling eddies the fluid moved northward along the outer surface and southward along the inner surface uniformly at all longitudes. In the shear zone where the fluid was involved in vortex motion, however, the north and south components of these vortices existed through the entire thickness of the fluid so that the meridional circulation was no longer the same at all longitudes; it was alternately to the north or to the south, depending upon which branch of the vortex was involved.

Summary of results--In this apparatus and for the range of speeds used, horizontal shear alone is not sufficient to generate the desired eddies. In addition to shear, a certain amount of meridional circulation is required. The necessary and sufficient conditions for vortex formation are: (1) A minimum speed difference at 70° latitude of 45 cm sec^{-1}. (2) The fluid nearest the axis of rotation must turn at a greater rate than the fluid more distant from the axis. (3) A relatively large north-south fluid displacement superimposed on the shear zone.

Interpretation--TAYLOR [1922] produced unstable and stable flow under conditions that appear to be similar to the unstable and stable cases of these experiments. The similarity lies in the fact that stability obtained when the fluid nearest the axis of rotation moved slower than the fluid at a greater distance from the axis, and conversely, instability set in at some critical magnitude of shear when the fluid near the axis of rotation had a greater angular velocity than the fluid more distant. The similarity of the experiments stops here, however, because the circulation that Taylor obtained with instability lies in a plane perpendicular to the eddies under investigation by the current work; in fact, a single annular ring of Taylor's vortices corresponds roughly in orientation and sense of rotation to the secondary (meridional) circulation described above and is obviously of an entirely different nature.

The results of the six experiments might be interpreted as illustrations of inertial stability and instability, although the application of particle mechanics to fluid dynamics is recognized as an approximation, at best. Following the notation of HAURWITZ [1941, pp. 292-295], dynamic equilibrium with respect to small disturbances can be expressed, STABLE $\eta^*(R + r)^2 > \eta R^2$; NEUTRAL $\eta^*(R + r)^2 = \eta R^2$; UNSTABLE $\eta^*(R + r)^2 < \eta R^2$, where η is the angular speed of the undisturbed fluid at a distance R from the axis of rotation and η^* is the angular speed of the fluid at a distance (R + r) from the axis. These criteria state that stability exists if the absolute angular momentum increases away from the axis and instability exists with the opposite condition. A fluid in solid rotation meets the condition for stability, while a VR-vortex satisfies the conditions for neutrality.

The stability criteria can be represented graphically by plotting the absolute angular momentum ηR^2 against distance R from the axis of rotation ($R = a \cos \phi$, where a is the radius of the sphere and ϕ is the latitude). In this coordinate system, dynamic stability is represented by a curve of positive slope, instability by a curve of negative slope, and neutrality by a horizontal line.

Each layer of the fluid within the model rotates at a rate proportional to the underlying driving surface (without lateral shear) except the fluid near the shear zone centered at 70°; therefore, dynamic stability must exist to the north and south of the region of shear. When the polar cap and zone rotate at different speeds (or in opposite directions), the band centered about 70° is a region of instability, neutrality, or stability, depending upon the slope of the angular momentum profile.

In Figure 6 the dotted line represents the angular momentum profile of the driving surfaces of the model where the polar cap is turning 1.75 times the angular velocity of the zone. The solid line represents a hypothetical profile of angular momentum of the fluid layer midway between the inner (driving) and outer (stationary) surfaces, under the assumptions that the fluid layer is moving

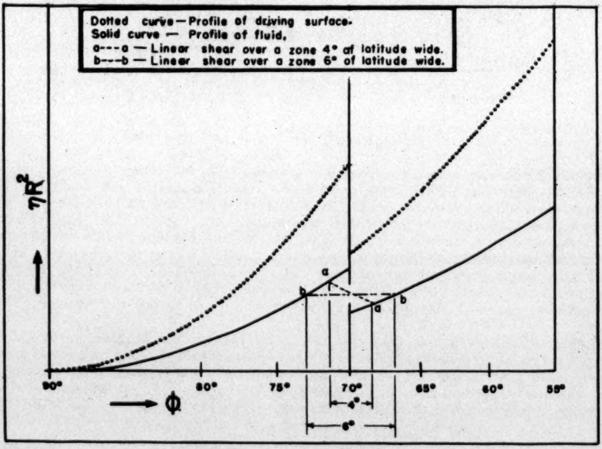

Fig. 6--Absolute angular momentum profiles at half the speed of the driving surface and that the width of the shear zone is zero. Since we are dealing with a real fluid, the shear will be distributed over a band of finite width and a-a represents a linear shear zone four degrees of latitude wide, while b-b represents a linear shear zone six degrees wide.

Figure 6 represents conditions typical of Experiment 5 in trials where the zone and polar cap rotate in the same direction. The experiments performed show that, although a zone of instability exists whenever the polar cap rotates faster than the zone, eddies do not form until the tangential velocity difference at 70° reaches a critical value. A speculative reason for this is offered. It has been observed (Exp. 5) that eddies were formed within a very narrow range of diameters and disappeared when conditions changed, without first decreasing in size. It therefore seems likely that some characteristic of this particular apparatus (say, for example, the relation between the radius of the boundary surfaces and the thickness of the working fluid) restricts the eddies to a certain diameter. Assuming that there are some such "natural sized" eddies in this particular model, the width of the band of instability must attain a latitudinal width nearly equal to that "natural size" before any eddies can form.

Now consider the mechanism that creates and widens the band of instability in the experiment represented by Figure 6. Suppose a particle of air is moving with the speed proportional to the driving surface, just south of 70° latitude, and is displaced a small distance northward by the meridional circulation and finds itself in an environment moving slightly faster. Since the momentum of the air is very small, the particle will assume the velocity of its environment before it can move far from its initial latitude, and consequently the width of the disturbed zone will remain narrow. When either the meridional circulation or the magnitude of the shear increases, the particles will move farther from their initial latitude before they assume the speed of their environment, thus increasing the width of the disturbed zone. (The equipment used in these preliminary experiments was not sufficiently flexible to permit a wide variation of the shear magnitude independent of the secondary circulation; when the shear was increased, the meridional circulation also increased, and therefore it was not possible to examine the separate roles of the north-south displacement of the fluid and of the shear magnitude in creating the instability zone.)

Referring again to Figure 6 it is evident that, with the magnitude of shear shown, the zone of instability can be no wider than six degrees of latitude. Increased mixing would produce a wider shear zone, but the profile would become neutral when the width became six degrees wide and stable when the zone increased to a greater width.

At first glance it would seem that the preceding discussion does not cover the case where the polar cap and zone rotate in opposite directions. It does in this manner; the criteria of the equations mentioned are actually concerned with the absolute magnitude, not the sign, of the angular momentum. When the fluid of the polar cap and zone are moving in opposite directions, the angular speed of the fluid will be zero near the shear line. As a result, the profile will have a large negative slope (unstable) immediately north of the shear line but an equally strong positive slope (stable) south of the shear line. Under such conditions an unstable profile can exist only in the region north of the latitude of zero velocity. The fact that the eddies formed under these conditions were centered near 70° latitude indicates that the zero velocity band was displaced southward rather than the zone of instability growing northward from 70°

The preliminary nature of this work makes it impossible to verify the explanation suggested here. Certainly the three-dimensional circulation that exists in the model is vastly more complicated than the particle instability discussed here. It is possible, however, that further work with this equipment will yield an insight into the type of instability that is manifest in these experiments,

Acknowledgments--These experiments were performed in the shop and laboratories of the Physical Research Division of the U. S. Weather Bureau. The work could not have been done without the loan of these facilities, and the author is indebted to Ross Gunn and his staff for their cooperation.

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Washington 25, D. C.

(Communicated manuscript received March 19, 1952, and, as revised, July 11, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

A TECHNIQUE FOR TAGGING AND TRACING AIR PARCELS

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Abstract--Successful tagging and tracing experiments have been carried out, using an aerosol of fluorescent pigment and exploring the aerosol "plumes" with an airplane carrying continuously-operating impaction equipment. Particles collected were identified and counted under the microscope, using ultraviolet illumination. Analysis of the data has yielded information on plume widths and concentrations up to 107 mi from the dispersal point.

Introduction

In connection with the general study of airborne particles of the New Mexico Institute of Mining and Technology, airplane equipment has been developed for quantitative collection of airborne particles, and chemical and optical means have been developed for recognition of individual small particles of some specific materials in the collections [CROZIER and SEELY, 1949]. This paper reports work done in the application of these techniques to tagging air parcels and tracing them through the meteorological process of turbulent diffusion.

The general method is similar to that used by several investigators. HEWSON [1945] used the gas SO_2 as a tagging material and measured its concentration at distances up to 27 mi from an industrial stack emitting 230 tons of SO_2 per day. SUTTON [1947] described experiments with both gases and particulate matter. CHURCH [1949] dispersed an oil fog through a stack and measured concentrations optically, apparently at distances up to 7000 ft from the stack. Recently, PERKINS and Others [1952], working independently from the authors, have used methods similar to those presently described.

It is believed that the method to be described, including recognition and counting of individual particles of a particulate tagging material, represents an advance over previous methods. The method permits the very rapid making of significant measurements of particle concentrations below the order of 100 per m^3, corresponding to a mass concentration from one part in 10^{16} to one part in 10^{13} parts of air, depending upon the particle size and recognition method used. Such concentrations are considerably below anything likely to be reached by conventional gas analysis under comparable sampling conditions, and, of course, are far below the limit attainable by any method using a collective property of an aerosol, such as optical scattering.

The earliest experiments with the present techniques used aerosols of copper compounds, largely because a microscopic chemical technique [CROZIER and SEELY, 1949] had been developed that made possible individual recognition of particles as small as 10^{-15} gm. Significant concentrations of a copper aerosol were detected at distances up to seven miles from a very weak source. Unfortunately, no method was found for dispersing the copper compounds in particles of the desired uniformity, and, in addition, it was discovered that copper-bearing particles of natural origin occasionally were abundant enough to cause confusion. Efforts to find a more suitable material resulted in choise of a fluorescent pigment, and field tests with this material were made in May, 1951.

Method and apparatus

The general method comprised dispersal of a fluorescent pigment through a military smoke generator or a special "dry duster," and sampling the resulting aerosol "plume" with impaction equipment installed in an airplane. Analysis of the impactor collections then made possible the calculation of particle concentrations from point to point along the flight path.

The fluorescent pigment--The pigment was a synthetic crystalline zinc sulfide manufactured by New Jersey Zinc Company under specification No. 2210. It fluoresces green, with maximum response under 3650 A illumination. When dispersed as an aerosol, it has a particle-size distribution shown by the histogram of Figure 1. The average particle diameter is about 2.3 microns, corresponding to an average mass of 2.6×10^{-11} gm. The settling velocity of the average particle in still air is about five feet per hour, which is negligible in comparison with the scale of the

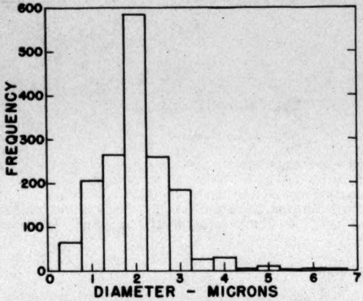

Fig. 1--Particle-size distribution of fluorescent pigment in collections from a smoke plume

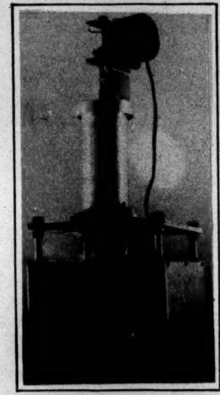

Fig. 2--The "dry duster"

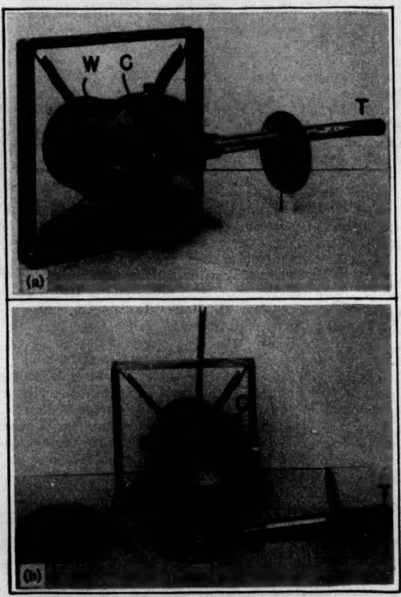

Fig. 3--The impactor, (a) assembled, (b) partially dismantled

experiments. Available information seems to indicate that the zinc sulfide pigment is not toxic, but this point should be further investigated before it is used in a populated area.

Dispersal--The dispersals were made in two ways; for most of the tests the pigment was mixed with an oil and fed into a military smoke generator (Besler, Model 374). The turbulence in the coils of the generator served to tear apart the pigment aggregates and quick dilution in the emitted smoke prevented re-aggregation. Most of the fluorescent particles are larger than the smoke droplets; some condensation of oil probably takes place directly on the fluorescent particles, but this appears to be unimportant. The same functions are performed, without the complications inherent in the smoke generator, by a "dry duster," shown in Figure 2, which was completed in time to be used in a few tests. The essential part of this apparatus is an eight-inch rotor spinning at 10,000 rpm, with its flat top separated from a stationary plate by a space of 0.030 inch. Air and pigment are admitted to this space near the center of the rotor and discharged at the edge after being subjected to strong shearing. Dilution is effected by the air admitted with the pigment and by a considerable centrifugal flow produced on the opposite side of the rotor. Pigment is fed by the hopper and stirrer arrangement shown on the top of the apparatus. The input to the smoke generator or the dry duster was adjusted to give an effective output of about 3×10^{10} particles per second.

Sampling--The impactor used for sampling is shown in Figure 3. It was installed in a Beachcraft twin-engine light transport aircraft, with the tube T projecting through the roof of the cabin.

The tube leads air from outside the airplane to a 1/8-inch nozzle N, which directs a jet of the particle-bearing air against a glycerin-gelatin coating on strip S, of transparent acetate. This strip is held in cylindrical form on a disk which is rotated by clockwork in W, so that the strip passes in front of the nozzle at a speed of 1/8 inch per minute. Particulate matter in the air processed is thus deposited in a ribbon lengthwise of strip S. Ram pressure on tube T, aided by an exhaust blower attached to outlet tube E, maintains a pressure head of 1200 ft of air (1.0 - 1.1 inches of Hg in the altitude range covered) across the nozzle, resulting in the processing of 38 liters of air per minute. Earlier studies of this impactor, which have been confirmed by recent work of RANZ and WONG [1952], indicated that practically all of the No. 2210 fluorescent particles would be collected except for those in the negligible smallest class shown in Figure 1.

Resolution--It is obvious that a resolution problem exists with this equipment. A momentary collection occupies a circle slightly more than 1/8 inch in diameter and the strip advances at 1/8 inch per minute. Briefly, the effect is to smooth out details of structure where dimensions of less than a mile or two are involved and, in general, in regions where particle concentration gradients are high, to distort somewhat the ratio between point-to-point particle density on the collecting strip and point-to-point particle concentration in the air sampled. Where dimensions of many miles are involved, the collection gives a fair picture of the plume, except for statistical fluctuations occasioned by the finite sampling rate in regions where concentrations are low.

Contamination--The results of the first few field tests were unsatisfactory because of confusion caused by stray particles. The first tests were made with an intake tube ten feet long on the impactor instead of the short tube T shown in Figure 3. In regions of high particle concentration, a small fraction of the incoming particles apparently became attached to the walls of the tube. In regions of low concentration some of these would be dislodged and deposited in the collection, constituting a serious source of error. The difficulty was overcome by substituting the short tube T, shown in Figure 3, for the long tube and by rigorous pre-flight scrubbing of the interior of the tube and impactor with a solvent, followed by coating with a film of vaseline, to retain any particles that happened to strike the walls. Precautions also were taken to avoid contamination of the strip with pigment particles in rooms where the pigment had been handled and from the clothing of persons handling the pigment.

Analysis--The fluorescent particles on the strip collection were counted under the microscope, with 60× magnification, using ultraviolet illumination. Points on the strip were correlated with positions of the airplane by appropriate marks made on the strip during flight. The counts were made in successive segments along the strip, the segment length being determined by a reticle width in the microscope. This segment length corresponded to a 25-second advance of the collecting strip as it passed in front of the impactor nozzle.

Using the particle counts, the plane speed, the strip speed, and the air flow, the particle concentrations in the air along the flight path were calculated, subject to the resolution limitation just mentioned. It happened that one fluorescent particle per segment represented a particle concentration of 63.7 particles/m^3 in the air sampled, explaining why concentrations shown in most of the figures occur in multiples of 63.7 (or 64).

Results

Eight of the field tests were particularly successful and examples of results obtained in three of the tests have been selected for the brief accounts below.

Test of May 18, 1951--The dispersal site was on the San Augustin Plains near Datil, New Mexico, at an altitude of 6900 ft. On this day the surface winds varied erratically five to 25 mph, swinging across an arc of more than 90°. The winds aloft, obtained by pilot-balloon observations during the test, were fairly uniform to 12,000 ft at eight to 16 mph. The lapse rate as measured by a free-air thermometer on the airplane was dry adiabatic up to 675 mb. Visual observation of the smoke plume indicated that vigorous mixing was taking place. There were scattered fair-weather cumulus clouds with bases at 12,000 - 13,000 ft. Many "dust devils" were present in the area.

A portion of the flight plan is mapped in Figure 4, the aerosol having been dispersed at the generator site at the left. Altitudes and times of day are shown along the flight path, as are the particle concentrations (in particles/m^3) for the latter part of the flight. The dispersal was from 14h 06m to 16h 30m MST. Besides the flight path shown in Figure 4, the exploration included a pattern of nine runs transverse to the radial runs, grouped about Section A_1 - A, approximately

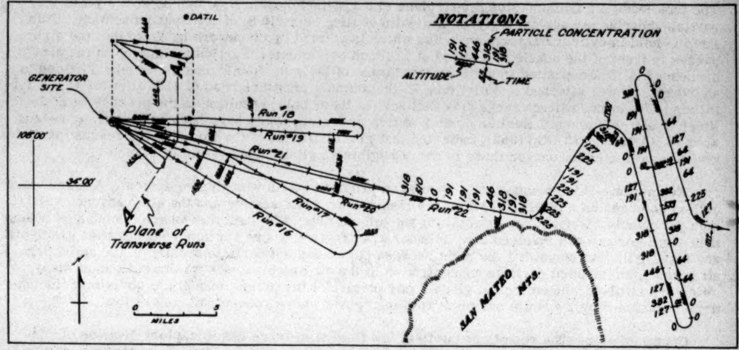
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Fig. 4--Flight plan of May 18, 1951, including particle concentration data for latter portion

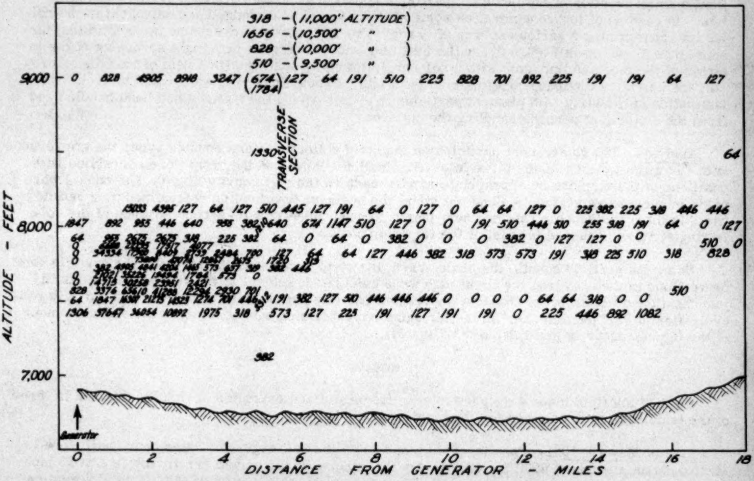

Fig. 5--Particle concentrations on radial runs of May 18, 1951

five miles from the generator site. The particle concentration data for all the radial runs have been compressed into a single section, shown in Figure 5, which also includes concentrations measured near the center of the plume on some of the transverse runs.

The particle concentrations found on the transverse runs near Section A$_1$ - A are shown in Figure 6. In this graph, the data for individual runs have been translated laterally for short distances to make the regions of highest concentration fall along one vertical line. This has the effect of eliminating large-scale meandering of the plume, giving what SUTTON [1947] calls the "instantaneous aspect" as distinguished from the "time-mean aspect." The time (44 min) required for

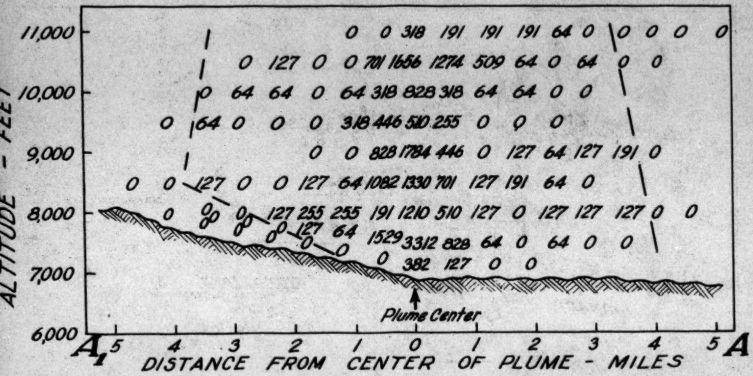

Fig. 6--Particle concentrations on the five-mile cross section (A_1 - A) of May 18, 1951

exploration of the section--even assuming all of the runs had been made in the same plane--was too long for a direct determination of the instantaneous aspect, and probably too short for a determination of the time-mean aspect.

The "instantaneous" width seen in this section is about 3.5 mi, and the height is about 4000 ft. The lateral distribution of concentration somewhat resembled the distribution that Sutton observed at shorter distances, but there is a tendency toward "bunching" vertically, one dense region being near the ground and another being about 3500 ft above the ground.

Using the concentration data of Figure 6, and estimating a general wind of 14 mph from the pilot-balloon data, the flux of particles through Section A_1 - A is calculated to have been approximately 2×10^{10} particles per second. Considering the nature of the measurements involved, this is regarded as consistent with the probable generator output of 3×10^{10} particles per second.

Test of May 24, 1951--This test is described because of the long distances involved. Otherwise, the experiment was not as satisfactory as some others.

Figure 7 shows the part of the flight pattern involving the longer distances. The dispersal was from a point ten miles west of Magdalena, New Mexico. The plume was carried down a valley between the San Mateo and Magdalena Mountains for about 16 mi, where this valley opens into a broad plain known as the Jornada del Muerto, interrupted only by the low Fra Cristobal Range. The dispersal point was at an altitude of 7000 ft, while most of the Jornada is at about 5000 ft.

Winds were stronger than on May 18, with considerable variations in speed aloft. The general direction was quite consistent, however. The dispersal extended from 10h 45m to 12h 23m MST, including a number of short shut-downs occasioned by malfunctioning of the equipment.

Two sets of transverse runs were made, near Sections C_1 - C and D_1 - D, 76 and 107 mi, respectively, from the source. The general features of particle concentration in the two sections were quite similar, and are illustrated by the 107-mi section in Figure 8. No lateral adjustments of the data have been made here, but the width probably should still be regarded as the "instantaneous width." Apparently neither the margins nor the top were quite reached in the exploration, but the particle flux through the section shown is about 2.4×10^{10} per second which suggests that not very much of the plume has been missed. The particle concentrations in this portion of the plume are so small that statistical fluctuations, in general, obscure any details of structure. It does seem, however, that a real feature is the wide gap between the dashed lines. This gap appears also in the 76-mi section, its position in both sections being indicated in Figure 7. No very adequate explanation is offered for the presence of this gap; conceivably, it may have been the result of a disturbance, such as a downdraft, near the Fra Cristobal Range.

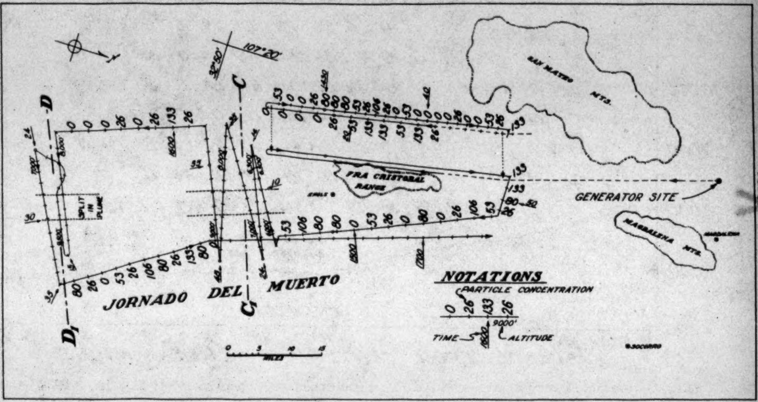
107°20'
32°50'
D
D1
C
C1
SAN MATEO MTS.
FRA CRISTOBAL RANGE
GENERATOR SITE
MAGDALENA MTS.
MAGDALENA
SPLIT IN PLUME
JORNADO DEL MUERTO
NOTATIONS
PARTICLE CONCENTRATION
0 26 133 26
9000'
TIME
1800
ALTITUDE
0 5 10 15
MILES
SOCORRO

Fig. 7--A portion of the flight plan of May 24, 1951, including particle concentration data

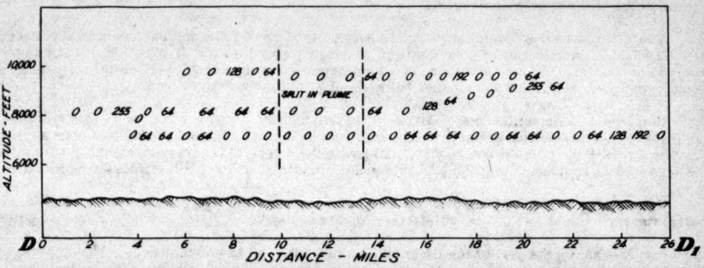

Fig. 8--Particle concentrations on the 107-mi cross section (D_1 - D) of May 24, 1951

Test of May 17, 1951--Figure 9 presents a single cross section, 15 mi from the dispersal point, from this test as an example of results obtained at a moderate distance. The plume was

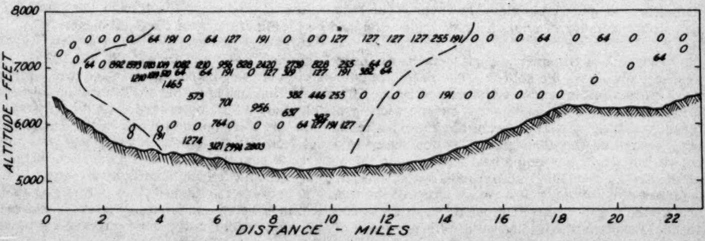

Fig. 9--Particle concentrations on the 15-mi cross section of May 17, 1951 not quite typical, however, in that less than usual vertical mixing was evident in both the visible plume and in the particle collections. The resultant particle concentrations are approximately double the values predicted on the basis of other tests at comparable distances. The dispersal was from a point 20 mi north of Socorro, and the plume traveled southward down the Rio Grande Valley. It apparently was under the influence of a gentle down-slope drainage following a thunderstorm. The lapse rate was still dry adiabatic, as far as could be ascertained by temperature measurements made at the aircraft. It is apparent in Figure 9 that the distribution of concentration has departed considerably from what might be called a "normal" distribution for plumes close to their sources. This is quite typical of the distributions generally found at distances greater than five or ten miles, the tendency being toward replacement of the symmetrical structure with a marked central peak by a more diffuse structure having minor spots of comparatively high concentration dispersed at random through the plume. This cross section is a particularly suitable one for calculation of particle flux; using the observed wind of 15 mph, the value is 3.6×10^{10} particles per second.

Summary of results--Although the primary purpose of this investigation was the evaluation of the technique, some organization of the data obtained may be of interest in connection with the general problem of turbulent diffusion.

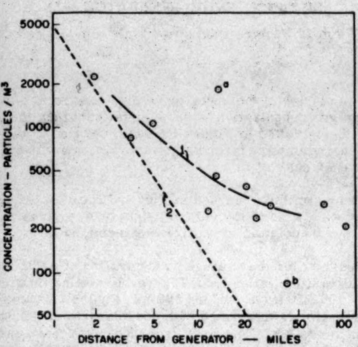

Fig. 10--The relation of peak particle concentrations to distance from source (corrected to 25-mph wind)

The plotted points in Figure 10 represent peak concentrations determined in 12 transverse cross sections at various distances from the dispersal point. The data were obtained in tests on seven different days and are lumped together for this graph only because of the small amount of data available from any one test. Conditions, except for wind speeds, were, in general, similar from day to day. The peak concentrations were taken arbitrarily as the highest averages over any three consecutive 25-second intervals in each plume section. For this reason, and also because the sampling procedure has smoothed out narrow peaks, the values plotted are considerably lower than the actual highest point concentrations in the plumes. At the shorter distances, where the distribution is fairly normal, the procedure gives an approximation to the kind of peak concentration at short distances observed and discussed by Sutton. At the longer distances the highest concentrations may be found anywhere in the plume, not necessarily near its center.

The actual concentrations have been corrected in all cases to a wind speed of 25 mph on the basis of the reciprocal law, that the concentration is inversely proportional to the wind speed, which was verified by SUTTON [1947].

Two of the points, marked a and b, seem to be definitely out of the class represented by the others. Point a represents the high concentration discussed above for the 15-mi section of May 17. In the case of b, at 43 mi, the wind pattern was such that the plume was dispersed over an unusually large cross section. A portion 25 mi wide and 3000 ft deep included only about one-third of the particle flux.

Curve 1 in Figure 10 has been fitted by eye to the experimental points. Curve 2 represents peak concentrations calculated on the basis of Sutton's observations for short distances, that is, the peak concentration is inversely proportional to the 1.76 power of the distance from the source. In this calculation, the reference point was taken at 0.001 mi (5.28 ft) from the source, where the average concentration over a plume six feet in diameter is 6×10^8 particles per cubic meter. The peak was assumed to be 10^9 per cubic meter. The experimental curve does converge upon this line at a distance of the order of a mile from the source. At distances beyond five or ten miles, the experimental peak concentrations, determined in the manner described, are much above curve 2; the actual peaks would be even higher.

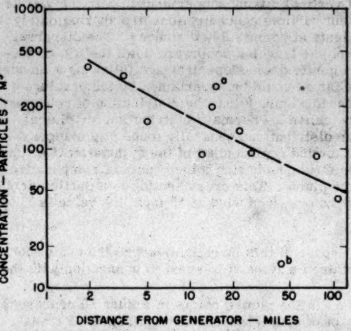

Fig. 11--The relation of average particle concentrations to distance from the source (corrected to 25-mph wind)

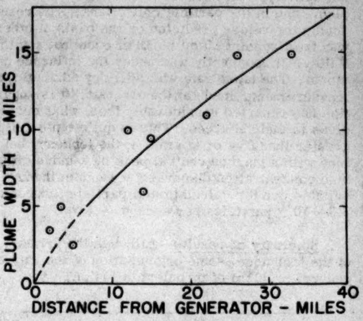

Fig. 12--Plume widths to 40 mi from the source

Because of the apparently random structure in plumes at distances more than approximately ten miles from the source, it may be that the average concentration is more significant than the peak concentration. Average concentrations have been plotted in Figure 11, again reduced to the 25-mph basis. The areas used in computing the averages were taken somewhat arbitrarily to include all but a few per cent of the particle flux in each case.

The points marked a and b correspond to those similarly marked in Figure 10 and the same remarks apply. The dashed line in Figure 11 has been fitted to the plotted points by eye. It is obvious that this line cannot be used as a basis for extrapolation to shorter distances, however.

Taking the figure used above, 6×10^8 particles/m^3, for the average concentration at 0.001 mi from the source and taking the average concentration at ten miles from Figure 11, we find that the dilution between 0.001 mi and 10 mi is about 3.6×10^6. Between ten and 100 mi, Figure 11 shows the dilution to be only three, however. This latter factor is mentioned with considerable diffidence, since it is based upon such a small amount of data and is doubtless quite dependent upon the atmospheric situation. Nevertheless, it is believed to be of the right order of magnitude for situations comparable to those existing during the tests.

The width of the plume as a function of distance is plotted in Figure 12 out to a distance of 40 mi from the source. The curve has been fitted to the plotted data by eye. The widths shown at 2, 3.5, and 5 mi are not very reliable because of the high concentration gradients encountered, so the curve could not be drawn with much confidence in the first ten miles. An attempt was made to define the margin of a plume as the point at which the concentration drops to one-tenth the peak value. The structures encountered beyond ten miles did not permit meaningful application of this sort of definition, however, and the widths used are merely those that were judged to include practically all of the plume. Only two points are available beyond 40 mi, those roughly estimated at 76 and 107 mi on May 24. The width at 100 mi probably was about 25 mi.

Appraisal

The results of the tests described indicate the feasibility of the method. With the rate of particle output used, the concentrations 100 mi from the dispersal point were somewhat marginal, and the best results were obtained with distances up to about 25 mi. However, dilution seems to be relatively slow beyond ten miles, and adequate concentrations could be obtained at 100 mi by merely increasing particle output by a factor of ten. This could be done by increasing the mass output, but the most satisfactory way would be to reduce the average particle diameter to about one micron, using about the same mass output.

Other desirable improvements, which can be made easily, include provision for increasing the resolution for close-in studies and provision for ultraviolet illumination and microscopic viewing of the particle collection as it is being made, so that the flight can be guided for best utilization of the flying time.

The fluorescent pigment appears to be an excellent tagging material, though the techniques are not necessarily limited to its use. Dispersing the particulate matter through a military smoke generator seems unnecessarily cumbersome, and further work should be done with some sort of "dry duster."

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(Manuscript received April 21, 1952, and, as revised, July 31, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 6, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

THE POWER SPECTRUM ANALYSIS OF OCEAN-WAVE RECORDS

Willard J. Pierson and Wilbur Marks

Abstract--Techniques of power spectrum analysis developed by Tukey and Hamming for the analysis of noise in electronic circuits are applied to ocean-wave records. The power spectrum of a sea-surface record is computed directly from the power spectrum of a pressure record. Statistical properties of the wave records are deduced and verified within the limitations of a linearized theory. The implications of the results with respect to the problem of wave forecasting and wave refraction are discussed. The relationship between the non-normalized autocorrelation function and the power spectrum is given, and the results of Deacon and Seiwell are interpreted in connection with the results of the analysis.

Data--The pressure record to be studied was obtained by a pressure gage submerged in 32.5 ft of water off the coast of Long Branch, New Jersey. Housed in the pressure gage is a transducer which converts water-pressure fluctuations into electric current or voltage signals. These signals are transmitted to a shore-based General Electric brush recorder which makes a continuous trace of the pressure variation on roll paper. The record was obtained during a minor coastal storm on October 18, 1951, and represents 25 min of continuous recording from 22h 58m to 23h 23m EST.

Once the pressure record is obtained, it is a matter of computing the properties of the free-surface disturbance from data available in the pressure record. Errors in past work of this sort arise from the assumption of a "characteristic period" for the wave motion, that is, the average period for the well-defined series of highest waves observed, and also from the consideration that the wave motion is sinusoidal in form. Observations have shown that actual wave heights are six to 25 pct greater than those computed theoretically [SNODGRASS, 1951], the error resulting partly from erroneous assumptions mentioned above. A more precise analysis is given both theoretically and quantitatively below.

The Lebesgue power integral for the Gaussian case--Consider a pressure record recorded as a function of time in shallow water by a pressure recorder located on the bottom. Neglect the static pressure. Let the depth of the water be H. Also consider the following formula for the pressure as a function of time at a fixed point.

$$P(t) = \rho g \int_0^\infty \cos(\mu t + \psi(\mu)) \sqrt{[A_{PH}(\mu)]^2 d\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The formula represents some kind of integration which is not usually found in the ordinary texts of mathematics. The integral can be called a Lebesgue power integral for the Gaussian case, and, in the same way that an ordinary integral can be expressed as the limit of a partial sum which approximates the area, this integral can be expressed as a limit of a partial sum.

First though, let $[A_{PH}(\mu)]^2$ be zero for μ negative or zero and everywhere positive or zero for μ greater than zero. Also let the integral over μ from zero to infinity of $[A_{PH}(\mu)]^2$ be E_{PHmax}. $[A_{PH}(\mu)]^2$ has the dimension of cm^2 sec. E_{PHmax} has the dimension of cm^2. Divide the positive μ axis into a net; μ_0 (=0), μ_1, μ_2, μ_3 . . . μ_{2r}. Then the limit of the partial sum given by (2) defines the integral given by (1).

$$P(t) = \lim_{\substack{\Delta\mu \to 0 \\ \mu_{2n} \to \infty}} \rho g \sum_{r=0}^{2n} \left[[A_{PH}(\mu_{2r+1})]^2 (\mu_{2r+2} - \mu_{2r}) \right]^{1/2} \cdot \cos\left[\mu_{2r+1} t + \psi(\mu_{2r+1})\right] \quad \ldots\ldots\ldots (2)$$

Each partial sum for a finite net is very easily written down once the rules for picking $\psi(\mu_{2r+1})$ are given. The rule is to pick a value for ψ between 0 and 2π in a completely random fashion such that (3) holds. Equation (3) is read: The probability that ψ lies between 0 and $2\pi\alpha$ equals α where $0 < \alpha < 1$.

$$p(0 < \psi(\mu_{2r+1}) < 2\pi\alpha) = \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Equation (1) is thus the limiting form of (2) as the mesh of the net approaches zero. The integral cannot be written down explicitly, but many of its properties are easily deduced.

Consider (4). It states that the square of the pressure record averaged over time equals $E_{PHmax}/2$ if the time over which the record is averaged approaches infinity. The equation can be proved by first treating a partial sum such as (2) and then letting the mesh of the net go to zero.

$$\lim_{T \to \infty} \frac{1}{(\rho g)^2} \frac{1}{T} \int_{t^*}^{t^*+T} [P(t)]^2 dt = \frac{E_{PHmax}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The Gaussian property of the record--The above derivation shows that the pressure record can be considered to be a special type of sum of infinitesimally high sine waves each with a different period and with random phases. High waves occur at phase reinforcement, and low waves occur at phase cancellation. A portion of a sine wave is no longer a sine wave, and methods of analysis based on counting the bumps in a pressure record and on attempting to extrapolate these up to the free surface are bound to fail.

The integral given in (1) has the very important property that points chosen at random from the pressure record P(t) will be distributed according to the Gaussian law or the normal probability curve. To prove this, consider the partial sum given by (2). Let the cosine be written as the real part of $\exp\left[i\left\{\mu_{2r+1} t + \psi(\mu_{2r+1})\right\}\right]$. Then the partial sum represents a number of vectors in the complex plane pointing in all different directions at random. In books on the theory of statistics it is proved that the projection on the real axis of the limit of this sum of random vectors is distributed according to the Gaussian law, and the final result is that (5) describes the distribution of points taken at random (from a long enough record) from P(t) if pressure is expressed in the engineering units of, say, centimeters of static water pressure.

$$p(K < P(t_1) < K + dK) = \frac{1}{\sqrt{\pi E_{PHmax}}} \int_{K}^{K+dK} e^{-\frac{\xi^2}{E_{PHmax}}} d\xi \quad \ldots\ldots\ldots\ldots \quad (5)$$

The equation is read: The probability that P(t) lies between K and K + dK is given by the area under a normal curve between the values K and K + dK where the variance of the distribution is given by $E_{PHmax}/2$.

If, then, several hundred points are taken from a wave record at random, they should be distributed according to a normal distribution with a mean of zero and a variance given by $E_{PHmax}/2$. This is indeed the case, as is shown in Table 1. From the 25-min pressure record described above, 200 points were chosen. A frequency distribution, based on the deviations from the mean of an arbitrary scale, was constructed and the resulting histogram appears to have the properties of a Gaussian distribution.

Next, the distribution was submitted to the chi-square test. The result was a probability factor of 0.5. This means in analogy with (5) that this set of points, P(t), chosen from this 25-min record, which contains an infinite population of points, could have come from a normal (Gaussian) distribution 50 pct of the time by chance.

To substantiate further this important property of the wave record, the total time during which the pressure record occupied each graduation of the recording chart was measured. A comparison with the theoretical results based on standard deviations was made and the results are tabulated in Table 2. Because of the convenience of measurement afforded by the chart scale units (col. 1), the theoretical probability (col. 3) that a portion of the wave record would occupy, say, that part of the scale from zero to plus one scale unit was computed by expressing the chart scale as fractional parts of $P(t_1)/\sqrt{E_{PHmax}/2}$ (col. 2). The predicted time that the record occupies each portion of the entire graph appears in column (4). Columns (5) and (6) comprise the actual time spent

Table 1--Frequency distribution of 200 observations from a pressure-wave record illustrating the Gaussian property of the wave record

Tally limits	Observed frequency (f_i)	Theoretical frequency (F_i)	$\frac{(f_i - F_i)^2}{F_i}$
-3.249 to -2.750	1.0	1.5	0.17
-2.749 to -2.250	2.0	4.1	1.08
-2.249 to -1.750	10.0	7.9	0.56
-1.749 to -1.250	15.0	15.0	0.00
-1.249 to -0.750	21.0	23.8	0.33
-0.749 to -0.250	39.0	31.2	1.95
-0.249 to +0.250	35.0	34.1	0.02
+0.251 to +0.750	31.0	31.1	0.00
+0.751 to +1.250	16.0	23.6	2.45
+1.251 to +1.750	16.0	14.9	0.08
+1.751 to +2.250	6.0	7.9	0.46
+2.251 to +2.750	5.0	3.4	0.75
+2.751 to +3.250	3.0	1.5	1.50

$\chi^2 = 9.35$

$P = \sim 0.5$

with ten degrees of freedom

Table 2--Predicted and observed total time during which pressure record occupies a portion of the graph of the 25-minute record

Chart scale units (1)	Ratio[a] (2)	Probability (3)	Predicted 25P (4)	Observed - (5)	Observed + (6)	Average $\frac{(+obs) + (-obs)}{2}$ (7)
			min	min	min	min
0.00 - ±1.00	0.00 - ±0.855	0.3034	7.58	7.35	8.03	7.69
±1.00 - ±2.00	±0.855 - ±1.71	0.1529	3.83	3.78	3.82	3.80
±2.00 - ±3.00	±1.71 - ±2.56	0.0382	0.931	1.12	0.64	0.88
±3.00 - ± ∞	±2.56 - ± ∞	0.0055	0.138	0	0.17	0.083
	Totals	0.5000	12.48	12.25	12.66	
			× 2		12.25	
			24.96		24.91	

[a] Ratio of chart scale units to standard deviation in scale units.

by the record above (+) and below (-) the mean in certain ranges of the chart scale, and column (7) is an average of the two. Columns (4), (5), and (6) are of interest, since column (4) represents the predicted behavior of a normal distribution, and columns (5) and (6) represent the observed behavior of the distribution under consideration. Comparison of these three columns leaves little doubt that the observations have been drawn from a normal distribution. Column (7) is the average of columns (5) and (6). The averaging takes out some of the left-over non-linearity of the record, and it agrees remarkably well with column (4).

The observed and theoretical elapsed times do not account for the full 25 min because of rounding error and measurement error from the record. The missing 5.4 sec are easily accounted for, when it is considered that the measurements of time from the pressure record were made visually to an accuracy of only 1/16 inch. Six inches of the record chart equals one minute of recording.

RUDNICK [1951] has also presented the cumulative distribution function of points taken from a wave record, and it is that of a normal distribution. This property is a very important property of wave records because it explains the various ratios of wave height values obtained in the literature as summarized by SNODGRASS [1951].

The power spectrum--The function $[A_{PH}(\mu)]^2$ is the power spectrum of the pressure record.

It varies from record to record. A narrow spectrum yields a "swell" record, a wide spectrum yields a "sea" record. If $[A_{PH}(\mu)]^2$ is given, then many properties of the waves can be deduced. If a more general function of the free surface obtained by PIERSON [1952] is given at the edge of a storm at sea, it is possible to forecast the power spectrum of the waves at any point outside of the storm. Thus one problem in wave-forecasting theory and in the theory of wave-record analysis is to find the power spectrum of a pressure record.

Various attempts to find this spectrum have been made. Machines described by KLEBBA [1946] and DEACON [1949] evaluate some sort of spectrum from a wave record. Great debate has occurred on the quantitative meaning of the results, and in particular SEIWELL [1949] was extremely dubious about their meaning. The spectra obtained (which were probably not true power spectra) were very jagged and irregular, and they did not seem to be very well related to the properties of the wave record.

These difficulties can be resolved by the techniques of analysis developed by TUKEY [1949] and TUKEY and HAMMING [1949] (originally for the analysis of colored noise). The jaggedness of the mechanical analysis can be explained on the basis of too sharply tuned and too narrow band-pass filters, on the basis of too short a record, and on the basis that the wheel of the analyzer slows down too rapidly. Minor modifications of these machines will probably make them reliable instruments for wave analysis after they have been calibrated.

The problem at present is to find $[A_{PH}(\mu)]^2$ given a pressure record. In words, the power spectrum is the Fourier cosine transform of the non-normalized autocorrelation function of the wave record. In symbols, the non-normalized autocorrelation function is given by (6).

$$Q(\tau) = \lim_{\bar{T}\to\infty} \frac{2}{\bar{T}} \int_{t^*}^{t^*+\bar{T}} P(t)P(t+\tau)\,dt = \int_0^\infty \cos\mu\tau \cdot [A_{PH}(\mu)]^2\,d\mu \quad \ldots\ldots\ldots\ldots (6)$$

Then the Fourier cosine transform yields the power spectrum as shown by (7).

$$[A_{PH}(\mu^*)]^2 = \frac{2}{\pi}\int_0^\infty Q(\tau)\cos\mu^*\tau\,d\tau = \lim_{M\to\infty}\frac{2}{\pi}\int_0^M\int_0^\infty \cos\mu\tau\,[A_{PH}(\mu)]^2\,d\mu\,\cos\mu^*\tau\,d\tau \quad \ldots\ldots (7)$$

The above equations imply records which are infinitely long and stationary in the sense that the fundamental character of the record never changes. Waves from a storm at sea do change their character slowly over time intervals of the order of hours. Now, infinity need not be a very big number for practical purposes, and by assuming that each pressure record is "temporarily" homogeneous it is possible to estimate the power spectrum of the pressure record and deduce the character of the waves passing overhead.

Numerical estimation of non-normalized autocorrelation function and the pressure power spectrum--In the practical applications of the above results, integrals are replaced by sums and a theoretically infinite record is replaced by a piece of a pressure record. TUKEY [1949] and TUKEY and HAMMING [1949] have considered the errors which enter into the numerical analysis of such a segment of a time series and they have found the corrections which must be applied to the "raw" results and the degree of reliability of the final results. The numerical method of analysis is analogous to the above integrals, and the final results can be stated quite precisely in statistical terms.

The formulas for numerical estimation of power spectra as presented by TUKEY [1949] and TUKEY and HAMMING [1949] are given by

$$t_2 - t_1 = t_3 - t_2 = \cdots\cdots t_N - t_{N-1} = \Delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$Q_p = \frac{2}{N-p}\sum_{n=1}^{N-p} P(t_n)P(t_{n-p}) \qquad (p = 0,1,2,\ldots m) \quad \ldots\ldots\ldots\ldots (9)$$

$$L_h = \frac{1}{m}\left(Q_0 + 2\sum_{p=1}^{m-1} Q_p \cos\frac{\pi ph}{m} + Q_m \cos\pi h\right) \qquad (h = 0,1,\ldots m) \quad \ldots\ldots\ldots\ldots (10)$$

$$U_h = 0.23L_{h-1} + 0.54L_h + 0.23L_{h+1} \; (h = 0, 1, \dots m) \qquad (11)$$

$$\mu_h = \frac{\pi h}{\Delta t m} \qquad (12)$$

$$[A_{PH}(\mu_h)]^2 = \frac{U_h \Delta t m}{\pi} \qquad (13)$$

Equation (8) simply states that N equally spaced time intervals are marked off on the record and that these points are spaced Δt units apart. The values of the record in terms of departure from the mean are then tabulated as $P(t_1)$, $P(t_2)$, . . .; $P(t_n)$, . . . $P(t_N)$, and thus there are N numbers to work with.

The non-normalized lag coefficients are then found by the use of (9). There are m + 1 numbers which result as Q_p ranges from Q_0 to Q_m. Note that Q_0 equals E_{PHmax}. An important part of the data is thrown away by normalizing the data and working with the autocorrelation function. Note the analogy to (6) where τ equals $\mu \Delta t$.

The pressure record was sent to Woods Hole Oceanographic Institution where their mechanical autocorrelator did the work of (9), thus eliminating much numerical work on a computing machine. The resulting lag coefficients, which were the non-normalized values multiplied by some arbitrary constant, were reduced to unit value by the division of all values by the value at lag zero, and the result was plotted against the lags. The resulting autocorrelation function appears in Figure 1. Multiplication of the normalized autocorrelation function by E_{PHmax} will yield the non-normalized autocorrelation function.

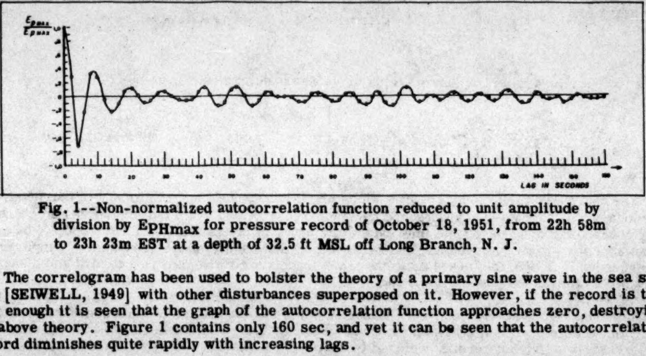

Fig. 1--Non-normalized autocorrelation function reduced to unit amplitude by division by E_{PHmax} for pressure record of October 18, 1951, from 22h 58m to 23h 23m EST at a depth of 32.5 ft MSL off Long Branch, N. J.

The correlogram has been used to bolster the theory of a primary sine wave in the sea surface [SEIWELL, 1949] with other disturbances superposed on it. However, if the record is taken long enough it is seen that the graph of the autocorrelation function approaches zero, destroying the above theory. Figure 1 contains only 160 sec, and yet it can be seen that the autocorrelation record diminishes quite rapidly with increasing lags.

The "raw" estimates of the power in the μ band from $\mu = \pi(h - 1/2)/\Delta tm$ to $\mu = \pi(h + 1/2)/\Delta tm$ are then found from (10). The result is again m + 1 numbers. This equation is the analogue of (7).

In the computation of the "raw" estimates, m = 30, and h goes from 0 to 30. The resulting graph is shown in Figure 2. Note the very jagged graph of the "raw" estimates. These raw estimates must be corrected for the effect of the distortions introduced by picking the discrete points and by the finite length of the record. The correction is given by (11) which filters the "raw" data. The auxiliary formulas that $L_{-1} = L_1$ and $L_{m+1} = L_{m-1}$ are needed to evaluate U_0 and U_m.

The final numbers, U_h, then estimate the mean value of the power spectrum from $\pi(h - 1/2)/\Delta tm$ to $\pi(h + 1/2)/\Delta tm$. At the point given by (12) the best estimate of the value of $[A_{PH}(\mu h)]^2$ is given by (13). The result of this smoothing process is shown in Figure 3 (continuous line). This then is the best estimate of the pressure power spectrum. Theoretically, no portion of the graph of the power spectrum should go below zero. TUKEY and HAMMING [1949] have shown that rounding and computation errors introduce "white" noise into the spectrum which

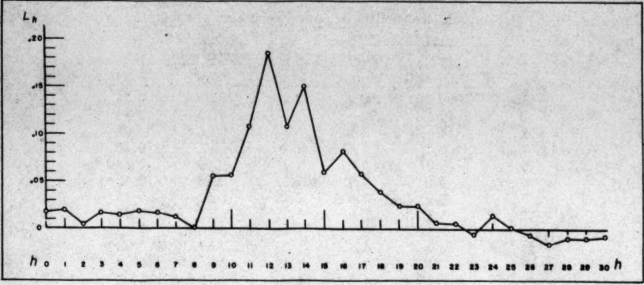

Fig. 2--"Raw" or unfiltered power spectrum of Figure 1

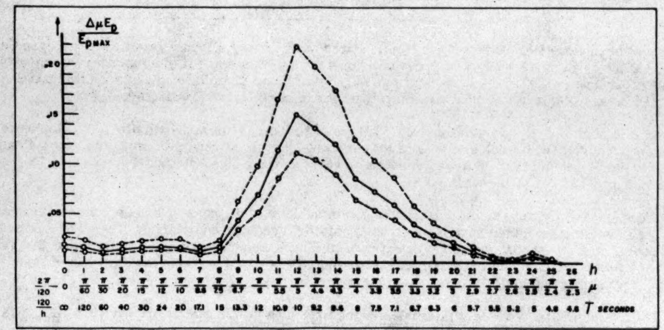

Fig. 3--Best estimate of the pressure power spectrum of the autocorrelation function in Figure 1 in terms of fraction of total power per unit band of the μ axis; sum of values at: $\mu = 2\pi h/120$, $h = 0, 1, 2, \ldots 30$ is equal to 1.018

accounts probably for the low values for high periods. The part below the axis on the far right is negligible, and represents possibly minor errors in some part of the computations.

The sum of the values measured at the points of the power spectrum is very close to one, and since the autocorrelation function is normalized this shows that the power under the spectrum is one and that the computations are reliable.

Reliability of results--The final results are estimates. Estimates can be in error, and the important part of this method of analysis is that it also yields a statistical analysis of how much in error the estimates can be. TUKEY [1949] and TUKEY and HAMMING [1949] have shown that the errors are distributed according to a χ^2 distribution with f degrees of freedom where f is given by (14).

$$f = \frac{N - \frac{1}{4}m}{\frac{1}{2}m} \qquad (14)$$

Given f, then the use of Table 3 yields information on the reliability of the one estimate for each band obtained by the analysis of one record.

If a given analysis has, say, ten degrees of freedom, then the number obtained from (11) can be multiplied by 2.6 and by 0.55. The true value of the power in the band under analysis will lie

Table 3--Reliability of power spectrum estimates

f	Possible error in observed value				
	2.5 pct	5 pct	50 pct	95 pct	97.5 pct
1	1000	250	3.1	0.26	0.2
2	40	20	1.5	0.33	0.21
3	14	8.5	1.3	0.36	0.32
4	8.3	5.63	1.2	0.42	0.36
5	6.0	4.37	1.15	0.46	0.39
6	4.8	3.6	1.12	0.48	0.42
8	3.8	2.8	1.10	0.51	0.46
10	3.1	2.6	1.07	0.55	0.49
15	2.4	2.1	1.05	0.60	0.55
20	2.1	1.8	1.02	0.63	0.59
50	1.55	1.45	1.02	0.74	0.69
100	1.35	1.29	1.00	0.79	0.78
Large		$10^{1/\sqrt{f}}$		$10^{-1/\sqrt{f}}$	

between these extremes 90 pct of the time. Stated another way, given a great many analyses of different short samples from a long run of a time series, then the set of bounds given above will enclose the true power in the band 90 times out of 100 if the sample length is such that ten degrees of freedom are obtained and if a very large number of samples are evaluated.

In this analysis, $N = 750$ and $m = 30$. The result from (14) is approximately 50 degrees of freedom. Therefore, if each ordinate of the smoothed power spectrum is multiplied by 1.45 and by 0.74, it can be said that the true pressure power spectrum lies between these bounds (dotted lines in Fig. 3) 90 pct of the time.

Were the record only seven minutes long, the number of degrees of freedom would be much less, and even the smoothed power spectrum would be irregular and jagged. The available evidence then seems to suggest that the power spectra of pressure records are quite smooth and that irregularities such as those found by KLEBBA [1946] and others are due to too few degrees of freedom and not to basic irregularity in the power spectrum.

The assignment of correct amplitudes to the pressure power spectrum--Since $E_{PH\max}$ is known to be equal to 601.5 cm^2 from direct computation of the standard deviation of the pressure record, a second scale is shown on the left of Figure 4. In units of cm^2 sec, this scale gives the true amplitude of the pressure power spectrum. The area under the solid curve then equals 601.5 cm^2. The total power present ought to be quite close to this value. It is possible, however, that the true power spectrum oscillates above and below the solid line quite rapidly within the dashed boundaries. To prove that it does though would require a very much longer record and a larger m.

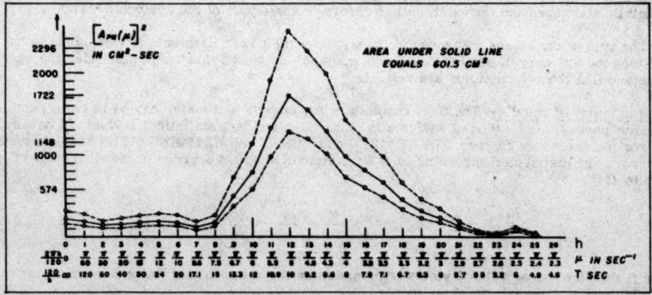

Fig. 4--Best estimate of the pressure power spectrum of the autocorrelation function plotted in terms of $[A_{PH}(\mu)]^2$ versus μ; true spectrum lies between dashed boundaries 90 pct of the time

Within the ability to analyze a 25-min record with two-second lags, the best estimate of the power spectrum as given by the solid lines is smooth and quite well behaved.

A curious feature--There is a very curious feature of the results that TUKEY [1949] and TUKEY and HAMMING [1949] have obtained which might be referred to as the invariance of the difficulty of obtaining a given degree of precision for a particular estimate. In (8), point values were chosen Δt units apart. From (12), the highest value of μ for which a power spectrum value is given occurs when h equals m. Thus $\mu = \pi/\Delta t$ (or $2\pi/2\Delta t$), which shows that the lowest period obtained is $2\Delta t$ sec. (The periods for the points under consideration are given by $2\Delta tm/h$.) One of the features of the numerical method of analysis is that values of μ greater than this value are aliased into values of μ which are less than this value. If the power spectrum contains important power contributions from periods less than $2\Delta t$, then Δt has been chosen too large for valid results. Similarly, if the power spectrum does not contain periods near or less than $2\Delta t$, then Δt is too small and great effort is being wasted.

It does not pay to pick Δt to be small in order to get more data from a wave record a given number of seconds long. Suppose, for example, that there is reason to believe that there are no important contributions from periods less than four seconds. Then, Δt can be chosen to be two seconds. Suppose that it is also desired to distinguish between the power in the bands given by $19\pi/120$ to $21\pi/120$ and $21\pi/120$ to $23\pi/120$ (or periods from 12.63 to 11.43 sec and from 11.43 to 10.43 sec). Then m has to be equal to 30. If 50 degrees of freedom are desired, then N has to be about 750 points, and if the points are two seconds apart the record must be 25 min long.

Now suppose that the value of Δt is halved. Then Δt is one second and in order to obtain the same resolution given above m must be chosen to be equal to 60. For a 25-min record the number of degrees of freedom, namely, $f = (1500 - 15)/30$, is still about 50. Were the original assumption that nothing of importance was present below a period of four seconds true, then all of the values of U_h in (11) (for $\Delta t = 1$ sec and $m = 60$) would be zero above $h = 30$. Thus doubling the value of N by halving Δt in this case yields 30 values of zero (or nearly zero). A study of (8) through (13) shows that the numerical work needed is far more than doubled, and yet final results are obtained which are no more reliable than the original analysis for $\Delta t = 2$ sec.

Application to a pressure record--The analysis of this pressure record is a perfect example of the above considerations. The spectral components from a sea-surface disturbance are filtered out with depth more rapidly for low periods than for high periods. Thus for a pressure recorder on the bottom at a depth of 30 ft, wave components with wave lengths less than 60 ft at the surface are not recorded. Waves with periods less than $\sqrt{12}$ are essentially undetected and spectral components with a period of four seconds cause a response of the pressure recorder proportional to about 0.025 times the power in the band actually present at the surface. Consequently, for all practical purposes the analysis of a pressure record made in 30 ft of water can be made with a Δt of two seconds, and this is what has been done.

A second example can be cited from the literature. This example will be discussed more thoroughly in a paper in preparation by Pierson. In the example, TUKEY and HAMMING [1949] analyzed an autocorrelation made by Seiwell with one-second lags of a pressure record taken in about 80 ft of water. As would be expected from the above remarks, the power in all of the bands from $2\pi/6$ to $2\pi/2$ (or two-thirds of the entire spectrum) was zero.

The power spectrum of the free surface--The waves passing over the pressure recorder produced the pressure record. As suggested by EWING and PRESS [1949], there is only one way to find the correct power spectrum of the free surface or actual wave record from the pressure record. Each spectral component of the pressure power spectrum must be multiplied by a different correction factor in order to find the power spectrum of the actual waves. The techniques described by SNODGRASS [1951] and, for that matter, many others are not correct because, as has been explained, the wave record is not one pure sine wave.

It can be shown that the wave number $(2\pi/L)$ of a wave in shallow water is expressible as

$$\frac{2\pi}{L} = \frac{\mu^2}{g}\coth\left(\frac{\mu^2 H}{g}\coth\left(\frac{\mu^2 H}{g}\coth\left(\frac{\mu^2 H}{g}\coth\ldots\right)\right)\right) = \frac{\mu}{g} I\left(\frac{\mu^2 H}{g}\right) \;.\;.\;.\;.\;.\;.\;. \quad (15)$$

The function $I(\mu^2H/g)$ will be named the Itcoth of μ^2H/g, short for the iterated hyperbolic cotangent of μ^2H/g. It can be evaluated by evaluating the hyperbolic cotangent of μ^2H/g from ordinary tables of the hyperbolic functions, multiplying the number obtained by μ^2H/g, evaluating the hyperbolic cotangent of the product, multiplying the number obtained by μ^2H/g, and so on until the value remains constant to the degree of accuracy of the table. The result is $I(\mu^2H/g)$. When $I(\mu^2H/g)$ is multiplied by μ^2/g, the result is the shallow water wave number $(2\pi/L)$ of a wave with a period given by T equal to $2\pi/\mu$ in water of depth H.

In classical theory multiplication of the pressure amplitude by cosh $(2\pi h/L)$ yields the amplitude of the free surface. Thus for these Gaussian records simply by considering any partial sum, as in (2), it follows that the pressure power spectrum must be multiplied by $\{\cosh[\mu^2H/g\ I(\mu^2H/g)]\}^2$ in order to obtain the power spectrum of the waves on the free surface. Define $[A(\mu)]^2$ to be the power spectrum of the free surface, and then in terms of the power spectrum of the pressure record $[A(\mu)]^2$ is given by (16). The power spectrum of the free surface computed from (16) is shown in Figure 5 together with its bounds obtained from Table 3 with 50 degrees of freedom. Knowledge of the power spectrum of a pressure record at a given point enables many useful properties of the sea surface to be deduced. The area under the solid curve equals 1263 cm², which would be E_{max} of the free-surface record. (This amount is equivalent to a pure sine wave 35.5 cm high from mean water level to the crests.)

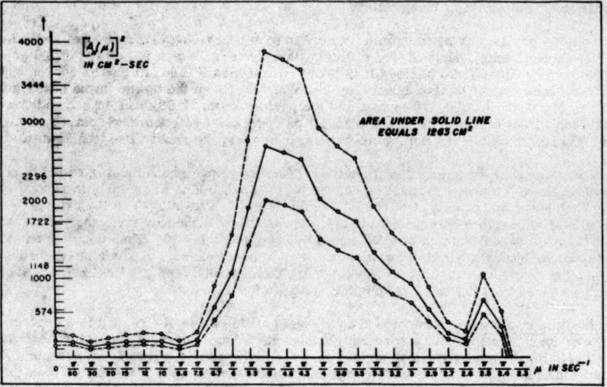

Fig. 5--Power spectrum of the free surface, $[A_H(\mu)]^2$

$$[A(\mu)]^2 = \left[\cosh\left[\frac{\mu^2H}{g} I\left(\frac{\mu^2H}{g}\right)\right]\right]^2 [A_{PH}(\mu)]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The Gaussian Lebesgue power integral for the free surface is given by (17), and it in turn has been studied in terms of equations analogous to (2), (3), (4), and (5) by PIERSON [1952]. Approximation of (17) by a formula similar to (2) would yield a record like that of the free surface.

$$\eta(t) = \int_0^\infty \cos(\mu t + \psi(\mu))\sqrt{[A(\mu)]^2 d\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Figure 6 shows the free-surface power spectrum plotted over the pressure power spectrum. Note that the free-surface power spectrum must be amplified considerably more for low periods (high μ) than for high periods. These considerations explain many of the seeming discrepancies reported in the literature on the relationship between pressure records and actual free-surface wave records, when the properties of the free-surface record are calculated from the pressure record and then compared with an actual free-surface record. Most of the analyses are carried out with the "significant" height and period (or characteristic period) as a starting point, and it is found that the computed results are from six to 25 pct too low compared with observations as

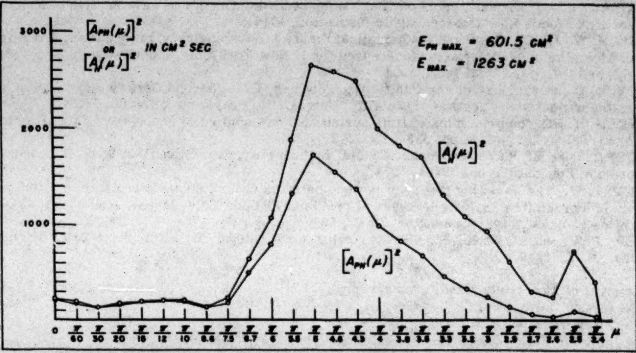

Fig. 6--Comparison of pressure power spectrum with the free-surface power spectrum; note the much greater amplification of the low-frequency components

reported by SNODGRASS [1951]. The explanation is that the low-period components of the pressure power spectrum have not been amplified sufficiently and that therefore the significant height values for the actual waves are too low and the significant period values are too high.

The significant period (or characteristic period) method of analysis (crudely) multiplies some number related to the square root of E_{PHmax} by one constant correction factor. If 9.2 sec, which from the autocorrelation function is the most probable time interval between successive crests in the pressure record, is used as the "significant" period, then it will be found that $\sqrt{E_{PHmax}}$ times the correction factor for this one period will be about ten per cent lower than the square root of E_{max} from Figure 6.

The wider the spread of the power spectrum, the more inaccurate the "significant" height and period method of analysis will be. All theoretical results based on significant or characteristic heights and periods will be in error if there are "sea" conditions present.

Conclusions--Other points of interest are discussed by PIERSON [1952]. For example, if the power spectrum in the generating area does not contain a certain band of frequencies, then waves with "significant" periods corresponding to this band can never be observed in the decay area. Computations of wave refraction must also involve the power spectrum, and even more refined analyses than those given here are needed to carry out such computations.

The work described herein is a preliminary step toward the explanation of the behavior of the sea surface and toward better methods of wave record analysis. More records need to be analyzed by these techniques, but it is believed that this one example explains many of the observed properties of the sea surface and that the methods shown are powerful, accurate, and useful.

Acknowledgments--The research reported herein on wave record and pressure record analysis has been sponsored by the Beach Erosion Board. The great interest and help of the members of the Board in this research is greatly appreciated. The authors are deeply indebted to J. W. Tukey for his help and suggestions. They also wish to thank E. H. Smith for his help in making available the Woods Hole mechanical autocorrelator, and Mary Hunt for carrying out the computations.

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(Manuscript received April 30, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 5, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

A LABORATORY INVESTIGATION OF WIND-GENERATED WAVES

J. W. Johnson and E. K. Rice

Abstract--Waves were generated by forcing wind through a covered laboratory flume. For steady-state conditions, measurements by means of an oscillograph were made of the wave characteristics at various points in the flume. The significant heights and periods were determined and related to the fetch and wind speed. The height and period of the individual waves for various fetch lengths were compared by means of joint frequency diagrams. The orbital motions of particles throughout the water depth were determined from motion pictures. Particle velocities below the crest are compared with theoretical relationships. Some information on wind set-up and shear-stress coefficients are presented.

Introduction--During the past half century, data on wind waves have been taken by several observers and under a great variety of conditions. Many of the available data were obtained from visual observations of the wave characteristics with the wind speeds being estimated either from scattered observations or from weather maps. SVERDRUP and MUNK [1946] compiled all available wind and wave data, and made plots representing wave characteristics as a function of fetch and of wind speed and duration.

Studies of wind waves in covered flumes have been made at the University of Minnesota [FLINSCH, 1946] and in England [STANTON, MARSHALL, and HOUGHTON, 1932; and FRANCIS, 1951]. These investigations were made by forcing air through a covered flume and measuring the wind speed, wave length, wave period, and, in some cases, the wave heights.

Wind-wave studies have been made on relatively small bodies of water with limited fetches at both Clear Lake, California [JOHNSON, 1948], and Abbotts Lagoon, California [JOHNSON, 1950]. In these experiments, wave data were obtained from pressure recorders, wind speed was obtained from anemometer readings, and the fetch was measured from a map for the observed wind direction. Relationships between wind speed, fetch, wave heights, and wave periods were plotted and compared with the curves of SVERDRUP and MUNK [1946]. After reviewing the investigations described above, and noting the rather large discrepancies in the measured values due to inexactness of measurement and to relatively uncontrolled conditions, and recognizing the differences in forecast and observed wave conditions [ISAACS and SAVILLE, 1949], it was thought desirable to make a study of wind waves under controlled laboratory conditions where a more accurate measurement of the variables could be made.

Experimental procedure--This investigation was made in a wave channel one foot wide, three feet deep, and 60 ft long, located in the Fluid Mechanics Laboratory of the University of California. The top of the channel was covered with plywood panels made air-tight by means of modeling clay. An exhaust fan was mounted at one end of the channel, and an air intake with straightening vanes was attached to the other end (see Fig. 1). A rock beach was placed at the exhaust end of the channel to absorb the wave energy and prevent reflections. The position of zero fetch was fixed by extending a flat plate for a short distance along the water surface from the bottom of the air intake. Records of the wave heights and periods were obtained by four resistance wire elements [MORISON, 1949]. The data from these elements were recorded simultaneously on an oscillograph, each element position defining a fetch length. Moving pictures of the waves were taken through a window in the side of the channel. A diaphragm pressure recorder, which recorded simultaneously with the resistance wire elements on an oscillograph, was used to measure wave pressure fluctuations in some of the test runs. The pressure diaphragm was located directly under resistance element No. 3 (see Fig. 1), the center of the diaphragm being 0.193 ft below the original still-water surface. The pressure drop in the flowing air was measured by draft gage type manometers installed as shown in Figure 1. Three piezometer tubes were used to determine the average stressed water level during the tests (see Fig. 1). The wind speed was measured by vertical center-line pitot-tube traverses at three positions in the channel, one near each end and one at the mid-section of the channel. The data from the traverses show that the wind speed varied by less than ten per cent throughout the length of the channel. A rough cross-channel traverse indicated that the wind-speed distribution was constant across most of the channel.

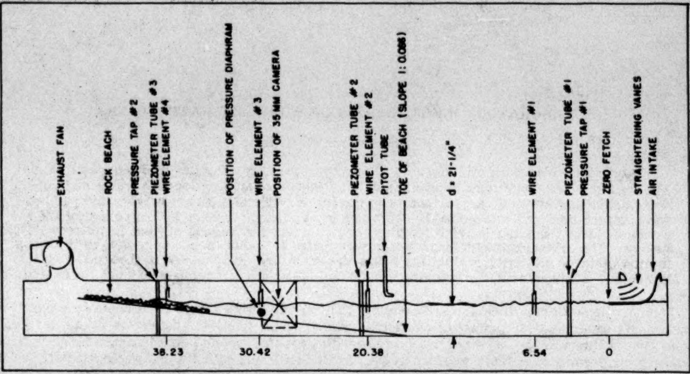
EXHAUST FAN
ROCK BEACH
PRESSURE TAP #2
PIEZOMETER TUBE #3
WIRE ELEMENT #4
POSITION OF PRESSURE DIAPHRAGM
WIRE ELEMENT #3
POSITION OF 35 MM CAMERA
PIEZOMETER TUBE #2
WIRE ELEMENT #2
PITOT TUBE
TOE OF BEACH (SLOPE 1:0.086)
d = 21-1/4"
WIRE ELEMENT #1
PIEZOMETER TUBE #1
PRESSURE TAP #1
ZERO FETCH
STRAIGHTENING VANES
AIR INTAKE
38.23
30.42
20.38
6.54
0

Fig. 1--Schematic diagram of 1 × 3 × 60 ft wave channel

Three experiments were made and coded as Tests 1, 2, and 3. In Test 1, three wind speeds were used, and wave heights and periods were measured with four resistance wire elements. In Tests 2 and 3, a single wind speed was employed, and the wave heights and periods were measured with four resistance wire elements and moving pictures. Use was made of a pressure device to measure subsurface pressure fluctuations due to wave action. Particle motions were measured by injecting a mixture having the same specific gravity as water (xylol and butyl phthalate colored with zinc oxide) and photographing the movement with a motion-picture camera. A grid of known dimensions was placed on the side of the channel and served as a reference from which the enlargement of the film was measured. A ten-second sweep clock was located so that it appeared in the pictures. Care was taken to have the camera at the same elevation as the water level. This was done in an effort to photograph a plane surface across the channel. It was noted in the analysis of the pictures that some of the wave fronts did not extend across the full width of the channel, making the surface appear warped.

Analysis of the data and experimental results--The mid-section vertical wind-speed profiles obtained during the various tests are shown in Figure 2. The average wind speed used in the analysis presented below is indicated on each profile. The pressure drops between Piezometers 1 and 2, Figure 1, are shown for each run in Table 1.

Table 1--Pressure differences between Piezometers 1 and 2[a]

Test	Run	Wind velocity	Pressure drop
		ft/sec	inches of water
1	1	42.70	0.31
1	2	32.50	0.13
1	3	36.44	0.18
2	1	41.3	0.36
2	2	41.3	0.39
2	3	41.3	0.34
3	1	42.6	0.36
3	2	42.6	0.35
3	3	42.6	0.39

[a] The distance between piezometers was 46.21 ft.

In Test 1, 100 successive waves were analyzed for each fetch and wind speed, making a total of 1200 waves. In Test 2, 100 successive waves from Wire Elements 2 and 3 and from the pressure-diaphragm record were analyzed. In Test 3, 100 successive waves from Wire Element 3 were analyzed. From each set of 100 waves, the average period and the significant wave height

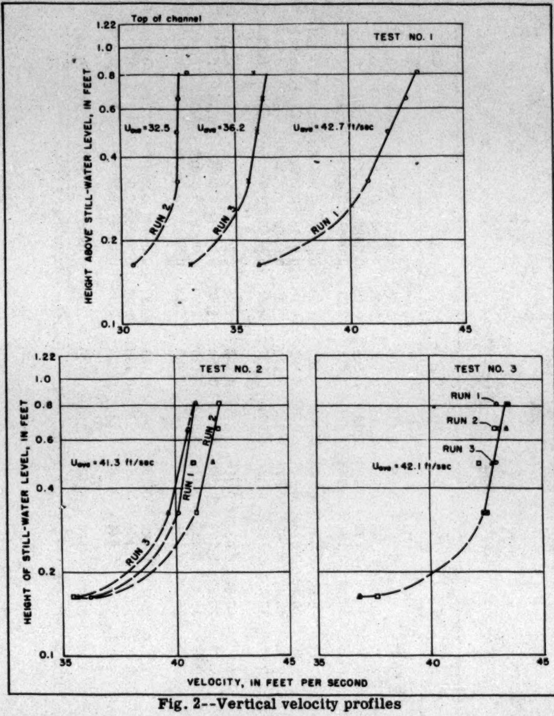

Fig. 2--Vertical velocity profiles

were determined. The wave period was defined as the time required for successive wave crests to pass a given point. Only those crests where the intervening trough was below the still-water level during the test were considered. The significant wave height was taken as the average height of the highest 1/3 of the waves of each group of 100 successive waves. Two diagrams, one representing wave period (or wave velocity) and the other wave height, were made using the same parameters as used by SVERDRUP and MUNK [1946], that is

$$C_0/U = f''(gF)/U^2$$

$$gH_0/U^2 = f'(gF)/U^2$$

where

H_0 = the significant wave height, ft
C_0 = the wave velocity, ft/sec = 5.12 T^2, where T = period, sec
g = acceleration of gravity, ft/sec^2
F = fetch, ft
U = wind speed, ft/sec

The significant wave heights, the average wave periods, lengths, steepness ratios, wave ages, and dimensionless parameters are tabulated in Table 2. It is noted that the heights and periods for the same fetch and approximately the same wind speed varied by less than five per cent from

Table 2--Summary of results

Test	Run	Element	Water depth	Fetch	H_o	T	L_o	H_o/L_o	C_o	C_o/U	gH_o/U^2	gT/U	gF/U^2
			ft	ft	ft	sec	ft		ft/sec				
1	1[a]	Wire 1	1.77	6.54	0.0689	0.285	0.416	0.166	1.46	0.0342	0.00122	0.215	0.116
		Wire 2	1.77	20.38	0.140	0.412	0.869	0.161	2.11	0.0494	0.00248	0.311	0.360
		Wire 3	0.969	30.42	0.187	0.489	1.22	0.153	2.49	0.0583	0.00331	0.369	0.537
		Wire 4	0.458	38.23	0.191	0.568	1.65	0.116	2.91	0.0681	0.00337	0.428	0.675
1	2[b]	Wire 1	1.77	6.54	0.0315	0.240	0.295	0.107	1.23	0.0378	0.000960	0.238	0.199
		Wire 2	1.77	20.38	0.0783	0.345	0.609	0.129	1.76	0.0543	0.00239	0.342	0.621
		Wire 3	0.969	30.42	0.0924	0.375	0.720	0.128	1.85	0.0569	0.00282	0.372	0.927
		Wire 4	0.458	38.23	0.123	0.434	0.964	0.127	2.22	0.0683	0.00375	0.430	1.17
1	3[c]	Wire 1	1.77	6.54	0.0545	0.258	0.341	0.160	1.32	0.0365	0.00134	0.230	0.161
		Wire 2	1.77	20.38	0.102	0.364	0.678	0.150	1.86	0.0514	0.00250	0.324	0.501
		Wire 3	0.969	30.42	0.132	0.429	0.942	0.126	2.20	0.0608	0.00326	0.382	0.747
		Wire 4	0.458	38.23	0.194	0.488	1.22	0.159	2.48	0.0685	0.00478	0.434	0.940
2	1[d]	Wire 2	1.77	20.38	0.149	0.455	1.06	0.141	2.33	0.0564	0.00282	0.355	0.385
		Wire 3	0.969	30.42	0.178	0.501	1.28	0.139	2.46	0.0596	0.00336	0.391	0.574
		Diaphragm	0.969	30.42	0.0246	0.467	1.12	. . . .	2.39	0.0579	0.000464	0.364	0.574
3	1[e]	Wire 3	0.969	30.42	0.170	0.488	1.220	0.140	2.50	0.0587	0.00302	0.369	0.540

[a]U = 42.7 ft/sec. [b]U = 32.5 ft/sec. [c]U = 36.2 ft/sec. [d]U = 41.3 ft/sec. [e]U = 42.6 ft/sec.

their respective average values. The significant wave height obtained from Wire Element 2 (Test 2) was 0.149 ft, and the calculated significant surface wave obtained from the pressure record was 0.134 ft, a deviation of 5.4 pct from the average value of 0.142 ft. The formula used to convert pressure fluctuations to surface wave height was that presented by **FOLSOM** [1947].

The data from these experiments are shown in Figure 3 compared with the flume studies of **STANTON** and Others [1932], **FLINSCH** [1946], **BRETSCHNEIDER** and **RICE** [1951], and **FRANCIS** [1951], and various field data compiled by **BRETSCHNEIDER** [1952]. Shown in Figure 3 are the original **SVERDRUP** and **MUNK** [1946] curves as well as the revised curves for wave height, wave velocity (or period), wave steepness, and minimum duration as proposed by **BRETSCHNEIDER** [1952]. In recent comparisons of observed and forecast wave conditions it has been found that the relationships of **BRETSCHNEIDER** [1952] permit a more accurate forecast than that obtained by

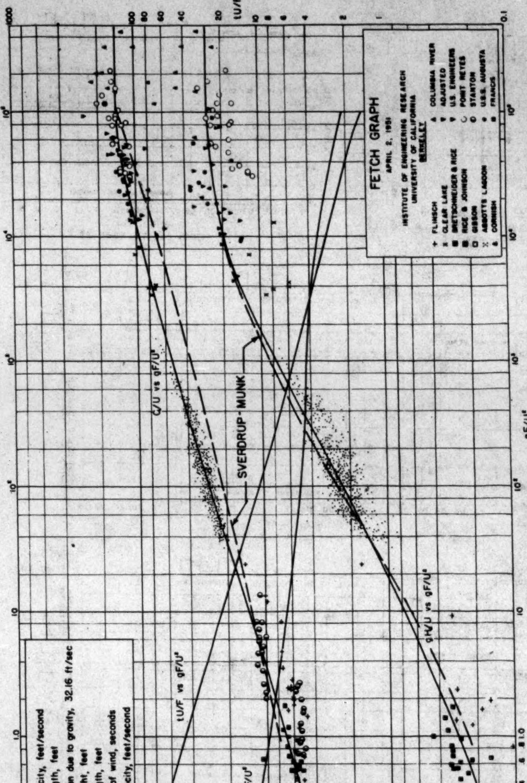

Fig. 3--Relationship between wind speed and duration, wave height and period, and fetch; see **BRETSCHNEIDER** [1952] for references

the Sverdrup-Munk curves, especially as to wave period and minimum duration. It should be recognized that, in studies of wind-generated waves in relatively narrow flumes, the waves approximate long-crested ones, whereas observations in natural bodies of water probably are mostly short-crested waves, as recently discussed by **FUCHS** [1951].

In addition to the information on average values of wave heights and periods as summarized in **Figure 3**, a statistical study of these terms was made for Test 3 in which the height and period of the individual waves were measured and tabulated. A plot of both the wave-height distribution and wave-period distribution on the same diagram (called a joint frequency distribution) is shown in **Figure 4** for a single wind speed of 42.6 ft/sec and four fetch lengths. The points in the various

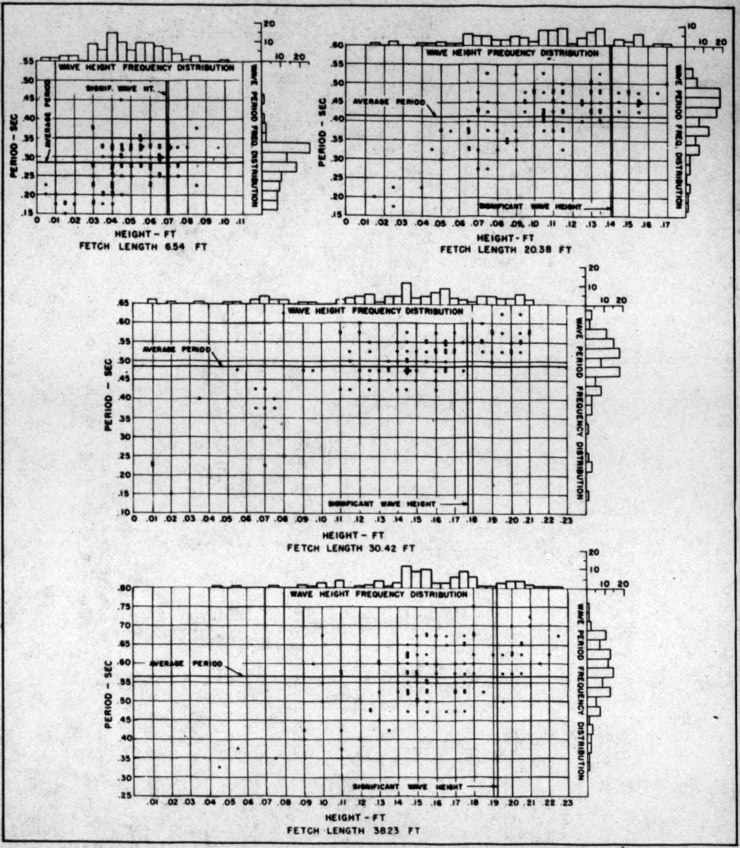

Fig. 4--Joint frequency distribution of wave period and wave height (U = 42.7 ft/sec)

diagrams give the height and period of individual waves which occurred in a sample of 100 waves. The amount of scatter for both the height distribution and the period distribution for the first fetch length is less than that for the other three fetch lengths. There is approximately the same amount of scatter of the heights and periods for the three longer fetch lengths. For all fetches, however, there is a relationship between height and period. These results apply to waves within the generating area and differ from the results obtained on studies of ocean swell [PUTZ, 1952]. In these latter studies no relationship between height and period was noted in the analysis of a large number of records from ocean swell recorders.

Of considerable interest are the results of the studies on particle motion. The 35-mm motion pictures taken in Test 3 were enlarged several times the actual size and the paths of seven particles located over the field of view were plotted (Fig. 5). Each point on the plot is the relative

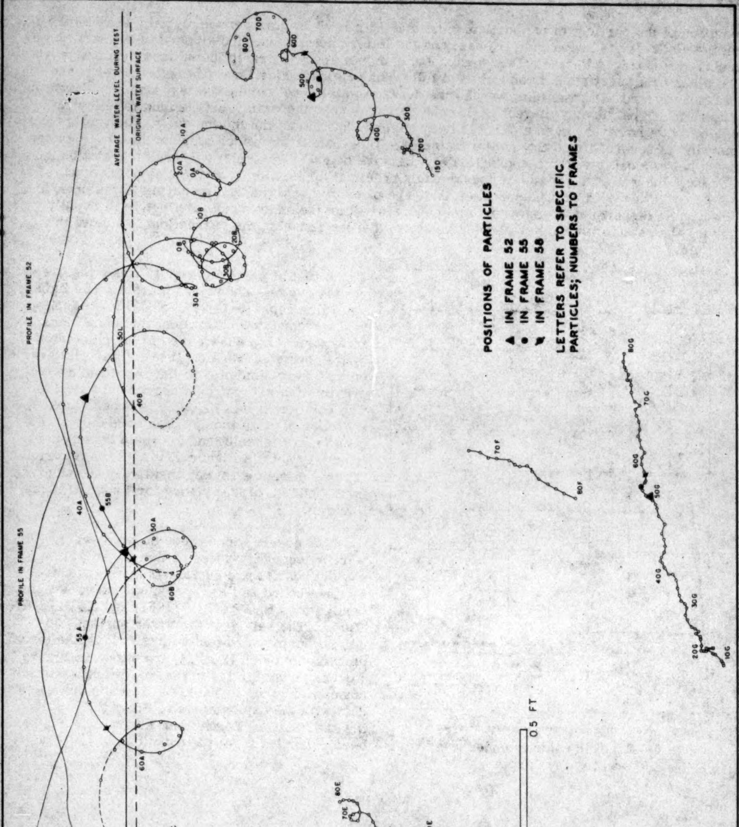

Fig. 5--Plot of particle paths, Test 3

position of the particle in the particular frame. In all, 80 frames were analyzed. The numbers on the plot refer to successive frames, and the letters refer to particular particles. Particles D and G are observed to rise. This was probably due to specific gravity being less than unity. On the other hand, particle F is observed to fall. The original water level (the water level in the channel when no wind was blowing) and the average water level during the test also are shown in Figure 5. Of particular interest is that the particles near the water surface tend to follow an orbital path; however, when a particle is at the crest of a wave the motion due to surface drag of the wind is several times the orbital diameter. This translation effect does not appear to extend much below a depth equal to approximately the wave height. This relatively high translation motion near the water surface is of considerably practical application in connection with ocean sewer outfalls. With an offshore wind, this type of motion, and the accompanying turbulence, are effective in dispersing the sewage. An onshore wind, on the other hand, is effective in rapidly moving the sewage toward neighboring beaches, thereby reducing the recreational value of the beaches.

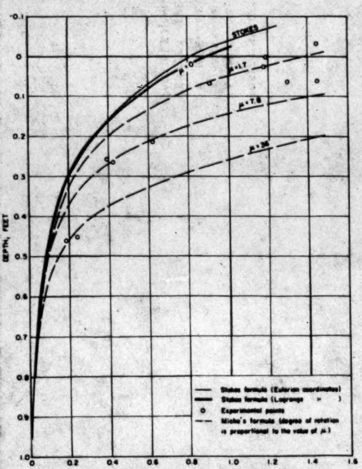

Fig. 6--Particle velocity under wave crest versus depth

A plot of particle velocity under the crest versus depth is shown in Figure 6. The light solid line represents STOKES' [1847] equation to the second approximation, including mass transport. Also shown are curves calculated by the theory of MICHE [1944]. These latter curves were computed to the second-order approximation with various degrees of rotation. The rotation in this instance is proportional to the value of μ as indicated on each curve. As seen from Figure 6, and as would be expected from the plots in Figure 5, it appears that rotational effects exist but apparently not as given by the MICHE [1944] theory for "normal rotational swell."

The observations made on wind set-up were a secondary phase of this investigation, and the results are summarized herein merely for the record and to supplement the recent investigations by FRANCIS [1951] and KEULEGAN [1951]. The set-up of the water surface, observed during each test with the aid of the three piezometer tubes (Fig. 1), are shown in Figure 7 uncorrected for the pressure difference in the channel as given in Table 1. To compare these data with the experiments of FRANCIS [1951] the shear stress coefficient, k, for each run has been computed from the equation

$$k = \tau/\rho U_{10}^2$$

where the shear stress τ is computed by the equation

$$\tau = w\,d\,i$$

In these equations

i = surface slope caused by the shear stress (corrected for pressure difference)
ρ = density of air = 0.002378 slugs/ft^3
w = unit weight of water = 62.3 lb/ft^3
d = depth of water = 1.77 ft
U_{10} = velocity at ten cm height, ft/sec

Values of shear-stress coefficient, k, have been computed and are summarized in Table 3. These values of k are an average for the entire channel. The data were insufficient to calculate the coefficients for the windward and lee parts of the channel separately as was done by FRANCIS [1951].

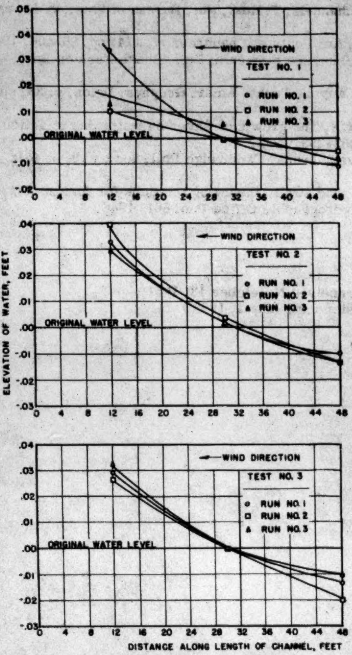

Fig. 7--Observed wind set-up in laboratory channel (uncorrected for pressure difference)

Table 3--Shear stress coefficients

Test	Run	U_{10}	1×10^4	$k \times 10^2$
		ft/sec		
1	1	40.8	5.1	1.42
	2	32.4	3.73	1.65
	3	35.7	5.08	1.86
2	1	40.1	3.5	1.01
	2	39.7	2.83	0.84
	3	40.8	3.92	0.11
3	1	42.3	3.64	0.95
	2	42.3	3.75	0.98
	3	42.3	3.72	0.97

The data in Table 3 compare with the data of FRANCIS [1951] in the lee part of the channel where the waves were more fully developed. The present experimental data are of interest in that the depth was appreciably greater than that employed by either FRANCIS [1951] or KEULEGAN [1951].

Conclusions--(1) The particle motion studies show that in general the particles make an orbital type path which follows the wave profile, but near the surface the particles show considerable translation in the direction of the wind. This translation decays rapidly with depth. At a depth of approximately one-half wave height below the average water level during test, little translation was observed.

(2) Joint frequency diagrams show that in the generating area a relationship appears to exist between the height and period of the individual waves.

Acknowledgment--The work summarized in this paper was part of an extensive investigation on waves and surf made at the University of California in a project sponsored by the Office of Naval Research.

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(Communicated manuscript received November 13, 1951;
open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

THE DIRECT CONSTRUCTION OF WAVE RAYS

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(Contribution from the Scripps Institution of Oceanography, New Series, No. 595)

Abstract--In the construction of refraction diagrams for ocean waves the rays (orthogonals) can be drawn more accurately if the approximate formula $\Delta\alpha = (\Delta L/L_{ave})\tan\alpha$, used by JOHNSON, O'BRIEN, and ISAACS [1948] in the preparation of refraction protractors, is replaced by $\Delta\alpha = \sin^{-1}[(1 + \Delta L/L)\sin\alpha] - \alpha$. In these formulas, α is the angle between wave crest and depth contour and L the wave length taken at the beginning of any contour interval; $\Delta\alpha$ and ΔL are the corresponding changes over the contour interval. A modified procedure for the construction of rays is presented.

Introduction--JOHNSON, O'BRIEN, and ISAACS [1948; henceforth referred to as JOI] have greatly improved the construction of refraction diagrams by devising a method for the direct construction of wave rays (orthogonals), which eliminates the need for first drawing the wave crests. The JOI method is based on an approximate formula, derived from refraction geometry, for the change in direction of a ray between adjacent contours. In the present paper the same problem is considered using as a starting point the differential equations of the ray path. The results lead to modifications which simplify and improve the accuracy of the JOI method.

First, a proposed procedure is developed without reference to JOI; then the two methods are compared. The accuracy of refraction diagrams can be improved either by following the JOI method but re-computing the protractors on the basis of the new equation (without added difficulties in their use), or by following the method proposed here.

The equation of the ray--Let $x = x(s)$ and $y = y(s)$ denote the coordinates of a point on a wave ray in terms of the distance s measured along the ray from some arbitrary point. Then if $\theta = \theta(s)$ denotes the angle between the tangent to the ray and the x-axis (Fig. 1), the differential equations of the ray are

$$\frac{Dx}{Ds} = \cos\theta \qquad \frac{Dy}{Ds} = \sin\theta \;..\; \text{(1a,b)}$$

$$\frac{D\theta}{Ds} = -\frac{1}{c}\frac{Dc}{Dn} \;.........\; \text{(1c)}$$

where $c = c(x,y)$ is an arbitrary, but known, two-dimensional field of wave velocity. The operators D/Ds and D/Dn refer to differentiation in the direction of the tangent and normal to the ray, respectively, and are defined by

$$\left.\begin{aligned} \frac{D}{Ds} &= (\cos\theta)\frac{\partial}{\partial x} + (\sin\theta)\frac{\partial}{\partial y} \\ \frac{D}{Dn} &= -(\sin\theta)\frac{\partial}{\partial x} + (\cos\theta)\frac{\partial}{\partial y} \end{aligned}\right\} \text{(2a,b)}$$

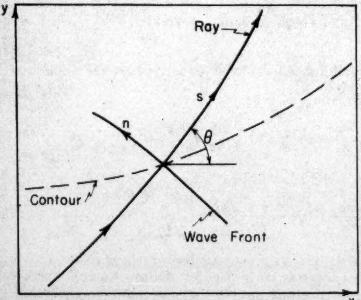


Fig. 1--Definition diagram for ray equation

Identical or equivalent forms of (1a,b,c) are derived in physics texts on light and sound [for example, FRANK, BERGMANN, and YASPAN, 1946]. Equations (1a,b) simply state that the ray makes an angle θ with the x-axis. Equation (1c) states that the curvature of the ray, Dθ/Ds, depends on Dc/Dn, the gradient of wave velocity in the direction of the normal. Equation (1c) is a consequence of Fermat's principle, according to which the ray is a path of minimum travel time. The derivation of (1c) requires that c, $\partial c/\partial x$, and $\partial c/\partial y$ be continuous. It is assumed that this condition holds in practice since we know c only through measurements of water depths at specified positions and interpolation therefrom.

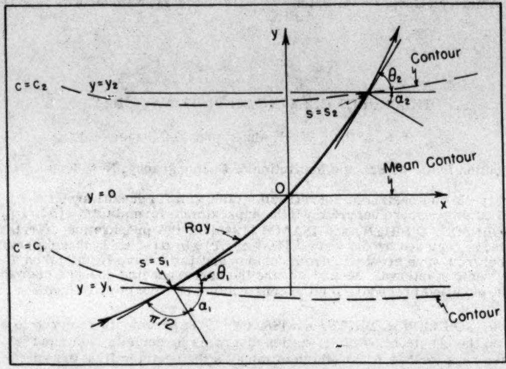


Fig. 2--Definition diagram for ray crossing contour interval

Integration between contours--The equation

$$\frac{D\theta}{Ds} = \frac{1}{c}\left[(\sin\theta)\frac{\partial c}{\partial x} - (\cos\theta)\frac{\partial c}{\partial y}\right]$$

is to be integrated between two adjacent bottom contours which are labeled in terms of the respective wave velocities c_1 and c_2 (Fig. 2). Since $c\,(x,y)$ is not known analytically, approximate methods must be used. If the x-axis is taken in the direction of the mean contour, as shown in Figure 2, then by Taylor's theorem

$$c = c_o + \left(\frac{\partial c}{\partial y}\right)_o (y - y_o) + \text{(higher order terms)}$$

where the subscript zero denotes a value at the origin. If the higher order terms are neglected, we have along the ray over the contour interval

$$\frac{\partial c}{\partial x} = 0 \qquad \frac{\partial c}{\partial y} = \frac{dc}{dy} = \text{const} = \left(\frac{\partial c}{\partial y}\right)_o = \frac{\Delta c}{\Delta y} \;\ldots\ldots\; \text{(3a,b)}$$

and

$$\frac{D\theta}{Ds} = -\frac{\cos\theta}{c}\frac{dc}{dy} = -\frac{\cos\theta}{c}\frac{\Delta c}{\Delta y} \;\ldots\ldots\ldots\ldots\ldots\; \text{(4)}$$

where the symbol Δ denotes the change in a variable across a contour interval, for example, $\Delta c = c_2 - c_1$ (the sign of Δc is negative if the ray proceeds from deep to shallow water). This "first" approximation is equivalent to the following assumptions: (1) The actual contours over a small interval are replaced by straight (parallel) contours with the mean trend. (2) The wave velocity varies linearly across the contour interval.

Equations (4) and (1b) may be combined to give an equation in c and θ, with the solution

$$\frac{\cos\theta}{c} = \text{const} = \frac{\cos\theta_1}{c_1} \;\ldots\ldots\ldots\ldots\ldots\; \text{(5)}$$

In terms of the angle $\alpha = \pi/2 - \theta$ between the wave front (crest) and the direction of the mean contour, we find

$$\frac{\sin \alpha_2}{c_2} = \frac{\sin \alpha_1}{c_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5a)$$

$$\frac{D\alpha}{Ds} = \frac{\sin \alpha_1}{c_1}\,\frac{\Delta c}{\Delta y} = \text{const} = \frac{\Delta \alpha}{\Delta s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4a)$$

The two equations (5a) and (4a) together form the basis for the construction of wave rays. Equation (5a) has the same form as Snell's law. Equation (4a) shows that under the given assumptions the ray between adjacent contours is a circular arc and the equation constitutes a formula for the calculation of the curvature.

It is interesting to note that condition (4a) of constant curvature over an interval is precisely the basis of Kelvin's graphical method [WILLERS, 1948] for solving a second-order differential equation (to which set (1a,b,c) is equivalent). In fact, it would have been more appropriate from the mathematical point of view to start with the assumption that the ray is approximated by smoothly joining together circular arcs of appropriate curvatures. The calculation of the constant curvatures, to a first approximation, leads to the condition that wave velocities vary linearly over the interval. An adaptation of Kelvin's method to the construction of a ray is now considered.

Proposed procedure--The usual small curvature of wave rays precludes convenient construction with a compass of the actual circular arcs across contour intervals. The difficulty can be avoided by drawing tangents (Fig. 3, AP and PB) to the arcs and turning them through angles $\Delta\alpha$, or by drawing chords and turning them through angles $\Delta\alpha/2$. The angle $\Delta\alpha$ is computed from (5a) using $\alpha_2 = \alpha_1 + \Delta\alpha$. The following procedure, which uses tangents, is proposed for crossing the interval from a smoothed contour c_1 to a smoothed contour c_2 (see Fig. 4).

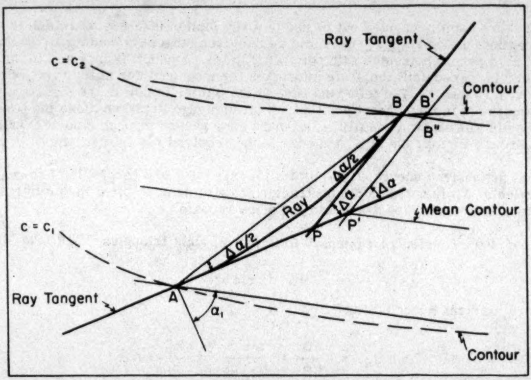


Fig. 3--Graphical solution by ray tangents

(1) Calculate the ratio c_1/c_2 by dividing the successive values of the ratio of wave speed at contour depth to wave speed in deep water which are obtained from tables [WIEGEL, 1948] or an alignment chart (see Fig. 5).

(2) Draw in a mean contour for the interval by eye.

(3) Extend the tangent at point A of c_1, which is known from the preceding interval, to intersection, P', with the mean contour.

(4) On a perpendicular to tangent AP' at P', mark a turning point R such that P'R = 1 on some arbitrary scale.

(5) From R swing an arc of radius c_1/c_2, measured on the same scale, intersecting the mean contour at S.

(6) Construct the ray tangent at contour c_2 by drawing a perpendicular to SR such that AP equals PB as judged by eye.

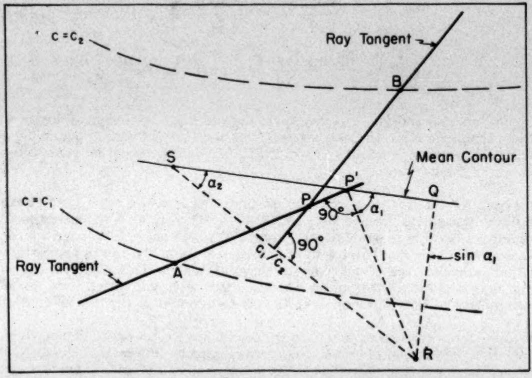


Fig. 4--Proposed method for construction of ray tangents

Steps 3-6 are rapidly carried out by use of a ray plotter (see Fig. 6), which is made of heavy plastic. The line labeled ray tangent is laid on the given tangent at contour c_1 so that the value of unity on the c_1/c_2 scale coincides with point P' (Fig. 4). A pencil is inserted in the turning point and the plotter is turned until the scale intersects the mean contour at the value of c_1/c_2 computed for this interval in step 1. The left-hand edge of the plotter is now in the direction of the new tangent at c_2, but is not in the correct position. A straightedge is placed along the base of the plotter and the device is slid sideways until the left-hand edge passes through a point P with AP = PB. A line drawn through P along the left-hand edge is the required ray tangent at c_2.

To attain the desired accuracy, the limits $|\Delta c/c_1| < 0.2$ and $|\Delta\alpha| < 15°$ are suggested. If an excessive value of $\Delta\alpha$ is noted on the protractor scale at point P' after the plotter is turned, an intermediate contour should be drawn and the steps repeated.

The proof of the construction (steps 1-6) is that in right triangles P'QR (Fig. 4)

$$RQ = \sin\alpha_1$$

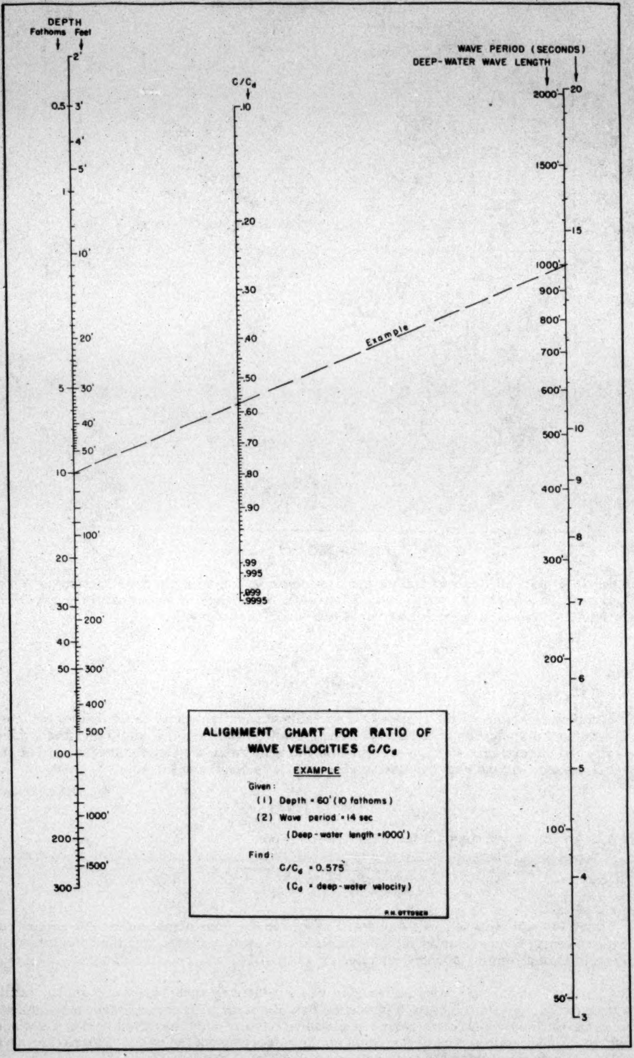
and, therefore, in right triangle SRQ

$$\sin\alpha_2 = \frac{RQ}{SR} = \frac{\sin\alpha_1}{c_1/c_2} = \frac{c_2}{c_1}\sin\alpha_1$$

as required by (5a). Tangents AP and PB are constructed equal as required for a ray of constant curvature (4a).

The ray plotter virtually eliminates the chances of turning off the tangents in the wrong direction [DUNHAM, 1951, p. 36], and no change in procedure is required to draw rays from shallow to deep water [SAVILLE, 1951], that is, for $c_2 > c_1$. However, special care must be exercised in applying (5a) when the ray becomes tangent to a bottom contour ($\alpha = 90°$) as in the following two cases: (1) where a ray proceeds from deep to shallow water and has a smaller curvature than the bottom contours, the ray reaches a point of tangency at some minimum depth (point T, Fig. 7), and then proceeds back to deeper water; (2) where a ray proceeds from shallow to deep water but is "totally reflected" by the deep water [ISAACS, WILLIAMS, and ECKART, 1951].

Whenever α approaches 90° (section ATB of the ray, Fig. 7), it becomes necessary to proceed in smaller steps by interpolating contours and to locate the point of tangency by successive approximations.



ALIGNMENT CHART FOR RATIO OF WAVE VELOCITIES C/C$_d$

EXAMPLE

Given:
(1) Depth = 60' (10 fathoms)
(2) Wave period = 14 sec
(Deep-water length = 1000')

Find:
C/C$_d$ = 0.575
(C$_d$ = deep-water velocity)

Fig. 5--Alignment chart for ratio of wave velocities c/c$_d$

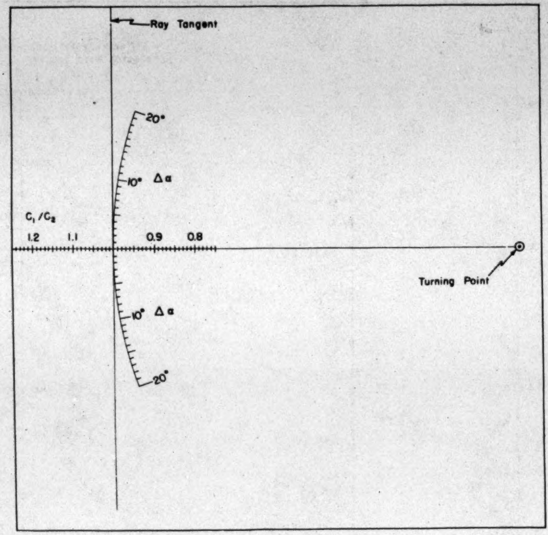


Fig. 6--SIO ray plotter

Since it is more difficult to maintain the precision of the triangle construction as α approaches 90°, an alternative method is suggested. From (4a), the change in direction of the ray over an arbitrary portion, δs, of the circular arc crossing a contour interval is

$$\delta\alpha = \frac{\delta s}{\Delta y} \cdot \frac{\Delta c}{c_1} \sin \alpha_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4b)}$$

where Δ refers as before to the change in a variable across the whole contour interval from c_1 to c_2. An assumed distance $\delta s/2$ is laid off along the initial tangent, the calculated angle $\delta\alpha$ is turned off, and the distance $\delta s/2$ is measured off to give the new point of tangency at the end of the portion of arc. As the construction is repeated, it is convenient to keep

$$\delta s = n \cdot \Delta y$$

where n is arbitrary but constant for the interval. Then

$$\delta\alpha \doteq n \cdot \frac{\Delta c}{c_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4c)}$$

and if $|\delta\alpha| < 15°$ and $\alpha_1 > 80°$, the combined errors in the approximation for $\delta\alpha$ amount to less than two per cent. The protractor on the ray plotter is used in turning off the calculated angle $\delta\alpha$. Further details of a similar construction are given in JOI.

The parallel movement of the plotter may be facilitated by incorporating a set of parallel rulers with the device. In the case of extensive use of charts with certain standard contours, it may be worth while to construct a series of plotters, one for each specified period, on which the c_1/c_2 scale is labelled in terms of the depth contours corresponding to c_1. Other time-saving devices will occur to the reader for any particular problem at hand.

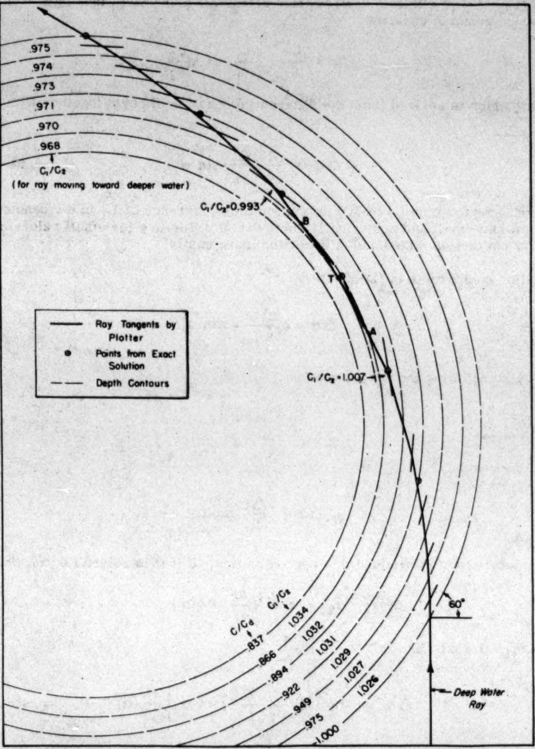


Fig. 7--Comparison of proposed method of ray construction with exact ray solution for a special case of circular countours; wave velocity is assumed to increase as the square root of the contour radius to the maximum value c_d appropriate to deep water; exact solution is a parabola [ARTHUR, 1946]

Comparison with JOI--The principles underlying the proposed method are not at all original, but follow largely from JOI. There are, however, certain differences which may become important. Separate consideration is given to those arising from the theoretical bases of the two methods, and those involving differences in construction.

Theory--The JOI method is based on the equation

$$\Delta\alpha = \frac{R}{J}\cdot\frac{\Delta L}{L_{ave}}\cdot\sin\alpha_1 \quad \text{(JOI 1)}$$

where

$$R = s_2 - s_1 \text{ and } J = y_2 - y_1 \quad (6)$$

and $L_{ave} = (L_1 + L_2)/2$. This is derived from refraction principles, making use of the fact that the wave period remains constant.

$$T = \frac{L}{c} = \frac{L_1}{c_1} = \frac{L_2}{c_2} \quad \text{(7)}$$

Our (4a), which is derived from the differential equation of rays, together with (6) and (7) yields

$$\Delta\alpha = \frac{R}{J} \cdot \frac{\Delta L}{L_1} \cdot \sin\alpha_1 \quad \text{(8)}$$

which has the same form as (1) of JOI except for the occurrence of L_1 in the denominator instead of L_{ave}. This is a small and numerically unimportant difference for small values of $\Delta L/L_1$, but (8) is actually correct as stated and is evaluated more easily.

An equation comparable to (2) of JOI

$$\Delta\alpha = \frac{\Delta L}{L_{ave}} \tan\alpha_1 \quad \text{(JOI 2)}$$

is obtained by substituting the approximation

$$\frac{R}{J} \doteq \sec\alpha_1 \quad \text{(9)}$$

into (8) with the result

$$\Delta\alpha \doteq \frac{\Delta L}{L_1} \tan\alpha_1 \quad \text{(10)}$$

It is not necessary to introduce this approximation. If (5a) is solved for α_2, then

$$\alpha_2 = \sin^{-1}\left(\frac{c_2}{c_1}\sin\alpha_1\right) \quad \text{(5b)}$$

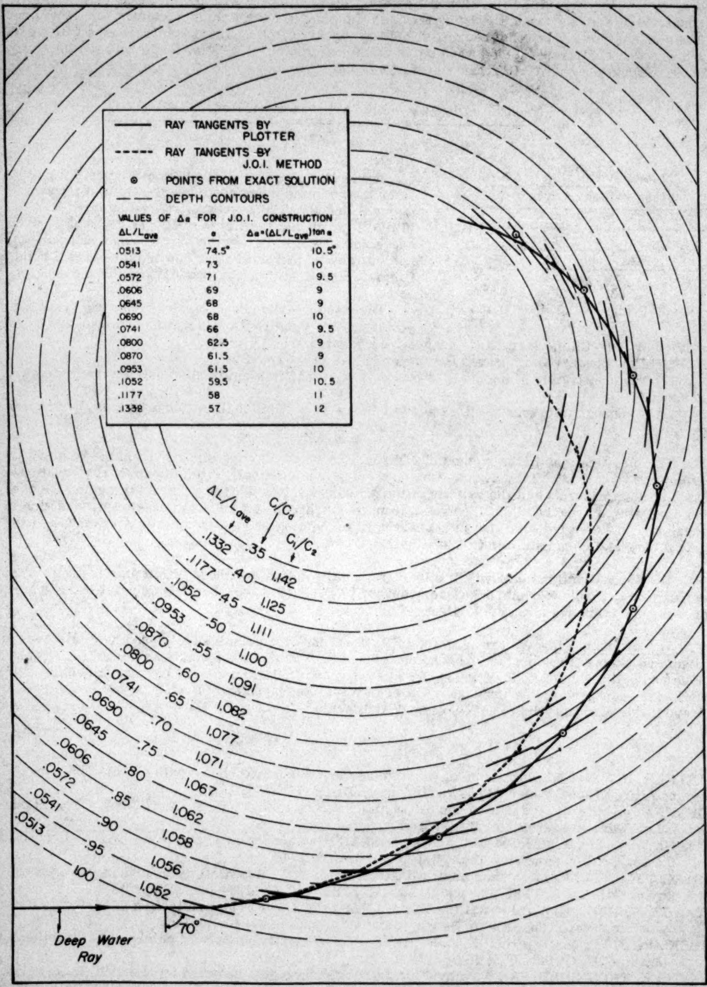
and since $c_2/c_1 = 1 + \Delta L/L_1$

$$\Delta\alpha = \sin^{-1}\left[\left(1 + \frac{\Delta L}{L_1}\right)\sin\alpha_1\right] - \alpha_1 \quad \text{(11)}$$

Table 1--Error in $\Delta\alpha$ resulting from approximate formula

$\Delta L/L_1$	α_1	$\Delta\alpha$ from (10)	Error
	°	°	pct
-0.05	77.6	-13	37
-0.10	66.2	-13	21
-0.15	56.6	-13	14

The errors in $\Delta\alpha$ resulting from the use of (JOI 2) may be excessive (see Table 1) even within the limits of $\alpha_1 < 80°$ and $|\Delta\alpha| < 13°$ stated in JOI. The error may be cumulative and becomes particularly important for extreme cases where α_1 remains large over many contour intervals (Fig. 8). The accuracy of the method is improved if the type I and type II protractors [JOI, 1948, pp. 30-31] are redesigned for the application of (8) and (11) instead of (JOI 1) and (JOI 2). (After presentation of this paper in May, 1952, the writers had the opportunity of examining a manuscript by T. Saville, Jr. and K. Kaplan, of the staff of the Beach Erosion Board. SAVILLE and KAPLAN [1952] have extended the derivation given by ISAACS [1944] to obtain equations which are more accurate than (JOI 1) and (JOI 2), and they have redesigned the refraction protractors.) As an alternative, an accurate construction can be made using only (JOI 1) or (8), but the protractor required is more complex [ISAACS, 1944]. Preparation of charts of bottom topography is simplified if the protractors are plotted in terms of c_1/c_2 instead of $\Delta L/L_1$.


RAY TANGENTS BY PLOTTER
RAY TANGENTS BY J.O.I. METHOD
POINTS FROM EXACT SOLUTION
DEPTH CONTOURS
ΔL/L_ave
C/C_d
C_1/C_2
.1332 .35 1.142
.1177 .40 1.125
.1052 .45 1.111
.0953 .50 1.100
.0870 .55 1.091
.0800 .60 1.082
.0741 .65 1.077
.0690 .70 1.071
.0645 .75 1.067
.0606 .80 1.062
.0572 .85 1.058
.0541 .90 1.056
.0513 .95 1.052
1.00
70°
Deep Water Ray

VALUES OF Δα FOR J.O.I. CONSTRUCTION

$\Delta L/L_{ave}$	α	$\Delta\alpha = (\Delta L/L_{ave})\tan\alpha$
.0513	74.5°	10.5°
.0541	73	10
.0572	71	9.5
.0606	69	9
.0645	68	9
.0690	68	10
.0741	66	9.5
.0800	62.5	9
.0870	61.5	9
.0953	61.5	10
.1052	59.5	10.5
.1177	58	11
.1338	57	12

Fig. 8--Comparison of proposed method of ray construction and JOI method with exact ray solution for a special case of circular contours; wave velocity is assumed to increase linearly with contour radius to the maximum value c_d appropriate to deep water; exact solution is a logarithmic spiral [ARTHUR, 1946]

Construction--The JOI procedure calls for extending the known tangent, that is, the known direction of the ray, at A (Fig. 3) to the point of intersection P′ with the mean contour, and then constructing the new tangent P′B′ by turning off the angle $\Delta\alpha$ which is calculated for the interval. The construction is not exact because AP′ is not equal to P′B′. The error increases with α_1 and JOI propose a different method when α_1 is greater than 80°. If ϵ is the angle between the true chord AB and the chord AB″, then a measure of the error is

$$\frac{\epsilon}{\Delta a} = \frac{(a_1 + \Delta a/2) - \tan^{-1}\frac{1}{2}\left[\tan a_1 + \tan(a_1 + \Delta a)\right]}{\Delta a} \quad \ldots\ldots. (12)$$

a relation which may be derived from Figure 3 if perpendiculars to the mean contour are extended through points A and P′. The results of an evaluation of $\epsilon/\Delta\alpha$ (Table 2) show that the limits of $|\Delta\alpha| < 13°$ and $\alpha_1 < 80°$ suggested in JOI for using the inexact construction are not sufficiently strict. The difficulty is avoided if point P is selected in such a way that AP and PB are equal.

Table 2--Values of $\epsilon/\Delta\alpha$ for given values of α_1 and Δ

$\Delta\alpha$	α_1		
	60°	70°	80°
°			
- 5	0.033	0.053	0.099
-10	0.063	0.094	0.163

Discussion--The advantages of either the proposed procedure for the direct construction of wave rays or the method of JOI with the suggested changes incorporated are:

(1) The accuracy is improved for larger values of α_1 and $\Delta\alpha$.

(2) The preparation of the chart of bottom topography is simplified as only the ratio c_1/c_2 is needed.

(3) No change in procedure is required to draw rays from shallow to deep water, that is, for $c_2 > c_1$.

The modifications in the method of JOI can be derived starting directly from the basic assumptions (1) and (2). The advantage in proceeding systematically from the exact ray equations (1a,b,c) is an assurance of the validity of the graphical solution. The results strengthen the conclusions already reached in JOI. Although improvements in accuracy and ease of construction are suggested, it is not believed that in general rays in existing refraction diagrams are invalid except for special cases similar to that illustrated (Fig. 8).

The approximations involved in either graphical solution are those of Kelvin's method. Techniques are available for improving the accuracy [WILLERS, 1948], but it is doubtful if the accuracy of bathymetric charts supports further refinement.

Acknowledgments--We are indebted to P. H. Ottosen for permission to use one of a series of alignment charts (Fig. 5) which he has constructed for various wave relationships. The suggestions and criticisms of D. R. Forrest, R. A. Fuchs, and D. L. Inman have been most helpful. This work was sponsored by the U. S. Navy Office of Naval Research and the U. S. Army Beach Erosion Board under contract with the University of California.

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(Manuscript received May 1, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 5, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

STATUS OF POSSIBILITIES OF ARTIFICIAL PRECIPITATION

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Abstract--Cloud seeding with dry ice or silver iodide is designed to increase the ice-crystal content of clouds containing supercooled liquid water droplets, and thereby to permit the growth of precipitation elements (snowflakes or raindrops) by diffusion from water droplets to the induced ice crystals. Uncertainties as to need for additional ice crystals in clouds of appreciable water content make it difficult to estimate the potential value of cloud seeding in producing increased precipitation. Tests thus far have shown that it is often possible to produce slight amounts of precipitation from the bases of clouds, the rain reaching the ground on some occasions. Further research, both in the field and in the laboratory, will be needed to determine quantitatively the significance of cloud seeding in augmenting the nation's water resources. A short bibliography is furnished.

During recent years much interest has been shown in the possibility of increasing precipitation by artificial means through the so-called "cloud-seeding" techniques. Of particular interest to many is the possibility that the snow pack in mountain regions might be increased, thus providing additional water for power and irrigation uses, among others. Although the question of the feasibility of producing additional precipitation in these areas is not essentially different from the question in general, which is far from being settled, nevertheless, the rather special cloud structures occurring over mountains present a slightly different aspect to the problem. A number of experiments, and a number of commercial operations which are not so much in the nature of experiments, have been carried out in mountain regions. Before discussing them, however, it might be advisable to consider briefly the present state of our knowledge of the precipitation process and the hypothesized influence which cloud seeding might have.

For many years meteorologists have been investigating the mechanism by which the very fine cloud droplets (1/2000 inch diameter) are converted into drizzle drops (1/200 inch diameter) or raindrops (1/20 inch diameter), or into snowflakes of equivalent size. This conversion process is the more startling when it is realized that for a cloud droplet to grow into a raindrop, it must increase in volume a million times. Many possible effects which might explain or account for this growth have been considered, but only two have appeared to permit the observed rates of growth. One of these mechanisms is the collision process by which drops or ice particles of different sizes will fall at different speeds, thus permitting frequent collisions with other drops or particles with resulting coalescence or attachment. The other possibility is that when ice particles and supercooled (below freezing) liquid water droplets are both present in a particular region of a cloud, the ice particles will grow at the expense of the droplets. If the number of ice particles is fairly small as compared with the number of drops, substantial growth will be possible. On the other hand, if the number of ice particles is of the same order of magnitude as the number of drops, little growth could be expected. Although the relative importance of these two effects is not clear, many investigators feel that the ice-particle mechanism may be the most important for the initial growth of the ice particles, and that collisions between drops or snowflakes may account for the later stages of growth. Still to be explained on this basis, however, is the heavy rain which falls from tropical clouds which do not reach the freezing level.

With this brief (and admittedly incomplete) background in mind, we can now see why it has been suggested that seeding methods can lead to increased precipitation. It is observed that by inserting dry ice, silver iodide, and other agents into supercooled clouds of water drops, the drops can be converted into ice particles. Such artificially created ice particles can then be expected to grow as mentioned above, and, if conditions are right, to become large enough by the various processes involved to fall out as rain or snow.

At first glance, therefore, it might appear that, within reason, increases in precipitation could always be expected as a result of cloud seeding. There is one matter that has been overlooked, however. In nature most of the clouds which are thick enough to contain substantial moisture also contain ice particles which form naturally, and which may account for natural rain or snow. Furthermore, most of the clouds which contain substantial moisture produce rain or snow naturally in varying amounts. The question of increasing the usable precipitation from cloud systems thus requires an inventory of the precipitable moisture content of non-precipitating supercooled clouds, together with definitive experiments showing to what extent usable precipitation can be obtained from them. We would also have to consider the possibility that the efficiency of naturally precipitating clouds could be increased.

Cloud systems can in general be classified as stratus (layer type) or cumulus (towering type). The non-precipitating supercooled stratus clouds are usually fairly thin and contain only slight amounts of moisture. Although they can be affected by seeding, any slight precipitation which falls is apt to evaporate before reaching the ground. Cumulus clouds range in size from the small "fair-weather" variety to the towering thunderstorm. The latter, of course, usually produces heavy rain naturally, while the fair-weather types contain relatively small amounts of moisture, and also may not reach the freezing level. In between are a large number of clouds which would seem to be suitable for seeding experimentation. A number of tests have been carried out on these clouds, but they have not been statistically designed, and have not provided much in the way of a satisfactory answer. A great deal of further investigation will be necessary before we can estimate the quantity of additional moisture, if at all substantial, that can be realized from them.

In the West, and particularly over the Sierra and Rockies, a third type of cloud system is found--the orographic. These clouds often contribute substantially to the precipitation over mountains, although usually when associated with general precipitation conditions. During extensive periods, however, it is noted that clouds of either the stratus or cumulus type develop over mountains when conditions in the surrounding lowlands do not favor precipitation, and account for some of the excess of rain or snow noted on the crest and windward side of ranges. Here again, however, there are many periods when clouds that do not precipitate form over mountains, and it is further known that the winds provide a continuous supply of moisture to such cloud systems, the wind blowing through the clouds. It has been thought that such clouds might be particularly suitable for cloud seeding provided, of course, that the other conditions are met.

It is difficult to speculate on whether it may be possible to increase the precipitation from clouds which precipitate naturally. Some evidence even indicates that such clouds overseed themselves soon after precipitation begins, so that we should need less rather than more ice particles. Experimental evidence of the effects of seeding on clouds already precipitating is lacking at the present time. Suggestions have also appeared that it might be possible to reduce precipitation through overseeding. Although this possibility seems to exist, there is no satisfactory theoretical or experimental basis for the suggestion at present. A recent article by HOUGHTON [1951, see third item of Bibliography following this paper] provides an excellent review of this and other aspects of the problem.

Because of the complex nature of precipitation processes, and our general lack of knowledge of the mechanisms and the conditions for natural precipitation, it is difficult at present to predict a priori to what extent cloud-seeding techniques may influence precipitation. There has consequently been an effort to answer this question by actual experimentation in the atmosphere. A number of tests have been carried out on both stratus and cumulus clouds, as already mentioned, using dry ice dropped from aircraft as the seeding agent. If the conditions are suitable, that is, if the clouds are supercooled and are of substantial thickness, it is often possible to produce precipitation falling from the bases of the clouds. In many cases the precipitation will evaporate in the drier air below and will not reach the ground in any quantity. In some cases, however, it appears that rather substantial showers were initiated. Seeding such clouds in mountainous regions has a distinct advantage in that the cloud bases are closer to the ground at the higher elevations and there is consequently less chance for evaporation.

The tests have usually been conducted on clouds that were not already precipitating. In the case of stratus-type clouds, as before mentioned, the total precipitable water has been low and it has not been expected that the seeding would produce very substantial results. With cumulus clouds the lack of statistical design has materially affected the reliability of the results obtained. Many of the "successes" were in areas where precipitation was occurring naturally from other clouds, a fact that made it extremely important to separate out the natural component of rainfall by statistical means. It is in just such cases that the chance of success of seeding may be the greatest, that is, where many clouds may be on the verge of precipitation but require a slight "urge" to complete their development.

As regards silver iodide smoke, tests similar to those using dry ice have been conducted in Australia. It was generally found that in most cases no visible results appeared, possibly because of the lower temperatures required for silver iodide to produce ice crystals. The Weather Bureau operated a silver iodide generator at Colfax, California, at the foot of Donner Pass, from February to June, 1949, in an attempt to increase precipitation in the orographic flow up the western side of the Sierra. No detectable results appeared. During the past year commercial operations with ground-based silver iodide generators have been undertaken on a wide scale throughout the West, and a number have been designed to increase the snow pack on the Sierra and Rockies. The operators are generally reticent about results, and it has been impossible to obtain a very clear picture of what has actually gone on. Several analyses of these undertakings have been made by independent persons using varied methods, and the results have ranged from indications of marked success to findings of no detectable effect. Private evaluations of the rainmakers themselves often are not convincing, especially when they appear to be based on arbitrarily selected methods or selected cases.

The California Electric Power Company has been making tests over the Bishop Creek, California, watershed on the eastern slope of the Sierra since February, 1948. Preliminary estimates by the company indicated modest, favorable results. The Weather Bureau is at present analyzing the operation in considerable detail, and a report will be published in the near future.

Experiments in the atmosphere are complicated by the occurrence of natural precipitation, and complex statistical procedures must be resorted to in an attempt to separate out the precipitation to be solely ascribed to the seeding. Available methods are relatively insensitive and will require many years of testing before estimates having reasonable limits of error can be made. The smaller the true effect, the longer must be the series of experiments to determine it to any given degree of confidence.

Other difficulties arise in the use of silver iodide smoke. A number of tests have shown that this substance loses much of its effect upon clouds upon a half hour's exposure to sunlight. Also, it is difficult to determine exactly where it has gone after release, thus introducing further uncertainties.

The present outlook, therefore, appears to be that many suitably controlled experiments in the atmosphere, accompanied by adequate research into cloud physics in the laboratory and in the field, and with further development of instrumentation, will be required to determine quantitatively the significance of cloud seeding in augmenting the nation's water resources, and to define the meteorological and regional conditions favorable and unfavorable for results of practical importance.

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(Communicated manuscript received April 30, 1952;
open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

ESTIMATED FREQUENCIES AND EXTREME VALUES OF SNOW-PACK WATER EQUIVALENT AT MAJOR CITIES IN THE UNITED STATES

Walter T. Wilson and Richard D. Tarble

Abstract--The limited number of direct observations of weight of snow pack, particularly in populated localities, has made it necessary to study relations between observed or estimated weight of snow pack and other parameters. Fair relations have been established at stations having suitable data, and have been generalized on the basis of the good areal distribution of two parameters that are available in convenient published form: depth of snow on the ground and accumulated seasonal snowfall. The results seem to be consistent with the limited snow-survey data available in the area of interest and with other sources.

The water equivalent of accumulated seasonal snow pack that may be equaled or exceeded once in 50 years (about the maximum of record) is about seven per cent of the maximum seasonal snowfall and about 20 pct of the maximum observed snow depth in inches. The values of water equivalent to be equaled or exceeded one year in ten are 75 pct, and one year in two are about 50 pct, of the maximum of record.

Introduction--At many stations there are long records of observed depth of snow pack, but not of its density, and of measured water equivalent of individual falls of snow, but not their sum, after their accumulating on the ground and being subject to melting and other weather influences. It has been necessary to estimate the water equivalent of the snow pack under various conditions and at various locations and to relate its magnitude to observed depth and total seasonal fall. To establish these relations, and to generalize the results over a large area, a detailed study was made of about 140 important station-years, selected from known record depths, falls, and accumulations in populated localities.

Nearly all direct measurements of weight of accumulated snow pack have been made in mountainous and other remote localities. Where valid comparisons can be made between these data and those of nearby precipitation gages, the results show fairly good average relationships which were useful to this project. Seasonal snow pack is defined as the maximum amount of snow on the ground during a particular season.

Figure 1 shows the maximum observed depth of snow, in inches, at about 140 Weather Bureau first-order stations in the United States. In the non-mountainous regions of the country this map would come close to indicating the maximum values of water equivalent of the snow pack if appropriate density factors were to be applied. In general, where the snow pack is built up from several storms, the maximum water equivalent occurs at a different time than the maximum depth and is usually slightly greater than at the time of maximum depth, both within and among seasons. In the South, the governing density for maximum water equivalent is that of new-fallen snow, about ten per cent. The snow-pack density at more northerly low-elevation sites usually ranges from less than ten per cent to more than 20 pct. It may be noted that an isoline of ten inches depth could be drawn at about 33° latitude in eastern United States on Figure 1. Such a line would very nearly coincide with the ten-inch line of maximum observed 24-hour snowfall. The only important exception to this is the phenomenal storm of February, 1895, along the Gulf Coast, when, for example, near Lake Charles, La., 24 inches of snow fell, with an estimated water equivalent of two inches. In general, the area south of latitude 33° is not subject to the accumulation of several storms, but the maximum depths represent single storms, whose deposits melt rapidly.

Figure 2 shows, on a time scale, schematic trends of four important snow-pack parameters. The example is hypothetical for a northern or high-mountain station, with the lines smoothed. Actually, the lines should be irregular because of the intermittent occurrence of precipitation and of sampling variation. At most regular snow courses, observations are made at monthly or less frequent intervals; thus, curves such as in Figure 2 are not regularly available. In the laboratories of the Weather Bureau - Corps of Engineers Cooperative Snow Investigations, data for such curves are obtained weekly, and even daily at a few stations, for the entire snow-cover season. In the example shown, which is typical of deep packs and long seasons, line B shows the trend in snow depth. It starts with an initial fall in December, rises to a maximum depth in early March,

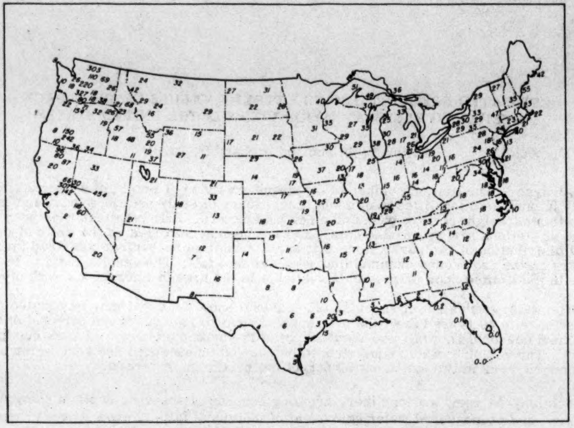

Fig. 1--Maximum recorded depth of snow on ground, in inches, at selected first-order stations of the Weather Bureau

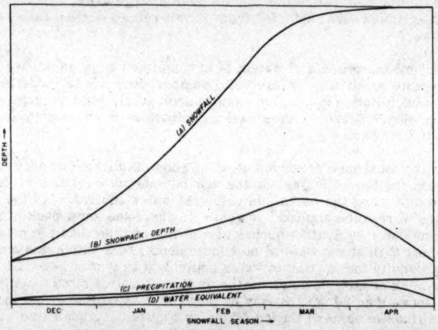

Fig. 2--Schematic trends of accumulation of some snow-pack parameters

and declines rapidly during April. The water-equivalent line, D, shows the corresponding trend of successive snow-pack measurements, starting in December, reaching a maximum in April, then declining rapidly as sustained melting occurs. The accumulated precipitation line, C, very nearly coincides with the water-equivalent line during the pre-melt period. The two lines being so close indicates that there may be negligible loss from a snow pack prior to its melting. The precipitation line continues to rise late in the season, when melting exceeds snowfall or when precipitation occurs as rain and is not stored in the snow.

Available evidence indicates that, with shorter seasons and smaller amounts of snow, the curves are similar, the principal difference being that the time scale is shortened, and the values of density, depth, and water equivalent do not reach the high maxima which typify the snow packs of the western mountains. The trend of snow-pack density varies from region to region and from

season to season. Without being able, as yet, to express these trends in precise quantitative terms, knowledge of their general shape was helpful in estimating the water equivalent from observations of pack depth and accumulated snow precipitation.

Analytical relations--Relations which are discussed qualitatively in the following paragraphs provided aids for estimating water equivalent in terms of the elements examined. Some of the relations established maximum limits, others established minimum limits; and by comparison and in combination they converged on values which seemed sufficiently valid to warrant their use. Pertinent available data were examined, in some cases hour by hour, and various relations were applied as they became appropriate.

The water equivalent of a snow pack measured by weighing a core bears a relation to the summation of precipitation after establishment of snow cover. As discussed earlier and as illustrated in Figure 2, this relation is generally good for deep accumulations prior to sustained melting. However, at stations where deep packs do not accumulate over long seasons, the ratio of snow-pack water equivalent to summation of precipitation may vary greatly. Possibly, with enough study, this ratio could be expressed as a function of melting temperature and other elements, but preliminary efforts showed no immediate promise of a direct precise relation other than establishing an upper limit for the water equivalent of the seasonal snow pack.

Snow-pack density is related to time of year, duration of cover, summation of degree days, and other variables; and increases as the season progresses. By the time of maximum water equivalent, the total error, without better definition of the relation than is now evident, is frequently excessive; but conservative density values help determine the minimum water equivalent of seasonal snow pack.

Expression of density of new-fallen snow as a function of air temperature and other conveniently available elements showed too low a correlation to substitute for mean station values in generalizing the summation of observed depths of new-fallen snow.

Consideration was given to estimating the residual snow pack by deducting such things as observed stream flow from the accumulated winter precipitation. With sparce data on change in soil-moisture storage, evaporation, and other important elements, the development of detailed storage relations was limited though this method was helpful in some instances.

The use of degree-day or degree-hour melting factors for estimating loss from total accumulated snow precipitation was useful as a guide, but had limited definitive value. Degree-hour factors vary greatly, especially during early stages of melting, and this project applies largely to areas where intermittent melting prevails and to periods before sustained melting becomes established. There seemed to be security in interpreting below-freezing periods as having no melt, especially if the persistence of an antecedent thin layer of snow showed no melting at the ground surface.

Evaporation from a natural snow surface prior to periods of continuous rapid melting for several days has been observed to about balance the accretion by condensation. The results of many studies indicate that evaporation may slightly exceed condensation over long periods, but the amount is well within the errors in estimating or measuring successive values of water equivalent of the snow pack itself.

The study concentrated on heavily populated areas. Areas excluded from detailed study are shown on the maps in Figures 3 and 4. Localities were chosen for study partly on the basis of their population, areal distribution, and availability of pertinent data.

Generalization and areal distribution--The technique of generalization was to study a portion of the record in detail, and to derive relationships by means of which readily available summarized data for a large number of stations could be applied to expressing the water equivalent of snow on the ground. The most readily available and most expressive parameters for this relationship are total seasonal snowfall and maximum depth on the ground.

Figure 5 shows the geographical distribution of the ratio of maximum observed seasonal snowfall to maximum observed snow depth for about 140 first-order weather stations. The two elements, whose ratio is shown here, are the two parameters necessarily used for estimating water equivalent, which was carefully studied at about 50 stations. The orderly areal pattern of these two elements and their ratio provides a climatological basis for the large-scale interpolation required in drawing the final maps.

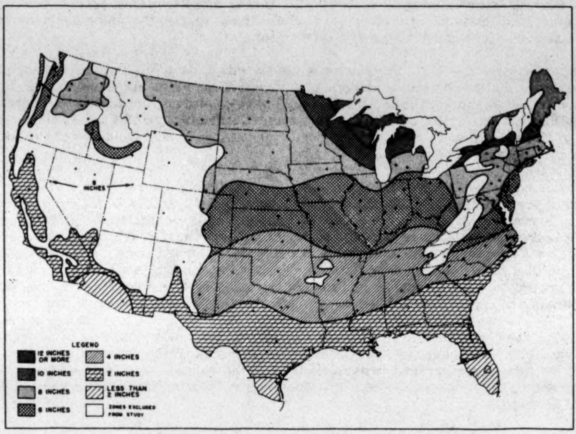
8 INCHES
LEGEND
12 INCHES OR MORE
10 INCHES
8 INCHES
6 INCHES
4 INCHES
2 INCHES
LESS THAN 2 INCHES
ZONES EXCLUDED FROM STUDY

Fig. 3--Maximum snow pack of record, inches of water

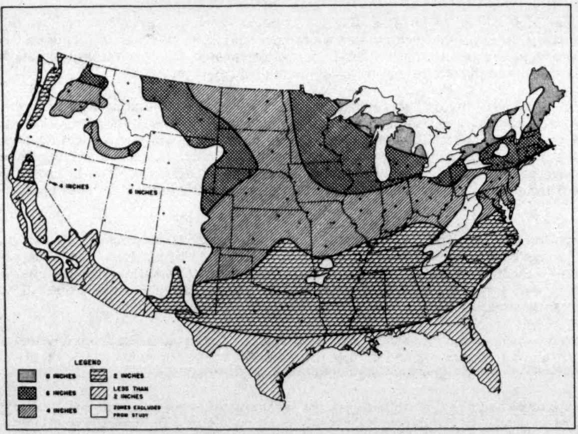
4 INCHES
6 INCHES
LEGEND
8 INCHES
6 INCHES
4 INCHES
2 INCHES
LESS THAN 2 INCHES
ZONES EXCLUDED FROM STUDY

Fig. 4--Water equivalent of seasonal snow pack equaled or exceeded one year in ten, inches

Figure 6 shows the relation between estimated or observed water equivalent of seasonal snow pack and total seasonal snowfall for about 120 station-seasons. The maximum water equivalent of the snow pack for the season was estimated, as indicated earlier, for those cases where there were accumulations, intermittent melting, and other complications. For other cases, mostly at southern stations, the water equivalent of the seasonal snow pack was observed after individual falls.

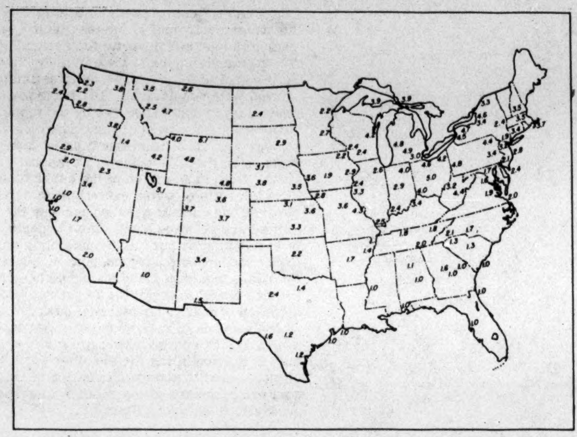
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Fig. 5--Ratio of maximum seasonal snowfall to maximum snow depth of record

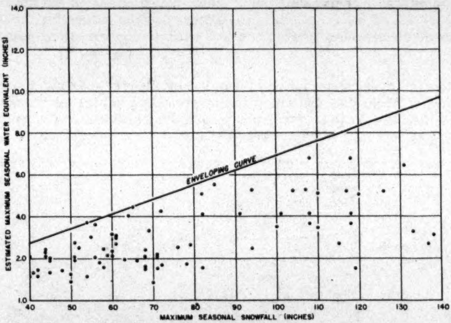

Fig. 6--Relation between maximum water equivalent and seasonal snowfall

The straight-line enveloping curve which has been drawn for values above 40 inches of fall and 2.5 inches of water equivalent shows that as much as 70 pct of the total seasonal snowfall may remain on the ground to make up the seasonal pack. The most that could remain is 100 pct. The difference, namely, 30 pct, is due to intermittent melting and to falls of snow occurring too early or too late in the season to contribute to the maximum pack. Because of uncertainties in estimating maximum seasonal water equivalent for extremely "snowy" stations such as Portland, Me., and Marquette, Mich., no attempt was made to generalize for seasonal pack exceeding 12 inches of water equivalent. Many of the points plot far below the enveloping curve, because they represent single great storms during seasons when the seasonal accumulation was not great. Obviously these values should be enveloped (rather than averaged in some sort of regression) for expressing the maximum of record. The smoothness of the envelopment and the fact that several years are repeated at different stations near the curve give confidence to the methods used in estimating water equivalent and to the implied areal transposition of seasonal pack in drawing the final maps.

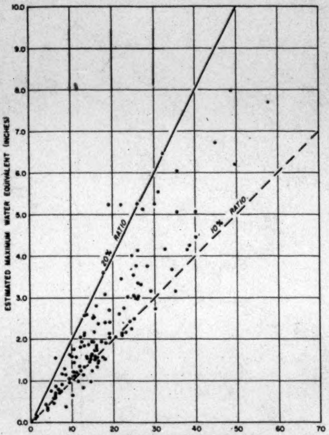

Fig. 7--Maximum depth of snow pack versus maximum water equivalent

Figure 7 corresponds to Figure 6 in relating the water equivalent of the maximum seasonal pack to a parameter useful for generalization. The parameter here is maximum depth of snow on the ground for each season of estimated or observed water equivalent. The relation of water equivalent to depth is shown by the ten and 20 pct lines which have been drawn. The points near the ten per cent line are single falls of snow, and those near the 20 pct line are seasonal accumulations. Only in a portion of the cases on this figure does the ratio of water equivalent to depth have the physical meaning of density, because the ratios plotted are not necessarily for the depth at the time of seasonal pack but for the maximum depth whenever it occurred during the season. Otherwise, the ratios would in many instances be greater. It seemed useful to apply the 20 pct ratio for generalization of maximum seasonal pack for stations where records of maximum observed depths were available. Of course, this ratio gives results too high in the South, but for the other parts of the country there is good confirmation of the results obtained by means of the relation to seasonal snowfall, as shown in Figure 6.

Figure 8 shows the estimated areal distribution of water equivalent of maximum seasonal pack, based on the 70 pct enveloping curve of Figure 6. The values for the South are obviously too low, as will be discussed later. Figure 8 was used as a guide, along with Figure 9, in drawing Figure 3.

Figure 9 shows the areal distribution of water equivalent of maximum seasonal pack, applying the 20 pct factor of Figure 7. This map shows excessive values for the South, but this is unimportant because the deep South is enveloped by a two-inch water-equivalent isopleth. An example of the failure of this map to represent southern stations adequately is the fictitious value of

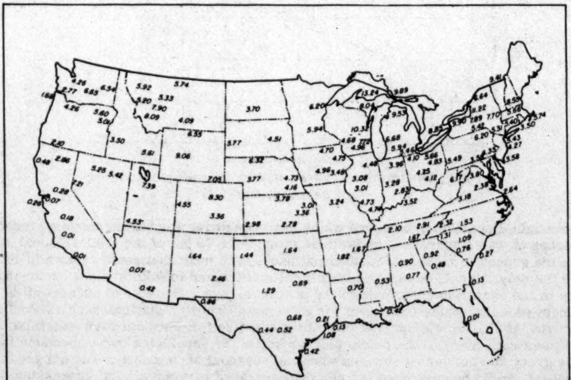

Fig. 8--Maximum estimated or observed water equivalent, inches, based on 70 pct of maximum seasonal snowfall and ten per cent average density of new snow

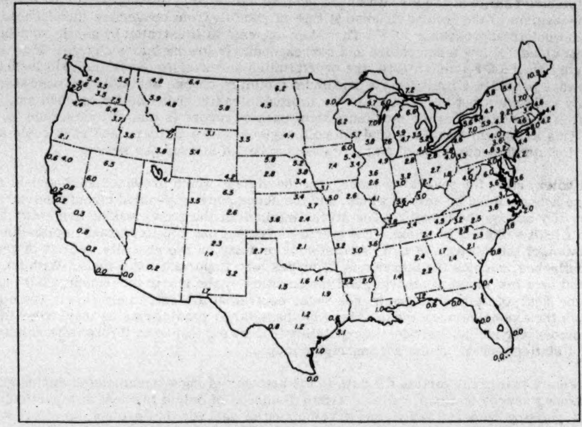

Fig. 9--Estimated water equivalent, inches, of maximum recorded snow depth based on 20 pct ratio of water equivalent to snow depth

3.1 inches water equivalent at Galveston, where the greatest water equivalent ever recorded on the ground there was 1.50, from the storm of February, 1895, during which 15.4 inches of snow fell. While any map of mean seasonal snowfall indicates a gradual decrease of values southward, such a decrease may not apply to extreme events, whose frequency defies definition in this case.

In generalizing over the South, another factor is the limited security of the very few observations upon which the extreme values, and their interpolation, must be based. Farther north, where snow is more common, it was possible to close in on a solution from several directions with mutual confirmation.

Study of the February, 1895, storm along the Gulf Coast and of other storms of that locality indicates that in the extreme the immediate Gulf Coast may be subject to some of the same type of influences that produce the well-known lake effect on the lee sides of the Great Lakes. Similar indications exist along the Atlantic Coast, where return flow and instability sometimes produce unusually heavy snowstorms, such as occurred December 26 and 27, 1947, at New York City.

While a comprehensive study of sleet or freezing rain was not made, examination of many such storms and of reports of the thickness of ice deposited soon indicated that accumulations of two-inch water equivalent were possible any place where freezing rain or snow has fallen.

There is some uncertainty about where to draw the line of zero snow, but it seems reasonable to exclude the southern extremities of Florida and Texas and parts of the extreme southwest, where snowfall has not been observed and where there is little climatological basis for expecting snow to fall.

Though it is beyond the scope of this project to consider the physics of the formation and precipitation of snow, certain physical and synoptic aspects must be considered in connection with the implied transposition involved in drawing isopleths and in attempts to appraise the frequencies of the recorded maximum occurrences. The amount of precipitation that storms produce is determined by two basic factors: the amount of moisture that the atmospheric circulation brings into the storm area, and the mechanism of the individual storm in its process of condensation and precipitation. The amount of moisture brought into the storm area is a function of the moisture charge of the air, and of the rate and duration of inflow to the storm area. Hydrometeorological studies have shown that the moisture charge is not only a limiting factor by itself, but also is related to the storm mechanism. Significantly, the amount of precipitation that storms produce is limited by the moisture charge of the air, which, in turn, is indicated by the dewpoint. At one extreme of storm mechanism is the violent cloudburst type of rainfall from convective thunderheads with surface dewpoints approaching 80°F. The other extreme is illustrated by gentle snowfall from stratiform clouds of low temperature and correspondingly low moisture charge. With saturated air of higher than 32°F temperature, the precipitation necessarily occurs as rain instead of snow. There is thus provided a limit for the maximum moisture charge available for snowstorms that is not only rather low but is reflected in the limited intensity of the storm mechanism. It is possible that in some instances the maximum snow pack of record is also the maximum to be expected. This statement would not apply in cold regions where an increased pack could accumulate from the fortuitous occurrence of an increased number of storms per season.

It is noteworthy, for several reasons, that the storms which produced the heaviest falls of snow were accompanied by severe winds, and the accompanying general circulation carried them rather swiftly across the country. The strong winds drift the snow, making observations of its depth a difficult sampling problem. The storm movement has two important implications. One is the limitation of the duration of most snowstorms to a day or two at a site. Areas of great orographic influence, such as the Sierra and Cascades, are important exceptions. With the rate of fall limited by a low moisture charge, and the duration by the storm movement, total individual falls, in the East, of more than two inches water equivalent are extremely rare. The other implication is the extensive area encompassed by these large snowstorms as they cross the country, which provides a synoptic basis for interpolation in drawing isopleths through the relatively sparse network of stations shown on the accompanying maps.

It is known that great variation exists in the amounts of snow accumulated during given periods, not only among nearby stations, but also within distances of only a few feet at a particular station. In the interpolation, generalization, and envelopment of data for this project, no effort was made to encompass isolated occurrences such as 40-ft drifts that have resulted from local peculiarities, and which certainly would be unreasonable as a basis for generalized maps. Many of the earlier records are less reliable than more recent ones. There is also the possibility that a few outstanding occurrences of water equivalent were missed in the selection of station-years on the basis of maximum depth and maximum monthly and seasonal accumulations of snowfall.

Studies of small-scale areal variability of snow accumulation have been made. Examples are the 1947 storm at New York and the 1918 storm at Chicago, where there is the possibility that the metropolitan station failed to experience the maximum snow conditions of its locality. It is thought that the enveloping processes and the large mass of data examined encompass most of this type of variation.

All the sources of error in the analysis and areal generalization are estimated, in the aggregate, to come within a total of 20 pct, or one inch of water equivalent, whichever is greater. This 20 pct is not the result of statistical summation of variances because there is some intercorrelation, though poorly defined, and because some of the sources of error can hardly be expressed in a manner amenable to formal analysis. That some of the error is cumulative rather than random is recognized, but it is accommodated to some extent by the envelopmental processes, such as drawing for the "high points" in preparing the maps of Figures 3 and 4.

Frequencies--Figure 10 shows examples of the frequency distribution of seasonal snowfall at six representative stations of the many whose frequencies were studied. There seemed to be no justification for using elaborate methods either for determining plotting positions or for locating and shaping curves through the plotted points. With 50 years of record, the maximum point was plotted at two per cent, the next highest at four per cent, the next at six per cent, and so on.

After preparing the frequency distributions for various stations, as illustrated in Figure 10, efforts were made to generalize them on a climatic or geographical basis. The most effective variables were found to be mean seasonal snowfall, and a measure of dispersion, which was taken at the probability level of one in ten. Referring again to Figure 10, most of the distributions were concave upward, as for Valentine. The points for Concord show very nearly a normal distribution. The record for Rapid City shows the three top values to have about the same magnitude, illustrating the difficulty (with only about 50 years of record) of defining the frequency of events much rarer than one in ten years. Havre shows a corresponding difficulty, namely, that the highest value far exceeds the next highest one. The ten per cent frequency for Havre was rounded off at 55 inches, this value being a little higher than the nearby points, so as to give some weight to the maximum value. The data for Portland show the characteristic asymmetry of the record in regions where occasional seasons experience no snowfall. The points for Duluth raise the question, if one is compelled to draw a curve, of how closely the curve should follow the points and how closely it should

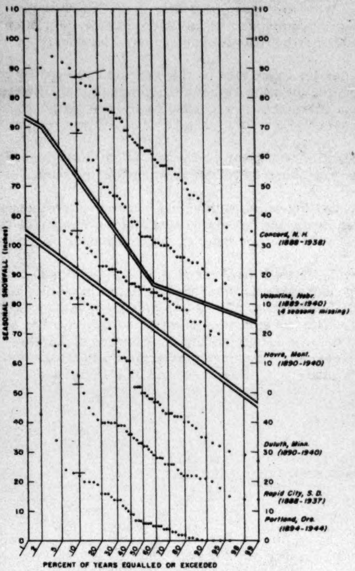

Fig. 10--Frequency distribution of seasonal snowfall at representative stations showing values chosen for ten per cent frequency

follow some relatively smooth a priori formula. As stated earlier, most of the distributions are concave upward, though each of the six examples in Figure 10 typify the records of several stations. The values for ten per cent frequency were determined by inspection and interpolation.

In Figure 11, the ten per cent frequency values are plotted against the mean seasonal snowfall, each point representing a station. The curve was drawn by inspection. The curve of Figure 11 was drawn for stations having available published annual or seasonal mean snowfall, and the relation thus defined was applied to the 200 stations on which the map of Figure 4 was based.

Some bias may possibly result from applying the frequency distribution of seasonal snowfall to the frequency of water equivalent of the residual seasonal snow pack. A few stations publish maximum depth for each year, and tests of these frequency distributions showed consistency with those for snowfall, thus supporting the assumed application of snowfall frequency to frequency of seasonal pack.

Conclusions--Figures 3 and 4 represent the conclusions of this project, showing zones of extreme values of seasonal snow pack, and of accumulations having a frequency of one year in ten. Figure 3 resulted from applying the relation of Figures 6 and 7 to snow of average initial density. Figure 4 includes the relation of Figures 6, 7, and 11.

Figure 3 was drawn, examined, and revised several times. The data supporting this map were compared with various maps, tabulations, reports of local climate and individual storms, and many other sources. The irregular-shaped areas of mountain regions are excluded from the generalization.

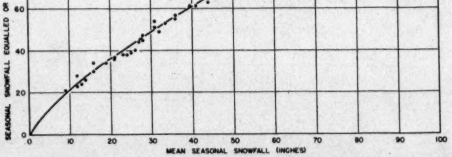

Fig. 11--Relation of ten-year seasonal snowfall to mean seasonal snowfall

The value for metropolitan areas in the Far West are based on scanty data. To show precisely the zone boundaries in those localities would require additional study, but for the cities near sea level the indicated values seem reasonable from examination of their individual records.

To some degree, the tolerance referred to earlier is carried on the safe side by specifying, for each zone on these maps, the maximum value for the zone rather than interpolating between zonal boundaries. The two-inch water-equivalent increment is regarded as the finest division that the data, sampling, and interpretation warrant.

While it may be observed that some consideration was given to the so-called lake effect in defining the areal pattern, no objective relations were derived for expressing this quantitatively.

A rule of thumb for remote mountain areas, and for areas with values exceeding ten inches of water equivalent, is for the maximum water equivalent to be about twice and the ten per cent frequency water equivalent to be about one and a half times the local average seasonal pack.

Acknowledgments--This paper is based largely on work performed by Weather Bureau employees in connection with a research project sponsored by the Housing and Home Finance Agency, Office of the Administrator, under Title III of the Housing Act of 1948, as amended, and carried out through agreement with the Weather Bureau. With the exception of snow-survey records of the Wisconsin Valley Improvement Company, all data were obtained by or prepared under the auspices of the Weather Bureau, Corps of Engineers, Soil Conservation Service, and Geological Survey. Helpful criticisms and suggestions were made by C. S. Gilman and J. L. Paulhus.

U. S. Weather Bureau,
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(Communicated manuscript received April 2, 1952;
open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

ARTIFICIAL ROUGHNESS STANDARD FOR OPEN CHANNELS

A. R. Robinson and M. L. Albertson

Abstract--In the formulas in present use the resistance coefficient has been considered constant for a particular type of material in a particular state of upkeep. The resistance coefficient, however, is not a constant for a given channel but varies with both velocity and depth. From the nature of these formulas in present use, it may be seen that the variable influence of viscosity is not taken into consideration.

Because of the need for more accurately determining the discharges in open channels a standard for roughness for open channels such as that which exists for pipes is desirable. The problem of developing a reproducible standard for roughness in open channels has previously received very little systematic study. Although several investigations have been made, each was made with a different type of roughness.

This study was conducted for the purpose of determining whether an artificial roughness standard could be established for a channel with a boundary so rough that the viscous effects were negligible. The equation

$$C = 26.65 \log_{10} (1.891\ d/a)$$

was established and is applicable over at least a fourfold variation in d/a and nearly a twofold variation in the resistance coefficient.

Although the conveyance of water in open channels is one of the earliest of engineering achievements, as yet no formulas for determining discharges have been developed that are without important limitations. The difficulty has been in determining the extent to which the flow is retarded for different boundary and flow conditions. In the discharge formulas in present use, the resistance coefficient is considered to be a constant for a particular type of material in a particular state of upkeep without regard to the other variables. Many investigators have shown that this is not the case as the resistance coefficient may vary as much as 50 pct for different depths in the same channel because of the effects of viscosity and shape. In the case of alluvial channels, this variation in resistance coefficient may also be effected by the formation of dunes on the bed of the channel.

It has become increasingly evident that an artificial standard for roughness in open channels, such as that which exists for pipes, is needed. Pipe roughness has been referred to an artificial standard in order to determine a relative roughness for each type of boundary material having various surface conditions. Using this standard as a basis, formulas have been developed experimentally so that the discharge through pipes can be predicted with considerable accuracy.

The problem of determining an artificial standard for roughness in open channels has hitherto received very little systematic study. Several investigators have made studies using artificial roughness, each using a different type of roughness for the purpose of determining the retardation of flow for the particular type of roughness chosen.

This study was for the purpose of choosing an artificial roughness and testing it in a range where the channel would be so rough that viscous forces would be relatively insignificant.

Previous developments--Two formulas for computing discharges in open channels are now widely used. The first of these is the Chezy formula which may be stated as

$$Q = CA\sqrt{RS} \qquad (1)$$

where Q is the discharge, C is the Chezy resistance coefficient, A is the cross-section area, R is the hydraulic radius, and S is the slope of the channel. The Manning formula which is more widely used in the United States at the present time is

$$Q = (1.486/n)\ AR^{2/3}\ S^{1/2} \qquad (2)$$

where n is the Manning coefficient of resistance. In both cases the resistance coefficient is considered to be constant for a particular type of boundary material in a particular state of repair regardless of the flow conditions.

In 1860, BAZIN [1888] and Darcy [GANGUILLET and KUTTER, 1889] made the first studies of artificial roughness in open channels. Experiments were conducted using transverse wood strips nailed to the bottom and sides of a channel to produce the roughness. Experiments were also conducted using gravel glued to the sides of the channel. No attempt was made to correlate the data obtained using artificial roughness with that available for natural roughness.

KEULEGAN [1938] applied the modern principles of fluid flow in circular pipes to the problem of turbulent flow in open channels by theoretical development. The formula which was developed by using the assumption that resistance to flow in a rough channel is the same as to flow in a rough pipe with the same hydraulic radius and degree of roughness, may be stated as

$$C/\sqrt{g} = 6.25 + 5.75 \log_{10} (R/k) \qquad (3)$$

The roughness k is the diameter of equivalent NIKURADSE [1933] sand particle.

POWELL [1946, 1950] made a series of experiments using square steel strips across the sides and bottom of a channel. With the fundamental formula given by Keulegan, he developed the equation

$$C = 1.263J + 17.9\,\beta_k + 41.2 \log_{10} (R/k) \qquad (4)$$

where $J = \sqrt{g}\,(a_r - 2.5)$, a_r being a function of k; and β_k is a function of the shape of the cross section. Through further simplification which included changing the scale by which the roughness was measured, the equation

$$C = 42 \log_{10} (R/\epsilon) \qquad (5)$$

was presented, where ϵ is the measure of roughness. This equation applies to that zone where the channel is so rough that the resistance to flow is caused by the roughness alone. In the transition zone between the rough and smooth channels the resistance depends both on the viscous effects and the relative roughness. For this condition, Powell developed the equation

$$C = -42 \log_{10} (C/Re + \epsilon/R) \qquad (6)$$

where Re is the Reynolds number.

The method of estimating the resistance coefficient for pipes is given by ROUSE [1943]. It is noted that the roughness of each type of pipe material is given in terms of k, which is the equivalent Nikuradse sand roughness. In that section in which the pipe is very rough the equation which determines the Darcy-Weisbach resistance coefficient f is given as

$$1/\sqrt{f} = 2 \log (D/k) + 1.14 \qquad (7)$$

where D is the diameter of the pipe.

Theoretical analysis--For the purpose of making a preliminary analysis of the problem, the forces which are acting on the free body as shown in Figure 1 must be considered, where τ_o is the shear along the boundary and WP is the wetted perimeter.

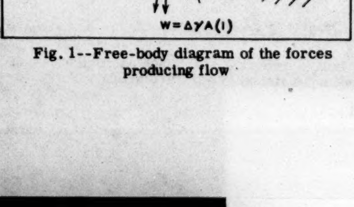

Fig. 1--Free-body diagram of the forces producing flow

If the flow is uniform and steady, the sum of the forces is equal to zero so that

$$\Sigma F = F_1 + W \sin \alpha - F_2 - WP\,\tau_o = 0 \qquad (8)$$

$$F_1 = F_2 \qquad (9)$$

and

$$\tau_o = (A/WP)\,\Delta\gamma \sin \alpha = R\,\Delta\gamma \sin \alpha \qquad (10)$$

Although it is possible to solve for the shear by analytical means, the use of dimensional analysis shows more clearly the influence of the variables involved. The variables which govern the intensity of shear along the boundary may be stated as

V = velocity of flow in the channel
k = roughness of the channel in terms of the equivalent Nikuradse sand roughness
sf = factor expressing the shape of the channel
R = hydraulic radius
ρ = density of the fluid
μ = fluid viscosity
$\Delta\gamma$ = difference in specific weight of air and water

The general relationship that exists may be stated as

$$\tau_o = \phi_1 (V, k, sf, R, \rho, \mu, \Delta\gamma) \tag{11}$$

Choosing V, R, and ρ as repeating variables, dimensional analysis yields

$$\tau_o/\rho V^2 = \phi_2 (R/k, sf, V/\sqrt{(\Delta\gamma/\rho) R}, VR\rho/\mu) \tag{12}$$

$$\tau_o = \rho V^2 \phi_2 (R/k, sf, V/\sqrt{(\Delta\gamma/\rho) R}, VR\rho/\mu) \tag{13}$$

in which $V/\sqrt{(\Delta\gamma/\rho) R}$ is a Froude number Fr and $VR\rho/\mu$ is a Reynolds number Re.

Equating (10) to (13) yields

$$R \, \Delta\gamma \sin \alpha = \rho V^2 \phi_2 (R/k, sf, Fr, Re) \tag{14}$$

so that

$$V = \sqrt{(\Delta\gamma/\rho)} \, \phi_2 (R/k, sf, Fr, Re) \sqrt{R \sin \alpha} \tag{15}$$

For small values of α, the slope S is approximately equal to $\sin \alpha$ so that

$$V = \sqrt{g} \, \phi_2 (R/k, sf, Fr, Re) \sqrt{RS} \tag{16}$$

Chezy's formula which is widely used for computing discharges in open channels is

$$V = C \sqrt{RS} \tag{17}$$

From (16) then,

$$C = \sqrt{g} \, \phi_2 (R/k, sf, Fr, Re) \tag{18}$$

or

$$C/\sqrt{g} = \phi_2 (R/k, sf, Fr, Re) \tag{19}$$

Because of the interdependence of R, sf, and depth of flow d as $d = \phi (R, sf)$, the Froude number and relative roughness may be expressed in terms of the mean depth. Further, if the channel to be studied is rectangular in shape, the shape factor sf may be replaced by the parameter B/d, where B is the width of the channel. Therefore,

$$C/\sqrt{g} = \phi_3 (d/k, B/d, Fr, Re) \tag{20}$$

The roughness k is dependent upon the geometric pattern of the type of roughness selected in the case of artificial roughness. In this study it was decided to use baffle plates as shown in Figure 2 so that

$$k = \phi_4 (a, t, b, x, e) \tag{21}$$

where

a = height of artificial roughness
t = thickness of the baffle plates
b = length of the plates
x = longitudinal spacing
e = transverse spacing

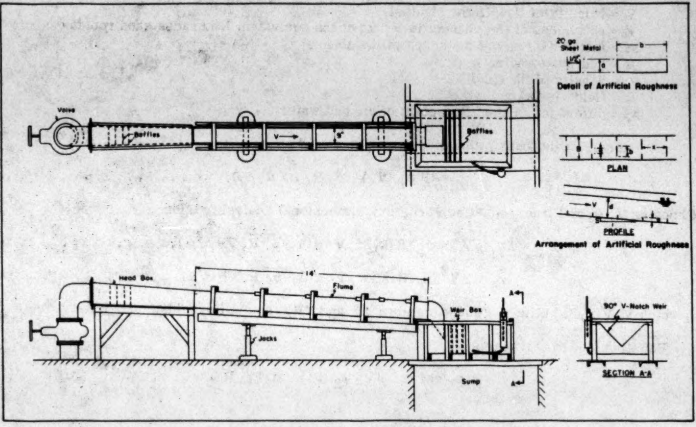

Fig. 2--Layout of experimental equipment

Dividing through by a, so that (21) will be in dimensionless form, results in

$$k/a = \phi_5 \,(t/a,\, x/a,\, b/a,\, e/a) \quad \ldots\ldots (22)$$

For this particular study the baffle plates used were made of very thin sheet metal and only two heights of roughness were used so t/a could be considered insignificant for all practical purposes. On the basis of previous experiments [ALBERTSON, 1948, 1951; JOHNSON, 1944; POWELL, 1946], maximum roughness should occur for the type of roughness used at a longitudinal spacing-to-height ratio x/a between 2 and 10. For this study this ratio was kept constant at 10. The arbitrary constants describing the roughness b/a and e/a were given values of 4 and 2. Eq. (22) then becomes

$$k/a = C_1 \text{ or } k = C_1\, a \quad \ldots\ldots (23)$$

where C_1 is a constant.

For this particular study C_1 was assumed to have a value of one so that k = a and (20) becomes

$$C/\sqrt{g} = \phi_6 \,(d/a,\, B/d,\, Fr,\, Re) \quad \ldots\ldots (24)$$

This equation is valid for all values and combination of values of relative roughness, relative width, Froude number, and Reynolds number. However, the Froude number is important only if surface waves or large irregularities are involved. For this study, large irregularities would not be involved and although surface waves possibly would exist, they should not be of sufficient magnitude that the Froude number would be of importance. This assumption simplifies (24) to

$$C/\sqrt{g} = \phi_7 \,(d/a,\, B/d,\, Re) \quad \ldots\ldots (25)$$

For large values of the Reynolds number or when the roughness is relatively large, the laminar sub-layer is destroyed so that the viscous effects are negligible and the Reynolds number may be omitted. Then

$$C/\sqrt{g} = \phi_8 \,(d/a,\, B/d) \quad \ldots\ldots (26)$$

When the resistance on the bottom is very great compared with the resistance on the sides, the effect of relative width becomes unimportant and B/d may be omitted.

$$C/\sqrt{g} = \phi_9 \text{ (d/a, Re)} \quad \ldots\ldots (27)$$

For large values of the Reynolds number and when the resistance on the bottom is great compared to that on the sides, the problem reduces to

$$C/\sqrt{g} = \phi_{10} \text{ (d/a)} \quad \ldots\ldots (28)$$

When the viscosity is of major importance compared with relative roughness, then d/a is unimportant and

$$C/\sqrt{g} = \phi_{11} \text{ (B/d, Re)} \quad \ldots\ldots (29)$$

If the relative width is great and the roughness is relatively small, then the function reduces to

$$C/\sqrt{g} = \phi_{12} \text{ (Re)} \quad \ldots\ldots (30)$$

On the basis of the theoretical development and previous research, an equation in logarithmic form should apply in determining the resistance coefficient for wide, rough channels. This equation should have the form of

$$C/\sqrt{g} = K_1 + K_2 \log_{10} \text{ (d/a)} \quad \ldots\ldots (31)$$

where K_1 and K_2 are constants to be determined experimentally.

Range of variables--In order to cover the widest possible range of variables with the limitations imposed by the equipment available, the following range was placed on those variables involved.

a = 1/2 and 1 inch
d = 2 to 8.5 inches
S = 0.001 to 0.04
B = 9 inches
V = 0.30 to 5.0 ft/sec (approx.)
d/a = 4 to 17
B/d = 1.1 to 4.5
Re = 3.5×10^3 to 2.5×10^5 (approx.)

Experimental equipment and procedure--The laboratory equipment used for this study consisted of a water-supply system, an adjustable slope flume to accommodate the artificial roughness, and a weir box to measure the discharge. The test section of the flume, which was made of wood, was 9 inches wide by 10 1/2 inches deep and 14 ft long. Upstream from the test section was an entrance section six feet long in which an attempt was made to establish the velocity distribution before the flow entered the test section. Adjustable jacks were used to set the flume at the desired slope. The artificial roughness was in the form of small sheet metal angles 1/2 and one inch high which were fastened to the floor with an arrangement as shown in Figure 1.

In order to adjust the depth of flow in the flume, the downstream end was constructed so that small slats could be placed across the flow to form a tailgate. To measure the depth of flow a movable point gage was used which could be placed at any one of six measuring points distributed along the length of the flume. The discharge was measured by a 90° V-notch weir placed at the end of the flume.

To begin an experiment a certain size of roughness was installed in the flume and the desired slope was set by adjusting the jacks and checking the drop for a known horizontal distance with a surveyor's level and rod.

Normally, for each slope of the flume, four runs were made with the depth of flow varying from very low to as high as possible. For each run the depth of flow was determined at each of the six measuring points and the discharge obtained with the weir. Since it was necessary that steady uniform flow be maintained, the slats at the downstream end of the flume were used to maintain a water surface that was parallel to the floor of the flume throughout most of its length.

Discussion of results--In the analysis of the problem, it was shown that there are at least 13 relatively important factors in the problem dealing with the effect of artificial roughness on flow

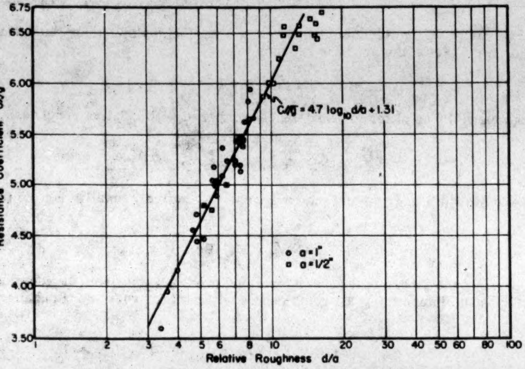

Fig. 3--Variation of resistance coefficient with relative roughness

in open channels. After combining these variables by dimensional analysis and eliminating some from consideration, only three parameters were chosen for study, the resistance coefficient $C/\sqrt{g}$, the relative roughness d/a, and the Reynolds number Re.

Figure 3 is a plot of $C/\sqrt{g}$ as a function of d/a as shown in (28) and (31) for rough channels. The equation of the line shown was found to be

$$C/\sqrt{g} = 1.31 + 4.7 \log_{10} d/a \ldots\ldots\ldots\ldots \quad (32)$$

or

$$C = 26.65 \log_{10} (1.891\ d/a)$$

The data for the 1/2 and one inch roughnesses agree very well for values of d/a less than 10. For values in excess of 10, the resistance coefficient determined experimentally is less than predicted by (32). The most plausible explanation for this is that the effect of relative width B/d or of viscosity becomes important at this point as shown in (29) and (30). In other words, the channel can not be considered as infinitely wide for values of d/a greater than 10, because at this point the effect of the side walls becomes important. The scatter of the data for values of d/a less than 10 can be attributed to experimental error and to the difficulty of obtaining uniform flow in some of the experiments.

The value of the coefficient in (32) is 26.65 as compared to 32.6 for Keulegan's Eq. (3), and 42 for Powell's Eq. (5). Keulegan used data from natural roughness to check his theoretical equation whereas Powell's equation was obtained by correlating data from his experiments using continuous metal strips for artificial roughness. For the case of Powell's roughness, only horizontal vortices would form because of the bottom roughness and only vertical vortices would form because of the wall roughness. Except at very shallow depths his channel must be considered to be narrow in which the side walls exert a considerable influence on the retardation of flow. Bazin's data, which Keulegan used to check his equation, were taken using a wide channel with gravel placed on the floor to serve as the roughness. Although Bazin's roughness is of a natural type, it does not give the same degree of roughness as that used in this study. Therefore, the comparison between (32) and (3) and (5) must consider the measure and type of roughness. A value of a for any given channel must be determined by first making actual discharge measurements in a rough channel in order to determine the other factors in (32). Then a constant value of a can be determined for a canal of a particular material and state of upkeep.

Figure 4 is a plot (based upon Figure 3) of the resistance coefficient $C/\sqrt{g}$ as a function of the Reynolds number Re for different values of the relative roughness d/a. This plot demonstrates the significant fact that for the roughness tested, the value of C remains constant over the range

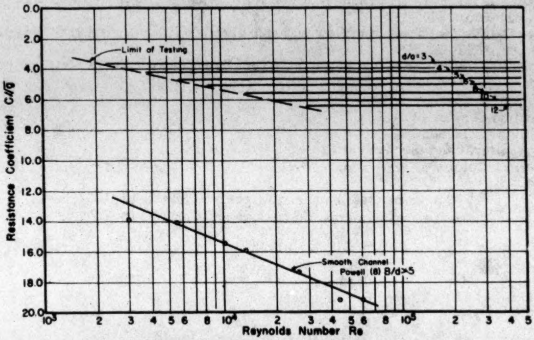

Fig. 4--Variation of resistance coefficient with Reynolds number and relative roughness

of Reynolds number observed for a particular value of d/a. In other words, the channel was acting as a rough channel. As mentioned previously this statement applies only for values of d/a equal to or less than 10. For values greater than 10, a study of Figure 3 leads one to believe that the channel was functioning in the transition zone between a rough and smooth channel or that the relative width has become important.

Also of interest in Figure 4 are the data shown for a smooth channel observed by POWELL [1946]. Based upon the theoretical analysis leading to (27) and (30), one would expect the curves shown for a rough channel for constant values of d/a to approach the curve for a smooth channel asymptotically for the lower ranges of Reynolds number. Similar curves are shown by ROUSE [1943] for pipes.

The foregoing analysis and experimental study demonstrate that a roughness standard such as exists for pipes may be set up for open channels with rough boundaries. The design of artificial roughness used herein could possibly be improved. This particular design was chosen because of ease of construction, ease of duplication in other laboratories, and the belief that the resulting resistance to flow would more nearly duplicate the type of resistance found for natural roughness.

Acknowledgments--This paper is based on the thesis submitted by A. R. Robinson under the direction of M. L. Albertson in partial fulfillment of the requirements for a Master's Degree in Irrigation Engineering at Colorado Agricultural and Mechanical College, Fort Collins, Colorado.

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(Communicated manuscript received March 15, 1952;
open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

STRUCTURE PLAN OF CLASTIC DIKES

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Abstract--Conclusions drawn from field observations are that vertical sandstone dikes of the White River Badlands, South Dakota, originated as open fractures that were filled with debris supplied by erosion of overlying formations. One existing explanation is that the dikes formed in deep desiccation cracks which developed at the surface as the silts and clays of the Oligocene White River group became consolidated. Evidence from a recent study of a small area of clastic dikes is incompatible with the desiccation-crack theory of origin. The pattern of intersecting dikes, when mapped, resembles a fracture system rather than having a hexagonal arrangement normally associated with desiccation cracks. Shear fractures appear on the map, intersecting at angles of 54° to 71°, the acute bisectrix varying in strike from N 45°W to N 54°W. One dike occupies a major tension fracture, strikes N 50°W, and extends vertically through two beds of strikingly different lithology, a condition unlikely to occur with desiccation cracks. South of the area, normal faults of small vertical displacement, trending northwest, suggest minor regional tension, which may well have caused the development of a fracture pattern as displayed by the dikes. If these fractures were open at the surface, subsequent filling from above would follow, resulting in numerous intersecting clastic dikes.

Sedimentary dikes occur in nearly all parts of the Big Badlands of South Dakota and appear to be confined stratigraphically to the Oligocene White River group in this region. The dikes are most abundant in the upper White River, or Brule formation, but a few occur in the lower portion, or Chadron formation. Dikes strike in several directions, intersecting in pronounced patterns, and are commonly vertical. They vary in thickness from a fraction of an inch to one or two feet.

The Badlands clastic dikes may be composed of various types of material, ranging from mixtures of sand and silt to clay, generally with large amounts of volcanic ash present. The presence of volcanic ash leads to the conclusion that the dikes represent open fissures which were filled from above by weathering and erosion of rocks higher in the stratigraphic column. In general, no volcanic ash beds are found below the present level of the dikes, and the major ash beds are in the upper part of the Brule formation, which is commonly referred to as the Whitney member or Whitney ash beds. In one locality, WANLESS [1923] recorded the occurrence of a dike filled with pure white volcanic ash, the source of which was found to be an ash layer nearly 200 ft higher in the series.

It was originally believed that the dikes were filled from below by compressional forces, or by hydrostatic pressure, in a manner similar to the formation of the clastic dikes in San Luis Obispo and Santa Cruz counties, California [NEWSOM, 1903]. HAY [1892] suggested this method of origin for similar clastic dikes in the Oligocene of northwestern Nebraska. He believed them to be related to the phenomena of mud volcanoes, intruded from below during the closing period of the Black Hills uplift. The nature of the material forming the dikes tends to render these theories doubtful, since the source was apparently higher than the present level at which the dikes are found.

A present concept concerning the origin of the Badlands clastic dikes assumes that they represent cracks which were filled from above, through the development of fissures by the gradual drying out of the White River clays after consolidation, forming desiccation cracks [BUMP, 1951; WANLESS, 1923].

WARD [1922] suggested that the clastic dikes may be related to structural features, but did not elaborate his ideas beyond a few general statements. Ward described four major and several minor normal faults in the vicinity of Cedar Pass. The maximum length of any one fault is one and one-half miles, with vertical displacement not exceeding 25 ft. The major faults trend approximately N 65°W - N 70°W, while the smaller faults are slightly more variable in trend. Ward suggested that the dikes represent tension cracks which developed at the time of faulting and were subsequently filled with material derived from sediments higher in the stratigraphic column.

While engaged in field work on a Columbia University research project in geomorphology, supported by the Office of Naval Research, the author had an opportunity to map a small area of

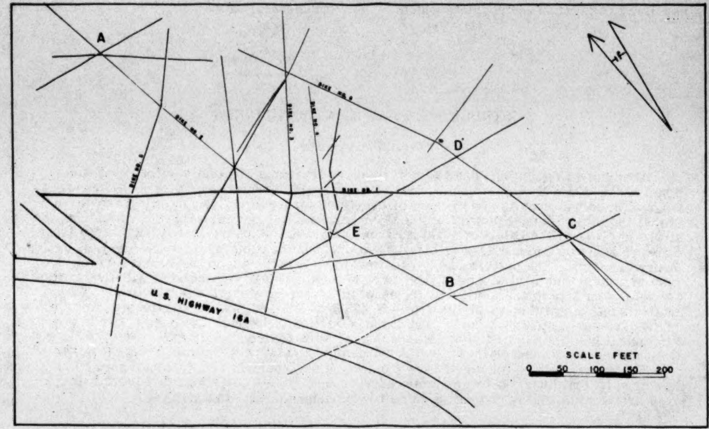

Fig. 1--Map of intersecting clastic dikes in small area north of Cedar Pass, Badlands National Monument, South Dakota

intersecting dikes and to make several observations bearing on their origin. The area mapped lies within the Badlands National Monument, just to the north of the Cedar Pass area, within that portion of the Badlands which is designated as the "Wall." Here 22 dikes occur within an area which is approximately 1000 ft long and 600 ft wide. The intersecting dikes form a pattern that resembles fractures related to deformational forces rather than having the polygonal arrangement normally associated with desiccation cracks (see Fig. 1). Dikes shown on the map vary from one inch to one foot in thickness.

Table 1--Angles of intersection of shear fractures and strikes of acute bisectrices

Point	Angle	Strike
A	71.5	N 44°W
B	54.5	N 50°W
C	60.0	N 44°W
D	65.0	N 47°W
E	56.5	N 54°W

Several of the dikes appear to occupy shear fractures which, on the map, intersect at points A, B, C, D, and E. The acute angle of intersection varies from 54 1/2° to 71 1/2°, averaging about 60°. The 60° acute angle is a phenomenon normally associated with that angle of intersecting shear fractures which is bisected by the axis of maximum compression. Acute angles of intersection at each point are shown in Table 1. The acute bisectrix at each intersection would thus be parallel to the axis of maximum compression and parallel to any tension crack resulting from the same deformational forces. The strikes of the acute bisectrices vary a maximum of ten degrees from one another, as shown in the last column of Table 1.

Dikes occupying tension cracks may also be recognized in the area. At point A on Figure 1, a tension-fracture dike nearly bisects the acute angle between two shear-fracture dikes. In addition, Dike 1 appears to occupy a major tension crack which extends the entire length of the map and strikes N 50°W. This major tension-crack dike cuts all the other dikes which it intersects, and its strike is approximately parallel with the strikes of the acute bisectrices of the shear-fracture intersections.

It is therefore suggested that the dikes represent fissure fillings which originally developed as a fracture pattern resulting from minor local deformation, including possible slight warping with tensional forces directed at right angles to the general strike of the acute bisectrices and Dike 1. The minor dikes, trending at right angles to Dike 1, may have originated as release fractures, which developed at right angles to the axis of maximum compression after the stresses ceased [BILLINGS, 1947, p. 103]. Assuming this to be true, Dike 1 would represent a tension crack, Dikes 2 and 6 would be shear fractures, while Dikes 3, 4, and 5 are release fractures.

The formations in this locality are essentially horizontal and relatively undisturbed and the proposed theory of dike origin may not at first seem plausible. WARD's [1922] observations on faulting in the Cedar Pass area tend to support the theory. The clastic dike area studied lies just north of the four major normal faults described by Ward as being present along the southern margin of the "Wall." The strikes of these faults are approximately parallel to the strikes of the main tension crack of the dike area and the acute bisectrices of the shear fractures. The possibility thus presents itself that the entire pattern may be the result of the same tensional stresses.

Some minor features were observed which are also considered as supporting evidence for the fracture-pattern theory of origin for the clastic dikes. Those dikes occupying what have been designated as shear fractures are commonly characterized by the presence of slickensides on their outer walls. This phenomenon suggests slight movement of one side with respect to the other when formed, a condition usually associated with shear fractures in any type of rock. There is no apparent displacement along opposite sides of the shear fractures, or any other indication of movement except the slickensides. According to Walter H. Bucher (personal communication) of Columbia University, this is not unusual and probably means that the total form change involved in the process which caused the development of the fractures was very small.

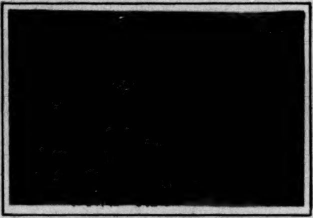

Fig. 2--View looking southeast along major tension-crack dike, showing general relation of dike to horizontal bedding of the area; intersection with minor dike shown in foreground

Another important field characteristic which is considered as supporting evidence for the fracture-pattern theory of origin is the manner in which a given dike may extend vertically through two or more beds of markedly different lithology. This relation is readily apparent in a southeastward-facing photograph along Dike 1 (Fig. 2). In the foreground of the photograph the dike cuts a soft nodular clay, while in the distant background it intersects alternating beds of clay and volcanic ash. Finally, on the skyline of the illustration the dike extends vertically through a bed of dense volcanic ash which is approximately 30 ft thick. The vertical rise along the dike exposure as described is about 100 ft. It seems rather unlikely that if the dikes represent desiccation cracks, as assumed under the present concept of origin, they would have the vertical extent which is shown by this dike and several others in the area. Nor would desiccation cracks tend to be as long and straight as are some of the Badlands clastic dikes.

Summary--Several facts appear to support a fracture-pattern theory of origin for the clastic dikes rather than the desiccation-crack theory, the fractures developing as a result of local deformational forces. These are:

(1) The absence of the polygonal pattern which is usually associated with desiccation cracks.

(2) The nature of the pattern of intersection displayed by the clastic dikes, suggesting shear fractures and tension cracks.

(3) The presence of slickensides on the walls of the dikes filling those fissures which appear to be shear fractures.

(4) The vertical extent of many of the dikes through two or more beds differing in lithology.

(5) The linearity of some of the dikes, many of which are quite straight and as long as 1000 ft.

(6) The presence, a short distance south of the area, of small displacement normal faults which trend approximately parallel with the strike of the main tension-crack dike and with the acute bisectrices of the shear fractures.

Conclusions which may be drawn from the field observations are that the fracture pattern, as expressed by the clastic dikes mapped, developed as a result of the same tensional forces which caused the formation of the normal faults in the vicinity of Cedar Pass, the tensional forces acting in a northeast-southwest direction. The resulting fractures were evidently open at the surface, which allowed an accumulation of debris to fill the fissures from above, thus forming the present pattern of numerous intersecting clastic dikes.

Acknowledgments--The author wishes to thank E. N. Goddard of the University of Michigan and Walter H. Bucher of Columbia University, both of whom read the manuscript and offered helpful suggestions concerning subject matter.

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(Manuscript received April 18, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 7, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

A METHOD OF ESTIMATING THE MELTING-POINT GRADIENT IN THE EARTH'S MANTLE

Robert J. Uffen

Abstract--Using the Einstein-Debye theory of solids and Lindemann's theory of fusion, the ratio of the melting point at various depths in the mantle to that at depth 100 km is computed from seismic data. It is found that the melting-point gradient decreases with depth. Using data for solid olivine, a value of 5300°K is found for the melting point at the core boundary, which gives an upper limit for the actual temperature there.

Introduction--The problem of the temperature distribution within the Earth is of major importance in the study of the Earth's internal constitution and has been investigated by many geologists and physicists [for summaries, see ELSASSER, 1950; SLICHTER, 1941]. Since seismological evidence shows that the Earth's mantle is in the solid state, an upper limit to the temperature at any depth will be given by the melting point at that depth. Elsasser has pointed out: ". . . there seems to exist only one effort at estimating the temperature that will stand close criticism, and even if it should not be correct it represents at least a model with closely specified conditions. This is the hypothesis of JEFFREYS [1929] that the temperature in the mantle is everywhere about that of the point of solidification, thus determined by the hydrostatic pressure alone. This is based on the fact that heat conduction in the solid mantle is an extremely slow and inefficient process, even over times as long as the age of the Earth. The dependence of the melting point upon depth in the Earth is described by the Clausius-Clapeyron equation." (This hypothesis actually dates back to the work of ADAMS [1924, pp. 459-472].)

We shall give a method of estimating the change of melting point with depth using seismic data. The basis of the method is the computation of the characteristic frequency of vibration of the crystal lattice as used in theories of solids due to Einstein and Debye from the measured velocities of longitudinal and transverse waves. The melting point is then computed using Lindemann's theory of fusion which relates the characteristic frequency to the melting point.

The characteristic frequencies of the material of the Earth's mantle--In the development of his theory of the specific heats of solids, DEBYE [1912, p. 789] treated the solid as a continuous elastic medium and took into account the discontinuous (molecular) structure of the solid by assuming that the allowable frequencies of vibration stop abruptly at an upper limit ν_{max} such that the total number of independent vibrations equals the 3N vibrational degrees of freedom of the N molecules. The expression for the Debye characteristic frequency is

$$\nu_{max} = (9N/4\pi V)^{1/3} \, (1/v_{\ell}^{3} + 2/v_{t}^{3}) \quad \ldots\ldots\ldots\ldots \quad (1)$$

where V is the volume occupied by the N molecules, and v_{ℓ} and v_t are the velocities of longitudinal and transverse waves, respectively.

For the material of the Earth's mantle we shall consider the molecules to be arranged in a simple cubic lattice. Equation (1) may be written in the form

$$\nu_{max} = (9\rho N_0/4\pi M)^{1/3} \, (1/v_{\ell}^{3} + 2/v_{t}^{3})^{-1/3} \quad \ldots\ldots\ldots\ldots \quad (2)$$

where ρ is the density, N_0 is Avogadro's number, and M is the molecular weight of the solid.

In the theory of specific heats it is customary to introduce the Debye characteristic temperature θ_D defined by

$$\theta_D = h\,\nu_{max}/k \quad \ldots\ldots\ldots\ldots \quad (3)$$

where h is Planck's constant and k is Boltzman's constant.

EINSTEIN [1911, p. 170; see RICHTMYER, 1934, pp. 275-277] derived an expression for a characteristic frequency ν on the assumption that the compressibility χ is directly proportional to the mean distance between molecules and inversely proportional to the force per unit displacement acting on the molecules. For molecular crystals his relation is

$$\nu = C_1 V_M^{1/6} (\chi M)^{-1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where V_M is the volume per mole, M is the molecular weight, and C_1 is a proportionality constant.

The relationship between melting point and characteristic frequency--When a crystal is heated to a sufficiently high temperature, the lattice breaks down and the crystal melts. LINDEMANN [1910, p. 609; see RICHTMYER, 1934, pp. 277-279, or ROBERTS, 1943, pp. 457-459] assumed that this occurs when the amplitude of oscillation becomes so great that "direct collisions" take place between neighboring molecules. He also assumed that the molecules of the solid are arranged in a simple cubic lattice and that the melting point γ is large compared with Θ_D so that the energy of an oscillator is given by its classical value, $3k\,\gamma$. This is the case for the mantle where the temperatures are of the order of 1000° K and the Θ_D's are of the order of 500°K. To show this we have computed ν_{max} and Θ_D for various depths in the mantle using (2) and BULLEN's [1947, chap. 13] data for ρ, v_ℓ, v_t and the value M = 140. This is the molecular weight of olivine, Mg_2SiO_4 (see Table 1).

Table 1--Computation of values of ν_{max} and Θ_D from Bullen's data for various depths, M = 140

Depth	Density	Bulk modulus K = $1/\chi$	v_ℓ	v_t	ν_{max}	Θ_D
km	gm/sec	dyn/cm^2	km/sec	km/sec	sec^{-1}	°K
100	3.38	1.24×10^{12}	7.95	4.45	7.50×10^{12}	360
200	3.47	1.38×10^{12}	8.26	4.60	7.82×10^{12}	375
300	3.55	1.54×10^{12}	8.58	4.76	8.16×10^{12}	392
413	3.64	1.73×10^{12}	8.97	4.96	8.58×10^{12}	412
500	3.88	2.15×10^{12}	9.75	5.30	9.37×10^{12}	440
600	4.11	2.57×10^{12}	10.25	5.66	10.19×10^{12}	458
800	4.46	3.19×10^{12}	11.00	6.13	11.33×10^{12}	510
1000	4.65	3.59×10^{12}	11.42	6.36	11.93×10^{12}	537
1400	4.88	4.20×10^{12}	11.99	6.62	12.62×10^{12}	568
1800	5.10	4.87×10^{12}	12.53	6.83	13.23×10^{12}	595
2200	5.31	5.57×10^{12}	13.03	7.02	13.79×10^{12}	620
2600	5.51	6.23×10^{12}	13.50	7.21	14.35×10^{12}	645
2898	5.66	6.51×10^{12}	13.64	7.30	14.66×10^{12}	659

For molecular crystals, Lindemann's expression is

$$\nu = C_2 M^{-1/2} V_M^{-1/3} \gamma^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where C_2 is a function of the lattice spacing and the diameter of the "effective sphere of action" of a molecule. Lindemann assumed that C_2 had the same value for a wide range of substances. Then the ratio of the characteristic frequencies of two solids is given by

$$\nu_1/\nu_2 = (M_2/M_1)^{1/2} (V_2/V_1)^{1/3} (\gamma_1/\gamma_2)^{1/2} \quad \ldots\ldots\ldots\ldots (6)$$

Using the known value of ν for platinum, obtained from specific heat measurements, he found good agreement between the frequencies determined from specific heat measurements and those obtained from melting-point data. Similar agreement is obtained for the Debye characteristic frequency ν_{max}, and this is the justification for identifying the Debye ν_{max} with the Lindemann ν.

Eliminating ν from (4) and (5), we obtain a simple expression for the ratio of the melting point at depth d in the mantle to the melting point at depth 100 km

$$\gamma_d/\gamma_{100} = (\rho\chi)_{100}/(\rho\chi)_d \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

which does not depend upon a knowledge of the chemical nature of the material of the mantle, provided it remains uniform throughout.

Table 2--Values of γ_d/γ_{100} for various depths

Depth	γ_d/γ_{100} Eq. (7)	γ_d/γ_{100} Eq. (8)	Mean
km			
100	1.00	1.00	1.00
200	1.08	1.07	1.08
300	1.18	1.14	1.16
413	1.30	1.24	1.27
500	1.51	1.42	1.47
600	1.70	1.62	1.66
800	1.95	1.90	1.92
1000	2.10	2.04	2.07
1400	2.34	2.22	2.28
1800	2.60	2.36	2.48
2200	2.86	2.50	2.68
2600	3.08	2.65	2.86
2898	3.13	2.71	2.92

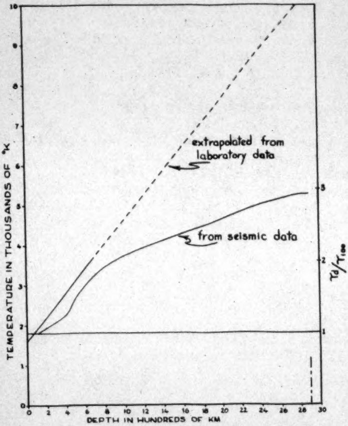

Fig. 1--The melting-point gradient in the Earth's mantle

Similarly, identifying ν_{max} with ν, we obtain from (2) and (5)

$$\gamma_d/\gamma_{100} = \left[\frac{(v_t^3 + 2\,v_\ell^3)_{100}}{(v_t^3 + 2\,v_\ell^3)_d}\right]^{2/3} \left[\frac{(v_\ell\ v_t)_d}{(v_\ell\ v_t)_{100}}\right]^2 \quad (8)$$

Values of γ_d/γ_{100} for various depths are given in Table 2 and Figure 1. We see that the melting-point gradient is not constant but decreases with depth. A decrease in the melting-point gradient with depth has been postulated by a number of authors [see ELSASSER, 1950; GUTENBERG, 1939, chap. 7].

It is widely accepted that the mantle is composed of ultra-basic rock, largely olivine or high-pressure modifications of olivine [BULLEN, 1947, p. 226; see also ADAMS and WILLIAMSON, 1923, and ADAMS and GIBSON, 1929]. Accepting this identification, we set γ_{100} = 1800°K as the value for the melting point appropriate to the pressure at a depth of 100 km [GUTENBERG, 1939, p. 160] and get γ_{2900} = 5300°K for the melting point at 2900 km, the boundary of the Earth's core. However, it should be noted that the melting point of olivines varies greatly with iron content [see BOWEN and SCHAIRER, 1935, pp. 151-217] and the figure used above corresponds to what is believed to be the most probable composition for the mantle. Figure 1 compares the temperatures in degrees absolute, obtained by our method, with JEFFREYS' [1929] values which were obtained by using the Clausius-Clapeyron equation

$$d\gamma/dr = -(g\gamma/L)\ (\Delta\rho/\rho)\ .\ .\ .\ .\ .\ .\ .\ .\ (9)$$

where L is the latent heat of fusion, g is the acceleration due to gravity, and $\Delta\rho$ is the density increase at fusion. Jeffreys considered that a linear extrapolation from the laboratory data for olivine to the pressure at the lower boundary of the mantle should give an upper limit for the temperature there, since one would expect $\Delta\rho$ to decrease more rapidly than L with increased pressure. Our values are in good agreement with DALY's [1943] estimate. The temperature in the upper portions of the mantle and the crust will be lower than that given by the melting point due to cooling. As has been shown by ADAMS [1924], SLICHTER [1941], and others, the cooling produced by conduction to the surface is significant only to a depth of 600 km at the present age of the Earth.

The most serious assumptions of our method are that the molecular weight remains constant throughout the mantle, and that the constant C_2 in Lindemann's expression (5) is the same for all pressures. However, it is believed that the present method, based on seismic data, offers a quantitative method of evaluating the melting-point gradient and is preferable to the extrapolation of laboratory data at relatively low pressures.

Acknowledgments--This work is part of a study being carried out with the aid of a scholarship from the Research Council of Ontario. The writer wishes to acknowledge the encouragement and criticisms of A. D. Misener, E. Brannen, and R. W. Nicholls of the Department of Physics, University of Western Ontario.

Note added October 30, 1952--It has been shown in a recent paper by BIRCH [1952] and independently by the writer [UFFEN, 1952] that it is preferable to treat the material of the mantle as a monatomic solid with a "mean atomic weight" equal to M/n, where M is the molecular weight and n is the number of atoms per molecule. For Mg_2SiO_4, n = 7 and the value of M/n is 20. This requires that the values for ν_{max} and θ_D in Table 1 be multiplied by $n^{1/3} = 1.9$. Further, BIRCH [1952, Table 11, p. 256] has shown that the "mean atomic weight" is close to 20 for most of the rock-forming silicates. This revision does not affect the values we have obtained for the melting-point ratios.

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(Manuscript received January 14, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C , May 5, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

THE APPLICATION OF RECENT COUNTING TECHNIQUES TO GEOPHYSICAL RESEARCH

PART 1--FIELD MEASUREMENT OF THE RADIOACTIVITY OF ROCKS

Howard A. Slack

Abstract--A gamma-ray counter suitable for measuring the radioactivity of ordinary rocks “in situ” is described. This instrument represents a considerable improvement both in sensitivity and portability on an earlier instrument [SLACK and WHITHAM, 1951].

The instrument was calibrated against prepared standard radioactive concrete blocks and was used to investigate the radioactivity of the Kirkland Lake area in northern Ontario. The results on and around the Round Lake batholith in this area are compared with previous results over the same batholith. In addition, graphs are presented showing the distribution of radioactivity across the contact zones of this batholith.

Introduction--It has been shown [KEEVIL and Others, 1943; SLACK and WHITHAM, 1951] that in order to obtain the radioactivity of rock formations with a minimum expenditure of time and with minimum sampling error, a field method in which a count is obtained from a representative fraction of the rock is superior to a laboratory method in which the count is obtained from a relatively small portion of a hand-collected specimen. A field gamma counter suitable for measuring weakly radioactive sources and the results obtained with this instrument over two batholiths in northern Ontario have been described [SLACK and WHITHAM, 1951]. Further development in this field has yielded an improved instrument of greater sensitivity and portability. This improved instrument and some additional results obtained with it over the previously studied Round Lake batholith in the Kirkland Lake area of northern Ontario are described in this paper.

Description of instrument--The improved instrument was, in essence, a three-channel, anticoincidence, gamma-ray counter. Figure 1 shows a block diagram of the apparatus used. The number on each one of the Geiger tubes, comprising the detector unit of the instrument, designates the channel to which that particular Geiger tube was connected. The characteristics and the conditions under which these Geiger tubes were operated are similar to those in the pilot instrument [SLACK and WHITHAM, 1951]. In this improved instrument, pulses obtained from the Geiger tubes were fed directly through the 100-ft shielded cable to the inputs of three monostable flip-flops. These flip-flops, one corresponding to each channel, produced uniform sized and shaped pulses, which were in turn fed directly to the common mixer stage. This mixer stage was composed entirely of a specially designed binary adder tube developed by the Computation Centre of the University of Toronto, and under operating conditions simultaneous pulses in any two or in all three of the channels, produced presumably by high-energy cosmic rays, did not produce an output pulse from the mixer stage. On the other hand, random pulses, produced by gamma radiation from the rock, in either Channels 2 or 3 produced an output pulse which was fed to a scaling unit. The tubes comprising Channel 1 were shield tubes, and random pulses produced by them were not counted. Thus, in effect, gamma-ray quanta from rocks were recorded and more energetic cosmic-ray particles capable of firing at least one tube in more than one channel were not recorded. The action can perhaps be better understood by reference to Table 1 where the symbol + denotes a pulse formed and the symbol 0 no pulse formed. The more efficient use of the Geiger tubes as actual gamma-ray recording tubes as well as a slight increase in the efficiency of cosmic-ray elimination in this instrument is responsible for the increase in sensitivity over that obtained with the pilot instrument [SLACK and WHITHAM, 1951]. The radio-frequency, high-voltage, power-supply unit and the regulated plate-supply unit were operated by a gasoline-

Table 1--Example of the action of the binary adder tube

Channel 1	Channel 2	Channel 3	Output
0	0	0	0
0	+	0	+
0	0	+	+
+	0	0	0
0	+	+	0
+	0	+	0
+	+	0	0
+	+	+	0

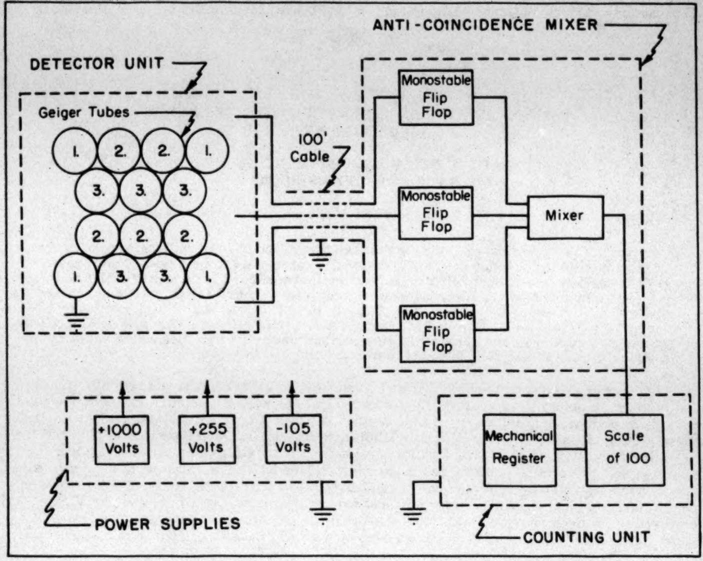
ANTI-COINCIDENCE MIXER
DETECTOR UNIT
Geiger Tubes
1.
2.
2.
1.
3.
3.
3.
2.
2.
2.
1.
3.
3.
1.
100' Cable
Monostable Flip Flop
Monostable Flip Flop
Monostable Flip Flop
Mixer
+1000 Volts
+255 Volts
-105 Volts
POWER SUPPLIES
Mechanical Register
Scale of 100
COUNTING UNIT

Fig. 1--Block diagram of the three-channel, anti-coincidence, gamma-ray counter for determining the radioactivity of weakly active rock bodies in the field

driven motor generator, which was mounted on a pack board, thus rendering the entire instrument conveniently portable.

Calibration of instrument--In a manner similar to that employed in the case of the pilot instrument, the background counting rate was observed to be 250 counts per minute, and the efficiency of cosmic-ray elimination determined to be 76 pct. These figures are to be compared to a background counting rate of 210 counts per minute, and an efficiency of cosmic-ray elimination of 70 pct for the pilot instrument [SLACK and WHITHAM, 1951].

For a rock containing U micrograms of uranium per gram of rock in equilibrium with its radioactive series, Th micrograms of thorium per gram of rock in equilibrium with its radioactive series, and K per cent potassium, calibration experiments showed that after correcting for background the signal counting rate per second was

$$C = 0.94\ U + 0.47\ Th + 1.9\ K$$

The estimated uncertainties in these constants were 3.7, 4.7, and 4.5 pct, respectively. These calibration experiments were carried out on standard radioactive rock bodies obtained by mixing known weights of uranium concentrate, thorium concentrate in radioactive equilibrium, and potassium carbonate into three concrete blocks approximately 4 × 5 × 1 ft. The size of these blocks was decided upon with due regard both to the counting losses to be encountered when the body is not semi-infinite, but finite in extent, and to the practical limitations of handling large amounts of concrete. A block of plain concrete was also included so that the background counting rate due to the concrete matrix alone could be obtained.

A comparison of the calibration constants obtained with the two instruments in the case of uranium and thorium shows that the sensitivity of the improved instrument is approximately 1.3 times that of the pilot instrument. The discrepancy between the calculated calibration constant for potassium [SLACK and WHITHAM, 1951] and the above experimentally determined constant is undoubtedly due to erroneous assumptions made in determining the calculated constant.

Since the alpha activity in α/mg hr is given by $a = 0.368\ U + 0.0886\ Th$ [KEEVIL, 1943] and assuming $Th/U = r_1 = 3.2$ [KEEVIL, 1944] and $K/U = r_2 = 1.4$ [KEEVIL, 1943], the alpha activity for the rocks can be obtained directly from the observed counting rate of this instrument by using the relationship

$$a = (0.13 \pm 4.3 \text{ pct})\ C$$

Whilst a/C is relatively uncritical to r_1, it is critically dependent on r_2. For any value of r_2 between 1 and 2 a change in r_1 from 3.0 to 4.0 changes a/C by only four per cent, while for values of r_1 between 3 and 4 a change in r_2 from 1.0 to 2.0 changes a/C by as much as 35 pct. If the potassium content of the rock is known, it will then be more accurate to use

$$a = 0.27\ (C - 1.9\ K)$$

in determining the alpha activity from the observed counting rate.

As shown previously [SLACK and WHITHAM, 1951], the rate of heat production in cal/gm per million years is given by

$$H = 2.6a$$

This constant was calculated from the measured energies of the alpha, beta, and gamma rays emitted together with the experimentally determined decay constants and represents the most accurate estimate obtainable from the literature at this time.

Discussion of results for the Round Lake batholith--To supplement the previous radioactive determinations [SLACK and WHITHAM, 1951] on and around the exposed eastern portion of the Round Lake batholith, situated approximately 16 miles south of Kirkland Lake, Ontario (Map 1934a of the Abitibi-Timiskaming area by the Ontario Department of Mines), an additional 175 readings were taken over the same area with the improved gamma-ray counter. The equivalent alpha activities were computed from the observed readings in this case using available figures [LAWTON, 1950] for the average potassium content of the rocks of this batholith.

The new results did not in any way alter the previous distribution of radioactivity across this batholith [SLACK and WHITHAM, 1951; SLACK, 1949]. The Round Lake batholith is characterized by an interior of low radioactivity ($\overline{0.74}^{34} \pm 0.12$ α/mg hr, the average radioactivity of 34 individual determinations), a marginal rim one to two miles wide of higher radioactivity ($\overline{1.2}^{24} \pm 0.2$ α/mg hr), and a roughly circular area to the north of still higher activity ($\overline{2.8}^{28} \pm 0.5$ α/mg hr). These figures are to be compared with the activities of $\overline{0.36}^{40} \pm 0.08$, $\overline{0.60}^{11} \pm 0.13$, and $\overline{1.6}^{17} \pm 0.3$, respectively, obtained with the pilot instrument [SLACK and WHITHAM, 1951]. Although the new figures for the radioactivity are consistently higher than the old figures, it is felt that an increase in activity is justified on the basis of the more accurate methods now employed, namely, determining the potassium constant experimentally under simulated field conditions and using the measured average potassium content of the rocks in computing the equivalent alpha activities from the observed readings. These new activities may, however, be slightly high due to the small counting losses encountered in calibrating the instrument on concrete blocks of finite size. For future work, it is recommended that an attempt be made to carry out calibrations on actual rock outcrops, the activities of which have been determined by a laboratory method similar to the one proposed in Part 2 of this paper.

This differentiation of the radioactive material can be accounted for by assuming the radioactive material to rise to the top of a crystallizing magma in a manner similar to that which JEFFREYS [1929, pp. 159-160] has suggested to take place in the Earth's crust as a whole. Subsequent erosion of the top surface of the batholith would then be expected to produce the observed ring effect. Although it is believed that the radioactive elements or compounds are associated with the accessory minerals apatite, zircon, and allanite [BILLINGS and KEEVIL, 1946], these minerals do not average over one per cent of the rocks and do not show any tendency to be concentrated in a particular part of the batholith [LAWTON, 1950]. The distribution of this radioactivity can only be explained then either on the assumption that the radioactive materials occur in some other form or that the small increases in accessory minerals necessary to account for the observed radioactivity distribution were undetectable in the analysis made. A more detailed analysis will be required before this point can be established definitely.

The area of higher radioactivity to the north (activity 2.8 α/mg hr) is now believed to be a separate geological structure. Recent geological work in this area [LAWTON, 1950] shows that both lithologically and petrographically this area may be distinguished from the Round Lake batholith proper. The rocks comprising this area are relatively rich in microcline and low in quartz, and it has been classified on this basis as a syenite stock. Virtually none of the post-Algoman intrusives, which are found cutting the batholith, are found within this syenite stock, and for this reason it is proposed that it is of later age than the Algoman batholith.

If the average activity of the Algoman granite making up the batholith is assumed to be equal to that of the low central region or 0.74 α/mg hr, it is of interest to note that the average radioactivity for Algoman granite in Ontario is higher and equal to $\overline{0.90}^{126}$ α/mg hr [WALTER, 1950]. Thus, average values obtained from surface measurements may, in many cases, be too high to represent the radioactivity of the Earth's crust as a whole.

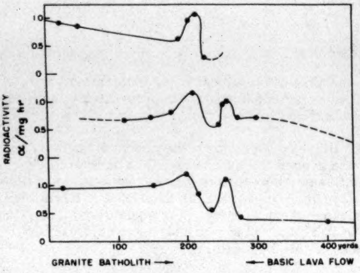

Fig. 2--The distribution of radioactivity across the northern contact of the Round Lake batholith as determined by three individual traverses

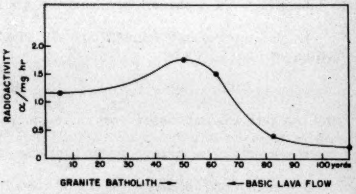

Fig. 3--The distribution of radioactivity across the southern contact of the Round Lake batholith as determined by a single traverse

An examination of Figure 2, which shows the distribution of radioactivity across the northern contact of the Round Lake batholith as determined from readings taken on the bedrock surface shows that the contact of the batholith with the surrounding basic lava flow is characterized by a minimum activity interspaced between two regions of maximum activity. The exact relation of this minimum to the actual contact could not be obtained due to the large amount of overburden present in this area, but it is certain that in all cases the contact occurred between the two maximum activities. This pattern is to be compared with that obtained by a traverse across the southern contact, shown in Figure 3, where a maximum region of activity was observed to occur, but where no evidence of a minimum was obtained. It is suggested that in both cases the pattern is due to the presence of highly radioactive pegmatitic or hydrothermal solutions, which circulated within the contact zone and diffused outward into the crystallized intrusive and into the country rock. The distance which these radioactive materials traveled from regions of high concentration to regions of lower concentration is in accord with the experimental data on the transfer of material through water-saturated rocks by solute diffusion [GARRELS and Others, 1949]. The minimum observed at the northern contact (Fig. 2) could then be explained as due to the action of non-radioactive solutions leaching the rocks in the immediate neighborhood of the highly sheared contact zone at a later time. This leaching out of the radioactive materials in rock by mineralizing solutions has been observed by others [WALTER, 1950; DAMON, 1950] and has been suggested as a possible method of locating regions of mineralization. The southern contact of the batholith is relatively unsheared and would therefore be expected to resist the movement of these leaching solutions.

The distribution of radioactivity across the northern contact of the syenite stock as shown in Figure 4 is much like that of the northern contact of the Round Lake batholith except that from the values obtained for the three traverses made the minimum does not appear to have developed to the same extent. In the light of the foregoing discussion, this would be expected since the syenite stock is relatively unsheared at its contact zone and, as in the case of the southern contact of the batholith, would resist the flow of any leaching solutions.

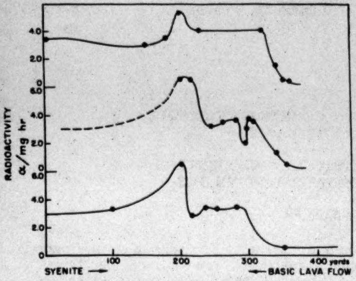

Fig. 4--The distribution of radioactivity across the northern contact of the syenite stock as determined by three individual traverses

It is considered desirable that further measurements of radioactivity across contacts be made in conjunction with petrographic studies. This would confirm whether the above results are typical and whether the explanations given can be supported by additional evidence.

Conclusions--It has been shown that quick reliable measurements can be made to determine the radioactive content of rocks in the field. The results of one such investigation indicate that the distribution of radioactivity in rocks is systematic and may be a useful tool in geological studies.

Previously, it had been shown [SLACK, 1949] how such a knowledge of the radioactivities of different rock types can be used to estimate the percentage of the total heat emanating from the Earth's surface, which is due to radioactive material in the Earth's crust. It was also suggested [SLACK, 1949] that the distribution of radioactivity across the surface of an intruded body of rock was dependent upon the attitude of the intruded mass and upon the depth of erosion. It is now shown that zones of alteration are characterized by anomalous radioactivity, but a petrographic study of such zones of alteration must still be made in conjunction with a radioactivity survey in order to determine the relation between the two.

Acknowledgments--I am indebted to J. Tuzo Wilson for originally suggesting this investigation and for his continued interest and support, to E. C. Bullard for his advice and assistance, and to the Research Council of Ontario for scholarships and funds to support this research. The assistance of K. Whitham and J. Katz is also gratefully acknowledged.

This work was conducted at the University of Toronto in Toronto, Canada, and is published in partial fulfillment of the requirements for a Ph.D. Degree, Department of Physics, University of Toronto.

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Transactions, American Geophysical Union Volume 33, Number 6 December 1952

THE APPLICATION OF RECENT COUNTING TECHNIQUES TO GEOPHYSICAL RESEARCH

PART 2--LABORATORY SCINTILLATION COUNTERS APPLIED TO SOME GEOPHYSICAL PROBLEMS

Kenneth Whitham

Abstract--The development and operation of a laboratory scintillation counter capable of analyzing the separate uranium and thorium contents of radioactive minerals are described. A gamma-gamma analysis, that is, a method depending on two measurements giving the relative proportion of high- and low-energy gamma rays, was used. The advantages of this technique have been proved for the analysis of non-equilibrium samples, and some comparisons with the results given by conventional radiometric methods have been made for a number of samples. The counter finally developed is capable of analyzing samples with concentrations of only 0.05 pct U_3O_8 equivalent and distinguishing between the two radioactive families when $r = U_3O_8/ThO_2$ is as low as 0.01. The absolute sensitivity of the counter is 0.005 pct U_3O_8 equivalent (estimated gamma activity equivalent 3×10^{-9} curies). The applications of uranium and thorium determinations to a variety of geophysical problems, including the determination of geological ages are discussed. An analysis of the laboratory results proved the advantages of multiplier phototube detectors in field instruments designed to investigate the problems outlined in Part 1 of this paper.

Introduction--Analysis of uranium and thorium contents of naturally occurring radioactive bodies is of considerable geophysical interest in connection with several methods of determining geological ages and investigations of the radioactive heat production in the Earth, and also of commercial interest in connection with the contractual purchase of ores.

The earlier determinations of geological ages by the method of total lead analyses required knowledge of the uranium and thorium contents of the mineral analyzed. More recent determinations using the lead isotope ratio, Pb 207/Pb 206 [COLLINS and Others, 1951], must be corrected for the presence of ordinary lead. In this connection and because it is desirable to use more than one method, uranium and thorium analyses are still necessary in age determinations.

It would also be of interest to have techniques available by which radiometric studies could be made of the proportion of radioactive equilibrium in various types of minerals. For example, ages which have been determined using the ratios Pb 208/Th 232, Pb 206/U 238, are seriously dependent on the state of radioactive equilibrium of the mineral used, and are consequently often regarded as unreliable.

Until recently, uranium and thorium analyses have only been possible by chemical or radiometric methods using ionization chambers, or Geiger tubes. Chemical methods, if performed with great care, are considered more reliable than radiation-assay methods. They are, however, very long and can give information about the primary member only of the series. Radiometric methods have been developed which measure the $\beta\, U_3O_8$ equivalent, or the γU_3O_8 equivalent of radioactive minerals, or both [LAPOINTE, 1950]. Empirical relationships have been used to estimate thorium from these two equivalents, but the method fails if the beta and gamma activities are from uranium which is not in secular equilibrium. Lack of equilibrium can occur from loss of radon, from leaching, or by the weathering of the surface rock. Uranium and thorium can also be determined by the use of radon and thoron methods, but reliable results are difficult to obtain. A number of other methods of limited applicability have been proposed, but it was felt that a different physical approach was required, and this paper describes an investigation of the application of a scintillation counter to the determination of the two radioactive families.

Principle of method investigated--A scintillation counter can distinguish between the radiations from the uranium and thorium families by virtue of the property of energy proportionality of scintillations. The basis of the gamma-gamma analysis suggested by the author is an analysis of the pulse heights obtained using suitable crystals for detecting the gamma radiation from the two

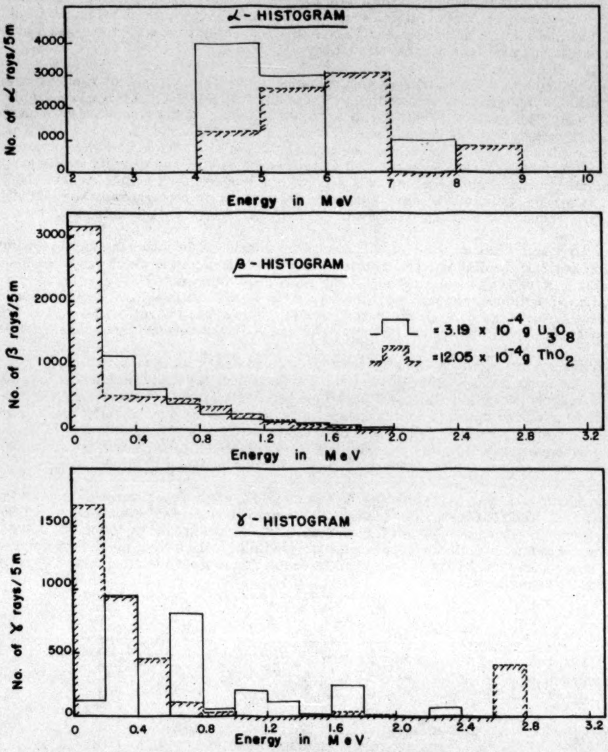


Fig. 1--α, β, γ activities of uranium and thorium families in secular equilibrium

series. It was first necessary to consider whether counting alpha, beta, or gamma radiation would give best resolution between the uranium and thorium series. To this end, a literature survey was made, and the results are shown in Figure 1 where the primary alpha, beta, and gamma spectra of the two families in secular equilibrium are shown in the form of histograms of number versus energy.

Nearly 90 pct of the total alpha emission from both families lies in the energy range four to seven mev per particle; the energy difference of the alpha rays from RaC^1, ThC^1, is only about one mev. The loss in energy of an alpha ray in passing through the emitting material makes it very unlikely that this difference could be usefully employed experimentally to analyze uranium and thorium in weak concentrations. In the case of beta emission, similar objections hold, and there is little difference in the beta-ray distribution histograms for the two series. Another objection is that to detect high-energy beta rays with good efficiency, large phosphor thicknesses must be used and hence primary gamma radiations would mask the effect sought. Appreciable self-absorption and the production of secondaries by gamma radiation would also complicate matters.

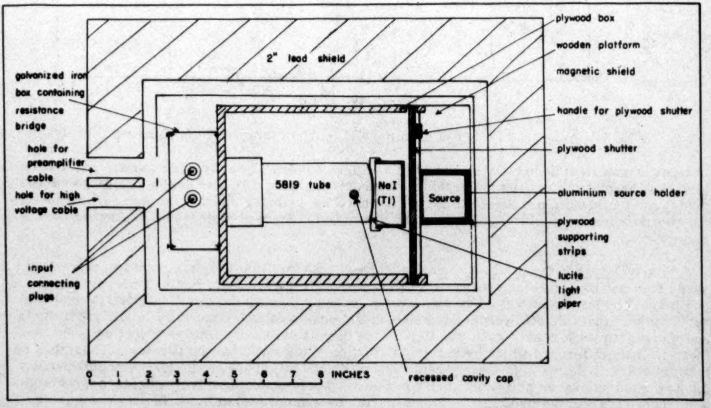
Consequently, it was decided to work with gamma radiation. The highest gamma-ray energy is the 2.62 mev line from ThC^{11}, which accounts for some ten per cent of the gamma radiation from the thorium family. In the uranium series, the nearest lines are 1.8 mev, and a much less frequent one at 2.2 mev; both of these are from RaC.

Other reasons for working with gamma radiation were: (1) This method removes some of the difficulties which are associated with the use of any beta method if the uranium family is not in equilibrium. This is because effectively all of the gamma radiation from the uranium series is emitted by members of the radium family, compared to about one-half of the usually measured beta activity. In the cases when the radium family is out of equilibrium with the earlier members of the uranium series, the method proposed estimates the radium and thorium families correctly. To be certain that the uranium series is in secular equilibrium, the uranium content given by a gamma-gamma analysis must be compared with a beta assay or preferably a chemical assay. (2) Errors introduced by self-absorption and geometrical effects are minimized.

Secondary properties of scintillation counters of considerable advantage in this type of investigation are: (1) the high efficiency for detecting gamma radiation which can be obtained, compared to the efficiency of a Geiger tube; (2) the short dead times easily obtained, thus avoiding the recalibrations for different geometries, corresponding to different levels of activity, necessary in conventional assays employing Geiger tubes; and (3) the apparatus is readily flexible in operating characteristics of sensitivity and selectivity, whereas a Geiger counter arrangement is not.

Experimental arrangement--The detector used consisted of an experimental type RCA 5819 multiplier phototube, and a sodium iodide (thallium activated) crystal. The largest crystal available was a roughly cut cylindrical piece of sodium iodide (two inches in diameter, one inch thick), and the detector was designed to accommodate this crystal mounted in its reflecting and moisture-proof housing. Figure 2 shows a plan view of a horizontal section through the multiplier phototube mounted horizontally in a light-tight wooden box. For increased sensitivity, "poor source-counter geometry" was used, and the source was as close as was convenient to the crystal.

Source holders were made of aluminum three mm thick, which is sufficient to stop all the primary beta rays. The source-holder volume was chosen arbitrarily to be approximately 60 ml; this corresponds to a mass of 100 gm for many minerals, when ground to 100 mesh, 100 gm being usually considered reasonable for representative sampling. All sides of the detector were covered with interlocking lead bricks two inches thick as protection against radiation from the walls of the room and soft cosmic rays.



2" lead shield
galvanized iron box containing resistance bridge
hole for preamplifier cable
hole for high voltage cable
input connecting plugs
5819 tube
NaI (Tl)
Source
plywood box
wooden platform
magnetic shield
handle for plywood shutter
plywood shutter
aluminium source holder
plywood supporting strips
lucite light piper
recessed cavity cap
0 1 2 3 4 5 6 7 8 INCHES

Fig. 2--Plan view of arrangement of source and detector

Experiments showed that in the region where the counter showed appreciable differentiation between uranium and thorium, that is, at comparatively steep bias curves, the reproducibility in slope of any bias curve to ±1 pct was only possible if a long-term stability of phototube high voltage to one part in 1000 was obtained. From a conventional electronically regulated high-voltage supply using dry batteries as the source of reference voltage, an output was obtained which was measured to be stable to one part in 2000 for more than ten hours after a warm-up period of about one hour.

An amplifier of a modified JORDAN and BELL [1947} type was constructed and used with a low-gain preamplifier. The amplifier was linear to positive output signals of 100 volts.

A cathode follower output fed into an integral discriminating unit, which had a maximum discriminating level of 100 volts positive and was free from drift after a short warm-up period. The output of the discriminator was fed into a scale of a 1000 counting unit. The amplifying and counting circuits were found to be stable for periods of 24 hours.

Operating characteristics of counter--All radioactive assay methods are non-absolute methods and require standards. In the determination of uranium and thorium, two standards are required, one of equilibrium uranium and one of equilibrium thorium. The uranium standard used was a pitchblende concentrate; chemical analysis gave 11.3 ± 0.1 pct U_3O_8. The evidence for its equilibrium was largely indirect, but seems well enough justified. For example, a beta assay gave 11.32 ± 0.02 pct U_3O_8 against a standard known to one part in 800. The thorium standard used was a monazite sand, assaying chemically 0.165 ± 0.005 pct U_3O_8 and 6.4 ± 0.1 pct ThO_2.

In order to determine positions of operation where distinct differences are found in the pulse heights obtained from the uranium and thorium families, integral bias curves were obtained for each of the two standards, for a number of different multiplier phototube voltages from -800 to -1000 volts overall, and a constant amplifier gain of 800. The results are shown plotted without dead-time corrections in Figure 3, where the uranium standard curves are normalized for 1.51 gm U_3O_8 to fit the thorium standard curve (3.24 gm ThO_2, 0.0831 gm U_3O_8) at ten volts bias, -1000 volts overall. The curves are shown corrected for all sources of background.

For comparison, a bias curve taken at -900 volts for a four-degree beam of Co 60 gamma rays (mean energy = 1.33 mev) is shown normalized to the 1.51 gm U_3O_8 curve at ten volts bias. The complexity of the gamma-ray spectra of the radioactive families is immediately obvious by visual inspection. Quantitative comparisons are pointless since the method of interaction of gamma rays with the crystal depends on the energy of the gamma ray. It can be seen, however, that both families emit gamma rays with energies greater than 1.33 mev.

The bias curves for the radioactive minerals showed no resolution, but did show clearly the difference in the gamma-ray spectra of the two series. At high discriminator bias, a substantial difference arises because of the 2.62-mev gamma ray in the thorium family, and this difference increases as the overall gain is reduced. In addition, examination of the curves in Figure 3 shows that the thorium family emits rather less energetic gamma rays in the low-energy region than does the uranium family.

Figure 4 shows the performance of the counter as a unit for discriminating between the two series. For accurate reproducibility of results, ten volts bias is the limiting low bias value used. Let (S_{10}) U_3O_8 be the counting rate at ten volts bias from equilibrium U_3O_8, and (S_{10}) ThO_2 be the counting rate at ten volts bias from equilibrium ThO_2, etc.

The ordinate, which represents the inverse of the selectivity of the counter, can be varied by a change in overall voltage. An ordinate value of 0.5 corresponds to the experimentally determined selectivity of the beta-gamma assay method mentioned earlier; values less than 0.5 are correspondingly more selective, and a value of 1.0 represents a completely non-selective counter. The background at 100 volts bias, shown in milligram U_3O_8 equivalent, is also plotted in Figure 4 with the same abscissa. This represents the effective limitation on the sensitivity of a selective counter. It is convenient to define in the usual way the limiting concentration that the counter can handle as that for which the signal counting rate equals the background counting rate. Then for the same selectivity as the beta-gamma method, the ordinate is 0.5 for the selectivity curve, and the corresponding limiting concentration is 55 mg U_3O_8 equivalent or, for a 100-gm source, 0.055 pct U_3O_8 equivalent. This compares well with other methods. For much stronger sources it is convenient to increase the selectivity.

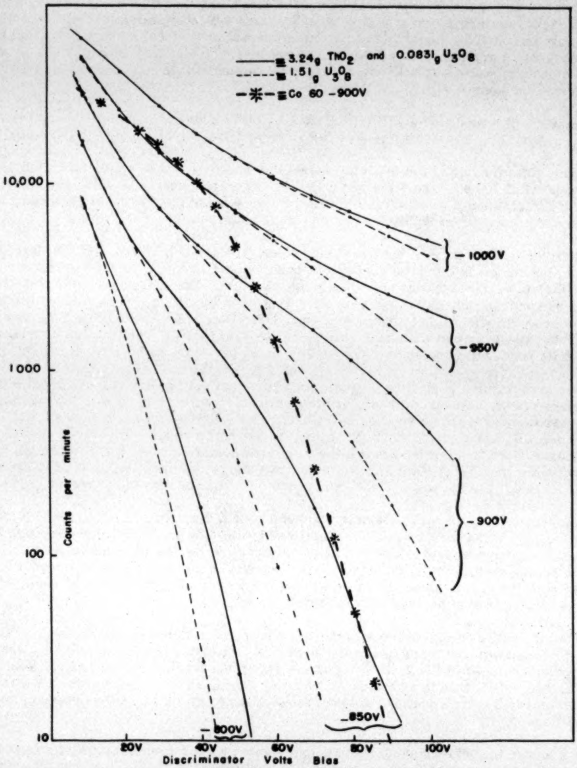


Fig. 3--Integral bias curves

In working with the counter, the following operating conditions were used.

0.1 pct < U_3O_8 equivalent < 1 pct, -930 volts, selectivity factor = 2.7
1 pct < U_3O_8 equivalent, -900 volts, selectivity factor = 6.2

The order of magnitude of the U_3O_8 equivalent was quickly found for any sample using a gamma Geiger counter.

The background at ten volts bias may conveniently be taken as a measure of the absolute sensitivity of the counter. This reached a maximum at -950 volts of 0.005 pct U_3O_8; more lead shielding would reduce this figure. Lower concentrations were measurable with careful counting, and the counter responded to an average granitic rock. It is not claimed, however, that the arrangement used is the best for measuring the U_3O_8 equivalent of ordinary rocks.

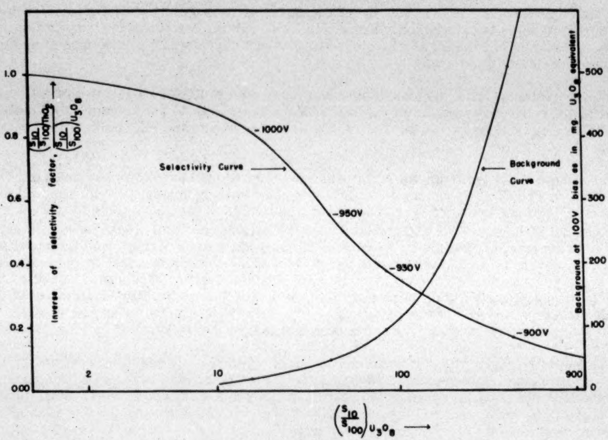


Fig. 4--Selectivity of counter

Using the relation (total gamma emission per minute) × (geometrical factor) × (average efficiency of detection) = extrapolated counts per minute, the source-counter geometry was estimated at ten per cent, which agrees with theory.

Estimation of uranium and thorium contents--By measuring the sample counting rate at ten and 100 volts bias, the contents of U_3O_8 and ThO_2 in the sample can easily be calculated by simple arithmetic. The calibration constants for the apparatus were determined by counting with the two standards.

The probable error of the estimated radioactive contents can be estimated in an orthodox manner using the theory of difference counting. Experimentally a difference counting rate between the background and source counting rate, and the background counting rate with no source present is always measured.

The elimination of systematic errors--A number of systematic errors are present, and these were eliminated or minimized as far as was conveniently possible. The dead time of the counter was measured experimentally using the two-source method to be 5.1 ± 0.2 microseconds, substantially that of the scaler as was expected. This should be contrasted with the dead time of a Geiger counter which, unless special electronic techniques are used, is 100 microseconds or more. In the work described, dead-time corrections, if appreciable, were always applied, but for most samples these were negligible.

Errors from drifting phototube voltage and drifting discriminator bias were shown by experiment to be negligibly small. Errors in standards could not be minimized; those used were the best available to the author.

Two series of experiments were performed to demonstrate that appreciable loss of radon did not occur on grinding the samples to 100 mesh. If the same geometry is accurately maintained for each sample, variations in the chemical composition of the minerals necessitate the use of somewhat different masses. This might introduce errors if self-absorption were appreciable. This effect was investigated experimentally and theoretically, and very small corrections were introduced when necessary.

Background limitations--Amplifier noise, multiplier phototube noise, cosmic radiation, radiation from the walls, ceiling, and floors of the room, and radiation from sources of radioactive contamination can all contribute to the background counting rate in any bias position.

In the experiments of the author, accurate measurements were not extended below ten volts bias; amplifier noise is considerably below this level. Multiplier phototube noise was almost entirely thermionic noise and, as expected, using an end cathode tube this source of noise was found experimentally to be negligible.

Cosmic radiation had a serious effect. Using the data of ROSSI [1948], it was estimated that the number of cosmic-ray counts in the unshielded counter was 14 per minute. Two inches of lead shielding removes some 40 pct only of the soft component, and affects the total cosmic-ray count by only 15 pct.

The gamma radiation from the structural materials of the laboratory was analyzed using a combination of scintillation and calibrated Geiger counter techniques. It was estimated that a content of $\sim 10^{-5}$ gm ThO_2/gm in the structural materials of the room could produce the background gamma radiation. The extrapolated counting rate at zero volts bias, measured using a scintillation counter, is reduced by a factor of 20 using the two-inch thick lead house. The lead shielding, however, improved the sensitivity of a selective counter by about 50 pct only, since this is largely determined by the effect of the shielding on the total background at 100 volts bias. Other experimental work in this laboratory has confirmed that radioactive contents of this magnitude in structural materials are reasonable. The background measured experimentally was very constant. Consequently, it was necessary to measure the background only occasionally.

The resolution of the counter--The resolution of a gamma-ray counter is defined by its ability to distinguish between gamma rays of different energies, and in the case of scintillation counters this depends on the types of interaction of the gamma rays with the crystal, the efficiency of the light collection, and the sensitivity of the multiplier phototube. The resolution of the counter described was investigated, using a Co 60 gamma-ray source, the gamma radiation from potassium sulphate, and a number of sodium iodide and anthracene crystals of different shapes and sizes.

The results of a systematic series of experiments proved that the tube used was some eight times less sensitive than the best reported values [PRINGLE and STANDIL, 1950; HOFSTADTER and McINTYRE, 1950]. Variations in sensitivity of this magnitude were found in other work with 1P21, 1P28 multiplier phototubes. No other 5819 tube was available to make a direct comparison.

It was not possible to detect the peak in the differential bias curve corresponding to the loss of 2.62 mev of energy in the large sodium iodide crystal. If a more sensitive phototube was used, it might prove possible to assay ThO_2 by a direct determination of the peak corresponding to the liberation of 2.62 mev in the crystal. The total energy must be used since pair annihilation pulses, Compton breaks, etc., would be confused with pulses from the interactions of gamma rays from the uranium family.

Two crystal methods have recently been proposed to increase the resolution of scintillation counters, but these require strong collimated sources and are not readily adaptable to the direct assay of thorium in the form of a weak naturally occurring thick source, using the high-energy gamma ray.

Results--Table 1 shows a number of results obtained with a wide variety of radioactive minerals. The U_3O_8 equivalents range from 0.1 to 50 pct, and $r = (U_3O_8/ThO_2)$ from ∞ to 0.01.

The counting time involved in any one assay depends on the accuracy required and the value of r for that sample. The results, accurate enough for many purposes, took about one-half to two hours counting time per determination depending on the sample concentration. The counter has been previously calibrated.

Discussion of results--Examination of Columns (3) and (4) shows that the internal consistency of the gamma-gamma results was quite satisfactory. In the 1P21, 1P28 counter results, the sensitivity and counting rates were lower and probable errors consequently higher. These counters depended on the same principle and are not described in this paper since the 5819 tube is superior in this application. The satisfactory agreement between Columns (3) and (4) and the low probable errors in the 5819 counter results suggest that systematic errors were kept satisfactorily small. The method compares very favorably in internal consistency with all other radioactive assay methods.

The samples listed were assayed for a variety of reasons. The assays included the precise determination of radioactive concentrations in secondary standards used in other geophysical

Table 1--Results

Sample (1)	Nature of sample (2)	Gamma-gamma analysis, 5819 tube and NaI (Tl) (3)		Gamma-gamma analysis, 1P21, 1P28 multiplier phototubes (4)		U_3O_8/ThO_2 from gamma-gamma analysis, 5819 tube and NaI (Tl) (5)
		U_3O_8	ThO_2	U_3O_8	ThO_2	
		pct	pct	pct	pct	
1	Monazite sand	0.19 ±0.02	5.88 ±0.06	0.19 ±0.07	5.8 ±0.2	0.03
2	Australian samarskite	10.8 ±0.1	1.2 ±0.2	10.7 ±0.3	1.5 ±0.7	9.0
3	P.S.[a] - allanite	0.37 ±0.01	1.92 ±0.02	....	....	0.19
4	Euxenite-bearing	7.45 ±0.03	0.92 ±0.06	7.0 ±0.2	0.9 ±0.4	8.1
5	Pitchblende	48.2 ±0.1	None	48.0 ±0.2	None	∞
6	P.S.[a]	0.13 ±0.02	0.60 ±0.04	0.11 ±0.03	0.68 ±0.06	0.22
7	Not known	0.007 ±0.004	0.66 ±0.01	....	....	0.01
8	P.S.[a]	0.01 ±0.004	0.73 ±0.01	....	....	0.01
9	Not known	0.008 ±0.002	0.38 ±0.01	....	....	0.02
10	Hatchettolite-bearing	....	....	5.2 ±0.1	3.6 ±0.3	...
11	Athabaskan pitchblende	13.9 ±0.3	None	14.1 ±0.1	None	∞
12	Belgian Congo pitchblende	49.4 ±0.2	3.8 ±0.7	49.3 ±0.6	4.3 ±1.8	13
13	Euxenite-bearing	6.45 ±0.03	1.88 ±0.06	6.4 ±0.1	2.0 ±0.3	3.4
14	P.S.[a]	0.113 ±0.004	0.67 ±0.01	0.13 ±0.04	0.63 ±0.09	0.17
15	P.S.[a]	2.49 ±0.01	None	2.51 ±0.01	None	∞
16	P.S.[a]	0.56 ±0.04	7.13 ±0.07	0.50 ±0.07	7.4 ±0.2	0.08
17	Uranothorite-bearing	6.9 ±0.1	27.1 ±0.3	....	....	0.26
18	P.S.[a]	0.025 ±0.002	0.385 ±0.005	0.03 ±0.02	0.36 ±0.06	0.07

[a] P.S. = prospector's sample.

research in this laboratory [SLACK, 1952], the investigation of uranium and thorium contents of samples whose geological ages were to be determined by the lead isotope method [COLLINS and Others, 1951], and assays performed on a number of samples investigated elsewhere by chemical and other radiometric methods. The latter samples were investigated in order to have a check on the method. A few examples of the use of these results are mentioned below.

The first sample is the thorium standard used by SLACK [1952] in geophysical investigations. A beta-gamma analysis of this material gave 5.8 pct ThO_2, 0.2 pct U_3O_8, and the agreement with the author's result is excellent.

Mass spectrographic data on the Belgian Congo pitchblende showed that not more than one per cent ThO_2 would be expected from the measured abundance of Pb 208, whereas the mineral assays 3.8 ± 0.7 pct ThO_2. This indicates that loss of lead from this material has been very appreciable, and this is supported by other evidence. Comparison with a beta assay showed that the Athabaskan pitchblende had a 20 pct excess of the radium family. A beta-gamma analysis of this material would therefore mistakenly indicate the presence of thorium; mass spectographic data showed that no thorium was present.

Samples 15 and 16 have been highly altered and a very large excess of the radium family is present. The beta-gamma method leads to results which are completely misleading, whereas the method investigated by the author correctly assays the ThO_2 and the radium family present. In the case of Sample 15, a careful chemical analysis for ThO_2 was performed elsewhere and none was found.

The first ten samples listed are almost certainly in secular equilibrium and the βU_3O_8 equivalent calculated from the author's results are in agreement with those measured by beta assays. Beta-gamma assay results are available for most of these samples, and the agreement is quite satisfactory. Values of r as low as 0.02 cannot, however, be estimated by the beta-gamma method. In the case of the highly radioactive Sample 5, the author's method agrees with a chemical assay to one per cent, whereas other radiometric methods are poorer, probably due to overloading the Geiger tubes at this high concentration.

Sample 2 is interesting in that the material was in equilibrium and the different radiometric methods agreed, but showed that the chemical analysis supplied with the specimen was grossly in error.

The application of this method in field instruments--SLACK and WHITHAM [1951] and SLACK [1952] have described portable instruments, using anticoincidence arrays of gamma Geiger counters. The counter described by SLACK [1952] is some four times more sensitive than a single Geiger tube (sensitivity being defined as the signal counting rate from rocks divided by cosmic-ray counting rate). This instrument has been used to investigate the radioactivity of geological formations and in estimating radioactive heat production in rocks. The uncertainties in the interpretation of the measured counting rates due to the presence of potassium in the rocks have been described, and the necessity of knowing regional values for the potassium content demonstrated in Part 1 of this paper. Using Geiger counter methods, this seems unfortunately unavoidable.

A portable scintillation counter employing a 5819 multiplier phototube and a large NaI (Tl) crystal could be used with increased sensitivity in such a way that the potassium problem could be solved. A comparison of the behavior of the scintillation counter described above with the Geiger tubes used by Slack was made, using naturally radioactive sources. Even with the phototube of low sensitivity available, a sensitivity of more than 20 times that obtained by Slack is possible. Bias curves were obtained by the author for a source of 81.6 gm potassium sulphate (3.5×10^{-9} curies) and compared with uranium and thorium bias curves. The possibility of discriminating between potassium and uranium (and thorium) by a gamma-gamma analysis was evident. For example, at the overall voltage -930 volts, a decrease in the uranium (and in the thorium) counting rate of about 11 was found between five and 40 volts bias. The potassium counting rate decreases by a factor of five between the same limits.

The discriminating factor between potassium and uranium (and thorium) is thus 2.2 to 1, whereas under these conditions that between uranium and thorium is negligible and less than five per cent. A decrease of counting rate between five and 11 would still leave a signal counting rate larger than the background counting rate over most Precambrian rocks. However, counting at two bias voltages to separate the uranium and thorium does not seem possible in any instrument designed for the investigation of weakly radioactive rocks. The counting rates left at operating levels where the difference is appreciable would be prohibitively small. Fortunately this is not important since the ratio of uranium to thorium assumed does not severely affect the interpretation of results [SLACK, 1952].

Since the gamma spectra are so nearly alike the ratios between the calibration constants for the evaluation of results found by Slack would not change very severely for a scintillation counter, and this was confirmed experimentally. Any change would be in the direction to decrease the relative proportion of counts from potassium, and this would be an advantage.

The greatest difficulty in such a field instrument would be that of voltage stability. One part in 1000 would be desirable, and this should be achievable. The size and weight of a scintillation instrument would not be greater than that of the instrument described by Slack.

Conclusions--The development of a laboratory gamma scintillation counter, capable of distinguishing between the radiations from the uranium and thorium families, has been described and its performance evaluated. The limitations of the method have been described and possible alternative procedures outlined. The advantages of a scintillation gamma-gamma analysis over other radioactive procedures when the uranium in the mineral is not in radioactive equilibrium have been discussed and illustrated by a number of results on such minerals.

An analysis of laboratory experimental data has shown that in measuring the distribution of radioactivity in geological formations, and in estimating the radioactive heat production in rocks from field observations, scintillation counters using the gamma-gamma counting technique could be used to eliminate the uncertainties due to the presence of potassium, and such a counter would be at least 20 times more sensitive than the three-channel anticoincidence Geiger counter apparatus described by Slack in Part 1 of this paper.

Acknowledgments--I am indebted to J. Tuzo Wilson and Ursula M. Martius for their suggestions and assistance during the course of this research. The generous cooperation of H. V. Ellsworth and his staff of the Radioactivity Division, Geological Survey of Canada, in certain phases of this work is also gratefully acknowledged. I also thank the Goldsmiths' Company of London for the award of an overseas research scholarship and the Research Council of Ontario for later financial assistance. This paper is published in partial fulfillment of the requirements for a Ph.D. Degree, Department of Physics, University of Toronto.

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(Communicated manuscript received March 7, 1952;
open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

COMPLICATED WELL PROBLEMS SOLVED BY THE MEMBRANE ANALOGY

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Abstract--The two general equations for well flow, whether confined or unconfined, are outlined for steady-state conditions. The membrane theory is presented and the experimental equipment and results outlined for the case of a well discharging from storage in an aquifer bounded by a circular line source. It is shown that with certain limitations the membrane analogy will yield solutions for steady-state problems of ground-water flow with complicated boundary or well-distribution geometry, whether flow is confined or unconfined. The principles of model design necessary to duplicate the sample boundary conditions are presented. The membrane analogy should be considered as another tool to augment present techniques used in the solution of well problems.

Introduction--The flow of ground water can be divided into two broad classifications. The first is confined flow, as, for example, the flow moving horizontally in a radial direction toward a well tapping an aquifer that is confined both above and below by impermeable layers. The second classification is unconfined flow where the water is moving radially toward the well through an aquifer underlain by a horizontal impermeable layer and has an unbounded upper surface at atmospheric pressure.

The steady-state case of confined flow with axial symmetry is readily susceptible to rather complete mathematical treatment. The equation [JACOB, 1950; MUSKAT, 1946] for this type of flow with a constant rate of discharge is

$$h_2 - h_1 = (Q/2\pi Kb) \log_e (r_2/r_1) \quad \ldots\ldots\ldots\ldots (1)$$

where

h_1, h_2	Piezometric or hydrostatic head
Q	Rate of discharge
K	Coefficient of permeability having the dimension of velocity
b	Thickness of the aquifer
r_1, r_2	Radial distance from the axis of the well

For the steady-state case of unconfined flow where the shape and position of the upper surface are a variable, a complete mathematical solution is more difficult. The solution proposed by Dupuit in 1863 for small water-table slopes

$$h_2^2 - h_1^2 = (Q/\pi K) \log_e (r_2/r_1) \quad \ldots\ldots\ldots\ldots (2)$$

has several assumptions involved in the derivation which restrict its application in certain regions of the flow. The errors involved in these assumptions are most serious in the region where the curvilinear flow is greatest, which occurs near the well in the vicinity of the free surface (or water table). Equation (2) describes the piezometric head along the impermeable base and describes the free surface only at those distances from the well where the flow is essentially horizontal [BABBITT and CALDWELL, 1948; HANSEN, 1949, 1951; JACOB, 1950; MUSKAT, 1946; WYCKOFF, BOTSET, and MUSKAT, 1932].

Of particular concern is the location of the free surface (or water table) near the well. Such information becomes particularly important in cases where wells are used to lower the water table or where one well is within the cone of depression of an adjacent well. It has been found [HANSEN, 1949, 1951] that the free surface near the well can be represented by a linear logarithmic solution of the type

$$h_2 - h_1 = m \log_e (r_2/r_1) \quad \ldots\ldots\ldots\ldots (3)$$

where m is a parameter relating the geometry of the system, the discharge and the permeability. The general functional relationship

$$Q/K\, r_w^2 = f\,(h_w/r_w,\, h_s/r_w) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

has been developed [HANSEN, 1949, 1951] for which curves have been presented [HANSEN, 1949, 1951; PETERSON, ISRAELSEN, and HANSEN, 1952] permitting a solution of (4). In (4), r_w = radius of the well, h_w = piezometric head at the well, and h_s = height above the impermeable layer of the intercept of the free surface with the edge of the well.

Since confined flow in two dimensions can be represented by a linear logarithmic solution of the Laplace equation, and the free surface for unconfined flow can likewise be represented in the vicinity of the well by a similar solution, the following investigation of a rubber membrane was made to ascertain if it would also yield a similar solution when the correct boundary conditions were applied. If comparable results were obtained, then the membrane could be used to give the shape of the piezometric surface for confined flow and the free surface near the wells for unconfined flow in any arrangement of wells. The visual nature of such a solution makes it particularly desirable.

Membrane analogy--Prandtl, in 1903, was one of the first to present the membrane theory. The concept has been applied by the Germans [von BAUERSFELD, 1934] to numerous problems of fluid flow such as flow around wing profiles and through radial flow turbines. In this country, exclusive of its use in stress analysis, the membrane theory has seen only limited application. A rubber membrane has been used by the radio industry [ZWORYKIN and Others, 1941; ZWORYKIN and RAJCHMAN, 1939) to predict the flow paths of electrons in the complicated multiple-anode tubes. Excellent results were obtained for a problem too complicated to be analyzed mathematically.

Comprehensive treatments of the membrane theory can be found in von BAUERSFELD [1934], ZWORYKIN and Others [1941], and ZWORYKIN and RAJCHMAN [1939]. The essence of these developments is that the surface of the membrane assumes the shape of either the potential or stream surfaces when the proper boundary conditions are applied. For a membrane weighing γt pounds per unit surface area when under a uniform tension T, it can be shown that when the slopes are small (so that the sine can be replaced by the tangent and the tension remains sensibly constant), the partial differential equation reduces to the form

$$\partial^2 z/\partial r^2 + (1/r)\,\partial z/\partial r = -\,\gamma t/T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The Laplace equation in two dimensions is obtained by making the right side of the equation approach zero.

Equipment for membrane analogy--In order to study the possible use of a thin rubber membrane to predict flow into wells, the equipment shown in Figure 1 was constructed. The rubber was clamped under uniform tension to a frame on the right side of the table, and a circular boundary 36 inches in diameter impressed upon the membrane from above. Small rods to serve

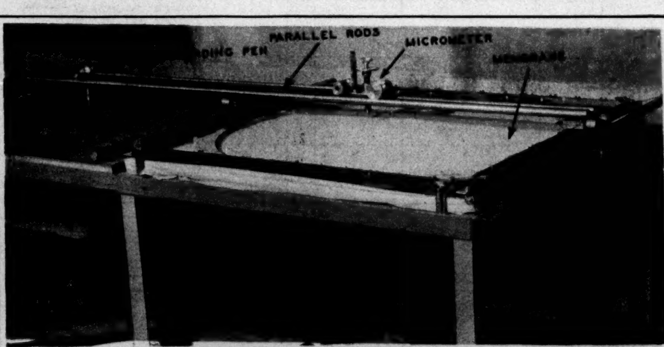

Fig. 1--Membrane-analogy equipment

as probes were screwed into adjustable bushings on the under side to deflect the membrane. The rubber was deflected from the under side rather than from above in order to facilitate the measurement of both the surface near the well and the elevation of the probe. In other words, the shape assumed by the membrane is just the inverse of that assumed by the piezometric surface for actual flow into wells. Three leveling rods were placed across the table to serve as guides for two parallel rails running across the table. The parallel rails were maintained in position very conveniently and accurately as they moved along the leveling rods by use of a cord arrangement similar to the design for straight-edges used in drafting. On top of the parallel rails was mounted a micrometer reading to 0.0004 inch which was used to measure the deflections of the membrane. Connected by a rod to the micrometer was a recording ball-point pen to transcribe at full scale the coordinates of the micrometer onto graph paper on the left portion of the table.

For limited practical tests, equipment as elaborate as that described above is surely not needed. The above equipment was designed, however, to reduce to a minimum the experimental error of the definitive tests herein described. It was felt that under the necessarily small permissible deflections the equipment must be very carefully designed in order to see clearly the extent of the limitations. The initial proof of the membrane analogy was obtained by stretching a sheet of rubber over a wooden frame made in an hour's time. Hence, it is felt that results of a sufficient accuracy for most practical tests can be secured with a minimum of equipment.

Results of membrane-analogy tests--Four factors which might possibly influence the shape of the membrane were investigated: kind of rubber, gravity, tension, and boundary conditions. Regarding the first of these, correspondence with the Bell Telephone Laboratories indicated that dental dam secured from uncut strips had proved very satisfactory in the solution of their electron-optical problems.

As indicated previously, the membrane equation approaches the Laplace equation as the ratio of $\gamma t/T$ approaches zero. Hence, the effect of both gravity and tension were studied by varying the ratio $\gamma t/T$. The most practical way of varying the ratio was by varying the tension. To accomplish this, the rubber was first placed on a flat surface and squares marked on it. Then the rubber was stretched over the frame and the tension adjusted until the lines comprising the squares were straight, indicating that a uniform stress had been obtained. The tension was expressed as a ratio of the area of the stretched surface to the unstretched surface, A_s/A_n.

For a given tension, the round probe (representing a well) was raised in the center of the confining circle and the resulting displacements recorded. The elevation of the top of the probe represented the potential existing at the well. When the displacements were plotted against the logarithm of the radius, a straight line resulted over a varying portion of the membrane (Fig. 2). The departure from a straight line for r/r_w greater than approximately 40 is due to the accumulative effect of gravity on the membrane. Placing the membrane in the vertical plane will minimize this departure.

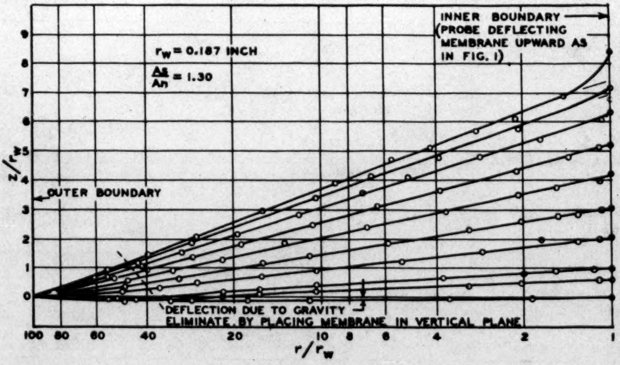

Fig. 2--Membrane-analogy test No. 1

Note on Figure 2 that the departure of the membrane surface from the linear logarithmic relationship (departure from a straight line on semilogarithmic paper) is most pronounced within approximately 1/4 radius of the well for large deflections. Recent studies indicate that the particular solution ($h \sim \log_e r$) of the Laplace equation may not be the correct solution for this section. However, the assumptions of small deflections of a perfectly elastic membrane used in the development of the differential equation are least accurate in this region. Hence, at present whether the departure from a straight line adjacent to the well is to be expected or whether it is a departure of the membrane from the Laplace equation is not known. However, the usual field problem does not generally involve such excessive drawdowns so that this uncertainly is usually not serious in practical applications.

Application of membrane analogy--To solve any well problem, the boundary conditions of the flow must be known. The outer boundary condition for confined flow will be the relative position of the piezometric head at some radial distance from the well, while for unconfined flow the position of the free surface must be used. The inner boundary conditions may be either another point at a smaller radial distance from the well or the respective surface at the well. The boundary condition at the well in confined flow will be the elevation of the water surface in the well plus well losses. (Excess energy loss due to turbulent flow near the well can be considered in the category of well losses.) For unconfined flow the well boundary condition will be the water surface in the well plus the well losses plus the seepage surface. This point is referred to as the position of the free surface at the edge of the well and is designated as h_s. By use of the dimensionless plots developed by HANSEN [1949, 1951] and by PETERSON, ISRAELSEN, and HANSEN [1952], the intercept h_s can be determined.

The use of a rubber membrane for the solution of confined flow to a single well has no practical application, since the mathematical equation is known and is easy to solve, and the resulting solution gives good results. Even the solution for unconfined flow to a single well usually can be approximated with sufficient accuracy by either the Dupuit or BABBITT and CALDWELL [1948] equation, depending upon the region under question.

The utility of the membrane analogy is that it permits a solution to the very knotty problem of wells in any arrangement with extreme variation in approach conditions [HANSEN, 1949, 1951]. Non-uniformity of outer boundary conditions can be duplicated in the membrane by forcing the outer periphery of the membrane to a corresponding condition.

Two general types of problems will arise, depending upon whether or not the drawdowns are known. If the drawdown at each well is known, the proper boundary conditions can readily be applied as described previously. If the discharge from each well is desired, it can be obtained by substituting the average surface deflections at some convenient radius (not greater than the distance to the nearest ground-water divide) into the single-well formula. However, if the discharges instead of the drawdowns are known, it is necessary to estimate the drawdowns resulting from the given discharges and then proceed as before, repeating by trial-and-error until the correct discharges are obtained.

The membrane analogy is particularly useful in any problem where the piezometric head in some area must be reduced below a certain limit - for instance, in the use of wells for drainage or even to secure pressure relief for levees and dams. It is readily apparent that the membrane analogy becomes of primary value in such cases, since the shape of the piezometric surface is visible and the effect of any change in the arrangement or design of a system of wells is readily seen.

It is evident that the precision of the method can be assayed only in the light of the precision of the actual field measurements and, consequently, the accuracy of the applied boundary conditions. Of the many variables which may be involved, the permeability alone cannot be determined with a high degree of accuracy unless extensive and expensive field tests are undertaken. If the results of such tests are available, then, of course, the variables can be defined with increased precision. Membrane-analogy equipment can be designed easily and the limitations controlled to the extent that results of an accuracy fully comparable to the field data can be obtained at a small fraction of the cost.

Even though the degree of agreement is within the accuracy of the field data, of far more significance is the fact that the membrane analogy produces a solution to problems of extreme complexity and yet general occurrence at a fraction of the cost of other methods.

Conclusions--A thin rubber membrane can be used to obtain the shape of the piezometric surface for confined flow and the free surface for unconfined flow near the well where the above assumptions are justified by the linear logarithmic solution.

The simplicity, wide applicability, low cost, accuracy, and adaptability, together with the visual nature of the solution, makes the membrane analogy useful in the solution of multiple-well problems with complex approach conditions.

The membrane analogy should not be considered as a cure-all for all well problems, but rather as another tool for the engineer to add to his kit. Naturally this new tool, like all others, will serve its purpose only after the engineer has learned to use it effectively to augment his present techniques.

Acknowledgments--Appreciation for his cooperation is extended to Hunter Rouse, Director of the Iowa Institute of Hydraulic Research, under whose direction this study was made. The equipment was provided by the University of Iowa's Department of Mechanics and Hydraulics, of which J. W. Howe is head. Both were generous with their assistance. To Emmett M. Laursen, Research Engineer at the Iowa Institute, the writer gives very sincere thanks for his many helpful suggestions.

This paper is based on a dissertation entitled "Evaluation of unconfined flow to multiple wells by the membrane analogy" submitted by the author in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanics and Hydraulics of the Graduate College of the State University of Iowa, June, 1949.

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(Manuscript received February 25, 1952; presented at the Thirty-Third Annual Meeting, Washington, D. C., May 7, 1952; open for formal discussion until May 1, 1953.)

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

REVIEWS, ABSTRACTS, AND BIBLIOGRAPHY

REVIEWS AND ABSTRACTS

THOMAS, HAROLD EDGAR, The Conservation of Ground Water: A Survey of the Present Ground-Water Situation in the United States, McGraw-Hill, 327 pp., maps, 1951.

The author and the Conservation Foundation are to be commended for producing the first factual, detailed survey of this important component of the water resources of this country. It is noted that roughly 30 billion gallons per day, or about 15 pct of the total United States daily water consumption, is supplied from ground-water sources. Thomas places in their proper light the numerous stories and reports from virtually every state in the country regarding the "perilous shortages" of ground water.

The problems concerning ground-water resources are categorized into reservoir, pipeline, and watercourse problems; depending on whether the entire reservoir, including the recharge area and conditions, are involved; whether the transmissivity of the aquifer is the primary consideration; or whether the relationship between the ground-water picture and surface streams is the dominant factor. Thomas then goes on to elucidate these problems through brief but fact-packed descriptions and discussions of more than 60 specific ground-water areas.

Several, general, but equally important deficiencies are also pointed out. The facts that in only about five per cent of the country are ground-water conditions reasonably well known, and that in 75 pct of the country hardly any worthwhile information regarding ground-water resources is available, certainly point up the status of the situation. The apparently ever-present lack of adequate hydrologic data invariably brought up by technical workers in the field is also mentioned. Though the reviewer agrees with and appreciates the necessity for such data in specific development projects, the thought does occur that perhaps, in an overall way, enough data and operating experience are available to at least permit improved "management decisions" and legal approaches to national water-resource problems.

The portion of the book devoted to the relation of ground-water problems to those of land use and land management aptly summarizes this often-discussed subject. Though the author correctly states in the introduction that the effects of man's activities are insignificant in comparison with the results of geologic stress and strain, he also states that "the greatest change wrought by man has been the change from the original forest and grassland to cultivated or barren areas." That these changes have resulted in shifts in the hydrologic cycle with concomitant serious, practical problems in land and water management is amply exemplified by certain areas in the southeastern United States.

The rest of the book deals with the increasing requirements for water and how the ground-water resources tie into this picture; current deficiencies and future needs for effective ground-water development including the legal and social aspects of the problem. A particularly lucid chapter on Better Ground-Water Management by Abel Wolman, Professor of Sanitary Engineering and head of the Department of Sanitary Engineering and Water Resources at the Johns Hopkins University, makes an admirable summary and conclusion to the book.

Clear, non-technical language characterizes the entire text and only one very minor typographical error was noticed. The lack of graphical presentation, which in numerous cases would have added much to the narrative explanations, is perhaps the only deficiency of the book. It can be read, studied, and used as a reference with equal profit by hydrologist, engineer, and layman, and is indeed so recommended.

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NOTES AND PERSONALIA

NOTES

Civil Service examinations in geology--The U. S. Civil Service Commission announces a written examination for geologists, grades GS-5 and 7, for service in the U. S. Geological Survey and other agencies, with closing date January 6, 1953. Interested persons should get in touch with the Board of U. S. Civil Service Examiners, U. S. Geological Survey, Washington 25, D. C., or the nearest office of the Civil Service Commission. Ask for announcement 342.

Meeting of the American Society of Photogrammetry, Washington, D. C., January 14-16, 1953--This Society will hold its Nineteenth Annual Meeting at the Shoreham Hotel in Washington, D. C., January 14-16, 1953. Included in the program is a panel on Arctic Mapping, and a full day will be devoted to Photo Interpretation. Further information may be secured from the Society, Box 18, Benjamin Franklin Station, Washington 4, D. C.

Geophysics seminars continuing at Geophysical Research Directorate--The seminars on various aspects of geophysics are continuing on an approximately bimonthly schedule. On January 16, 1953, P. D. McTaggart-Cowan of the Meteorological Service of Canada will hold a seminar on Progress and Problems in Meteorology. The seminars are held in the Seminar Room of the Directorage at 415 Summer Street, South Boston, Massachusetts, at 14h 45m.

Fellowship program of the Humble Oil and Refining Co.--The fellowship program of this Company for grants to graduate students has been extended from 12 to 20 fellowships at nine different schools. The total grant to the student is $1,600 per year plus $300 for tuition and other fees. Further information may be secured from the Company, P. O. Box 2180, Houston 1, Texas.

Meeting of the Eastern Snow Conference--The Tenth Annual Meeting of the Eastern Snow Conference will be held at the Ten Eyck Hotel in Albany, N. Y., February 12-13, 1953. Among the features of the meeting will be a tour of the General Electric Research Laboratory and an address by Irving Langmuir. For further information, write to the Secretary, Dean B. Bogart, U. S. Geological Survey, P. O. Box 948, Albany 1, N. Y.

Extensive oceanographic survey in the Pacific--The Scripps Institution of Oceanography has an extensive research program under way in the Pacific. Two vessels, the *Horizon* and the *Baird*, are being used. The purpose of the expedition is to study the basin of the deep Pacific in its broadest aspects, paying particular attention to the Tonga Deep. The route will pass Hawaii, the Marshall Islands, the Fiji Islands, the Samoan Islands, the Marquesa Islands, and San Diego, returning in February, 1953. Work will include the gathering of seismic and magnetic data, deep sea cores, and meteorological data, in addition to continuous sounding records. Included in the staff are Roger Revelle, the expedition's leader, Russell Raitt, Clarence Palmer, Walter H. Munk, and John D. Isaacs.

New Italian Society of Geophysics--In May, 1952, the Società Italiana di Geofisica e Meteorologia was established in Italy. Its current membership exceeds 150. The Secretary is Mario Bossolasco of the Istituto Geofisico Italiano in Milan.

Seventeenth International Geographical Congress--The Eighth General Assembly and Seventeenth International Congress of the International Geographical Union was held in Washington, D. C., August 8-15, 1952. It was attended by 1200 persons, including 350 from abroad. Representatives and delegates from 55 countries were in attendance. Representatives from the American Geophysical Union were K. Hilding Beij, Leo O. Colbert, F. W. Reichelderfer, Waldo E. Smith, and C. Warren Thornthwaite. This was the first international geographical congress held in the United States since 1904. There were 23 technical sessions, two symposia, 12 commission meetings, and four major excursions, aside from exhibits and other special events. The following officers were elected: President, L. Dudley Stamp (Great Britain); First Vice-President, Orlando Ribeiro (Portugal); Vice-Presidents, Hans W:son Ahlmann (Sweden), Hans Boesch (Switzerland), George B. Cressey (United States), George Kuriyan (India), Max Sorre (France), and Hilgard O'Reilly Sternberg (Brazil); Secretary-Treasurer, George H. T. Kimble (United States). The next Assembly will be held in Brazil in 1956.

Eighth General Assembly of the International Astronomical Union--This Assembly was held in Rome, Italy, September 4-13, 1952. There were about 430 delegates in attendance from 35 countries, including 58 from the United States. Among those in attendance from the United States were Dirk Brouwer, G. P. Kuiper, O. Struve, and F. L. Whipple. Otto Struve of the United States was elected President, and P. T. Oosterhoff was elected General Secretary of the Union for the period 1952-1955. The next General Assembly will be held in Dublin, Ireland, in 1955.

Myojin Reef and the loss of the Kaiyo Maru No. 5--On September 17, 1952, the fishing vessel *Myojin Maru* of Yaizu reported to the Japanese Hydrographic Office a new island being created by volcanic activity near the Bayonnaise Rocks, south of Honshu. On the 23rd the 211-ton survey ship *Kaiyo Maru No. 5*, with a crew of 22 and a scientific party of nine, was dispatched to the spot. The ship was reported missing on the 24th, and rescue vessels later picked up a lifering, deck planking, and other wreckage. It is believed that the *Kaiyo Maru No. 5* was overwhelmed by blast and wave action from a sudden eruption of the volcano. Among those lost were Risaburo Tayama, Chief of the Surveying Section of the Japanese Hydrographic Office; Minoru Tsuchiya, Assistant Chief; Teratoshi Nakamiya, Chief of the Oceanographic Section; Kiyosuke Kawada, Assistant Professor at Tokyo Education University; and Harukichi Hamamoto, master of the vessel.

New publication in meteorology--The Weather Bureau of South Africa has begun the issuance of a new publication entitled *Notos*. This will include data gathered in South Africa and serves also as a medium for original contributions. Volume 1, number 1, contains three articles, all relating to the southern hemisphere, and extensive data on the circulation in the southern hemisphere during the first quarter of 1951. Inquiries should be addressed to Weather Bureau, P. O. Box 1135, Pretoria, Union of South Africa.

Petroleum test-well location map of Florida--A petroleum test-well location map, scale ten miles to the inch, covering adjacent counties in Alabama and Georgia as well as the entire state of Florida, is now available through the Florida Geological Survey in Tallahassee. All wildcat wells drilled in search for oil are located, and the operator, owner, elevation, and total depth are recorded.

Curriculum changes at Massachusetts Institute of Technology--The Department of Geology, recently renamed the Department of Geology and Geophysics, at MIT has introduced two separate courses, Course XIIA, Geology, leading to a Bachelor of Science degree in Geology, and Course XIIB, Geophysics, leading to a Bachelor of Science degree in Geophysics. The first two years are quite similar, including a summer field camp in Nova Scotia at the end of the second year. In the last two years, the course in Geophysics emphasizes work in mathematics, physics, and electrical engineering. Further information can be secured through the Department.

Oceanographic expedition--The National Geographic Society is sponsoring an extensive four-year oceanographic expedition which will begin its work in Caribbean waters, moving southward along the east coast of South America, thence working over to New Zealand, the Ross Sea, and the coast of Africa. The expedition will be under the direction of Jacques-Yves Cousteau, and will have the official backing of the French Government and the French Academy of Sciences. They will use the Calypso, a converted American mine sweeper, as their expedition ship.

Hydrologic and geological studies in the Marshall Islands--Studies of the hydrology and geology of more than 60 islands of this group have been under way for some time, and recent plans extend the work to Taongi and Bikar. These atolls have never been explored, except from shipboard, at a distance, and are believed never to have been inhabited. Theodore Arnow and Charles G. Johnson have been making the hydrologic surveys.

Inauguration of President at Wagner Lutheran College--Charles H. Behre was the representative of the American Geophysical Union at the inauguration of David M. Delo as President of Wagner Lutheran College on Staten Island.

Hydraulic lecture series in Colombia--John S. McNown, Associate Director of the Iowa Institute of Hydraulic Research, and A. L. Alin, Professor of Hydraulic Engineering at the State University of Iowa and consulting engineer, presented a series of lectures on hydraulics at the National University at Bogota, Colombia.

PERSONALIA

Edwin J. Chernosky, formerly of the Naval Ordnance Laboratory, has taken up duties with the Geophysics Directorate of the Air Force Cambridge Research Center.

Philip F. Clapp, who has devoted the last year to the study of meteorology at the University of Stockholm, has returned to his duties in the Extended Forecast Section of the U. S. Weather Bureau in Washington, D. C.

Robert I. B. Cooper, who had planned to devote the academic year 1952-1953 to work at the University of Toronto, as noted in Personalia in the October issue, has changed his plans and is remaining at Cambridge University.

Carl Eckart, Director of the Marine Physical Laboratory of the University of California in La Jolla, is devoting a sabbatical year to work at the Institute for Advanced Study at Princeton.

C. T. Elvey, Director of the Geophysical Institute of the University of Alaska, has been elected President of the Alaska Division of the American Association for the Advancement of Science for the year 1952-1953. He is also President of the Arctic Section of the Division with headquarters at Fairbanks, Alaska.

J. R. Fulks, Chief of the Analysis Center of the U. S. Weather Bureau, is devoting a year to meteorological study at the University of Stockholm.

Beno Gutenberg, Director of the Seismological Laboratory at California Institute of Technology, has been awarded the Charles LaGrange Prize of the Classe des Sciences of the Belgian Royal Academy for his work in Geophysics.

H. W. Hemple, Captain, U. S. Coast and Geodetic Survey, retired on October 31, 1952, after 35 years of active duty. His post as Chief, Division of Geodesy, is now occupied by Albert J Hoskinson, Captain, U. S. Coast and Geodetic Survey.

Walter L. Huber, Consulting Engineer of San Francisco, has been named President of the American Society of Civil Engineers.

John O. Nigra, until recently research geologist for the Arabian American Oil Co., has taken up duties as Associate Professor of Geology at Tulane University.

Clarence S. Ross of the Geochemistry and Petrology Branch of the U. S. Geological Survey was recently given the Distinguished Service Award of the Department of the Interior.

Kenneth G. Smith, who completed his studies at the University of Michigan in June, 1952, has taken up duties with the Continental Oil Co. with headquarters in Midland, Texas.

Roscoe H. Suttie, Professor of Civil Engineering at Yale University, has retired after nearly 40 years. He joined the staff in 1913, and, except for one year with the U. S. Geological Survey, served the University without interruption.

Herbert D. Vogel, Colonel, Corps of Engineers, who has recently become Division Engineer of the Southwestern Division at Dallas, Texas, has become Chairman of the Arkansas-White-Red Basins Inter-Agency Committee, and a member of the International Boundary and Water Commission for the Rio Grande.

Harold E. McComb, recently retired as Chief of the Geomagnetism Branch, Division of Geophysics, U. S. Coast and Geodetic Survey, died October 11, 1952, in Washington, D. C., at the age of 65. He was born at Wilsonville, Nebraska, and took his baccalaureate and master's degrees from the University of Nebraska. He entered the service of the Coast and Geodetic Survey in 1909, and, except for service in World War I, he worked continuously with the Survey. He was Secretary of the Section of Seismology 1930-1936, Vice President 1938-1941, President 1941-1944, and served on various committees of the Union. He had been a member of AGU since 1929.

Fran Ramovš, President of the Slovene Academy of Sciences and Arts, died September 16, 1952, at Ljubljana, Yugoslavia.

Franklin Thomas, Dean of Students and Professor of Civil Engineering at California Institute of Technology, died August 27, 1952, after a serious operation. He was born May 19, 1885, at Red Oak, Iowa. He received his baccalaureate degree from the University of Iowa in 1908, attended McGill University in 1908-09, was an instructor at the University of Michigan from 1910 to 1912, and worked with the Alabama Power Co. in 1912 and 1913. Since 1912 he had been a member of the staff of California Institute of Technology, but had engaged in many other important activities in addition, including work as a consultant on flood-control and sanitation work for Los Angeles County. He was a member of the Board of Directors of the Metropolitan Water District of Southern California, and was recognized as one of the outstanding hydraulic engineers of the country. He was active in the arrangement of the regional meetings of AGU held in California. He had been a member of AGU since 1935.

Franklin Peter Ulrich, Chief of the Seismological Field Survey of the U. S. Coast and Geodetic Survey, died in San Francisco, July 3, 1952, at the age of 61. He was born in Schenectady, N. Y., and took his baccalaureate degree from Union College in 1913. He worked with the New York State Engineering Department for a year following his graduation and in 1914 became a member of the staff of the Coast and Geodetic Survey, working chiefly on magnetic and seismic research. He had been a member of AGU since 1930.

Ferdinand von Wolff, Professor of Mineralogy and Petrography at the University of Halle, died April 7, 1952, at 77 years of age. He was considered an "Altmeister der deutschen Mineralogen und Petrographen," and ranked with the foremost volcanologists.

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MINUTES OF EXECUTIVE COMMITTEE, AMERICAN GEOPHYSICAL UNION

Meeting of November 7, 1952

The meeting was called to order in the Secretary's Conference Room of the Department of the Interior (Room 5160), Interior Building, Washington, D. C., at 13h 35m by President Walter H. Bucher. Twenty-five persons were present, namely: L. H. Adams, Wallace W. Atwood, Jr., K. Hilding Beij, Walter H. Bucher, Wilbur Burbank, Perry Byerly, Felix Chayes, Morgan Dubrow, H. N. Eaton, E. A. Eckhardt, Jno. A. Fleming, Woodrow C. Jacobs, J. Wallace Joyce, Walter D. Lambert, Walter B. Langbein, J. Putnam Marble, Leonard M. Murphy, A. C. Redfield, F. W. Reichelderfer, Irvin Roman, McClain B. Smith, Waldo E. Smith, Harlan T. Stetson, Thomas P. Thayer, and R. Hanson Weightman.

Approval of Minutes--The minutes of the Executive Committee meeting on May 7, 1952, published in the August 1952 issue of the *Transactions*, pages 624-638, were approved.

General Secretary's Report

General Activities

Executive Committee--There has been no change since our last meeting.

Representation in other organizations--On November 30, 1952, the term of W. T. Thom, Jr., as one of our representatives on the Board of the American Geological Institute expires. E. A. Eckhardt has been appointed for a two-year term in his place. Harry H. Hess has been designated representative of AGU on the Division of Geology and Geography of the National Research Council for a three-year term ending June 30, 1955. He succeeds W. T. Thom, Jr.

Financial report--A detailed financial statement has been presented by the Committee on Audit (see Appendix 2). As of November 1, 1952, the net amount of our account at the Riggs National Bank was $7,978.01. In addition, our firm assets include our account at Northwestern Federal Savings and Loan Association ($5,023.06, the Permanent Fund), Series F government bonds ($2,368 cost, $3,200 maturity value, Permanent Fund), and our account at Prudential Building Association ($1,000, Emergency Fund), making a total of $16,369.07. Although our current bank balance is below last year's, this is the result of two causes: (1) We published a directory of members in October at considerable expense, and (2) the Permanent Fund has been increased from $3,469.82 to $7,391.06, and the Emergency Fund ($1,000) has been established and separated from our bank account.

Expenses under budgeted funds indicate that they are adequate, except in two cases. (1) The directory of members exceeded the budgeted cost ($1,500) by several hundred dollars because of increased costs, continuation of the geographical part of the directory, and the ordering of a surplus stock over immediate needs to provide copies to incoming members and for other purposes. (2) The officers felt that it was essential that AGU be represented by the Executive Secretary at the Chicago meeting in September, 1952. The expenditure involved exceeded the item of $50 budgeted for travel of officers.

Membership--A summary of the membership changes since April 30, 1952, is as follows:

Membership, April 30, 1952	4775	
New members, May 1 to November 5, 1952	290	
Reactivated	3	5068
Resignations	23	
Deceased	9	
Dropped	224	
Inactive	5	261
Membership, November 5, 1952		4807

We have never before had as large a number of members dropped as shown above. This large number is probably the result of two causes: (1) Dues were increased from $5.00 to $7.00 per year at the beginning of 1950, to which date these persons paid; (2) many of these were among the 1500 new members that became members only about a year earlier, some of whom were likely to have no permanent interest in AGU.

Only 263 members have not paid dues for 1952 (the comparable number a year ago was 399) and 210 have not paid for 1951 and 1952 (the comparable number last year was 273).

Those reported as deceased since our last meeting are as follows: John S. Douds (Nov. 2, 1950), Dana Coman (Jan. 29, 1952), Oscar Rosenzweig (March 4, 1952), Joseph W. Gross (April 26, 1952), Andrew C. Lawson (June 16, 1952), Allyn C. Swinnerton (July 6, 1952), Franklin Thomas (Aug. 27, 1952), H. E. McComb (Oct. 11, 1952), and Jake Moon (date unknown).

Meetings and regionalization--The Twenty-Eighth Regional Meeting was held in Chicago, September 11-12, 1952, jointly with the American Meteorological Society and in connection with the celebration of the Centennial of Engineering. Nine sessions were held, partly at the Conrad Hilton Hotel and partly at the Del Prado Hotel. Registration was about 250, with many attending who were registered in connection with the Centennial but not for these sessions.

In the early summer, we received a request from the Institute of Mathematical Statistics to co-sponsor a session they were planning on the evolution of cloud-seeding operations. After discussion among the officers of the Union and the Section of Meteorology, it was decided that we should aid. We helped in arranging papers to complete the program, and sent word of the meeting to members residing in that region. Reports are that the meeting was very good.

In accordance with the action of the Executive Committee in May, the AGU is co-sponsoring the sessions bearing on geophysics being arranged by the AAAS for their annual meeting in St. Louis in December, 1952. Two of our members, E. M. Brooks and Ross R. Heinrich, have been active in these arrangements, though acting only informally for the AGU.

The Twenty-Ninth Regional Meeting is scheduled to be held by the Pacific Southwest Regional Group at Stanford University, February 6-7, 1953. J. W. Johnson is Chairman of the Regional Committee. David K. Todd is Chairman for the meeting and Ray K. Linsley is Chairman of the Committee on Local Arrangements. Presumably Hydrology and Meteorology will participate. Other Sections that may be interested should get into touch with the Committee.

The Thirty-Fourth Annual Meeting will be held in Washington, May 4-6, 1953.

A regional meeting is planned for August 31, 1953, in Minneapolis by the recently appointed Regional Committee for the North Central Region, now organizing. The initial Committee appointed to work out the organization are Martin E. Nelson (Chairman), Paul C. Benedict, M. C. Boyer, Lorenz G. Straub, and Roy F. Warner. Other groups meeting on successive days are the International Association for Hydraulic Research and the Hydraulics Division of the American Society of Civil Engineers. At present, plans involve only our Section of Hydrology, but other interested Sections should take immediate steps to join in this activity.

A suggestion has come from R. M. Field that a Symposium on William Bowie be arranged, perhaps at Trinity College in Connecticut of which Bowie was an alumnus. The establishment of the Bowie Medal of the AGU was supported by Trinity College.

Activity toward organization of the Pacific Northwest Region is quiescent at this time. Comments are invited.

The AAAS has suggested that our regional meeting in 1955 be moved ahead and joined with theirs to be held in San Francisco in December, 1954. This is under study by the Regional Committee.

Suggestion of combination of facilities--Suggestions have come from the American Congress on Surveying and Mapping and from the American Society of Photogrammetry that it might be advantageous and possible to combine certain facilities of some of the smaller professional societies with ours. The Executive Secretary has heard from three different people in this regard, but while he has indicated, in principle, that such might be desirable, he has otherwise lent no particular support or attempted to work out any plan toward this end.

Publications--We are still currently publishing the Transactions in an edition of 5200 in the second half of the year, and 5400 in the first half of the year, the difference being due to the fact that those delinquent in dues receive Transactions only through the first half of the year. The October issue was recently mailed to the membership and subscribers in the total number of 4887, in addition to nearly 200 being sent to our foreign exchange list and 100 to the sixth list of new members. Part 2 of the October Transactions (the directory) was published in an edition of 6000.

We now have about 20 student subscriptions. It is recommended that this type of subscription be continued indefinitely.

Recently authority was given to the Library of Congress to reproduce papers in our Transactions by means of a facsimile transmitter located in the Library of the Atomic Energy Commission on an experimental basis to June 30, 1954. On June 30, 1953, and again a year later, they are to advise us what papers have been so transmitted.

The Society of Exploration Geophysicists have under way a project to microcard various geophysical publications to make complete sets available to libraries and others. Our Transactions are included. This matter was referred to the Editorial and Publication Committee for study and action. Though no formal proposition has yet been forthcoming from the SEG, the Committee has worked out a trial plan which seems to be satisfactory to the Chairman of the SEG Committee. The plan is as follows:

(1) We will give permission for the microcarding and sale of complete volumes or sets of volumes only, up to and including the Transactions of 1951. No microcards for partial volumes or particular papers will be sold.

(2) No exclusive rights will be given by the American Geophysical Union.

(3) The arrangement, being exploratory in nature, will run up to July 1, 1954.

(4) There is no implied promise of future action beyond that date, and sales beyond that date of volumes already microcarded will not be undertaken without specific authorization beyond a total of ten.

(5) As of June 30, 1953, and again at the end of the project on July 1, 1954, a report will be made on the sales made, including the names of purchasers and volumes supplied.

As the project is moving so slowly, the closing date may be advanced in order to give a full trial period of two years.

Elections--Preparations for the triennial elections are in progress in accordance with the time schedule adopted in 1946. Nominations to offices of the Union and of the Sections are due and most of them have been received.

Administration of the Union--Our arrangements for quarters in the Administration Building of the Carnegie Institution of Washington continue through next January. Prior to that time the officers of the Union will make request for the continuation of the present arrangement. We have recently been advised that it will probably be necessary to reorganize our activities to make arrangements for an office for one of the retired members of the Institution's staff. Plans to do this are being worked out. On our staff, Samuel Pepper left on October 31. Mrs. Elert is again being employed on a temporary basis.

Fund for medal in Terrestrial Magnetism--Honorary President Fleming has conceived of the establishment of a mdeal in Terrestrial Magnetism and Electricity, and has secured a pledge of $1,000 from the C. F. Kettering Foundation providing $9,000 more can be raised.

News items--The International Geographical Union had its Eighth General Assembly and Seventeenth International Congress in Washington, August 8-15. The AGU sent representatives.

Considerable publicity has recently been given to the October issue of the Bulletin of Atomic Scientists which is devoted to American Visa Policy and Foreign Scientists -- the effects of the McCarran Act. AGU was invited to contribute, but did not because (1) it was felt that we should not act in view of our relationship to NRC; (2) the NRC and the Science Office of the State Department are doing what can be done to alleviate the situation; and (3) we believe that proper changes may better be made through quiet work to this end rather than through noisy clammer that may react unfavorably.

Time magazine requested a copy of our list of registrants at our annual meeting in order to have the names of leaders in geophysics over the country. We advised that the registration cards are available for consultation in our office. Probably no action will result.

The Maruzen Company of Japan, whose prewar debt of about $85 was forgiven at our last meeting, has written acknowledging this action with gratitude.

Activities as Committee on Geophysics of the National Research Council

Education in Geophysics--The National Scientific Register of the Office of Education had planned to bring our "Education in Geophysics," published in 1945, up to date so we could publish a revised and more complete analysis in the Transactions. Some functions have been disrupted at the Scientific Register, however, and others have been transferred to the National Science Foundation, so the project is now at a standstill. There is hope that we may get it under way again.

American Committee for the Third Geophysical (Polar) Year--This Committee has not yet been appointed but action by the National Research Council is expected soon.

Meeting of International Council of Scientific Unions in Amsterdam--The International Council of Scientific Unions held a meeting in Amsterdam in early October. Among those present who attended are President Bucher, J. W. Joyce, and W. W. Atwood.

Meeting of the Pan American Institute of Geography and History in Haiti--General Secretary Laclavère, representing the IUGG, and several from the United States attended a meeting held recently in Haiti. This group is becoming increasingly interested in Geophysics as it affects cartographic and geographic work, but the interest is chiefly in this hemisphere. It is important that, on the international level and in the various countries associated with both IUGG and the Institute where both groups are active, care be taken to avoid duplication of effort or working at cross purposes.

Association of Seismology and the Physics of the Earth's Interior, IUGG--In accordance with the action taken at the last meeting of the Executive Committee, the officers of the Association have been advised that Perry Byerly has been designated as the American representative until June 30, 1953, and thereafter until his successor's appointment.

Plan to compile an international roster of scientific geophysicists--While at the Amsterdam meeting of ICSU, the President sent word of plans of IUGG to prepare a roster of geophysical scientists in the world. Presidents and Secretaries of Sections have been cooperating in the preparation of the American part of the roster.

UNESCO--UNESCO recently requested recommendations for a hydrogeologist (ground-water hydrologist) for work in Turkey. While certain names have been sent, further suggestions may still be forwarded.

Response to resolutions--The resolutions passed at our Thirty-Third Annual Meeting have been sent out. A number of acknowledgments and words of appreciation have been received.

Bibliography of Hydrology--In 1951, a contract was made between the National Academy of Sciences, as our parent corporate body, and the Bureau of Reclamation of the Department of the Interior, to compile an Annotated Bibliography of Hydrology for the United States for the years 1941-1950. This was completed June 30, 1952, as provided by the contract, and the master copy was delivered to the Bureau. It has since been published and 300 copies have been sent to the International Association of Hydrology for international distribution. Personal copies may be secured through the Government Printing Office in Washington.

K. Hilding Beij, General Secretary

Action on the General Secretary's Report--The report was accepted.

Issuance of the directory of members--It was pointed out that some organizations have not issued directories of members in recent years, largely because of the expense involved, while others have issued these after longer intervals. The AGU, during the 30's, issued lists of members annually. During the war years, issuance was omitted for some time. Later, issuance was on 18-month intervals, and subsequently extended to two years. A three-year interval was suggested to keep down the average annual cost. Several expressed the thought that the list is of great value to members, and an advisory vote indicated preference for the maintaining of the present two-year schedule.

Suggested Bowie Symposium--Richard M. Field, former President of the Union, has suggested the development of an international symposium relating to the works of William Bowie who did so much in the development of geophysics as a science. It was voted that we should support this symposium in principle, as it was agreed that such recognition of William Bowie is in order, and that such a symposium would be especially helpful while many of those who worked with him are active.

Combination of facilities--This matter as outlined by the General Secretary was discussed, but no action was taken.

Size of edition of the Transactions--In view of the need for microcarding and the use of a facsimile transmitter, the question was raised as to whether or not we are ordering a large enough edition of the Transactions. It was pointed out that the matter of storage is the critical problem, and though we are out of certain issues of 1947, 1950, and 1951, it is impossible to avoid this entirely because of the unknown magnitude of the number of new members that come in during the period immediately following publication and the demand that sometimes develops for a particular issue.

Medal in Terrestrial Magnetism and Electricity--Honorary President Fleming expressed the thought that over the years we should develop plans for medals in the various fields, and that he had recently taken preliminary steps to that end in his particular field, Terrestrial Magnetism and Electricity. Although $1,000 has been pledged by the C. F. Kettering Foundation of Detroit, this is contingent upon the receipt of $9,000 more. (A promise of $1,000 more has been received since the meeting from the Vulcan Copper and Supply Co. of Cincinnati.) It is estimated that the design, dies, and slugs of the medal will cost $5,000, with an addition of $5,000 on deposit with earnings for the prizes and expenses of the recipients. The thought was that it might be awarded every two or three years.

International Council of Scientific Unions (ICSU)--J. W. Joyce, W. W. Atwood, and the President reported briefly on the meeting of the International Council of Scientific Unions held in Amsterdam in early October. Eight from the United States attended. It is the desire of the Council to develop a balanced program independent of UNESCO as far as usual functions are concerned. The Third International Geophysical (formerly Polar) Year was discussed, and a small commission of the Council is being established. Latitude determinations will be a part of the program. President Bucher noted the importance of and need for cross-fertilization among the Unions, and felt that more time should have been devoted to the work of the several International Unions. He mentioned the importance of these Unions in organizing international enterprises.

J. A. Fleming, Past President of the ICSU, reported that at the Amsterdam meeting the new officers elected were as follows: President, B. Lindblad (Sweden); Vice Presidents, H. Solberg (Norway) and E. Herbays (France); General Secretary, A. V. Hill (United Kingdom); Treasurer, W. A. Noyes, Jr. (U.S.A.); Members of the Bureau, J. Peres (France) for three years, Naoto Kameyama (Japan) for six years, and A. von Muralt (Switzerland) as Past President.

He also reported that his papers and accounts relating to ICSU had been turned over to the Administrative Assistant to newly elected Treasurer Noyes as of October 31, 1952, and that the total in these dollar accounts was $15,429.09. He also advised that the dollar account of IUGG, currently totaling $9566.25, is still in his hands, at the request of General Secretary Laclavère.

International roster of geophysicists--The President outlined the plan of the IUGG to compile and publish a roster of scientifically active geophysicists, and that work is in progress now. For the United States, he has been gathering the material from the officers of the eight Sections of AGU which he plans to turn over soon to the International Secretariat. It was pointed out that this is just a start and that supplementary lists will be issued annually with the expectation that in about five years a thoroughly comprehensive roster may be developed.

Pan American Institute of Geography and History--The matters relating to the Institute were discussed briefly, but no further suggestions were made.

Student subscriptions--On the recommendation of the General Secretary, it was approved that the plan for such subscriptions, as originally adopted in 1950 [see Transactions, v. 32, p. 146, 1951], be extended indefinitely.

Reports of Standing Committees

Statutes and By-Laws--Chairman McClain B. Smith presented a report (see Appendix 1) relating to life membership provisions, which was approved. It was agreed that in Article 3(a)(3) of the Statutes "retirement from active professional service because of age" should be interpreted to mean official retirement.

Meetings--Chairman Marble reported briefly on meetings past and in prospect, mentioning especially the schedule of sessions set for the Thirty-Fourth Annual Meeting and requesting that officers of the Sections advise promptly if any changes are felt desirable. He expects to attend the St. Louis meeting of the AAAS in December, 1952, at which we are co-sponsoring sessions on geophysics.

Editorial and Publication--Chairman Beij requested that foreign news items -- as well as others -- for Notes and Personalia be sent in whenever one has newsworthy items.

Nominations--Chairman L. H. Adams presented the report of the Committee (other members are Ernst Cloos, L. B. Slichter, Lorenz G. Straub, and Charles A. Whitten) giving the nominations for officers of the Union for the triennium 1953-1956, as follows: for President, F. W. Reichelderfer and James B. Macelwane; for Vice President, Perry Byerly and Maurice Ewing; for General Secretary, John P. Marble and J. Frank Schairer. Attention was called to the fact that the Executive Committee was to act on this in accordance with item (3) of the procedure established in 1946 [Transactions, v. 27, p. 910, 1946] "that such nominations then be submitted by the Nominating Committee to the Executive Committee, which body may accept the report, or add other nominees for such officers. It will then proceed to select by secret ballot not less than two official nominees for each office." The report was accepted.

Audit--Chairman Weightman reported that the financial statement and the books of the Union for the year ending June 30, 1952, had been audited. The report of the Committee is published as Appendix 2.

National Research Council

Office of International Relations--Director W. W. Atwood said that the appointment of the American Committee to cooperate with the Commission on the Third Geophysical (Polar) year which ICSU is sponsoring, is pending and that he desires counsel on this matter. He mentioned that WMO (World Meteorological Organization) is especially interested in the tropics phase of the work, and that the work concerns various sections of AGU, but perhaps more particularly the International Scientific Radio Union. He desires that some one from our Executive Committee be designated to act on an informal committee.

He raised the question, too, whether the Executive Committee of the Union as now constituted is the best form for the American National Committee of IUGG. He pointed out that these National Committees take various forms, but ours is the only one in our form. Officers are not primarily elected because of their international interests. He raised the question as to whether we ourselves are satisfied. It was moved and seconded that the President appoint a committee to survey the situation, including our relationship with our parent body generally, the National Research Council.

Affiliated Organizations

American Geological Institute--A report (see Appendix 3) by W. T. Thom, Jr., retiring at the end of November as one of AGU's two representatives on the American Geological Institute, was read and accepted.

New and Unfinished Business

Nominating procedures--Chairman L. H. Adams of the Committee on Nominations expressed the thought that while it may be desirable to have at least two nominees for the offices of Vice President and General Secretary, he felt that it is perhaps usually undesirable to have two nominees for President. He suggested that the difficulty might also be avoided by electing a President-Elect and possibly eliminating the office of Vice President, though it was pointed out that this would involve a change in our Statutes and By-Laws, whereas the number of nominees was established by the regulations adopted for elections in 1946 [Transactions, v. 27, p. 910, 1946] and could be easily changed. It was mentioned that the Nominating Committees of the Sections, where more than one nominee is required, had sometimes offered similar comments. The Executive Committee of the

Section of Geodesy recently modified its regulations to permit single nominees, and in some Sections provisions have been made for "one or more nominees" for each office.

It was voted to refer this matter to a Committee to consider the possibilities of changes in the regulations.

Adjournment--The meeting adjourned at 16h 20m.

K. Hilding Beij, General Secretary
Waldo E. Smith, Executive Secretary

APPENDIX 1--REPORT OF THE COMMITTEE ON STATUTES AND BY-LAWS ON THE MATTER OF LIFE MEMBERSHIP FOLLOWING RETIREMENT

McClain B. Smith, Chairman; Lewis V. Judson

The matter of life membership following retirement was referred to this Committee on October 1, 1952, for study and report. Some 18 scientific societies were contacted for comparison purposes. Four of these do not supply a journal to retired members, and for purposes of this study were not considered, as it was assumed we would wish to supply the Transactions.

Of the remaining 14, membership requirements for remission of dues after retirement ranged from 10 to 40 years by five-year groups. The greatest number in any one group totaled four societies and these are more liberal than we are, requiring only ten years' membership.

We feel, however, that regardless of what other societies do, we should err on the side of being too liberal in view of our experience so far with members on the retired list, who in general contribute to the Union far more than the cost (estimated at about $2 a year by our General Secretary) of retaining them on the membership rolls. We should not risk the loss of members who have been active for at least 15 years who find that their retirement allowances are insufficient to permit paying dues to AGU. From a purely dollars-and-cents standpoint we do not feel this would be a good policy. We recommend therefore that no change be made in the present 15-year requirement.

APPENDIX 2--REPORT OF THE COMMITTEE ON AUDIT

Fiscal Year July 1, 1951 to June 30, 1952

R. H. Weightman, Chairman
A. G. McNish A. Nelson Sayre

Your Committee on Audit has examined the books of the Union for the fiscal year ending June 30, 1952, and finds them in agreement with the statement of the Union's finances, prepared by the General Secretary, which is attached.

At the beginning of the year, the cash assets amounted to $15,161.47, of which $13,895.92 was held as cash reserve, $1,180.91 in the Robert E. Horton Fund, and $84.64 in the William Bowie Medal Fund. All three funds are carried in a single account at the Riggs National Bank. The Permanent Fund, in reality the Invested Fund, amounted to $3,469.82, consisting of $2,368.00 in Series F bonds (maturity value $3,200) and $1,101.82 in the Northwestern Federal Savings and Loan Association. [See Trans., v. 32, p. 940, Dec. 1951, for statement of assets and liabilities as of June 30, 1951.]

Receipts for the year totaled $47,833.75, while expenditures totaled $46,791.15, an increase in liquid assets of $1,042.60. It was found possible to increase the Permanent Fund by additions of $3,758.75 in cash and accumulated interest of $62.49 to our account with the Northwestern Federal Savings and Loan Association, making a total in that Association of $4,923.06. The bond investment remained the same.

At the end of the year, the cash assets of the Union were $16,204.07, an increase of $1,042.60, $1,180.91 being held in the Horton Fund and $63.15 in the William Bowie Medal Fund. The total of the Permanent or Invested Fund was increased to $7,291.06.

Operating Statement

(Fiscal year ending June 30, 1952)

Receipts		
Grant		
National Research Council		$ 400.00
Income		
Dues		
1949	$ 120.00	
1950	288.00	
1951	3,762.24	
1952	26,120.37	
1953 et seq.	10.90	
Corporation Memberships	1,500.00	
Life Memberships	100.00	31,997.51
Sales of publications		
Transactions	$ 7,532.83	
Reprints from Transactions	2,160.09	
Index of Transactions (1920-1944)	2,756.88	
Bibliography of Hydrology (1941-1950)	2,610.26	15,060.06
Interest, Northwestern Savings and Loan Association from July 1, 1951 to June 30, 1952		62.49
Miscellaneous and contributions		313.69
		$47,833.75
Disbursements		
Publication of Transactions		
Printing	$ 9,374.04	
Preparation of master copy	7,916.00	
Mailing	694.44	
Honoraria to co-editors	1,350.00	$19,334.54
Other publications		
Index of Transactions (1920-1944)[a]	$ 1,639.95	
Bibliography of Hydrology (1941-1950)[a]	3,705.22	
Reprints of Transactions	2,047.51	7,392.68
General expenditures (office of the Union)		
Salaries[b]	$12,478.21	
Stationery and supplies[b]	835.91	
Postage and communications[b]	1,241.66	
Office equipment	239.66	14,795.44
Expenses, meetings		
Annual meeting	$667.24	
Regional meetings	79.83	747.07
Expenses, Sections		
Terrestrial Magnetism and Electricity	$4.00	
Hydrology	3.78	7.78
Expenses, Committees (Membership)		308.26
William Bowie Medal (paid from William Bowie Medal Fund)		21.49
Association of Volcanology, IUGG		300.00
Travel		32.44
Investments: $3,758.75 deposited in the Northwestern Federal Savings and Loan Association, as part of the Permanent Fund, plus interest July 1, 1951 to June 30, 1952, $62.49		3,821.24
Miscellaneous		30.21
		$46,791.15

Excess of receipts over disbursements

Receipts	$47,833.75	
Disbursements	46,791.15	
Increase in liquid funds		$ 1,042.60

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[a] Postage included in postage and communications under general expenditures (office of the Union).

[b] Includes some minor items involved in the preparation of the Bibliography of Hydrology.

Statement of assets and liabilities as of June 30, 1952

Assets			
Cash Reserve	$14,960.01		
Robert E. Horton Fund	1,180.91		
William Bowie Medal Fund	63.15	$16,204.07[a]	
Permanent Fund			
Series F bonds purchased prior to July 1, 1948 (maturity value $3,200) at cost	$ 2,368.00		
On deposit in Northwestern Federal Savings and Loan Association as shown for July 1, 1952	4,923.06	7,291.06	$23,495.13
Bills receivable			
Dues			
1952	$ 3,879.20		
1951 and 1952	3,122.00	$ 7,001.20	
Publications			
Transactions (including subscriptions)	$243.62		
Reprints	352.58		
Index (1920-1944)	116.00	712.20	7,713.40
Bibliography of Hydrology (1941-1950)			1,397.43
			$32,605.96
Liabilities			
Bills payable and advance payments			
Dues in advance (1953 et seq.)		$331.23	
Subscriptions in advance (1953 et seq.)		90.25	
Reprints paid in advance		80.00	$ 501.48
Surplus (Capital Account)			32,104.48
			$32,605.96

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[a] Riggs National Bank statement of June 30, 1952	$21,170.02
Less checks outstanding	4,965.95
Net balance June 30, 1952, as above	$16,204.07

The Committee is in receipt of a letter dated September 29, 1952, from Honorary President Fleming, who inaugurated the present system of bookkeeping used by AGU, proposing some change in the financial procedures and the keeping of records of the same.

Your Committee, after a preliminary study, feels that some changes in procedures are desirable. The Auditing Committee will make further study of the system now in use looking to the making of recommendations to the General Secretary.

We will bear in mind that an extremely elaborate system is not necessary and would not be justified but should be one better adapted to the present requirements of the Union. It is our opinion that it would not be desirable to introduce procedures which would require additional personnel.

Corrigendum--In the audited financial statement for the year ending June 30, 1949, published in the December, 1949, issue of the *Transactions*, page 933, the second item under expenditures for the *Transactions* (preparing manuscript) should be $2,722.36 instead of $2,762.36.

APPENDIX 3--REPORT BY RETIRING REPRESENTATIVE OF AGU ON THE BOARD OF THE AMERICAN GEOLOGICAL INSTITUTE

This brief memorandum is being submitted as my report to the Executive Committee covering my agency as one of the Union's two representatives on the Directorate of the American Geological Institute. There are, I believe, two matters to be covered.

First, in accordance with my understanding of the Union's policy and position with respect to the Institute, I have sought to stress the point that the Union had agreed to take part in the Institute in order to determine the validity of the proposition that an Institute, broadly representative of the field of Earth Science, could render national services above and beyond those which were within the capacities of the several component societies, but with the further proviso and understanding that the Institute would neither seek to duplicate society activities nor attempt to limit the autonomy of thought and action on the part of the component societies. Because of these understandings I have joined others in opposing various suggestions which have been made to the Board of Directors of the American Geological Institute.

My second point deals with the present status of the American Geological Institute. Dr. Delo, who was Executive Director of the Institute (and Vice Chairman of the Division of Geology and Geography of the National Research Council), has resigned these posts to become a college president, and the appointment of a new Executive Director is being considered by Dr. Croneis, President of the American Geological Institute, and Dr. Cloos of the National Research Council. Naturally the whole policy and program of the Institute is being simultaneously reconsidered and will be taken up in detail at a meeting of Institute Directorate to be held in Boston on November 13, 1952.

W. T. Thom, Jr.

Meeting in Chicago, September 11-12, 1952--The Twenty-Eighth Regional Meeting was held at the Del Prado and Conrad Hilton Hotels in Chicago, September 11-12, 1952, jointly with the American Meteorological Society as a part of the celebration of the Centennial of Engineering, September 3-13, 1952. The Centennial Celebration, marking the anniversary of the first of the American non-military engineering societies, that is, the American Society of Civil Engineers, drew thousands of engineers, scientists, and technologists to Chicago where some 60 societies and other organizations held sessions. Perhaps the high point of the 11-day celebration was the luncheon on Convocation Day, September 10, which was attended by 1500. The program included addresses by Herbert Hoover, Past President of the United States; Clarence Decatur Howe, Minister of Trade, Commerce, and Defense Production of Canada; and Benjamin Franklin Fairless, President of the U. S. Steel Corporation.

There were about 60 papers on the eight sessions of the AGU-AMS joint meeting, including a Symposium on the Natural Setting and Engineering Problems of the Great Lakes, and sessions on Applied Climatology, Applied Forecasting, Air Pollution, Evaporation from Lakes and Reservoirs (including the major portion of a session on the new Lake Hefner Studies), Ground Water in the Great Lakes Region, and Seismology as it is Applied to the Design of Structures. The program as planned (there were a few late changes) can be found in the Bulletin of the American Meteorological Society [v. 33, no. 5, May 1952, pp. 217-219]. The additions included a paper on Applied Climatology by C. W. Thornthwaite and M. H. Halstead, one on air pollution by G. B. Bell, E. Robinson, and D. H. Hutchinson, and the presentation of two films, one relating to the formation of snow crystals and the other to snow surveys in Japan by U. Nakaya. There were nearly 300 registrants, but as many general Centennial registrants also attended, the total attendance probably exceeded 400.

A joint AGU-AMS banquet, held on the evening of September 11, was attended by about 50 and was addressed by Harry J. Fuller of the University of Illinois on the origin of our domesticated food plants.

Transactions, American Geophysical Union Volume 33, Number 6 December 1952

SPECIAL ANNOUNCEMENTS

LIST OF OFFICIAL NOMINEES FOR OFFICERS OF THE UNION AND OF THE SECTIONS FOR THE TRIENNIUM JULY 1, 1953 TO JUNE 30, 1956

Union or Section	President	Vice President	Secretary[a]
Union	James B. Macelwane F. W. Reichelderfer	Perry Byerly Maurice Ewing	John P. Marble[a] J. Frank Schairer[a]
Geodesy	Albert J Hoskinson	Milton O. Schmidt	Frank L. Culley
Seismology	Ross R. Heinrich Leonard M. Murphy	Hugo Benioff James T. Wilson	Henry F. Birkenhauer James A. Peoples, Jr.
Meteorology	Phil E. Church Bernard Haurwitz	Jerome Namias	Woodrow C. Jacobs
Terrestrial Magnetism and Electricity	C. W. Gartlein Victor Vacquier	David G. Knapp H. R. Joesting	L. R. Alldredge R. R. Bodle
Oceanography	Richard H. Fleming Eugene C. LaFond	Walter H. Munk Dale F. Leipper	John Lyman Donald W. Pritchard
Volcanology	William F. Foshag Francis G. Wells	Gordon A. Macdonald Aaron C. Waters	Felix Chayes George Phair
Hydrology	H. G. Wilm	Ray K. Linsley E. N. Munns	Harry F. Blaney E. A. Colman
Tectonophysics	Francis Birch Chester R. Longwell	Harold W. Fairbairn L. B. Slichter	James R. Balsley, Jr. James T. Wilson

In addition, two members are to be elected to the Executive Committee of the Section of Tectonophysics. The nominees are Wilbur S. Burbank and Thomas P. Thayer.

Nominees to the Pacific Southwest Regional Committee
Hydrology (three to be elected): Donald M. Baker, M. R. Huberty, Leonard Longacre, J. C. Marr, H. McV. Stafford, Vito A. Vanoni, and K. Q. Volk.
Meteorology (three to be elected): R. C. Counts, Jr., H. H. Engelbrecht, and C. F. Woffinden.

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[a] For General Secretary in the case of the Union.

Explanatory note--In accordance with the Statutes and By-Laws as amended to July 31, 1950, elections by mail ballot are to be held in the first part of 1953. The Committees on Nominations for the Sections were published in the Transactions of June 1952 in the minutes of the Sections, pages 481-485. The Executive Committee of the Union has established procedures for nomination and election of officers of the Union, as published in the December 1946 issue of the Transactions, page 910. These are as follows: (1) Nominations shall be initiated by a small Nominating Committee consisting of three or five members, appointed by the President. (2) At least two persons shall be nominated for each of the offices of President, Vice President, and General Secretary. (3) Such nominations shall then be submitted by the Nominating Committee to the Executive Committee, which body may accept the report, or add other nominees for such offices. It will then proceed to select by secret ballot not less than two official nominees for each office. (4) Such nominations shall then be announced to the membership, sufficient time being provided before election to permit further nominations by petition when signed by one per cent (49 at present) of entire membership, it being mandatory that nominations by petition be added to the official nominations. (5) Ballots containing official nominees and those submitted by petition shall be mailed to the active membership and such ballots, postmarked before a specified date, shall be marked and forwarded by the membership to the General Secretary. (6) The General Secretary within ten days after the specified date shall turn over all such ballots for tallying to a Committee of Tellers consisting of one member selected from each of the eight sections and one member from the Executive Committee. (7) The Committee of Tellers shall make report to the General Secretary within ten days, showing the result of the ballot. (8) A plurality of votes cast shall elect. (9) Consent of each candidate is to be obtained before placing his name on the ballot. (10) The Sections shall be urged to adopt similar procedures.

Similar Section procedures were established and were published in the Transactions of December 1946, with amendments covering Seismology and Hydrology in the Transactions of August 1949, page 627. In the summer of 1952, the Executive Committee of the Section of Geodesy voted to permit the nomination of one candidate for each office of that Section. The procedures will not be repeated in full here. All provide for nominations by petition, the number of member petitioners affiliated with the Section being as follows: Geodesy, ten members; Seismology, 13 members; Meteorology, 25 members; Terrestrial Magnetism and Electricity, one per cent of those affiliated with the Section (11 members at present); Oceanography, ten members; Volcanology, one per cent of those affiliated with the Section (nine members at present); Hydrology, 25 members; and Tectonophysics, 15 members.

Please note that nominations by petition must include written consent from the nominee, to be transmitted to the office of AGU with the petition.

The following schedule for nominations and elections has been established.

By November 10, 1952, steps (1) to (3) and step (9) of procedure must be complete for the official nominees.

In Transactions of December, 1952, publish list of official nominees with explanatory note.

By February 1, 1953, nominations by petition must be in the office of AGU with written consent of nominees.

By March 1, 1953, ballots carrying names of official and petition nominees, will be mailed to the membership. A brief biographical sketch relating to each nominee will accompany, but not be a part of, the ballot.

On March 31, 1953, balloting will close. Ballots must be postmarked not later than that date and be received in the office of AGU not later than April 4, 1953, in order to be counted.

By April 10, 1953, ballots will be turned over to the Committee of Tellers.

By April 20, 1953, results of tally will be turned over to the General Secretary, results to be announced at the Annual Meeting May 4-6, 1953, and published in the Transactions in June, 1953.

In addition to the election of Union and Section officers, the Pacific Southwest Regional Branch will hold its first elections by mail ballot, the nominees to be voted upon only by members in that Region. The names are also shown in the accompanying table.

Waldo E. Smith, Executive Secretary

Pacific Southwest Meeting at Stanford University, February 6-7, 1953--Attention is called to this annual meeting which will be held on this occasion at Stanford University on February 6-7, 1953. It is anticipated that the Sections of Meteorology and Hydrology will join in this, but if there is desire among the members of the Region for other Sections to participate also, arrangements can be made. Send word of papers or suggestions for the program to David K. Todd, Chairman of the Program Committee (Division of Civil Engineering and Irrigation, University of California, Berkeley 4, Calif.) without delay. Local arrangements are under the chairmanship of Ray K. Linsley, Department of Civil Engineering, Stanford University, Stanford, Calif. J. W. Johnson, University of California at Berkeley, is Chairman of the Regional Committee.

Arrangements are being made to include the nominees for the Regional Committee for the triennium from July 1, 1953 to June 30, 1956, with the ballot for National Union and Section officers, so that elections by mail ballot may be held in accordance with the By-Laws of the Region [see p. 629 of the issue of August 1952]. The nominees are shown in another Special Announcement in this issue.

Thirty-Fourth Annual Meeting, Washington, D. C., May 4-6, 1953--Attention is called to the extended notice concerning the Thirty-Fourth Annual Meeting on page 794 of the issue of the Transactions of October, 1952. Note that titles and abstracts of papers for the program should be sent in as soon as possible, and not later than March 10, 1953. Abstracts should be informative, and not more than 150 words. Longer abstracts will be omitted or edited to length. Section Secretaries reserve the right to close programs filled in advance of March 10. The complete manuscripts should be in the office of AGU by April 15, 1953, with advice from the authors as to whether the paper is submitted for publication in the Transactions. Also, send in word concerning projection equipment needed. Only a standard slide projector will be provided unless other word is received. Communications should be sent to the appropriate Secretary, listed in the October announcement, or to the office of AGU.

In preparing your paper for presentation, bear in mind that requirements are quite different than for publication. We might borrow the succinct instructions of the Association of Seismology, IUGG, at the Brussels Meeting in the summer of 1951: "Speak slowly, clearly, and briefly; avoid mathematics."

Block bookings have been made at three hotels as follows (they are listed in the decreasing order of the number of rooms booked):

Roger Smith Hotel, Pennsylvania Avenue at 18th Street, Northwest
Raleigh Hotel, Pennsylvania Avenue at 12th Street, Northwest
Washington Hotel, Pennsylvania Avenue at 15th Street, Northwest

Make your hotel or room reservation early directly with the hotel of your choice. If you request block-booked space, be sure to cite the AGU meeting.

Meeting in Minneapolis, Minn., August 31, 1953--Steps have been taken to organize a Committee for the North Central Region, embracing the states of Minnesota, Iowa, Missouri, Kansas, Nebraska, the Dakotas, and adjacent areas in Canada. A Regional Committee consisting of Martin E. Nelson (Chairman), Roy F. Warner (Secretary), Paul C. Benedict, M. C. Boyer, and Lorenz G. Straub has been appointed to work out this organization, and to arrange a meeting that will be held on August 31, 1953, immediately preceding a meeting of the International Association for Hydraulic Research and the Division of Hydraulics of the American Society of Civil Engineers. While at present only the Section of Hydrology is sponsoring activity in this Region, others may join if there is desire among the members of the Region.

Persons interested in the meeting on August 31 or in other aspects of the program should get in touch with the Secretary of the Regional Committee, Roy F. Warner, 1872 N. Howard St., St. Paul 9, Minn.

Publication of Bibliography on Hydrology--An Annotated Bibliography on Hydrology, 1941-1950, United States and Canada, has recently been published by the Bureau of Reclamation. This was compiled by the American Geophysical Union under contract between the National Academy of Sciences acting on behalf of the AGU and the Bureau of Reclamation acting on behalf of several interested Federal Agencies. It contains over 4500 items. Copies may be secured from the Superintendent of Documents, Washington 25, D. C.

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